Rheology of Complex Fluids
Rheology of Complex Fluids
Preface

In complex fluids, the atoms and molecules are organized in a hierarchy of structures from nanoscopic to mesoscopic scales, which in turn make up the bulk material. Consideration of these intermediate scales of organization is essential to understand the behaviour of the complex fluids. These fluids, an important part of what is generally called soft matter, have very complex rheological responses. Many industrial substances encountered in chemical, personal care, food and other processing industries, such as suspensions, colloidal dispersions, emulsions, powders, foams, polymeric liquids and gels, exhibit this complex behaviour. In addition, understanding the behaviour of these complex fluids is also very important in biological systems.

The area of complex fluids and soft matter has been evolving with rapid advances in experimental and computational techniques. The main aim of this book is to introduce these advanced techniques, after a review of fundamental aspects. Since a study of complex fluids involves multidisciplinary tools, contributors with different backgrounds have contributed chapters to this book.

The chapters in this book are based on lectures delivered in the School on Rheology of Complex Fluids, held at Indian Institute of Technology Madras, Chennai during January 4–9, 2010. This school is a part of such series, earlier held at different institutions over the last decade in India. The aim of these schools has been to bring together young researchers and teachers from educational and R&D institutions, and expose them to the basic concepts and research techniques used in the study of rheological behaviour of complex fluids. These schools have been sponsored by the Department of Science and Technology, India.

The book begins with introductory chapters on non-Newtonian fluids, rheological response and fluid mechanics. This is followed by an exposition on how to understand multicomponent and multiphase systems, of which a lot of complex fluids are examples. Analysis of rheological behaviour has been facilitated by experimental and theoretical techniques. The next section of the book gives examples of these
in the form of large amplitude oscillatory shear, flow visualizations and stability analysis. The remaining chapters of the book cover application areas of polymers, active fluids and granular materials and their rheology.

IIT Madras, India
April 2010

Abhijit P. Deshpande
J. Murali Krishnan
P. B. Sunil Kumar
Acknowledgements

The chapters in this book are based on lectures delivered in the School on Rheology of Complex Fluids, held at Indian Institute of Technology Madras, Chennai during January 4–9, 2010. The school was sponsored by Science and Engineering Research Council, Department of Science and Technology, Government of India. We acknowledge all the participants for active discussions during the school which had helped in shaping the contents of this book.

Devang Khakhar, R P Chhabra, K S Gandhi and Sriram Ramaswamy played an important role in deciding the topics presented here. We thank them for their advice.

We thank R P Chhabra, V Kumaran, K R Rajagopal, S Pushpavanam, Rama Govindarajan, Arti Dua, P Sunthar, Gautam Menon and Mehrdad Massoudi for their contributions. Their help in providing lecture notes, draft chapters and proofreading is acknowledged.

We thank the patient typesetting support provided by VALARDOCS and their team through several modifications of the chapters. Editorial assistance from Santosh was very crucial in bringing the book to its present form. Brett Kurzman and Amanda Davis from Springer helped us in getting through different stages of the book preparation.

Murali Krishnan would like to thank Mary Yvonne Lanzerotti for her encouragement in the earlier stages of this book preparation.

We also acknowledge the support provided by Indian Institute of Technology Madras during different stages of this book preparation.

IIT Madras, India
April 2010

Abhijit P. Deshpande
J. Murali Krishnan
P. B. Sunil Kumar
## Contents

Part I  Background

1  Non-Newtonian Fluids: An Introduction .......................... 3  
   Rajendra P. Chhabra

2  Fundamentals of Rheology ........................................ 35  
   V. Kumaran

3  Mechanics of Liquid Mixtures .................................... 67  
   Kumbakonam Ramamani Rajagopal

Part II  Rheology

4  Oscillatory Shear Rheology for Probing Nonlinear Viscoelasticity of Complex Fluids: Large Amplitude Oscillatory Shear ............................................................. 87  
   Abhijit P. Deshpande

5  PIV Techniques in Experimental Measurement of Two Phase (Gas–Liquid) Systems ......................................................... 111  
   Basheer Ashraf Ali and Subramaniam Pushpavanam

6  An Introduction to Hydrodynamic Stability ......................... 131  
   Anubhab Roy and Rama Govindarajan

Part III  Applications

7  Statics and Dynamics of Dilute Polymer Solutions ................ 151  
   Arti Dua

8  Polymer Rheology ..................................................... 171  
   P. Sunthar
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>Active Matter</td>
<td>193</td>
</tr>
<tr>
<td></td>
<td>Gautam I. Menon</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Mathematical Modelling of Granular Materials</td>
<td>219</td>
</tr>
<tr>
<td></td>
<td>Mehrdad Massoudi</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Author Index</strong></td>
<td>247</td>
</tr>
<tr>
<td></td>
<td><strong>Subject Index</strong></td>
<td>253</td>
</tr>
</tbody>
</table>
Contributors

**Basheer Ashraf Ali**  Research Scholar, Indian Institute of Technology Madras, Chennai 600036, India, ashrafmchem@gmail.com

**Rajendra P. Chhabra**  Department of Chemical Engineering, Indian Institute of Technology Kanpur 208016, India, chhabra@iitk.ac.in

**Abhijit P. Deshpande**  Indian Institute of Technology Madras, Chennai 600036, India, abhijit@iitm.ac.in

**Arti Dua**  Department of Chemistry, Indian Institute of Technology Madras, Chennai 600036, India, arti@iitm.ac.in

**Rama Govindarajan**  Engineering Mechanics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Jakkur, Bangalore 560064, India, rama@jncasr.ac.in

**V. Kumaran**  Department of Chemical Engineering, Indian Institute of Science, Bangalore 560 012, India, kumaran@chemeng.iisc.ernet.in

**Mehrdad Massoudi**  U.S. Department of Energy, National Energy Technology Laboratory (NETL), Pittsburgh, PA 15236, USA, massoudi@netl.doe.gov

**Gautam I. Menon**  The Institute of Mathematical Sciences, CIT Campus, Taramani, Chennai 600113, India, menon@imsc.res.in

**Subramaniam Pushpavanam**  Professor, Indian Institute of Technology Madras, Chennai 600036, India, spush@iitm.ac.in

**Kumbakonam Ramamani Rajagopal**  Department of Mechanical Engineering, Texas A&M University, College Station, TX 77845, USA, krajagopal@tamu.edu

**Anubhab Roy**  Engineering Mechanics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Jakkur, Bangalore 560064, India, anubhab@jncasr.ac.in

**P. Sunthar**  Department of Chemical Engineering, Indian Institute of Technology Bombay, Mumbai 400076, India, sunthar@che.iitb.ac.in
Acronyms

BSW  Baumgärtel, Schausberger and Winter spectrum
CCD  Charge Coupled Device
CFD  Computational Fluid Dynamics
FENE Finitely Extensible Nonlinear Elasticity
GNF  Generalized Newtonian Fluids
LASER Light Amplification by Stimulated Emission of Light Radiation
LAOS  Large Amplitude Oscillatory Shear
LDV  Laser Doppler Velocimetry
PIB  Polyisobutylene
PIV  Particle Image Velocimetry
PM  Parsimonious Model Spectrum
PMFI Principle of Material Frame-Indifference
PMMA Polymethylmethacrylate
SAOS Small Amplitude Oscillatory Shear
SSP  Self-Sustaining Process
VIBGYOR Violet, Indigo, Blue, Green, Yellow, Orange, Red
WLC  Wormlike Chain
Part I

Background
Chapter 1
Non-Newtonian Fluids: An Introduction

Rajendra P. Chhabra

Abstract The objective of this chapter is to introduce and to illustrate the frequent and wide occurrence of non-Newtonian fluid behaviour in a diverse range of applications, both in nature and in technology. Starting with the definition of a non-Newtonian fluid, different types of non-Newtonian characteristics are briefly described. Representative examples of materials (foams, suspensions, polymer solutions and melts), which, under appropriate circumstances, display shear-thinning, shear-thickening, visco-plastic, time-dependent and viscoelastic behaviour are presented. Each type of non-Newtonian fluid behaviour has been illustrated via experimental data on real materials. This is followed by a short discussion on how to engineer non-Newtonian flow characteristics of a product for its satisfactory end use by manipulating its microstructure by controlling physico-chemical aspects of the system. Finally, we touch upon the ultimate question about the role of non-Newtonian characteristics on the analysis and modelling of the processes of pragmatic engineering significance.

1.1 Introduction

Most low-molecular-weight substances such as organic and inorganic liquids, solutions of low-molecular-weight inorganic salts, molten metals and salts and gases exhibit Newtonian flow characteristics, i.e., at constant temperature and pressure, in simple shear, the shear stress $\tau$ is proportional to the rate of shear $\dot{\gamma}$ and the constant of proportionality is the familiar dynamic viscosity $\eta$. Such fluids are classically known as the Newtonian fluids, albeit the notion of flow and of viscosity predates Newton [40]. For most liquids, the viscosity decreases with temperature...
Table 1.1 Values of viscosity for common fluids at room temperature

<table>
<thead>
<tr>
<th>Substance</th>
<th>$\eta$(Pa s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>Water</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Ethyl alcohol</td>
<td>$1.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>Mercury</td>
<td>$1.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>Ethylene glycol</td>
<td>$20 \times 10^{-3}$</td>
</tr>
<tr>
<td>Olive oil</td>
<td>0.1</td>
</tr>
<tr>
<td>100% Glycerol</td>
<td>1.5</td>
</tr>
<tr>
<td>Honey</td>
<td>10</td>
</tr>
<tr>
<td>Corn syrup</td>
<td>100</td>
</tr>
<tr>
<td>Bitumen</td>
<td>$10^8$</td>
</tr>
<tr>
<td>Molten glass</td>
<td>$10^{12}$</td>
</tr>
</tbody>
</table>

and increases with pressure. For gases, it increases with both temperature and pressure [35]. Broadly, higher the viscosity of a substance, more the resistance it presents to flow (and hence more difficult to pump!). Table 1.1 provides typical values of viscosity for scores of common fluids [12]. As we go down in the table, the viscosity increases by several orders of magnitude, and thus one can argue that a solid can be treated as a fluid whose viscosity tends towards infinity, $\eta \rightarrow \infty$. Thus, the distinction between a fluid and a solid is not as sharp as we would like to think! Ever since the formulation of the equations of continuity (mass) and momentum (Cauchy, Navier–Stokes), the fluid dynamics of Newtonian fluids has come a long way during the past 300 or so years, albeit significant challenges especially in the field of turbulence and multiphase flows still remain. During the past 50–60 years, there has been a growing recognition of the fact that many substances of industrial significance, especially of multiphase nature (foams, emulsions, dispersions and suspensions, slurries, for instance) and polymeric melts and solutions (both natural and manmade) do not conform to the Newtonian postulate of the linear relationship between $\sigma$ and $\dot{\gamma}$ in simple shear, for instance. Accordingly, these fluids are variously known as non-Newtonian, non-linear, complex or rheologically complex fluids. Table 1.2 gives a representative list of fluids, which exhibit different kinds and varying severity of non-Newtonian flow behaviour [12]. Indeed, so widespread is the non-Newtonian fluid behaviour in nature and in technology that it would be no exaggeration to say that the Newtonian fluid behaviour is an exception rather than the rule! This chapter endeavours to provide a brief introduction to the different kinds of non-Newtonian flow characteristics, their characterization and implications in engineering applications. The material presented herein is mainly drawn from our recent books [11, 12]. The assumptions of material isotropy and incompressibility are implicit throughout our discussion.
Table 1.2 Examples of substances exhibiting non-Newtonian fluid behaviour

| Adhesives (wall paper paste, carpet adhesive, for instance) | Food stuffs (Fruit/vegetable purees and concentrates, sauces, salad dressings, mayonnaise, jams and marmalades, ice-cream, soups, cake mixes and cake toppings, egg white, bread mixes, snacks) |
| Ales (beer, liqueurs, etc.) | Greases and lubricating oils |
| Animal waste slurries from cattle farms | Mine tailings and mineral suspensions |
| Biological fluids (blood, synovial fluid, saliva, etc.) | Molten lava and magmas |
| Bitumen | Paints, polishes and varnishes |
| Cement paste and slurries | Paper pulp suspensions |
| Chalk slurries | Peat and lignite slurries |
| Chocolates | Polymer melts and solutions, reinforced plastics, rubber |
| Coal slurries | Printing colors and inks |
| Cosmetics and personal care products (nail polish, lotions and creams, lipsticks, shampoos, shaving foams and creams, toothpaste, etc.) | Pharmaceutical products (creams, foams, suspensions, for instance) |
| Dairy products and dairy waste streams (cheese, butter, yogurt, fresh cream, whey, for instance) | Sewage sludge |
| Drilling muds | Wet beach sand |
| Fire fighting foams | Waxy crude oils |

1.2 Classification of Fluid Behaviour

1.2.1 Definition of a Newtonian Fluid

It is useful to begin with the definition of a Newtonian fluid. In simple shear (Fig. 1.1), the response of a Newtonian fluid is characterized by a linear relationship between the applied shear stress and the rate of shear, i.e.,

$$\sigma_{yx} = \frac{F}{A} = \eta \dot{y}_{yx}. \quad (1.1)$$

Figure 1.2 shows experimental results for corn syrup and for cooking oil confirming their Newtonian fluid behaviour; the flow curves pass through the origin and the viscosity values are $\eta = 11.6 \text{ Pa s}$ for corn syrup and $\eta = 64 \text{ mPa s}$ for the cooking oil. Figure 1.1 and (1.1), of course, represent the simplest case wherein there is only one non-zero component of velocity, $V_x$, which is a function of $y$. For the general case of three-dimensional flow (Fig. 1.3), clearly there are six shearing and three normal components of the stress tensor, $S$. It is customary to split the total stress into an isotropic part (pressure, $p$) and a deviatoric part as

$$S = -p I + \sigma, \quad (1.2)$$
where \( \sigma \) is traceless, i.e., \( tr \cdot \sigma = 0 \), and pressure is consistent with the continuity equation. The trace-free requirement together with the physical requirement of symmetry \( \sigma = \sigma^T \) implies that there are only three independent shear components (off-diagonal elements) and two normal stress differences (diagonal elements) of the deviatoric stress. Thus, in Cartesian coordinates, these are \( \sigma_{xy} (= \sigma_{yx}) \), \( \sigma_{xz} (= \sigma_{zx}) \) and \( \sigma_{yz} (= \sigma_{zy}) \), and the two normal stress differences are defined as

Primary normal stress difference, \( N_1 = \sigma_{xx} - \sigma_{yy} \) \hspace{1cm} (1.3)

Secondary normal stress difference, \( N_2 = \sigma_{yy} - \sigma_{zz} \) \hspace{1cm} (1.4)

For Newtonian fluids, these components are related linearly to the rate of deformation of tensor components via the scalar viscosity. For instance, the three stress
components acting on the \( x \)-face (oriented normal to the \( x \)-axis) in Fig. 1.3 are written as follows:

\[
\sigma_{xx} = -2\eta \frac{\partial V_x}{\partial x}, \quad (1.5)
\]
\[
\sigma_{xy} = -\eta \left( \frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right), \quad (1.6)
\]
\[
\sigma_{xz} = -\eta \left( \frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x} \right). \quad (1.7)
\]

Similar sets of equations can be set up for the stress components relevant to the \( y \)- and \( z \)-planes. For a Newtonian fluid, in simple shear, \( \sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0 \), because \( V_x \) only varies in the \( y \)-direction. Thus, the complete definition of a Newtonian fluid requires it to satisfy the complete Navier–Stokes equations rather than simply exhibit a constant value of shear viscosity.

\subsection*{1.2.2 Non-Newtonian Fluid Behaviour}

The simplest possible deviation from the Newtonian fluid behaviour occurs when the simple shear data \( \sigma - \dot{\gamma} \) do not pass through the origin and/or does not result in a linear relationship between \( \sigma \) and \( \dot{\gamma} \). Conversely, the apparent viscosity, defined as \( \sigma / \dot{\gamma} \), is not constant and is a function of \( \sigma \) or \( \dot{\gamma} \). Indeed, under appropriate circumstances, the apparent viscosity of certain materials is not only a function of flow conditions (geometry, rate of shear, etc.), but it also depends on the kinematic history of the fluid element under consideration. It is convenient, though arbitrary
(and probably unscientific too), to group such materials into the following three categories:

1. Systems for which the value of \( \gamma \) at a point within the fluid is determined only by the current value of \( \sigma \) at that point, or vice versa, these substances are variously known as purely viscous, inelastic, time-independent or generalized Newtonian fluids (GNF);
2. Systems for which the relation between \( \sigma \) and \( \gamma \) shows further dependence on the duration of shearing and kinematic history; these are called time-dependent fluids, and finally,
3. Systems which exhibit a blend of viscous fluid-like behaviour and of elastic solid-like behaviour. For instance, this class of materials shows partial elastic recovery, recoil, creep, etc. Accordingly, these are called viscoelastic or elastico-viscous fluids.

As noted earlier, the aforementioned classification scheme is quite arbitrary, though convenient, because most real materials often display a combination of two or even all these types of features under appropriate circumstances. For instance, it is not uncommon for a polymer melt to show time-independent (shear-thinning) and viscoelastic behaviour simultaneously and for a china clay suspension to exhibit a combination of time-independent (shear-thinning or shear-thickening) and time-dependent (thixotropic) features at certain concentrations and/or at appropriate shear rates. Generally, it is, however, possible to identify the dominant non-Newtonian aspect and to use it as a basis for the subsequent process calculations. Each type of non-Newtonian fluid behaviour is now dealt with in more detail.

### 1.3 Time-Independent Fluid Behaviour

As noted above, in simple unidirectional shear, this subset of fluids is characterized by the fact that the current value of the rate of shear at a point in the fluid is determined only by the corresponding current value of the shear stress and vice versa. Conversely, one can say that such fluids have no memory of their past history. Thus, their steady shear behaviour may be described by a relation of the form,

\[
\dot{\gamma}_{yx} = f(\sigma_{yx}),
\]  

(1.8)

or, its inverse form,

\[
\sigma_{yx} = f^{-1}(\dot{\gamma}_{yx}).
\]  

(1.9)

Depending upon the form of (1.8) or (1.9), three possibilities exist:

1. Shear-thinning or pseudoplastic behavior
2. Visco-plastic behaviour with or without shear-thinning behaviour
3. Shear-thickening or dilatant behaviour.
Figure 1.4 shows qualitatively the flow curves (also called rheograms) on linear coordinates for the above-noted three categories of fluid behaviour; the linear relation typical of Newtonian fluids is also included in Fig. 1.4.

1.3.1 **Shear-Thinning Fluids**

This is perhaps the most widely encountered type of time-independent non-Newtonian fluid behaviour in engineering practice. It is characterized by an apparent viscosity $\eta$ (defined as $\sigma_{yx}/\dot{y}_{yx}$), which gradually decreases with increasing shear rate. In polymeric systems (melts and solutions), at low shear rates, the apparent viscosity approaches a Newtonian plateau, where the viscosity is independent of shear rate (zero shear viscosity, $\eta_0$).

$$\lim_{\dot{y}_{yx} \to 0} \frac{\sigma_{yx}}{\dot{y}_{yx}} = \eta_0.$$  \hspace{1cm} (1.10)

Furthermore, polymer solutions also exhibit a similar plateau at very high shear rates (infinite shear viscosity, $\eta_\infty$), i.e.,

$$\lim_{\dot{y}_{yx} \to \infty} \frac{\sigma_{yx}}{\dot{y}_{yx}} = \eta_\infty.$$  \hspace{1cm} (1.11)

In most cases, the value of $\eta_\infty$ is only slightly higher than the solvent viscosity $\eta_s$. Figure 1.5 shows this behaviour in a polymer solution embracing the full spectrum
of values going from $\eta_0$ to $\eta_\infty$. Obviously, the infinite-shear limit is not seen for polymer melts and blends, or foams or emulsions or suspensions. Thus, the apparent viscosity of a pseudoplastic substance decreases with the increasing shear rate, as shown in Fig. 1.6 for three polymer solutions where not only the values of $\eta_0$ are seen to be different in each case, but the rate of decrease of viscosity with shear rate is also seen to vary from one system to another as well as with the shear rate interval considered. Finally, the value of shear rate marking the onset of shear-thinning is influenced by several factors such as the nature and concentration of polymer, the nature of solvent, etc. for polymer solutions and particle size, shape, concentration of solids in suspensions, for instance. Therefore, it is impossible to suggest valid generalizations, but many polymeric systems exhibit the zero-shear viscosity region below $\dot{\gamma} < 10^{-2}$ s$^{-1}$. Usually, the zero-shear viscosity region expands as the molecular weight of polymer falls, or its molecular weight distribution becomes narrower, as the concentration of polymer in the solution is reduced.

The next question which immediately comes to mind is that how do we approximate this type of fluid behaviour? Over the past 100 years or so, many mathematical equations of varying complexity and forms have been reported in the literature; some of these are straightforward attempts at curve fitting the experimental data ($\sigma$–$\dot{\gamma}$), while others have some theoretical basis (blended with empiricism) in statistical mechanics as an extension of the application of kinetic theory to the liquid state, etc. [9]. While extensive listing of viscosity models is available in several books, for e.g., see Ibarz and Barbosa–Canovas [23] and Govier and Aziz [19], a representative selection of widely used expressions is given here.
1.3.1.1 Power Law or Ostwald de Waele Equation

Often the relationship between shear stress ($\sigma$) and shear rate ($\dot{\gamma}$) plotted on log–log coordinates for a shear-thinning fluid can be approximated by a straight line over an interval of shear rate, i.e.,

$$\sigma = m(\dot{\gamma})^n,$$

or, in terms of the apparent viscosity,

$$\eta = m(\dot{\gamma})^{n-1}.$$ 

(1.12)\hspace{1cm} (1.13)

Obviously, $0 < n < 1$ will yield $(d\eta/d\dot{\gamma}) < 0$, i.e., shear-thinning behaviour of fluids is characterized by value of $n$ (power-law index) smaller than unity. Many polymer melts and solutions exhibit the value of $n$ in the range 0.3–0.7 depending upon the concentration and molecular weight of the polymer, etc. Even smaller values of power-law index ($n \sim 0.1–0.15$) are encountered with fine particle suspensions like kaolin-in-water, bentonite-in-water, etc. Naturally, smaller the value of $n$, more shear-thinning is the material. The other constant, $m$ (consistency index), is a measure of the consistency of the substance.

Although (1.12) or (1.13) offers the simplest approximation of shear-thinning behaviour, it predicts neither the upper nor the lower Newtonian plateau in the limits of $\dot{\gamma} \to 0$ or $\dot{\gamma} \to \infty$. Besides, the values of $m$ and $n$ are reasonably constant only over a narrow interval of shear rate range whence one needs to know a priori the likely range of shear rate to be encountered in an envisaged application.

---

**Fig. 1.6** Representative shear stress and apparent viscosity behaviour for three pseudoplastic polymer solutions
1.3.1.2 The Cross Viscosity Equation

In order to rectify some of the weaknesses of the power-law, Cross [14] presented the following empirical form, which has gained wide acceptance in the literature. In simple shear, it is written as

\[ \frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{1}{1 + m(\dot{\gamma})^n}. \]  

(1.14)

It is readily seen that for \( n < 1 \), this model also predicts shear-thinning behaviour. Furthermore, the Newtonian limit is recovered here when \( m \to 0 \). Though initially Cross [14] proposed that \( n = 2/3 \) was satisfactory for numerous substances, it is now thought that treating it as an adjustable parameter offers significant improvement in terms of the degree of fit [5]. Evidently, (1.14) correctly predicts \( \eta = \eta_0 \) and \( \eta = \eta_\infty \) in the limits of \( \dot{\gamma} \to 0 \) and \( \dot{\gamma} \to \infty \), respectively.

1.3.1.3 The Ellis Fluid Model

While the two viscosity models presented thus far are examples of the form of (1.9), the Ellis model is an illustration of the inverse form, (1.8). In unidirectional simple shear, it is written as

\[ \eta = \frac{\eta_0}{1 + \left( \frac{\sigma}{\sigma_{1/2}} \right)^{\frac{1}{\alpha-1}}}. \]  

(1.15)

In (1.15), \( \eta_0 \) is the zero-shear viscosity and the remaining two parameters \( \sigma_{1/2} \) and \( \alpha > 1 \) are adjusted to obtain the best fit to a given set of data. Clearly, \( \alpha > 1 \) yields the decreasing values of shear viscosity with increasing shear rate. It is readily seen that the Newtonian limit is recovered by setting \( \sigma_{1/2} \to \infty \). Furthermore, when \( \sigma/\sigma_{1/2} \gg 1 \), (1.15) reduces to the power-law model, (1.12) or (1.13).

1.3.2 Visco-Plastic Fluid Behaviour

This type of non-Newtonian fluid behaviour is characterized by the existence of a threshold stress (called yield stress or apparent yield stress, \( \sigma_0 \)), which must be exceeded for the fluid to deform (shear) or flow. Conversely, such a substance will behave like an elastic solid (or flow en masse like a rigid body) when the externally applied stress is less than the yield stress, \( \sigma_0 \). Of course, once the magnitude of the external yield stress exceeds the value of \( \sigma_0 \), the fluid may exhibit Newtonian behaviour (constant value of \( \eta \)) or shear-thinning characteristics, i.e., \( \eta(\dot{\gamma}) \). It therefore stands to reason that, in the absence of surface tension effects, such a material will not level out under gravity to form an absolutely flat free surface. Quantitatively, this type of behaviour can be hypothesized as follows: such a substance at rest consists of three-dimensional structures of sufficient rigidity to resist any external stress less
than $|\sigma_0|$ and therefore offers an enormous resistance to flow, albeit it still might deform elastically. For stress levels above $|\sigma_0|$, however, the structure breaks down and the substance behaves like a viscous material. In some cases, the build-up and breakdown of structure has been found to be reversible, i.e., the substance may regain its (initial or somewhat lower) value of the yield stress following a long period of rest.

A fluid with a linear flow curve for $|\sigma| > |\sigma_0|$ is called a Bingham plastic fluid, and is characterized by a constant value of viscosity $\eta_b$. Thus, in one-dimensional shear, the Bingham model is written as:

$$\sigma_{yx} = \sigma_0^B + \eta_b \dot{\gamma}_{yx} \quad |\sigma_{yx}| > |\sigma_0^B| \quad (1.16a)$$
$$\dot{\gamma}_{yx} = 0 \quad |\sigma_{yx}| < |\sigma_0^B| \quad (1.16b)$$

On the other hand, a visco-plastic material showing shear-thinning behaviour at stress levels exceeding $|\sigma_0|$ is known as a yield-pseudoplastic fluid, and its behaviour is frequently approximated by the so-called Herschel–Bulkley fluid model written for 1-D shear flow as follows:

$$\sigma_{yx} = \sigma_0^H + m (\dot{\gamma}_{yx})^n \quad |\sigma_{yx}| > |\sigma_0^H| \quad (1.17a)$$
$$\dot{\gamma}_{yx} = 0 \quad |\sigma_{yx}| < |\sigma_0^H| \quad (1.17b)$$

Another commonly used viscosity model for visco-plastic fluids is the so-called Casson model, which has its origins in modelling the flow of blood, but it has been found a good approximation for many other substances also [5, 6]. It is written as:

$$\sqrt{|\sigma_{yx}|} = \sqrt{|\sigma_0^C|} + \sqrt{\eta_c |\dot{\gamma}_{yx}|} \quad |\sigma_{yx}| > |\sigma_0^C| \quad (1.18a)$$
$$\dot{\gamma}_{yx} = 0 \quad |\sigma_{yx}| < |\sigma_0^C| \quad (1.18b)$$

While qualitative flow curves for a Bingham fluid and for a yield-pseudoplastic fluid are included in Fig. 1.4, experimental data for a synthetic polymer solution and a meat extract are shown in Fig. 1.7. The meat extract ($\sigma_0 = 17$ Pa) conforms to (1.16), whereas the carbopol solution ($\sigma_0 = 68$ Pa) shows yield-pseudoplastic behaviour.

Typical examples of yield-stress fluids include blood, yoghurt, tomato puree, molten chocolate, tomato sauce, cosmetics, nail polishes, foams, suspensions, etc. Thorough reviews on the rheology and fluid mechanics of visco-plastic fluids are available in the literature [3, 6].

Finally, before leaving this subsection, it is appropriate to mention here that it has long been a matter of debate and discussion in the literature whether a true yield stress exists or not, e.g., see the trail-blazing paper of Barnes and Walters [4] and the review of Barnes [3] for different viewpoints on this matter. Many workers view the
yield stress in terms of a transition from solid-like behaviour to fluid-like behaviour which manifests itself in terms of an abrupt decrease in viscosity (by several orders of magnitude in many substances) over an extremely narrow range of shear rate [43]. Evidently, the answer to the question whether a substance has a yield stress or not seems to be closely related to the choice of a time scale of observation. In spite of this fundamental difficulty, the notion of an apparent yield stress is of considerable value in the context of engineering applications, especially for product development and design in food, pharmaceutical and health care sectors [3, 36].

### 1.3.3 Shear-Thickening or Dilatant Behaviour

This class of fluids is similar to pseudoplastic systems in that it shows no yield stress, but its apparent viscosity increases with the increasing shear rate and hence the name shear-thickening. Originally, this type of behaviour was observed in concentrated suspensions, and one can qualitatively explain it as follows: at rest, the voidage of the suspension is minimum and the liquid present in the sample is sufficient to fill the voids completely. At low shearing levels, the liquid lubricates the motion of each particle past another thereby minimizing solid–solid friction. Consequently, the resulting stresses are small. At high shear rates, however, the mixture expands (dilates) slightly (similar to that seen in sand dunes) so that the available liquid is no longer sufficient to fill the increased void space and to prevent direct solid–solid contacts (and friction). This leads to the development of much larger shear stresses