Thermo-Fluid Dynamics of Two-Phase Flow
Dedication

This book is dedicated to our parents and wives.
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Preface
Thermo-Fluid Dynamics of Two-Phase Flow discussed in detail. The macroscopic formulations are presented in terms of the two-fluid model and drift-flux model which are two of the most accurate and useful formulations for practical engineering problems. The change of the interfacial structures in two-phase flow is dynamically modeled through the interfacial area transport equation. This is a new approach which can replace the static and inaccurate approach based on the flow regime transition criteria. The interfacial momentum transfer models are discussed in great detail, because for most two-phase flow, thermo-fluid dynamics are dominated by the interfacial structures and interfacial momentum transfer. Some other necessary constitutive relations such as the turbulence modeling, transient forces and lift forces are also discussed.

Mamoru Ishii, Ph.D.

Takashi Hibiki, Ph.D.
Foreword

Thermo-Fluid Dynamics of Two-Phase Flow takes a major step forward in our quest for understanding fluids as they metamorphose through change of phase, properties and structure. Like Janus, the mythical Roman God with two faces, fluids separating into liquid and gas, each state sufficiently understood on its own, present a major challenge to the most astute and insightful scientific minds when it comes to deciphering their dynamic entanglement.

The challenge stems in part from the vastness of scale where two-phase phenomena can be encountered. Between the microscopic nano-scale of molecular dynamics and deeply submerged modeling assumptions and the macro-scale of measurements, there is a meso-scale as broad as it is nebulous and elusive. This is the scale where everything is in a permanent state of exchange, a Heraclitean state of flux, where nothing ever stays the same and where knowledge can only be achieved by firmly grasping the underlying principles of things.

The subject matter has sprung from the authors' own firm grasp of fundamentals. Their bibliographical contributions on two-phase principles reflect a scientific tradition that considers theory and experiment a duality as fundamental as that of appearance and reality. In this it differs from other topical works in the science of fluids. For example, the leading notion that runs through two-phase flow is that of interfacial velocity. It is a concept that requires, amongst other things, continuous improvements in both modeling and measurement. In the meso-scale, this gives rise to new science of the interface which, besides the complexity of its problems and the fuzziness of its structure, affords ample scope for the creation of elegant, parsimonious formulations, as well as promising engineering applications.
Thermo-Fluid Dynamics of Two-Phase Flow

The two-phase flow theoretical discourse and experimental inquiry are closely linked. The synthesis that arises from this connection generates immense technological potential for measurements informing and validating dynamic models and conversely. The resulting technology finds growing utility in a broad spectrum of applications, ranging from next generation nuclear machinery and space engines to pharmaceutical manufacturing, food technology, energy and environmental remediation.

This is an intriguing subject and its proper understanding calls for exercising the rigorous tools of advanced mathematics. The authors, with enormous care and intellectual affection for the subject reach out and invite an inclusive audience of scientists, engineers, technologists, professors and students.

It is a great privilege to include the Thermo-Fluid Dynamics of Two-Phase Flow in the series Smart Energy Systems: Nanowatts to Terawatts.

This is work that will stand the test of time for its scientific value as well as its elegance and aesthetic character.

Lefteri H. Tsoukalas, Ph.D.

Smart Energy Systems: Nanowatts to Terawatts

Lefteri H. Tsoukalas, Ph.D.
Acknowledgments
INTRODUCTION

1.1 Relevance of the problem
Transport Systems
Air-lift pump; ejectors; pipeline transport of gas and oil mixtures, of slurries, of fibers, of wheat, and of pulverized solid particles; pumps and hydrofoils with cavitations; pneumatic conveyors; highway traffic flows and controls, etc.

Information Systems
Superfluidity of liquid helium; conducting or charged liquid film; liquid crystals, etc.

Lubrication Systems
Two-phase flow lubrication; bearing cooling by cryogenics, etc.

Environmental Control
Air conditioners; refrigerators and coolers; dust collectors; sewage treatment plants; pollutant separators; air pollution controls; life support systems for space application, etc.

Geo-Meteorological Phenomena
Sedimentation; soil erosion and transport by wind; ocean waves; snow drifts; sand dune formations; formation and motion of rain droplets; ice formations; river floodings, landslides, and snowslides; physics of clouds, rivers or seas covered by drift ice; fallout, etc.

Biological Systems
Cardiovascular system; respiratory system; gastrointestinal tract; blood flow; bronchus flow and nasal cavity flow; capillary transport; body temperature control by perspiration, etc.

It can be said that all systems and components listed above are governed by essentially the same physical laws of transport of mass, momentum and energy. It is evident that with our rapid advances in engineering technology, the demands for progressively accurate predictions of the systems in interest have increased. As the size of engineering systems becomes larger and the operational conditions are being pushed to new limits, the precise understanding of the physics governing these multiphase flow systems is indispensable for safe as well as economically sound operations. This means a shift of design methods from the ones exclusively based on static experimental correlations to the ones based on mathematical models that can predict dynamical behaviors of systems such as transient responses and stabilities. It is clear that the subject of multiphase flow has immense...
1.2 Characteristic of multiphase flow
major groups which can be called flow regimes or patterns such as separated flow, transitional or mixed flow and dispersed flow. It can be expected that many of two-phase flow systems should exhibit certain degree of physical similarity when the flow regimes are same. However, in general, the concept of two-phase flow regimes is defined based on a macroscopic volume or length scale which is often comparative to the system length scale. This implies that the concept of two-phase flow regimes and regime-dependent model require an introduction of a large length scale and associated limitations. Therefore, regime-dependent models may lead to an analysis that cannot mechanistically address the physics and phenomena occurring below the reference length scale.

For most two-phase flow problems, the local instant formulation based on the single-phase flow formulation with explicit moving interfaces encounters insurmountable mathematical and numerical difficulties, and therefore it is not a realistic or practical approach. This leads to the need of a macroscopic formulation based on proper averaging which gives a two-phase flow continuum formulation by effectively eliminating the interfacial discontinuities. The essence of the formulation is to take into account for the various multi-scale physics by a cascading modeling approach, bringing the micro and meso-scale physics into the macroscopic continuum formulation.

The above discussion indicates the origin of the difficulties encountered in developing broad understanding of multiphase flow and the generalized method for analyzing such flow. The two-phase flow physics are fundamentally multi-scale in nature. It is necessary to take into account these cascading effects of various physics at different scales in the two-phase flow formulation and closure relations. At least four different scales can be important in multiphase flow. These are 1) system scale, 2) macroscopic scale required for continuum assumption, 3) mesoscale related to local structures, and 4) microscopic scale related to fine structures and molecular transport. At the highest level, the scale is the system where system transients and component interactions are the primary focus. For example, nuclear reactor accidents and transient analysis requires specialized system analysis codes. At the next level, macro physics such as the structure of interface and the transport of mass, momentum and energy are addressed. However, the multiphase flow field equations describing the conservation principles require additional constitutive relations for bulk transfer. This encompasses the turbulence effects for momentum and energy as well as for interfacial exchanges for mass, momentum and energy transfer. These are meso-scale physical phenomena that require concentrated research efforts. Since the interfacial transfer rates can be considered as the product of the interfacial flux and the available interfacial area, the modeling of the interfacial area concentration is essential. In two-phase flow analysis, the
1. Introduction

void fraction and the interfacial area concentration represent the two fundamental first-order geometrical parameters and, therefore, they are closely related to two-phase flow regimes. However, the concept of the two-phase flow regimes is difficult to quantify mathematically at the local point because it is often defined at the scale close to the system scale. This may indicate that the modeling of the changes of the interfacial area concentration directly by a transport equation is a better approach than the conventional method using the flow regime transitions criteria and regime-dependent constitutive relations for interfacial area concentration. This is particularly true for a three-dimensional formulation of two-phase flow. The next lower level of physics in multiphase flow is related to the local microscopic phenomena, such as: the wall nucleation or condensation; bubble coalescence and break-up; and entrainment and deposition.

1.3 Classification of two-phase flow

There are a variety of two-phase flows depending on combinations of two phases as well as on interface structures. Two-phase mixtures are characterized by the existence of one or several interfaces and discontinuities at the interface. It is easy to classify two-phase mixtures according to the combinations of two phases, since in standard conditions we have only three states of matters and at most four, namely, solid, liquid, and gas phases and possibly plasma (Pai, 1972). Here, we consider only the first three phases, therefore we have:

1. Gas-solid mixture;
2. Gas-liquid mixture;
3. Liquid-solid mixture;
4. Immiscible-liquid mixture.

It is evident that the fourth group is not a two-phase flow, however, for all practical purposes it can be treated as if it is a two-phase mixture.

The second classification based on the interface structures and the topographical distribution of each phase is far more difficult to make, since these interface structure changes occur continuously. Here we follow the standard flow regimes reviewed by Wallis (1969), Hewitt and Hall Taylor (1970), Collier (1972), Govier and Aziz (1972) and the major classification of Zuber (1971), Ishii (1971) and Kocamustafaogullari (1971). The two-phase flow can be classified according to the geometry of the interfaces into three main classes, namely, separated flow, transitional or mixed flow and dispersed flow as shown in Table 1-1.
Table 1-1. Classification of two-phase flow (Ishii, 1975)

<table>
<thead>
<tr>
<th>Classification</th>
<th>Examples</th>
<th>Configuration</th>
<th>Geometry</th>
<th>Typical regimes</th>
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<td>Transportation of powder</td>
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<td>Solid particles in gas or liquid</td>
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<td>Particulate flow</td>
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<td>Spray cooling</td>
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<td>Liquid droplets in gas</td>
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<td>Droplet flow</td>
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<td>Chemical reactors</td>
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<td>Gas bubbles in liquid</td>
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<td>Bubbly flow</td>
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<td>Dispersed flows</td>
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<td>Boiling nuclear reactor channel</td>
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<td>Gas core with droplets and liquid</td>
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<td>Bubbly droplet annular flow</td>
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<td>Steam generator</td>
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<td>Gas core with droplets and liquid</td>
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<td>Droplet annular flow</td>
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<td>Evaporators with wall nucleation</td>
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<td>Gas bubbles in liquid film with gas core</td>
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<td>Bubbly annular flow</td>
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<td>Sodium boiling in forced convection</td>
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<td>Gas pocket in liquid</td>
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<td>Cap, Slug or Churn-turbulent flow</td>
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<td>Mixed or Transitional flows</td>
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<td>Jet condenser</td>
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<td>Liquid jet in gas</td>
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<td>Gas jet in liquid</td>
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<td>Film boiling</td>
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<td>Gas film in liquid</td>
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<td>Film flow</td>
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<td>Separated flows</td>
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Depending upon the type of the interface, the class of separated flow can be divided into plane flow and quasi-axisymmetric flow each of which can be subdivided into two regimes. Thus, the plane flow includes film and stratified flow, whereas the quasi-axisymmetric flow consists of the annular flow.
1. Introduction

and the jet-flow regimes. The various configurations of the two phases and of the immiscible liquids are shown in Table 1-1.

The class of dispersed flow can also be divided into several types. Depending upon the geometry of the interface, one can consider spherical, elliptical, granular particles, etc. However, it is more convenient to subdivide the class of dispersed flows by considering the phase of the dispersion. Accordingly, we can distinguish three regimes: bubbly, droplet or mist, and particulate flow. In each regime the geometry of the dispersion can be spherical, spheroidal, distorted, etc. The various configurations between the phases and mixture components are shown in Table 1-1.

As it has been noted above, the change of interfacial structures occurs gradually, thus we have the third class which is characterized by the presence of both separated and dispersed flow. The transition happens frequently for liquid-vapor mixtures as a phase change progresses along a channel. Here too, it is more convenient to subdivide the class of mixed flow according to the phase of dispersion. Consequently, we can distinguish five regimes, i.e., cap, slug or churn-turbulent flow, bubbly-annular flow, bubbly annular-droplet flow and film flow with entrainment. The various configurations between the phases and mixture components are shown in Table 1-1.

Figures 1-1 and 1-2 show typical air-water flow regimes observed in vertical 25.4 mm and 50.8 mm diameter pipes, respectively. The flow regimes in the first, second, third, fourth, and fifth figures from the left are bubbly, cap-bubbly, slug, churn-turbulent, and annular flows, respectively. Figure 1-3 also shows typical air-water flow regimes observed in a vertical rectangular channel with the gap of 10 mm and the width of 200 mm. The flow regimes in the first, second, third, and fourth figures from the left are bubbly, cap-bubbly, churn-turbulent, and annular flows, respectively. Figure 1-4 shows inverted annular flow simulated adiabatically with turbulent water jets, issuing downward from large aspect ratio nozzles, enclosed in gas annuli (De Jarlais et al., 1986). The first, second, third and fourth images from the left indicate symmetric jet instability, sinuous jet instability, large surface waves and skirt formation, and highly turbulent jet instability, respectively. Figure 1-5 shows typical images of inverted annular flow at inlet liquid velocity 10.5 cm/s, inlet gas velocity 43.7 cm/s (nitrogen gas) and inlet Freon-113 temperature 23 ºC with wall temperature of near 200 ºC (Ishii and De Jarlais, 1987). Inverted annular flow was formed by introducing the test fluid into the test section core through thin-walled, tubular nozzles coaxially centered within the heater quartz tubing, while vapor or gas is introduced in the annular gap between the liquid nozzle and the heated quartz tubing. The absolute vertical size of each image is 12.5 cm. The visualized elevation is higher from the left figure to the right figure.
Figure 1-1. Typical air-water flow images observed in a vertical 25.4 mm diameter pipe

Figure 1-2. Typical air-water flow images observed in a vertical 50.8 mm diameter pipe

Figure 1-3. Typical air-water flow images observed in a rectangular channel of $200 \times 10$ mm
1. Introduction

Figure 1-4. Typical images of simulated air-water inverted annular flow (It is cocurrent down flow)

Figure 1-5. Axial development of Inverted annular flow (It is cocurrent up flow)
1.4 Outline of the book
The singular characteristic of two-phase or of two immiscible mixtures is the presence of one or several interfaces separating the phases or components. Examples of such flow systems can be found in a large number of engineering systems as well as in a wide variety of natural phenomena. The understanding of the flow and heat transfer processes of two-phase systems has become increasingly important in nuclear, mechanical and chemical engineering, as well as in environmental and medical science.

In analyzing two-phase flow, it is evident that we first follow the standard method of continuum mechanics. Thus, a two-phase flow is considered as a field that is subdivided into single-phase regions with moving boundaries between phases. The standard differential balance equations hold for each subregion with appropriate jump and boundary conditions to match the solutions of these differential equations at the interfaces. Hence, in theory, it is possible to formulate a two-phase flow problem in terms of the local instant variable, namely, $F = F(\mathbf{x}, t)$. This formulation is called a local instant formulation in order to distinguish it from formulations based on various methods of averaging. Such a formulation would result in a multiboundary problem with the positions of the interface being unknown due to the coupling of the fields and the boundary conditions. Indeed, mathematical difficulties encountered by using this local instant formulation can be considerable and, in many cases, they may be insurmountable. However, there are two fundamental importances in the local instant formulation. The first importance is the direct application to study the separated flows such as film, stratified, annular and jet flow, see Table 1-1. The formulation can be used there to study pressure drops, heat transfer, phase changes, the dynamic and stability of an interface, and the critical heat flux. In addition to the above applications, important examples of when this formulation can be used.
include: the problems of single or several bubble dynamics, the growth or collapse of a single bubble or a droplet, and ice formation and melting.

The second importance of the local instant formulation is as a fundamental base of the macroscopic two-phase flow models using various averaging. When each subregion bounded by interfaces can be considered as a continuum, the local instant formulation is mathematically rigorous. Consequently, two-phase flow models should be derived from this formulation by proper averaging methods. In the following, the general formulation of two-phase flow systems based on the local instant variables is presented and discussed. It should be noted here that the balance equations for a single-phase one component flow were firmly established for some time (Truesdell and Toupin, 1960; Bird et al, 1960). However, the axiomatic construction of the general constitutive laws including the equations of state was put into mathematical rigor by specialists (Coleman, 1964; Bowen, 1973; Truesdell, 1969). A similar approach was also used for a single-phase diffusive mixture by Muller (1968).

Before going into the detailed derivation and discussion of the local instant formulation, we review the method of mathematical physics in connection with the continuum mechanics. The next diagram shows the basic procedures used to obtain a mathematical model for a physical system.

As it can be seen from the diagram, a physical system is first replaced by a mathematical system by introducing mathematical concepts, general axioms and constitutive axioms. In the continuum mechanics they correspond to variables, field equations and constitutive equations, whereas at the singular surface the mathematical system requires the interfacial conditions. The latter can be applied not only at the interface between two phases, but also at the outer boundaries which limit the system. It is clear from the diagram that the continuum formulation consists of three essential parts, namely: the derivations of field equations, constitutive equations, and interfacial conditions.

Now let us examine the basic procedure used to solve a particular problem. The following diagram summarizes the standard method. Using the continuum formulation, the physical problem is represented by idealized boundary geometries, boundary conditions, initial conditions, field and...
1.1 Single-phase flow conservation equations

1.1.1 General balance equations
\[
\frac{d}{dt} \int_{V_m} \rho_k \psi_k dV = \int_{A_m} \mathbf{n}_k \cdot J_k dA + \int_{V_m} \rho \phi_k dV
\]

where \( \rho_k \psi_k \) is a material volume with a material surface \( A_m \). It states that the time rate of change of \( \rho_k \psi_k \) in \( V_m \) is equal to the influx through \( A_m \) plus the body source. The subscript \( k \) denotes the \( k \)th-phase. If the functions appearing in the Eq. (2-1) are sufficient such that the Jacobian transformation between material and spatial coordinates exists, then the familiar differential form of the balance equation can be obtained. This is done by using the Reynolds transport theorem (Aris, 1962) expressed as

\[
\frac{d}{dt} \int_{V_m} F_k dV = \int_{V_m} \frac{\partial F_k}{\partial t} dV + \int_{A_m} \mathbf{F}_k \mathbf{v}_k \cdot \mathbf{n} dA
\]

\[
\int_V \nabla \cdot F_k dV = \int_A \mathbf{n} \cdot F_k dA.
\]

\[
\frac{d}{dt} \int_{V_m} F_k dV = \int_{V_m} \left[ \frac{\partial F_k}{\partial t} + \nabla \cdot (\mathbf{v}_k F_k) \right] dV.
\]

\[
\frac{d}{dt} \int_V F_k dV = \int_V \frac{\partial F_k}{\partial t} dV + \int_A F_k \mathbf{u} \cdot \mathbf{n} dA
\]
2. Local Instant Formulation

where $t V$ is an arbitrary volume bounded by $t A$ and $\mathbf{u} \cdot \mathbf{n}$ is the surface displacement velocity of $t A$.

In view of Eqs.(2-1), (2-3) and (2-4) we obtain a differential balance equation.

$$\frac{\partial \rho_k \psi_k}{\partial t} + \nabla \cdot (\mathbf{v}_k \rho_k \psi_k) = -\nabla \cdot \mathbf{J}_k + \rho_k \phi_k.$$ 

1.1.2 Conservation equation

$$\psi_k = \phi_k = J_k =$$

$$\frac{\partial \rho_k}{\partial t} + \nabla \cdot (\rho_k \mathbf{v}_k) = .$$

$$\psi_k = \mathbf{v}_k$$

$$J_k = -\mathbf{T}_k = p_k \mathbf{I} - \mathbf{C}_k$$

$$\phi_k = \mathbf{g}_k$$

$$\mathbf{I}$$

$$\mathbf{T}$$

$$\mathbf{g}_k$$

$$\frac{\partial \rho_k \mathbf{v}_k}{\partial t} + \nabla \cdot (\rho_k \mathbf{v}_k \mathbf{v}_k) = -\nabla p_k + \nabla \cdot \mathbf{C}_k + \rho_k \mathbf{g}_k.$$