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Josef Kallrath · Eugene F. Milone

Eclipsing Binary Stars: Modeling and Analysis

Second Edition

Springer

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Professori Dr. Hans Schmidt,

magisto doctissimo ac clarissimo, patri et amico
hoc opus votis optimis D.D.D.[1]

To Harlan Smith and Adriaan Wesselink,

teachers extraordinaire, and, to my great fortune,
my graduate advisors.[2]

[1] Josef Kallrath.

[2] Eugene F. Milone.

Foreword

Have you ever stopped at a construction project on the way to your office and the day's astrophysics? Remember the other onlookers – folks just enjoying the spectacle, as we all do in following developments away from our areas of active work? We are excited and thrilled when the Hubble Space Telescope discovers an Einstein Cross, when the marvelous pulsars enter our lives, and when computer scientists put a little box on our desk that outperforms yesterday's giant machines. We are free to make use of such achievements and we respect the imagination and discipline needed to bring them about, just as onlookers respect the abilities and planning needed to create a building they may later use. After all, each of us contributes in our own areas as best as we can.

In addition to the serious onlookers there will be passersby who take only a casual look at the site. They may use the building later, but have little or no interest in its construction and give no thought to the resources needed to bring it to completion. Upon arriving at work, those persons write astronomy and astrophysics books at various levels, in which they must say something about close binary stars. Usually a page or two will do, and the emphasis is on the MLR (mass, luminosity, radius) data obtained only from binaries. The role of binaries in stellar evolution also may be awarded a page or so, perhaps meshed with binaries being homes of black holes and neutron stars. We live in an era of ever more applied research, with national priorities set by the interests and judgments of select committees. Consequently, most authors tell us the answer to one central question: What have binaries done for us lately? Well, of course, binaries are alive and well as sources of fundamental information on many fronts. But what of the fun, intrigue, and beauty of close binaries?

However, I do not want to be hard on the generic text authors because I remember my initial reaction to binaries. A fellow graduate student (later a very accomplished researcher) was doing a binary star project and I could not imagine why he was so interested. He tried to explain it all, but it just was not working: subject = nonexciting. However, I soon was into binaries anyway, just due to being surrounded by binary star work (yes, with appalling conformity). Time went by and then something happened to turn the view around – it was Su Shu Huang's work on ϵ Aurigae and also on β Lyrae. Here was pure distilled cleverness and insight. Huang looked at problems that had been examined exhaustively by several of the most celebrated astrophysicists and saw things that had eluded everyone. Suddenly binaries were locales where mystery could transform into understanding if one looked in the right way. But where does one learn to do this sort of thing, in a course? Not at most schools. Can one learn from a book? Well, there are books on binary stars, but they mainly serve as repositories of formulas, derivations, and diagrams, and some follow the ideas of only one person or "school." A few books give recipes for procedures developed by their authors, usually without providing insight. There has long been a need for a book that takes a wide view of binary star models and their interface with observations, and that is the goal set by Josef Kallrath and Eugene F. Milone, who together have broad experience in binary star models and observations. Their creation has conscientious coverage throughout most of the "models versus observations" field. It can guide interested persons into the overall field and be a helpful companion as they explore new examples, such as in the *initial approach* (what is going on?), a *settling-in stage* (is it a standard situation or are there complications?), *getting up to speed* (developing intuition and extracting maximum information), and finally *evaluation of results*.

Examine the Contents to see a variety of topics not found in the few preceding books in this general area. Here we find extensive treatment of history, terminology, observational methods, accuracy, binary models *from the ground up*, system morphology, a sense of where things are going, perspectives for long-range development, guides to exploration of the literature, and even philosophy. Although not all important categories of binaries are covered, nor are all individual binaries of special interest, the coverage in this first edition is remarkable. Protest marches for inclusion of symbiotics, ultra-compact X-ray binaries, etc., in future editions may well be successful. For now the emphasis is on more general considerations. We see a balance between *hands-on* and *automated* analysis. Extreme *hands-on* advocates typically get things roughly right and can recognize novel features but fail to extract all available information. Extreme advocates of the *automated* school can reach optimal solutions for standard cases but miss anything new (there is more to astrophysics than parameter adjustment). We need to navigate between these extremes.

The names of some luminaries of the binary star field may seem to be underrepresented, for example, those in structure and evolution. Should we not be reading more about Eggleton, Kippenhahn, Lucy, Ostriker, Paczynski, Plavec, Taam, van den Heuvel, Webbink, etc.? But remember that it is a book about direct representation of observations through models and must be kept to a reasonable size, and there are excellent books on structure and evolution.

Although the scope is limited to models, observations, and related mathematics, there is something here for everyone. Thus we learn that the Kolmogorov–Smirnoff test is not, after all, a way to distinguish vodkas. And there are binaries for everyone. Game players will like the one that stays in eclipse 90% of the time and comes out for only 10% (*PK Boo*). Gadget afficianados prefer the remote paging device, *b Per*. We have a thing to play in *TV Cet* and a place to stay in *HO Tel*. And then there is the only star with a question mark in its name (*Y Sex?*). So peruse the book, learn from it, and enjoy close binaries. If you happen to find some MLR data along the way, so much the better.

R. E. Wilson

Preface to the Second Edition

Di, coeptis . . . *adspirate meis* (*Gods, aid my undertaking*)
Ovid (43 B.C.–A.D. 17), *Metamorphoses* 1,2-4

The second edition arose from the authors' and the publisher's observations that 10 years after the first edition a new edition is needed to cover the impressively long list of new physical features and analyzing methods in eclipsing binary (*EB*) star analysis. Direct distance estimation through *EB* analysis is one of the highlights. Complete derivation of the ephemerides and third-body orbital parameters from light and radial velocity curves is another. Incorporation of interpolation-based approximations to stellar atmospheres has become common practice. Limb-darkening coefficients do not need to be entered explicitly but are locally computed as function of temperature, gravity, wavelength, and chemical composition.

EB research has made great contributions to stellar astrophysics for over a century, e.g., resolution of the Algol paradox, insights into the physics of cataclysmic variables, and improved understanding of W UMa stars. Bright *EB*s can be observed and analyzed for orbital and physical properties to high accuracy with even modest equipment. The advent of larger telescopes and powerful instrumentation also allows analysis and distance estimations of *EB*s in Local Group galaxies even as far as M31 and M33. Large telescopes also allows the observation and study of eclipsing very-low-mass stars, brown dwarfs and planets, and even planets in *EB*s. The detection of extra-solar planets by transit methods is a field not entirely outside the *EB* research where *EB* techniques have been used successfully.

The Kepler mission[1] (cf. Koch et al. (2006)) launched on March 6th, 2009, the GAIA mission to be launched in 2012 or the *Large Synoptic Survey Telescope* (LSST; http://www.lsst.org/), in discussion for after 2015, will add a new challenge to the field: The analysis of large numbers of *EB* light curves from surveys. Finally, enhanced or completely new software is available for *EB* research. `PHOEBE` (Sect. 8.2) is an example of a platform-independent *EB* software with an attractive graphical user interface.

[1] Updated details are at http://www.kepler.arc.nasa.gov.

The proceedings of IAU Symposium No. 240 (2006) [entitled *Binary Stars as Critical Tools & Tests in Contemporary Astrophysics*, edited by Hartkopf et al. (2007)] provide an excellent overview on state-of-the-art and ongoing activities in close binary research. They briefly review major advances in instrumentation and techniques, new observing and reduction methods, and discuss binary stars as critical tools and tests for studying a wide variety of astrophysical problems. *Tools of the Trade and the Products they Produce: Modeling of Eclipsing Binary Observables* edited by Milone et al. (2008) is another source highlighting recent advances. We see strong enhancements both in physics and in *EB* software:

- additional physical features
  1. an alternative method to derive a binary's ephemeris;
  2. effects of third bodies on light curves and radial velocity curves;
  3. *EB*s with intrinsically variable components;
  4. stellar atmosphere approximation functions;
  5. direct distance estimation;
  6. color indices as indicators of individual temperatures;
  7. spectral energy distribution as independent data source; and
  8. main sequence constraints;
- enhanced programs and new software;
- techniques for analyzing large numbers of light curves, and;
- *EB*s in extra-solar planet research.

We largely retain the structure of the first edition. Some sections have been added to the existing chapters, especially the *Eclipsing Binary Guide for Researchers in Other Fields* in Chap. 1. What is now called Chap. 5 hosts most of the new material. Chapters 5, 6, and 7 of the first edition are now Chaps. 6, 7, and 8 of this second edition.

Last but not least: The book is now part of Springer's *Astronomy and Astrophysics Library Series* which both indicates and acknowledges its wider relevance for astronomy and astrophysics.

Gainesville, FL, US — Josef Kallrath
Calgary, AB, Canada — Eugene F. Milone

Preface to the First Edition

Di, coeptis . . . *adspirate meis* (*Gods, aid my undertaking*)
Ovid (43 B.C.–A.D. 17), *Metamorphoses* 1,2-4

This book arose from the realization that light curve modeling has not had a full expository treatment in 40 years. The last major exposition was that of Russell & Merrill (1952), and that treatment dealt exclusively with the Russell–Merrill spherical star model and with the process of light curve rectification. The present work may be the first comprehensive exposition of the merits of the major modern light curve analysis methods, notwithstanding the pioneer efforts of many investigators beginning with Kopal (1950), who again described mainly his own efforts. The need for a sourcebook and didactic presentation on the subject was recognized in the course of planning a conference on light curve modeling which was held in Buenos Aires and Cordoba in July–August, 1991. The proceedings of the Argentina meetings (Milone 1993) review the current state of light curve modeling methods and focus on special topics but do not give a general review. The only previous meeting devoted exclusively to the topic of comparative light curve modeling methods was IAU Colloquium No. 16 in Philadelphia in September 1971.

Although there is an extensive literature on eclipsing binary research, the paucity of instructional materials in the area of light curve analysis is striking. The graduate student, the researcher, or the advanced amateur astronomer must struggle through journal articles or conference proceedings and must read between the lines to glean the details of the modeling process. Most of the didactic books on light curve modeling date to the presynthetic light curve era. The monograph by Russell & Merrill (1952) is one major example and that by Kopal (1959) is another. Yet the need is acute: As with many other areas of science, the computer revolution has given many astronomers, amateur as well as professional, the tools to attempt light curve solutions. In this work, we provide a suitable background for the new modeler, a useful source book for the experienced modeler, and a springboard for the development of new modeling ideas. For the Wilson–Devinney approach in particular, we elaborate on some of the subtle details that determine the success or failure of light curve computation.

Methods of analyzing eclipsing binary data involve

1. the specification of an astrophysical model;
2. the selection of an algorithm to determine the parameters; and
3. the estimation of the errors of the parameters.

The focus of the book is primarily on models and algorithms rather than on model applications and individual binaries. Nevertheless, models and algorithms are illustrated through investigations of individual stars. The review by Wilson (1994) deals with the intuitive connection between binary star models and light curves and the historical development of the field. Here we concentrate on the mathematical formulation of models and the mathematical background of the algorithms. We present a self-contained, elementary treatment of the subject wherever possible. The book is written for the reader who is familiar with the basic concepts and techniques of calculus and linear algebra. As an aid to the exploration of higher-level material, a brief introduction into the theory of optimization and least squares methods is provided in Appendix IV. Besides presenting the physical and mathematical framework, we have tried to present the material so that it is not far from actual implementation.

The book consists of four major parts: Introductory material (Chaps. 1 and 2); the physical and mathematical core (Chaps. 3 and 4); practical approaches (Chaps. 5 and 6); and the authors' views on the structure of future light curve programs (Chap. 7). The appendices provide further mathematical details on specific topics and applications and point to other sources. The structure of the book should assist readers who are taking their first steps into the field to get a sound overview and also experienced researchers who are seeking a source book of formulas and references.

Chapter 1 gives a nonmathematical overview of the field. In particular, the issues of what can be derived from eclipsing binary stars and why these data are relevant to astrophysics in general are considered. Here we introduce the general concept of equipotential surfaces. Because an eclipsing binary analyst needs understanding of observational data, some background on the database and methods of data acquisition is necessary. Therefore, in Chap. 2 we review observational methods relevant to data analysis. Chapter 3 contains a general approach to light curve modeling. The chief concern is the solution of the direct problem: computing light curves, radial velocity curves, and other observables for a given set of parameters. To this end, a mathematical framework is presented for the relevant astrophysics. The inverse problem of the determination of eclipsing binary parameters is discussed in Chap. 4. As in Chap. 3, we present a formal approach that may serve as a platform for further developments in data analysis. Besides attending to critical issues in light curve analysis, the formulation allows various methods to be related and discussed from a common point of view. Chapter 5 gives characteristics of existing light curve programs and coding details for some of them. Here, the purpose is to provide the new light curve analyst with an overview to explore concepts relevant to the field, including the astrophysics of particular programs and how they work. Because the Wilson–Devinney program is the most frequently used light curve analysis tool, Chap. 6 discusses special eclipsing binary cases analyzed with the Wilson–Devinney program and related programs. In order not to overburden Chaps. 5 and 6, most of the practical details of the Wilson–Devinney program are collected in Appendix D.1. Chapter 7 summarizes ideas and strategies for building light curve programs and previews the coming decade. It is intended as a stimulant to further eclipsing binary research and light curve program development.

Some topics that are covered only briefly will have more extensive treatment in some later edition. A first group covers further model extensions requiring time rather than phase as input quantity, e.g., variable periods, apsidal motion, eclipsing binaries with intrinsically variable components, spot migration, circumstellar flows. Other extensions will focus on particular types of binaries: cataclysmic variables, symbiotic stars and other red-giant binaries, W Serpentis stars, high, low, and ultralow mass X-ray binaries, binaries with atmospheric eclipses, and individual strange objects such as *ε Aurigae*, *β Lyrae*, and *Cygnus X-1*.

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Part II Modeling and Analysis

Part III Light Curve Programs and Software Packages

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Journal Abbreviations

ACM Trans. Math. Software	*ACM Transactions on Mathematical Software*
Acta Astron.	*Acta Astronomica*
AJ	*The Astronomical Journal*
Ann. Rev. Astron. Astrophys.	*Annual Review of Astronomy and Astrophysics*
ApJ	*The Astrophysical Journal*
ApJ Suppl	*The Astrophysical Journal Supplement Series*
Ap. Sp. Sci.	*Astrophysics and Space Science*
A&A	*Astronomy and Astrophysics*
A&A Suppl.	*Astronomy and Astrophysics Supplement Series*
Astron. Nachr.	*Astronomische Nachrichten*
Astron. Rep.	*Astronomical Report*
BAAS	*Bulletin of the American Astronomical Society*
Bull. Math. Biol.	*Bulletin of Mathematical Biology*
CMDA	*Celestial Mechanics and Dynamical Astronomy*
Comp. J.	*The Computer Journal*
J. Comp. Phys.	*Journal of Computational Physics*
Mem. R. Astron. Soc.	*Memoirs of the Royal Astronomical Society*
MNASSA	*Monthly Notices of the Royal Astronomical Society of Southern Africa*
MNRAS	*Monthly Notices of the Royal Astronomical Society*
Quart. Appl. Math.	*Quarterly of Applied Mathematics*
Observatory	*The Observatory*
PASP	*Publications of the Astronomical Society of the Pacific*

Sov. Astron.	*Soviet Astronomy*
Space Sci. Rev.	*Space Science Reviews*
Vistas	*Vistas in Astronomy*
Zeitschr. f. Astrophys.	*Zeitschrift für Astrophysik*

Acronyms and Abbreviations

ως συνελῶς ειπεῖν (*to say it short*)

APT	automatic photometric telescope
CCD	charge coupled device
CCF	cross-correlation function
CDM	conjugate direction method
CLV	center-to-limb variation
CMD	color-magnitude diagram
DLS	damped least squares
EB	eclipsing binary
HST	Hubble Space Telescope
IAU	International Astronomical Union
IPMs	interior point methods
JDN	Julian Day Number
MACHO	massive compact halo object
MSC	main sequence constraint
NDE	Nelson–Davis–Etzel; usually used to refer to the NDE model
NLP	nonlinear programming
OGLE	optical gravitational lensing experiment
PMT	photomultiplier tube
RADS	Rapid Alternate Detection System
RV	radial velocity
SB1	single-lined spectroscopic binary
SB2	double-lined spectroscopic binary
SED	spectral energy distribution
SI	physical units according to the Système International
SQP	sequential quadratic programming
WD	Wilson–Devinney; used to refer to the Wilson–Devinney model or program
w.r.t.	with respect to
ZAMS	zero-age Main Sequence

Mathematical Nomenclature and Symbols, Physical Units

"What's in a name?" Shakespeare: *Romeo and Juliet*, ii, 2

A few general rules are observed: vectors are marked as bold characters, *e.g.*, $\mathbf{x}$, $\mathbf{n}$, or $\mathbf{r}$. The product $\mathbf{a} \cdot \mathbf{b}$ of two vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$ is always understood as the scalar product $\mathbf{a}^{\mathrm{T}}\mathbf{b} = \sum_{i=1}^{n} a_i b_i$. Matrices are indicated with sans serif font, *e.g.*, A. The list below gives our mathematical symbols and operators.

$\mathbb{R}^n$	the n-dimensional vector space of real (column) vectors with n components
∇	gradient operator $\nabla := \nabla_x = \frac{\partial}{\partial \mathbf{x}} = \left(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}\right)^{\mathrm{T}}$ applied to a scalar-valued function f
$:=$	defines the quantity on the left side of an equation by the term on the right side of the equation
$\equiv$	quantity on the left side of the equation is set identically to the term on the right side of the equation
$\doteq$	indicates approximation, as in a first-order Taylor series expansion
$\mathbf{x}^{\mathrm{T}}$	the transposed vector, $\mathbf{x}^{\mathrm{T}} := (x_1, \ldots, x_n)$ is a row vector
$\mathbf{e}_i$	unit vector associated with the ith coordinate axis
$\mathbb{1}$	identity matrix of appropriate dimension
$\mathcal{M}(m, n)$	set of matrices with m rows and n columns
$\forall$	it represents "for all"

Superscripts indicate attributes of a quantity, e.g., L^{bol}, a bolometric luminosity. Subscripts are used for indexing and counting. The subscript j is always used to refer to one of the binary components. To avoid confusion with the symbols R and L, the radii and luminosities of the binary components are written in calligraphic style $\mathcal{R}$ and $\mathcal{L}$ if they are in absolute (or solar) units. Symbols used in this book are listed in Appendix F.

Throughout this book we mostly use SI units. However, when referring to original papers or figures, CGS or even special units such as Å cannot be avoided. Where possible we give only generic physical dimensions of quantities, such as mass, length, time, energy.

Part I
Introduction