I. N. Melnikova
A. V. Vasilyev
**Short-Wave Solar Radiation in the Earth's Atmosphere**
Calculation, Observation, Interpretation

Irina N. Melnikova
Alexander V. Vasilyev

# Short-Wave Solar Radiation in the Earth's Atmosphere

## Calculation, Observation, Interpretation

with 60 Figures, 3 in color, and 19 Tables

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# Preface

Solar radiation has a decisive influence on climate and weather formation when passing through the atmosphere and interacting with the atmospheric components (gases, atmospheric aerosols, and clouds). The part of solar radiation that reaches the surface is a source of the existence and development of the biosphere because it regulates all biological processes. It should be mentioned that the part of solar radiation energy corresponding to the spectral region 0.35–1.0 μm is about 66% and to the spectral region 0.25–2.5 μm is more than 96% according to (Makarova et al. 1991). Thus, the study of the interaction between the atmosphere and the clouds and solar radiation in the short-wave range is especially interesting.

Numerous spectral solar radiation measurements have been made by the Atmospheric Physics Department, the Physics Faculty of Leningrad (now St. Petersburg) State University and in the Voeykov Main Geophysical Observatory under the guidance of academician Kirill Kondratyev for about 30 years from 1960. The majority of radiation observations were made during airborne experiments under clear sky condition (Kondratyev et al. 1974; Vasilyev O et al. 1987a; Kondratyev et al. 1975; Kondratyev et al. 1973; Vasilyev O et al. 1987b; Kondratyev and Ter-Markaryants 1976) and only 10 experiments were accomplished with an overcast sky (Kondratyev, Ter-Markaryants 1976); Vasilyev 1994 et al.; Kondratyev, Binenko 1984; Kondratyev, Binenko (1981). The results obtained have received international acknowledgment and currently this research direction is of special interest all over the world (King 1987; King et al. 1990; Asano 1994; Hayasaka et al. 1994; Kostadinov et al. 2000).

The airborne radiative observations were made over desert and water surfaces using the improved spectral instrument in the 1980s. As a result of 10-years of observations volume of the data set became very large. However, computer resources were not adequate for the instantaneous processing at that time. All the data were finally processed only at the end of the 1990s and now we have a rich database of the spectral values of the radiative characteristics (semispherical fluxes, intensity and spectral brightness coefficients) obtained under different atmospheric conditions. The database contains about 30,000 spectra including 2203 spectra of the upward and downward semispherical fluxes obtained during the airborne atmospheric sounding.

The inverse problem of atmospheric optics has been solved using the numerical method in the case of the interpretation of the observational results of the clear sky measurements and using the analytical method of the theory of radiation transfer in the case of overcast skies.

The interpretation of the radiative experiments under clear and overcast sky conditions is discussed in different sections because the mathematical methods of the description differ extensively. In addition, the extended (hundreds of kilometers) and stable (up to several days) cloudiness is worthy of special consideration because of its strong influence on the energy budget of the atmosphere and on climate formation.

It is necessary to set adequate optical parameters of the atmosphere for the practical problems of climatology, for distinguishing backgrounds and contrasts in the atmosphere and on the surface, and for the problems of the radiative regime of artificial and natural surfaces. The values obtained from the observational data are highly suitable in these cases. Unfortunately, to the present, theoretical values of the initial parameters are mostly used in the numerical simulations which leads to an incorrect estimation of the absorption of solar radiation in the atmosphere (especially when cloudy). The influence of the interaction of the atmospheric aerosols and cloudiness with solar radiation is taken into account in the numerical simulations of the global changes of the surface temperature only as the rest term for the coincidence between the calculated and observed values. The analysis of the database convinces us that solar radiation absorption in the dust and cloudy atmosphere is more significant than has been considered. Many authors have classified the experimental excess values of solar shortwave radiation absorption in clouds they obtained as an effect of "anomalous" absorption. This terminology indicates an underestimation of this absorption. Thus, the correct interpretation of the observational data, based on radiation transfer theory and the construction of the optical and radiative atmospheric models is of great importance.

Our results provide the spectral data of the solar irradiance measurements in the energetic units, the spectral values of the atmospheric optical parameters obtained from these experimental data and the spectral brightness coefficients of the surfaces of different types in figures and tables.

Let us point out the main results indicating the chapters where they are presented:

Chapter 1 reviews the definition of the characteristics of solar radiation and optical parameters describing the atmosphere and surface. The basic information about the interaction between solar radiation and atmospheric components (gases, aerosols and clouds) is cited as well.

In Chap. 2, the details of the radiative characteristic calculations in the atmosphere are considered. For the radiance and irradiance calculation, the Monte-Carlo method is chosen in the clear sky cases and the analytical method of the asymptotic formulas of the theory of radiation transfer is used for the overcast sky cases. Special attention is paid to the error analysis and applicability ranges of the methods. Different initial conditions of the cloudy atmosphere (the one-layer cloudiness, vertically homogeneous and heterogeneous, multilayer, the conservative scattering, accounting for the true absorption of radiation) are discussed as well.

In Chap. 3, the results of solar shortwave radiance and irradiance observation in the atmosphere are shown in detail. The authors have described both the instruments were used, as well as the special features of the measurements. Observational error analysis with the ways to minimizing the errors have been scrutinized. The methods of the data processing for obtaining the characteristics of solar radiation in the energetic units are elucidated. The examples of the vertical profiles of the spectral semispherical (upward and downward) fluxes observed under different atmospheric conditions are presented in figures in the text and in tables in Appendix 1. The results of the airborne, ground and satellite observations for the overcast skies are considered together with the contemporary views on the effect of the anomalous absorption of shortwave radiation in clouds.

In Chap. 4, the basic methods of procuring atmospheric optical parameters from the observational data of solar radiation are summarized. The application of the least-square technique for solving the atmospheric optics inverse problem is fully discussed. The influence of the observational errors on the accuracy of the solution is described and the methodology for its regularization is proposed. It is also shown how to choose the atmospheric parameters which are possible to retrieve from the radiative observations.

Chapter 5 is concerned with the methods and conditions of the inverse problem solving for clear sky conditions considered together with the results obtained. The vertical profiles and the spectral dependencies of the relevant parameters of the atmosphere and surface are shown in figures in the text and in tables in Appendix 1.

In Chap. 6, the analytical method for the retrieval of the stratus cloud optical parameters from the data of the ground, airborne and satellite radiance and irradiance observations including the full set of necessary formulas is elaborated. The example of the relevant formulas derivation for the case of using the data of the irradiance at the cloud top and bottom is demonstrated in Appendix 2. The analysis of the correctness of the inverse problem, existence, uniqueness and stability of the solution is performed and the uncertainties of the method are studied.

Chapter 7 provides the actual conditions of the cloud optical parameter retrieval from the data of the ground, airborne and satellite (ADEOS-1) observations. The spectral and vertical dependencies of the optical parameters are presented in figures in the text and in tables in Appendix 1. The analysis of the numerical values is accomplished, and the empirical hypothesis, which explains both the features revealed by the results and the anomalous absorption in clouds, is proposed. The book concludes with a summary of the results obtained.

The authors have wrote Chaps. 1 and 3 together, Sect. 2.1 and Chaps. 4 and 5 was written by Alexander Vasilyev, Chaps. 2 (excluding Sect. 2.1), 6 and 7 – by Irina Melnikova. The authors' intention was to present the material clearly for this book so that it would be useful for a large range of readers, including students, involved in the fields of atmospheric optics, the physics of the atmosphere, meteorology, climatology, the remote sounding of the atmosphere and surface and the distinguishing of backgrounds and contrasts of the natural and artificial objects in the atmosphere and on the surface.

It should be emphasized that the majority of the observations were made by the team headed by Vladimir Grishechkin (the Laboratory of Shortwave Solar Radiation of the Atmospheric Department of the Faculty of Physics, St. Petersburg State University). The authors would like to express their profound gratitude to Anatoly Kovalenko, Natalya Maltseva, Victor Ovcharenko, Lyudmila Poberovskaya, Igor Tovstenko and others who took part in the preparation of the instruments, the carrying out of the observations and the data processing. Unfortunately, our colleagues Pavel Baldin, Vladimir Grishechkin, Alexei Nikiforov and Oleg Vasilyev prematurely passed away. We dedicate the book to the memory of our friends and colleagues.

The authors very grateful to academician Kirill Kondratyev, Professors Vladislav Donchenko and Lev Ivlev, Victor Binenko and Vladimir Mikhailov for the fruitful discussions and valuable recommendations.

## References

Asano S (1994) Cloud and radiation studies in Japan. Cloud radiation interactions and their parameterisation in climate models. In: WCRP-86 (WMO/TD no 648). WMO, Geneva, pp 72–73

Hayasaka T, Kikuchi N, Tanaka M (1994) Absorption of solar radiation by stratocumulus clouds: aircraft measurements and theoretical calculations. J Appl Meteor 33:1047–1055

King MD (1987) Determination of the scaled optical thickness of cloud from reflected solar radiation measurements. J Atmos Sci 44:1734–1751

King MD, Radke L, Hobbs PV (1990) Determination of the spectral absorption of solar radiation by marine stratocumulus clouds from airborne measurements within clouds. J Atmos Sci 47:894–907

Kondratyev KYa, Ter-Markaryants NE (eds) (1976) Complex radiation experiments (in Russian). Gydrometeoizdat, Leningrad

Kondratyev KYa, Vasilyev OB, Grishechkin VS et al (1973) Spectral shortwave radiation inflow in the troposphere within spectral ranges 0.4–2.4 μm. I. Observational and processing methodology (in Russian). In Main geophysical observatory studies 322:12–23

Kondratyev KYa, Vasilyev OB, Grishechkin VS et al (1974) Spectral shortwave radiation inflow in the troposphere and their variability (in Russian). Izv. RAS, Atmospheric and Ocean Physics 10:453–503

Kondratyev KYa, Vasilyev OB, Ivlev LS et al (1975) Complex observational studies above the Caspian Sea (CAENEX-73) (in Russian). Meteorology and Hydrology, pp 3–10

Kondratyev KYa, Binenko VI (eds) (1981) The first Global Experiment PIGAP. vol 2. Polar aerosol, extended cloudiness and radiation. Gidrometeoizdat, Leningrad

Kondratyev KYa, Binenko VI (1984) Impact of Clouds on Radiation and Climate (in Russian). Gidrometeoizdat, Leningrad

Kostadinov I, Giovanelli G, Ravegnani F, Bortoli D, Petritoli A, Bonafè U, Rastello ML, Pisoni P (2000) Upward and downward irradiation measurements on board "Geophysica" aircraft during the APE-THESEO and APE-GAIA field campaigns. In: IRS'2000. Current problems in Atmospheric Radiation. Proceedings of the International Radiation Symposium, St.Petersburg, Russia, pp 1185–1188

Makarova EA, Kharitonov AV, Kazachevskaya TV (1991) Solar irradiance (in Russian). Nauka, Moscow

Vasilyev AV, Melnikova IN, Mikhailov VV (1994) Vertical profile of spectral fluxes of scattered solar radiation within stratus clouds from airborne measurements (Bilingual). Izv RAS, Atmosphere and Ocean. Physics 30:630–635

Vasilyev OB, Grishechkin VS, Kondratyev KYa (1987a) Spectral radiation characteristics of the free atmosphere above Lake Ladoga (in Russian). In: Complex remote lakes monitoring. Nauka, Leningrad, pp 187–207

Vasilyev OB, Grishechkin VS, Kovalenko AP et al (1987b) Spectral informatics – measuring system for airborne and ground study of shortwave radiation field in the atmosphere (in Russian). In Complex remote lakes monitoring. Nauka, Leningrad, pp 225–228

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# Solar Radiation in the Atmosphere

## 1.1
## Characteristics of the Radiation Field in the Atmosphere

In accordance with the contemporary conceptions, light (radiation) is an electromagnetic wave showing quantum properties. Thus, strictly speaking, the processes of light propagation in the atmosphere should be described within the ranges of electrodynamics and quantum mechanics. Nevertheless, it is suitable to abstract from the electromagnetic nature of light to solve a number of problems (including the problems described in this book) and to consider radiation as an energy flux. Light characteristics governed by energy are called *the radiative characteristics*. This approach is usual for optics because the frequency of the electromagnetic waves within the optical ranges is huge and the receiver registers only energy, received during many wave periods (not a simultaneous value of the electro-magnetic intensity). The electromagnetic nature of solar radiation including the property of the electromagnetic waves to be transverse is bound up with the phenomenon of *polarization*, which is revealing in the relationship of the process of the interaction between radiation and substance (refraction, scattering and reflection) and configuration of the electric vector oscillations on a plane, which is normal to the wave propagation direction. Further, we are using *the approximation of unpolarized radiation*. The evaluation of the accuracy of this approximation will be discussed further concerning the specific problems considered in this book.

The following main types of radiation (and their energy) are distinguished in radiation transferring throughout the atmosphere: *direct* radiation (radiation coming to the point immediately from the Sun); *diffused solar* radiation (solar radiation scattered in the atmosphere); *reflected solar* radiation from surface; *self-atmospheric* radiation (*heat atmospheric* radiation) and *self-surface* radiation (*heat* radiation). The total combination of these radiations creates the *radiation field* in the Earth atmosphere, which is characterized with energy of radiation coming from different directions within different spectral ranges. As is seen from above, it is possible to divide all radiation into solar and self (heat) radiation. In this book, we are considering only solar radiation in the spectral ranges 0.3–1.0 µm, where it is possible to neglect the energy of heat radiation of the atmosphere and surface, comparing with solar energy. Further with this spectral range we will be specifying *the short-wave spectral range*. Solar radiation integrated with respect to the wavelength over the considered

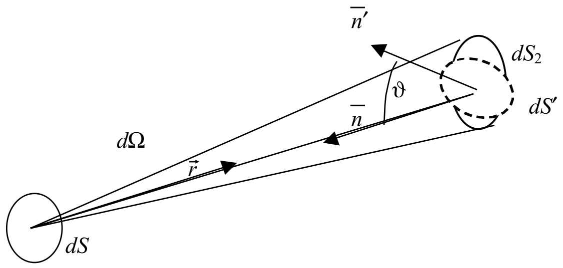


**Fig. 1.1.** To the definition of the intensity and to the flux of radiation (radiance and irradiance)

spectral region will be called *total radiation*. Meanwhile, it should be noted that further definitions of the radiative characteristics are not linked within this limitation and could be used either for heat or for microwave ranges.

The notion of a monochromatic parallel beam (the plane electromagnetic wave of one concrete wavelength and one strict direction) is widely used in optics for the theoretical description of different processes (Sivukhin 1980). Usually solar radiation is set just in that form to describe its interactions with different objects. The principle of an independency of the monochromatic beams under their superposition is postulated, i. e. the interaction of the radiation beams coming from different directions with the object is considered as a sum of independent interactions along all directions. The physical base of the independency principle is an incoherence of the natural radiation sources[1] (Sivukhin 1980).

This standard operation is naturally used for the radiation field, i. e. the consideration of it as a sum of non-interacted parallel monochromatic beams. Furthermore, radiation energy can't be attributed to a single beam, because if energy were finite in the wavelength and direction intervals, it would be infinitesimal for the single wavelength and for the single direction. For characterizing radiation, it is necessary to pass from energy to its distribution over spectrum and directions.

Consider an emitting object (Fig. 1.1) implying not only the radiation source but also an object reflecting or scattering external radiation. Pick out a surface element $dS$, encircle the solid angle $d\Omega$ around the normal $\boldsymbol{r}$ to the surface. Then radiation energy would be proportional to the area $dS$, the solid angle $d\Omega$, as well as to the wavelength ranges $[\lambda, \lambda + d\lambda]$ and the time interval $[t, t + dt]$. The factor of the proportionality of radiation energy to the values $dS$, $d\Omega$, $d\lambda$ and $dt$ would be specified *an intensity of the radiation* or *radiance* $I_\lambda(\boldsymbol{r}, t)$ at the wavelength $\lambda$ to the direction $\boldsymbol{r}$ at the moment $t$ according to (Sobolev 1972;

[1] It should be noted that monochromatic radiation is impossible in principle. It follows from the mathematical properties of the Fourier transformation: a spectrum consisting of one frequency is possible only with the time-infinite signal. Furthermore, the principle of the independency is not valid for the monochromatic beams because they always interfere. It is possible to remove both these contradictions if we consider monochromatic radiation not as a physical but as a mathematical object, i. e. as a real radiation expansion into a sum (integral Fourier) of the harmonic terms. The separate item of this expansion is interpreted as monochromatic radiation.

Hulst 1980; Minin 1988), namely:

$$I_\lambda(\boldsymbol{r}, t) = \frac{dE}{dS d\Omega d\lambda dt} \tag{1.1}$$

In many cases, we are interested not in energy emitted by the object but in energy of the radiation field that is coming to the object (for example to the instrument input). Then it would be easy to convert the above specification of radiance. Consider the emitting object and set the second surface element of the equal area $dS_2 = dS$ at an arbitrary distance (Fig. 1.1). Let the system to be situated in a vacuum, i. e. radiation is not interacting during the path from $dS$ to $dS_2$. Let the element $dS_2$ to be perpendicular to the direction $\boldsymbol{r}$, then the solid angle at which the element $dS_2$ is seen from $dS$ at the direction $\boldsymbol{r}$ is equal to the solid angle at which the element $dS$ is seen from $dS_2$ at the opposite direction ($-\boldsymbol{r}$). The energies incoming to the surface elements $dS$ and $dS_2$ are equal too thus; we are getting the consequence from the above definition of the intensity. The factor of the proportionality of emitted energy $dE$ to the values $dS$, $d\Omega$, $d\lambda$ and $dt$ is called an intensity (radiance) $I_\lambda(\boldsymbol{r}, t)$ incoming from the direction $\boldsymbol{r}$ to the surface element $dS$ perpendicular to $\boldsymbol{r}$ at the wavelength $\lambda$ at the time $t$, i. e. (1.1). Point out the important demand of the perpendicularity of the element $dS$ to the direction $\boldsymbol{r}$ in the definition of both the emitting and incoming intensity.

The definition of the intensity as a factor of the proportionality tends to have some formal character. Thus, the "physical" definition is often given: the intensity (radiance) is energy that incomes per unit time, per unit solid angle, per unit wavelength, per unit area perpendicular to the direction of incoming radiation, which has the units of watts per square meter per micron per steradian. This definition is correct if we specify energy to correspond not to the real unit scale (sec, sterad, µm, cm$^2$) but to the differential scale $dt$, $d\Omega$, $d\lambda$, $dS$, which is reduced then to the unit scale. Equation (1.1) is reflecting this obstacle.

Let the surface element $dS'$, which radiation incomes to, not be perpendicular to the direction $\boldsymbol{r}$ but form the angle $\vartheta$ with it (Fig. 1.1). Specify *the incident angle* (the angle between the inverse direction $-\boldsymbol{r}$ and the normal to the surface) as $\vartheta = \angle(\boldsymbol{n}, -\boldsymbol{r})$. In that case defining the intensity as a factor of the proportionality we have to use the projection of the element $dS'$ on a plane perpendicular to the direction of the radiation propagation in the capacity of the surface element $dS$. This projection is equal to $dS = dS' \cos\vartheta$. Then the following could be obtained from (1.1):

$$dE = I_\lambda(\boldsymbol{r}, t)\, dt\, d\lambda\, d\Omega\, dS' \cos\vartheta\ . \tag{1.2}$$

It is suitable to attribute the sign to energy defined above. Actually, if we fix one concrete side of the surface $dS'$ and assume the normal just to this side as a normal $\boldsymbol{n}$ then the angle $\vartheta$ varies from 0 to $\pi$, and the cosine from +1 to −1. Thus, incoming energy is positive and emitted energy is negative. It has transparent physical sense of the positive source and the negative sink of energy for the surface $dS'$. Now specify *the irradiance (the radiation flux* *of energy)* $F_\lambda(t)$ according to (Sobolev 1972; Hulst 1980; Minin 1988) (often it is specified as net spectral energy flux) as a factor of the proportionality of radiation energy $dE'$ incoming within a particular infinitesimal interval of wavelength $[\lambda, \lambda + d\lambda]$ and time $[t, t + dt]$ to the surface $dS'$ from *the all directions* to values $dt$, $d\lambda$, $dS'$s i. e.:

$$F_\lambda(t) = \frac{dE'}{dt\, d\lambda dS'} \,. \tag{1.3}$$

Adduce the "physical" definition of the irradiance that is often used instead of the "formal" one expressed by (1.3). Radiation energy incoming per unit area per unit time, per unit wavelength is called a radiation flux or irradiance. This definition corresponds correctly to (1.3) provided the meaning that energy is equivalent to the difference of incoming and emitted energy and uses the differential scale of area, time and wavelength. Proceeding from this interpretation, we will further use the term *energy* as a synonym of the *flux* implying the value of energy incoming per unit area, time and wavelength.

To characterize the direction of incoming radiation to the element $dS'$ in addition to the angle $\vartheta$, introduce the azimuth angle $\varphi$, which is counted off as an angle between the projection of the vector $\boldsymbol{r}$ to the plane $dS$ and any direction on this plane $(0 \leq \varphi \leq 2\pi)$. That is to say in fact that we are using the spherical coordinates system. Energy $dE'$ incoming to the surface $dS'$ from all directions is expressed in terms of energy from a concrete direction $dE(\vartheta, \varphi)$ as:

$$dE' = \int\limits_{\Omega=4\pi} dE(\vartheta, \varphi) d\Omega \,,$$

where the integration is accomplished over the whole sphere. Using the well-known expression for an element of the solid angle in the spherical coordinates $d\Omega = d\varphi \sin\vartheta d\vartheta$ we will get:

$$dE' = \int\limits_0^{2\pi} d\varphi \int\limits_0^{\pi} dE(\vartheta, \varphi) \sin\vartheta d\vartheta \,.$$

After the substituting of this expression to (1.3) with accounting (1.2) we will get the formula to express the irradiance:

$$F_\lambda(t) = \int\limits_0^{2\pi} d\varphi \int\limits_0^{\pi} I_\lambda(\vartheta, \varphi, t) \cos\vartheta \sin\vartheta d\vartheta \,. \tag{1.4}$$

In addition to direction $(\vartheta, \varphi)$, wavelength $\lambda$ and time $t$ the solar radiance in the atmosphere depends on placement of the element $dS$. Owing to the sphericity of the Earth and its atmosphere, it is convenient to put the position of this element in the spherical coordinate system with its beginning in the Earth's center.

Nevertheless, taking into account that the thickness of the atmosphere is much less than the Earth's radius is, in a number of problems the atmosphere could be considered by convention as a plane limited with two infinite boundaries: the bottom – a ground surface and the top – a level, above which the interaction between radiation and atmosphere could be neglected. Further, we are considering only *the plane-parallel atmosphere approximation*. The grounds of the approximation for the specific problems are given in Sect. 1.3. Then the position of the element $dS$ could be characterized with Cartesian coordinates $(x, y, z)$ choosing the altitude as axis $z$ (to put the $z$ axis perpendicular to the top and bottom planes from the bottom to the top). Thus, in a general case the radiance in the atmosphere could be written as $I_\lambda(x, y, z, \vartheta, \varphi, t)$. Under the natural radiation sources (in particular – the solar one) we could neglect the behavior of the radiance in the time domain comparing with the time scales considered in the concrete problems (e. g. comparing with the instrument registration time). The radiation field under such conditions is called a *stationary* one. Further, it is possible to ignore the influence of the horizontal heterogeneity of the atmosphere on the radiation field comparing with the vertical one, i. e. don't consider the dependence of the radiance upon axes $x$ and $y$. This radiation field is called a *horizontally homogeneous* one. Further, we are considering only stationary and horizontally homogeneous radiation fields. Besides, following the traditions (Sobolev 1972; Hulst 1980; Minin 1988) the subscript $\lambda$ is omitted at the monochromatic values if the obvious wavelength dependence is not mentioned. Taking into account the above-mentioned assumptions, the formula linking the radiance and irradiance (1.4) is written as:

$$F(z) = \int_0^{2\pi} d\varphi \int_0^{\pi} I(z, \vartheta, \varphi) \cos\vartheta \sin\vartheta d\vartheta \,. \tag{1.5}$$

It is natural to count off the angle $\vartheta$ from the selected direction $z$ in the atmosphere. This angle is called the zenith incident angle (it characterizes the inclination of incident radiation from the zenith). The angle $\vartheta$ is equal to zero if radiation comes from the zenith, and it is equal to $\pi$ if the radiation comes from nadir. As before we are counting off the azimuth angle from an arbitrary direction on the plane, parallel to the boundaries of the atmosphere. Then the integral (1.5) could be written as a sum of two integrals: over upper and lower hemisphere:

$$\begin{aligned}
F(z) &= F^{\downarrow}(z) + F^{\uparrow}(z) \,, \\
F^{\downarrow}(z) &= \int_0^{2\pi} d\varphi \int_0^{\pi/2} I(z, \vartheta, \varphi) \cos\vartheta \sin\vartheta d\vartheta \,, \\
F^{\uparrow}(z) &= \int_0^{2\pi} d\varphi \int_{\pi/2}^{\pi} I(z, \vartheta, \varphi) \cos\vartheta \sin\vartheta d\vartheta \,.
\end{aligned} \tag{1.6}$$

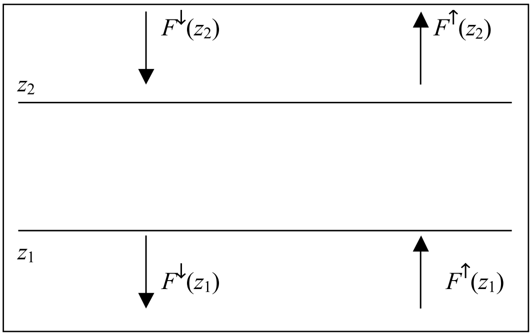


**Fig. 1.2.** Definition of net radiant flux

The value $F^{\downarrow}(z)$ is called the *downward flux* (*downwelling irradiance*), the value $F^{\uparrow}(z)$ – an *upward flux* (*upwelling irradiance*), both are also called *semispherical fluxes* expressed in watts per square meter (per micron). The physical sense of these definitions is evident. The downward flux is radiation energy passing through the level $z$ down to the ground surface and the upward flux is energy passing up from the ground surface. The downward flux is always positive ($\cos\vartheta > 0$), upward is always negative ($\cos\vartheta < 0$). In practice (for example during measurements) it is advisable to consider both fluxes as positive ones. We will follow this tradition. Then for the upward flux in (1.6) the value of $\cos\vartheta$ is to be taken in magnitude, and the total flux will be equal to the difference of the semispherical fluxes $F(z) = F^{\downarrow}(z) - F^{\uparrow}(z)$. This value is often called a (*spectral*) net *radiant flux* expressed in watts per square meter (per micron).

Consider two levels in the atmosphere, defined by the altitudes $z_1$ and $z_2$ (Fig. 1.2). Obtain solar radiation energy $B(z_1, z_2)$ (per unit area, time and wavelength) absorbed by the atmosphere between these levels. Manifestly, it is necessary to subtract outcoming energy from the incoming:

$$B(z_1, z_2) = F^{\downarrow}(z_2) + F^{\uparrow}(z_1) - F^{\downarrow}(z_1) - F^{\uparrow}(z_2) = F(z_2) - F(z_1) \,. \tag{1.7}$$

The value $B(z_1, z_2)$ is called a *radiative flux divergence in the layer between levels $z_1$ and $z_2$*. It is extremely important value for studying atmospheric energetics because it determines the warming of the atmosphere, and it is also important for studying the atmospheric composition because the spectral dependence of $B(z_1, z_2)$ allows us to estimate the type and the content of specific absorbing materials (atmospheric gases and aerosols) within the layer in question. Hence, the values of the semispherical fluxes determining the radiative flux divergence are also of greatest importance for the mentioned class of problems.

To provide the possibility of comparing the radiative flux divergences in different atmospheric layers we need to normalize the value $B(z_1, z_2)$ to the thickness of the layer:

$$b(z_1, z_2) = B(z_1, z_2)/(z_2 - z_1) \,. \tag{1.8}$$

We would like to point out that the definition of the normalized radiative flux divergences (1.8) with taking into account (1.7) gives the possibility of its theoretical consideration as a continuous function of the altitude after its writing as a derivation of the net flux $b(z) = \partial F(z)/\partial z$.

When we have defined the intensity and the flux above, we scrutinized the radiation field, i. e. the situation when radiation spreads on different directions. Actually, it is possible to amount to nothing more than this definition because no strictly parallel beam exists owing to the wave properties of light (Sivukhin 1980). Nevertheless, radiation emitted by some objects could be often approximated as one directional beam without losses of the accuracy. Incident solar radiation incoming to the top of the atmosphere is practically always considered as one-directional radiation in the problems in question. Actually, it is possible to neglect the angular spread of the solar beam because of the infinitesimal radiuses of the Earth and the Sun compared with the distance between them. Thus, we are considering the case of the plane parallel horizontally homogeneous atmosphere illuminated by a parallel solar beam. Some difficulties are appearing during the application of the above definitions to this case because we must attribute certain energy to the one-directional beam.

The radiance definition corresponding to (1.1) is not applicable in this case because it does not show the dependence of energy $dE$ upon solid angle $d\Omega$ [formally following (1.1) we would get the zero intensity]. As for the irradiance definition (1.3), it is applicable. Thus, it makes sense to examine the irradiance of the strictly one-directional beams. Then the dependence of energy $dE'$ upon the area of the surfaces $dS'$ projection in (1.3) appears for differently orientated surfaces $dS'$, which gives the follows:

$$F(\vartheta) = F_0 \cos\vartheta \,, \tag{1.9}$$

where $F_0$ is the irradiance for the perpendicular incident beam, $F(\vartheta)$ is the irradiance for the incident angle $\vartheta$.

The incident flux $F_0$ is of fundamental importance for atmospheric optics and energetics. This flux is radiation energy incoming to the top of the atmosphere per unit area, per unit intervals of the wavelength and time in the case of the average distance between the Sun and the Earth, and it is called a *spectral solar constant*. Figure 1.3 illustrates the *solar constant* $F_0$ as a function of wavelength. Concerning the radiance of the parallel incident beam, we can define it formally using (1.5). Actually, for accomplishing (1.5) and (1.9), it is necessary to assume the following:

$$I(\vartheta, \varphi) = F_0 \delta(\vartheta - \vartheta_0)\delta(\varphi - \varphi_0) \,, \tag{1.10}$$

where $\delta()$ is the delta function (Kolmogorov and Fomin 1999), $\vartheta_0$, $\varphi_0$ are the solar zenith angle and the azimuth angle which are determining the direction of the incident parallel beam. Remember that the delta function is defined as:

$$\int_a^b f(x)\delta(x - x_0)dx = f(x_0) \,.$$

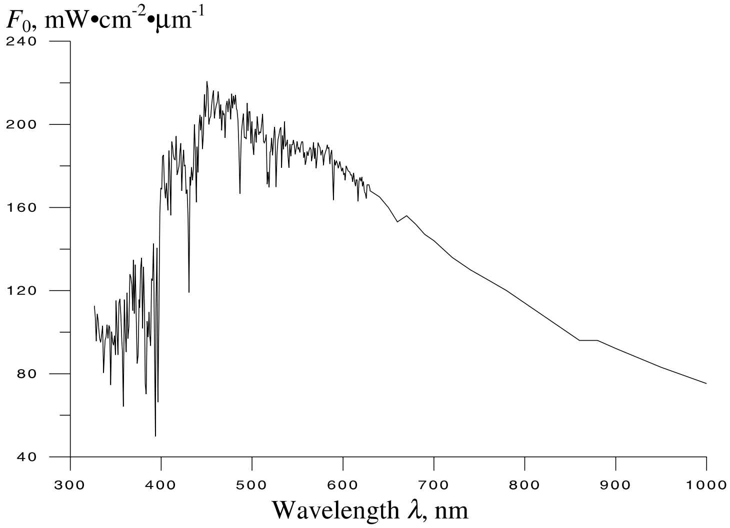


**Fig. 1.3.** Spectral extraterrestrial solar flux according to Makarova et al. (1991)

No real function can have such a property, thus the delta function is just a symbolic record. Roughly speaking it does not exist without the integrals.

Basing on (1.10) in the case of the parallel beam it could be said that the irradiance incoming to the perpendicular surface is numerically equal to the radiance, however this equality is truly formal because the radiance and the irradiance have different dimensions [that's all right with dimensions in (1.10)].

In conclusion consider the theoretical aspects of the procedures of radiance and irradiance measurements. It is radiation energy that influences the register element of an instrument. It could be written as:

$$E = \int_{t_1}^{t_2} dt \int_{\lambda_1}^{\lambda_2} d\lambda \int_S dxdy$$

$$\times \int_\Omega \sin\vartheta d\vartheta d\varphi I_\lambda(x,y,\vartheta,\varphi,t) f_i^*(t) f_\lambda^*(\lambda) f_S^*(x,y) f_\Omega^*(\vartheta,\varphi) ,$$

where $I_\lambda(x,y,\vartheta,\varphi,t)$ is the radiance incoming to the point of the input element (input slit) of an instrument with coordinates $(x,y)$; $[t_1,t_2]$ is the time interval of the input signal registration; $[\lambda_1,\lambda_2]$ is the registration wavelength interval; $f_t^*(t)$, $f_\lambda^*(\lambda)$, $f_S^*(x,y)$, $f_\Omega^*(\vartheta,\varphi)$ are the *instrumental functions*, which characterize a signal transformation by the instrument and they depend on time $t$, wavelength $\lambda$, input element point $(x,y)$, and direction of incoming radiation $(\vartheta,\varphi)$ correspondingly. The integration over the area $S$ is accomplished over the instrument input element surface, and the integration over the solid angle $\Omega$ is accomplished over the instrument-viewing angle. The instruments are calibrated so that the measured value of the radiance would be outputting instantaneously. From the theoretical point it means the normalization of the instrumental functions.

$$f_t(t) = f_t^*(t) \Big/ \int_{t_1}^{t_2} f_t^*(t)dt \, , \quad f_\lambda(\lambda) = f_\lambda^*(\lambda) \Big/ \int_{\lambda_1}^{\lambda_2} f_\lambda^*(\lambda)d\lambda \, ,$$

$$f_S(x,y) = f_S^*(x,y) \Big/ \int_S f_S^*(x,y)dxdy \, ,$$

$$f_\Omega(\vartheta,\varphi) = f_\Omega^*(\vartheta,\varphi) \Big/ \int_\Omega f_\Omega^*(\vartheta,\varphi)\sin\vartheta d\vartheta d\varphi$$

Then the measured value of radiance $I$ is expressed through the real radiance $I_\lambda(x,y,\vartheta,\varphi,t)$ by the following:

$$I = \int_{t_1}^{t_2} dt \int_{\lambda_1}^{\lambda_2} d\lambda \int_S dxdy \times \int_\Omega \sin\vartheta d\vartheta d\varphi I_\lambda(x,y,\vartheta,\varphi,t) f_t(t) f_\lambda(\lambda) f_S(x,y) f_\Omega(\vartheta,\varphi) \, . \tag{1.11}$$

Actually, the equality $I = I_0$ is valid according to (1.11) for normalized instrumental functions if $I_\lambda(x,y,\vartheta,\varphi,t) = I_0 = \text{const}$.

For the radiance measurements, the instrument viewing angle is chosen as small as possible. In this case, all the factors except the wavelength are neglected. Then the following is correct:

$$I = \int_{\lambda_1}^{\lambda_2} I_\lambda f_\lambda(\lambda)d\lambda$$

and the main instrument characteristic would be a *spectral instrumental function* $f_\lambda(\lambda)$, that will be simply called the *instrumental function*. If the radiance is slightly variable in the wavelength interval $[\lambda_1,\lambda_2]$ the influence of the specific features of the instrument on the observational process are possible not to take into account.

The function $f_\lambda(\lambda)$ plays an important role in the observation of the semi-spherical fluxes because the radiance at the instrument input changes evidently along the direction $(\vartheta,\varphi)$. However, comparing (1.4) and (1.11) it is easy to see that condition $f_\Omega^*(\vartheta,\varphi) = \cos\vartheta$ must be implemented specifically during the measurement of the irradiance. This demand to the instruments, which are measuring the solar irradiance, is called a Lambert's cosine law.

## 1.2 Interaction of the Radiation and the Atmosphere

Consider a symbolic particle (a gas molecule, an aerosol particle) that is illuminated by the parallel beam $F_0$ (Fig. 1.4). The process of the interaction of radiation and this particle is assembled from the *radiation scattering* on the particle and the *radiation absorption* by the particle. Together these processes constitute *the radiation extinction* (the irradiance after interaction with the particle is attenuated by the processes of scattering and absorption along the incident beam direction $\boldsymbol{r}_0$). Let the absorbed energy be equal to $E_a$, scattered in all directions energy be equal to $E_s$, and the total attenuated energy be equal to $E_e = E_a + E_s$. If the particle interacted with radiation according to geometric optics laws and was a non-transparent one (i. e. attenuated all incoming radiation), attenuated energy would correspond to energy incoming to the projection of the particle on the plane perpendicular to the direction of incoming radiation $\boldsymbol{r}_0$. Otherwise, this projection is called the *cross-section of the particle by plane* and its area is simply called a *cross-section*. Measuring attenuated energy $E_a$ per wavelength and time intervals $[\lambda, \lambda + d\lambda]$, $[t, t + dt]$ according to the irradiance definition (1.3) we could find the extinction cross-section as $dE_e/(F_0 d\lambda dt)$.

However, owing to the wave quantum nature of light its interaction with the substance does not submit to the laws of geometric optics. Nevertheless, it is very convenient to introduce the relation $dE_e/(F_0 d\lambda dt)$ that has the dimension and the meaning of the area, implying the equivalence of the energy of the real interaction and the energy of the interaction with a nontransparent particle possessing the cross-section equal to $dE_e/(F_0 d\lambda dt)$ in accordance with the laws of geometric optics. Besides, it is also convenient to consider such a cross-section separately for the different interaction processes. Thus, according to the definition, the ratio of absorption energy $dE_a$, measured within the intervals $[\lambda, \lambda + d\lambda]$, $[t, t + dt]$, to the incident radiation flux $F_0$ is called an *absorption cross-section* $C_a$. The ratio of scattering energy $dE_s$ to the incident radiation flux is called *a scattering cross-section* $C_s$ and the ratio of total attenuated energy $dE_s$ to the incident radiation flux is called *an extinction cross-section* $C_e$:

$$C_a = \frac{dE_a}{F_0 d\lambda dt}, \quad C_s = \frac{dE_s}{F_0 d\lambda dt}, \quad C_e = \frac{dE_e}{F_0 d\lambda dt} = C_a + C_s . \tag{1.12}$$

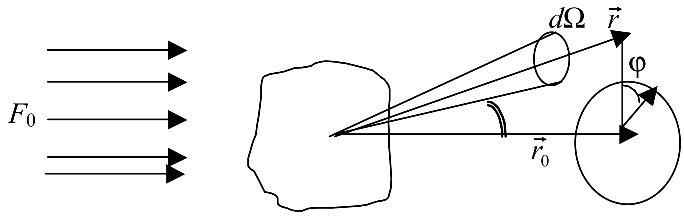


**Fig. 1.4.** Definition of the cross-section of the interaction

In addition to the above-mentioned, the cross-sections are defined as monochromatic ones at wavelength $\lambda$ (for the non-stationary case – at time $t$ as well).

Consider the process of the light scattering along direction $\boldsymbol{r}$ (Fig. 1.4). Here the value $dE_d(\boldsymbol{r})$ is the energy of scattered radiation (per intervals $[\lambda, \lambda + d\lambda]$, $[t, t+dt]$) per solid angle $d\Omega$ encircled around direction $\boldsymbol{r}$. Define *the directional scattering cross-section* analogously to the scattering cross-section expressed by (1.12).

$$C_d(\boldsymbol{r}) = \frac{dE_d(\boldsymbol{r})}{F_0 d\lambda dt d\Omega} . \tag{1.13}$$

Wavelength $\lambda$ and time $t$ are corresponding to the cross-section $C_d(\boldsymbol{r})$.

Total scattering energy is equal to the integral from $dE_d(\boldsymbol{r})$ over all directions $dE_s = \int_{4\pi} dE_d d\Omega$. Obtain the link between the cross-sections of scattering and directed scattering after substituting of $dE_d(\boldsymbol{r})$ to this integral:

$$C_s = \int\limits_{4\pi} C_d d\Omega . \tag{1.14}$$

Passing to a spherical coordinate system as in Sect. 1.1, introduce two parameters: *the scattering angle* $\gamma$ defined as an angle between directions of the incident and scattered radiation ($\gamma = \angle(\boldsymbol{r}_0, \boldsymbol{r})$) and *the scattering azimuth* $\varphi$ counted off an angle between the projection of vector $\boldsymbol{r}$ to the plane perpendicular to $\boldsymbol{r}_0$ and an arbitrary direction on this plane. Then rewrite (1.14) as follows:[2]

$$C_s = \int\limits_0^{2\pi} d\varphi \int\limits_0^{\pi} C_d(\gamma, \varphi) \sin\gamma d\gamma . \tag{1.15}$$

The directional scattering cross-section $C_d(\gamma, \varphi)$ according to its definition could be treated as follows: as the value $C_d(\gamma, \varphi)$ is higher, then light scatters stronger to the very direction $(\gamma, \varphi)$ comparing to other directions. It is necessary to pass to a dimensionless value for comparison of the different particles using the directional scattering cross-section. For that the value $C_d(\gamma, \varphi)$ has to be normalized to the integral $C_s$ expressed by (1.15) and the result has to be multiplied by a solid angle. The resulting characteristic is called *a phase function* and specified with the following relation:

$$x(\gamma, \varphi) = 4\pi \frac{C_d(\gamma, \varphi)}{C_s} . \tag{1.16}$$

[2] It is called also "differential scattering cross-section" in another terminology and the scattering cross-section is called "integral scattering cross-section". The sense of these names is evident from (1.12)–(1.15).

The substitution of the value $C_d(\gamma, \varphi)$ from (1.15) to (1.16) gives a *normalization condition of the phase function:*

$$\frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^{\pi} x(\gamma, \varphi) \sin \gamma d\gamma = 1 \,. \tag{1.17}$$

If the scattering is equal over all directions, i. e. $C_d(\gamma, \varphi) = const$, it is called *isotropic* and the relation $x(\gamma, \varphi) \equiv 1$ follows from the normalization (1.17). Thus, the multiplier $4\pi$ is used in (1.16) for convenience. In many cases, (for example the molecular scattering, the scattering on spherical aerosol particles) the phase function does not depend on the scattering azimuth. Further, we are considering only such phase functions. Then the normalization condition converts to:

$$\frac{1}{2} \int_0^{\pi} x(\gamma) \sin \gamma d\gamma = 1 \,. \tag{1.18}$$

The integral from the phase function in limits between zero and scattering angle $\gamma$ $\frac{1}{2} \int_0^{\gamma} x(\gamma) \sin \gamma d\gamma$ could be interpreted as a *probability of scattering to the angle interval* $[0, \gamma]$. It is easy to test this integral for satisfying all demands of the notion of the "probability". Hence the phase function $x(\gamma)$ is the *probability density of radiation scattering to the angle* $\gamma$. Often this assertion is accepted as a definition of the phase function.[3]

The real atmosphere contains different particles interacting with solar radiation: gas molecules, aerosol particles of different size, shape and chemical composition, and cloud droplets. Therefore, we are interested in the interaction not with the separate particles but with a total combination of them. In the theory of radiative transfer and in atmospheric optics it is usual to abstract from the interaction with a separate particle and to consider the atmosphere as a continuous medium for simplifying the description of the interaction between solar radiation and all atmospheric components. It is possible to attribute the special characteristics of the interaction between the atmosphere and radiation to an elementary volume (formally infinitesimal) of this continuous medium.

Scrutinize *the elementary volume* of this continuous medium $dV = dSdl$ (Fig. 1.5), on which the parallel flux of solar radiation $F_0$ incomes normally to the side $dS$. The interaction of radiation and elementary volume is reduced to the processes of scattering, absorption and radiation extenuation after radiation transfers through the elementary volume. Specify the radiation flux

[3] Point out that the phase function determines scattering only in the case of unpolarized incident radiation. After the scattering (both molecular and aerosol), light becomes the polarized one and the consequent scattering orders (secondary and so on) can't be described only by the phase function notion. Thus the theory of scattering, which doesn't take into account the polarization, is an approximation. In a general case, the accuracy of this approximation is estimated within 5% according to Hulst (1980). In special cases, it is necessary to test the accuracy that will be done in the following sections.

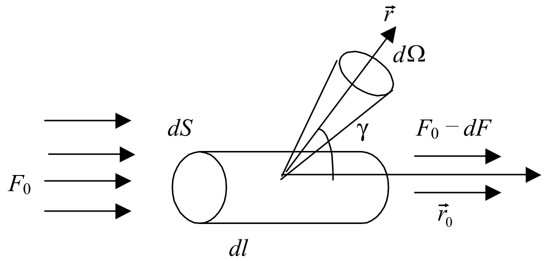


**Fig. 1.5.** Interaction between radiation and elementary volume of the scattering medium

as $F = F_0 - dF$ after its penetrating the elementary volume (along the incident direction $\boldsymbol{r}_0$). Take the relative change of incident energy as an extinction characteristic:

$$\frac{dE_e}{E_0} = \frac{(F_0 - F)dSd\lambda dt}{F_0 dSd\lambda dt} = \frac{dF}{F_0} \,.$$

As it is manifestly proportional to the length $dl$ in the extenuating medium, then it is possible to take the *volume extinction coefficient* $\alpha$ as a characteristic of radiation, attenuated by the elementary volume. This coefficient is equal to a relative change of incident energy (measured in intervals $[\lambda, \lambda + d\lambda]$, $[t, t + dt]$) normalized to the length $dl$ (i. e. reduced to the unit length) according to the definition:

$$\alpha = \frac{dE_e}{E_0 dl} = \frac{dF}{F_0 dl} \,. \tag{1.19}$$

The analogous definitions of *the volume scattering $\sigma$ and absorption $\kappa$ coefficients* follow from the equality of extinction energy and the sum of the scattering and absorption energies.[4]

$$\sigma = \frac{dE_s}{E_0 dl} \,, \quad \kappa = \frac{dE_a}{E_0 dl} \,, \quad \alpha = \sigma + \kappa \,. \tag{1.20}$$

It would be possible to introduce *a volume coefficient of the directional scattering* $s(\boldsymbol{r})$ considering energy $dE_d(\boldsymbol{r})$ scattered along direction $\boldsymbol{r}$ in solid angle $d\Omega$ analogously to (1.20): $s(\boldsymbol{r}) = dE_d(\boldsymbol{r})/(E_0 d\Omega dl)$. However, it is not done to use this characteristic. Actually, after accounting (1.20) we are obtaining $dE_d(\boldsymbol{r}) = \frac{1}{\sigma}s(\boldsymbol{r})dE_s d\Omega$ and substituting it to the relation $dE_s = \int_{4\pi} dE_d d\Omega$ that leads to the expression $\frac{1}{\sigma}\int_{4\pi} s d\Omega = 1$. It exactly corresponds to the normalizing relation (1.17) for the phase function in the spherical coordinates (Figs. 1.4 and 1.5) after the setting $s(\gamma, \varphi) = \frac{1}{4\pi}\sigma x(\gamma, \varphi)$, where $x(\gamma, \varphi)$ is the phase function of the elementary volume. As has been mentioned above, we are considering

[4]Notice, that the introduced volume coefficients have the dimension of the inverse length ($m^{-1}$, $km^{-1}$) and such values are usually called "linear" not "volume". Further, we will substantiate this terminological contradiction.

further the phase functions depending only upon the scattering angle $\gamma$ with the normalization relation (1.18). Thus, we obtain the following relation for energy scattered along direction $\gamma$

$$dE_d(\gamma) = \frac{\sigma}{4\pi} x(\gamma) E_0 d\Omega dl \, . \tag{1.21}$$

This relation may be accepted as a definition of phase function $x(\gamma)$ of the elementary volume of the medium (however, owing to the definition formality it is often used the definition of the phase function as a probability density of radiation scattering to angle $\gamma$).

Let us link the characteristics of the interaction between radiation and a separate particle with the elementary volume. Let every particle interact with radiation independently of others. Then extinction energy of the elementary volume is equal to a sum of extinction energies of all particles in the volume. Suppose firstly that all particles are similar; they have an extinction cross-section $C_e$ and their number concentration (number of particle in the unit volume) is equal to $n$. The particle number in the elementary volume is $ndV$. Substituting the sum of extinction energies to the extinction coefficient definition (1.19) in accordance with (1.12) and accounting the definition of the irradiance (1.3) we obtain the following:

$$\alpha = \frac{ndVC_eF_0d\lambda dt}{F_0dSd\lambda dtdl} = nC_e \, .$$

Thus, the volume extinction coefficient is equal to the product of particle number concentration by the extinction cross-section of one particle.[5]

If there are extenuating particles of $M$ kinds with concentrations $n_i$ and cross-sections $C_{e,i}$ in the elementary volume of the medium then it is valid: $dE_e = \sum_{i=1}^{M} n_i dVC_{e,i}F_0d\lambda dt$. Analogously considering the energies of scattering, absorption and directional scattering, we are obtaining the formulas, which link the volume coefficients and cross-sections of the interaction:

$$\begin{gathered} \alpha = \sum_{i=1}^{M} n_i C_{e,i} \, , \quad \sigma = \sum_{i=1}^{M} n_i C_{s,i} \, , \\ \kappa = \sum_{i=1}^{M} n_i C_{a,i} \, , \quad \sigma x(\gamma) = \sum_{i=1}^{M} n_i C_{s,i} x_i(\gamma) \, . \end{gathered} \tag{1.22}$$

We would like to point out that the separate items in (1.22) make sense of the volume coefficients of the interaction for the separate kinds of particles. Therefore, highly important for practical problems are the "rules of summarizing" following from (1.22). These rules allow us to derive separately the coefficients

[5]Just by this reason, the term "volume" and not "linear" is used for the coefficient. It is defined by numerical concentration in the unit volume of the air.

of the interaction and the phase function for each of $M$ components and then to calculate the total characteristics of the elementary volume with the formulas:

$$
\begin{aligned}
&\alpha = \sum_{i=1}^{M} \alpha_i \,, \quad \sigma = \sum_{i=1}^{M} \sigma_i \,, \\
&\kappa = \sum_{i=1}^{M} \kappa_i \,, \quad x(\gamma) = \sum_{i=1}^{M} \sigma_i x_i(\gamma) \Big/ \sum_{i=1}^{M} \sigma_i \,.
\end{aligned}
\tag{1.23}
$$

These rules also allow calculating characteristics of the molecular and aerosol scattering and absorption of radiation in the atmosphere separately. Then (1.23) is transformed to the following:

$$
\begin{aligned}
&\alpha = \sigma_m + \sigma_a + \kappa_m + \kappa_a \,, \\
&\sigma = \sigma_m + \sigma_a \,, \\
&\kappa = \kappa_m + \kappa_a \,, \\
&x(\gamma) = \frac{\sigma_m x_m(\gamma) + \sigma_a \, x_a(\gamma)}{\sigma_m + \sigma_a} \,,
\end{aligned}
\tag{1.24}
$$

where $\sigma_m$, $\kappa_m$, $x_m(\gamma)$ are the volume coefficients of the molecular scattering, absorption and molecular phase function for the atmospheric gases correspondingly and $\sigma_a$, $\kappa_a$, $x_a(\gamma)$ are the analogous aerosol characteristics.

The rules of summarizing expressed by (1.22)–(1.24) have been derived with the assumption that the particles are interacting with radiation independently. Here the following question is pertinent: is this assumption correct? From the view of geometrical optics, which we have appealed to, when introducing the cross-sections of the interaction, their areas (sections) mustn't intersect within the elementary volume, i. e. the total area of its projection to the side $dS$ must be equal to the sum of the areas of all particles. It would be accomplished if the distances between particles were much larger than the linear sizes of the cross-sections of the interaction or, roughly speaking, much larger than the particle sizes. Dividing the elementary volume to small cubes with side $d$, where $d$ is the distinctive size of the particle we are concluding that for this condition the particle number in the volume $dV$ has to be much less than the number of cubes – $ndV \ll dV/d^3$, i. e. $n \ll 1/d^3$, where $n$ is the particle number concentration. The second condition – the independency of the interaction between the particles and radiation – follows from the points of wave optics, according to which the independency of the interaction occurs if the distances between the particles are much larger than radiation wavelength $\lambda$ and that leads to the inequality $n \ll 1/\lambda^3$. Using the values of the real molecules and aerosol particle concentrations in the atmosphere it is easy to test that the condition $n \ll 1/d^3$ is always correct, the condition $n \ll 1/\lambda^3$ is correct in the short-wave range for aerosol particles and is broken for molecules of the atmospheric gases. Nevertheless, it is assumed that light scatters not on molecules but on the air density fluctuations (thus, the air is considered as a continuous medium) and it is possible to ignore this violation (Sivukhin 1980). For the calculation of the radiation field the elementary volume is chosen so that only one interaction act may happen within the elementary volume. Such volume is different for particles of different sizes (cloud droplets size is close to 10–20 μm, for atmospheric gases molecules (more exactly – density fluctuations) – the size is about $0.5 \times 10^{-3}$ μm). Thus, the diffusive medium is turned out non continuous. The violation of both conditions could occur when there are big particles in the air (for example cloud droplets). Actually taking into account the large size of the droplets (tens and hundreds of microns), there are a lot of gas molecules and small aerosol particles around these droplets and the both conditions are violated for them. Therefore, the question about the applicability of the summarizing rules in the cases mentioned above needs a special discussion.

The volume coefficients of the interaction between radiation and atmosphere are expressed through the scattering and absorption cross-sections according to relations (1.22). Thus, the most important problem will be the theoretical calculation of these cross-sections. The methods of their calculation are based on the description of the physical processes of the interaction between radiation and substance (Zuev et al. 1997). However, as we are not considering them here, the resulting formulas are adduced only, referring the reader to the cited literature.

The volume coefficient and the phase function of the molecular scattering are expressed as follows:

$$\begin{aligned}\sigma_m &= \frac{8}{3}\pi^3\frac{(m^2-1)^2}{n\lambda^4}\frac{6+3\delta}{6-7\delta}\,,\\ x_m(\gamma) &= \frac{3}{4+2\delta}[1+\delta+(1-\delta)\cos^2\gamma]\,,\end{aligned} \tag{1.25}$$

where $m$ is the refractive index of the air, $n$ is the number concentration of the air molecules, $\lambda$ is the radiation wavelength, $\delta$ is the depolarization factor (for the air it is equal to $\delta = 0.035$). The derivation of (1.25) is presented for example in the books by Kondratyev (1965) and Goody and Yung (1996) (the theory of the molecular scattering that is traditional for atmospheric optics) and in the book by Sivukhin (1980) (the scattering theory on the fluctuations of the air density). Using the known thermodynamic relation it is easy to obtain the number concentration:

$$n = \frac{P}{kT}\,, \tag{1.26}$$

where $P$ is the air pressure, $T$ is the air temperature, $k$ is the Boltzmann constant. For assuming the dependence of the air refractive index upon wavelength, pressure, temperature, and moisture, we are using the semi-empiric relation from the book by Goody and Yung (1996):

$$m - 1 = 10^{-6}\left(b(\lambda)\frac{P[+10^{-6}P(139.855 - 2.093T)]}{5.407(1 + 0.003661T)} - P_w\frac{8.319 - 0.0907\lambda^{-2}}{1 + 0.003661T}\right), \qquad (1.27)$$

$$b(\lambda) = 64.328 + \frac{29498.1}{146 - \lambda^{-2}} + \frac{255.4}{41 - \lambda^{-2}},$$

where $P_w$ is the partial pressure of the water vapor. It should be noted that in (1.27) wavelength is measured in microns (μm), pressure – in Pascals (Pa), temperature – in degrees Celsius (°C).

Two kinds of the input data for calculating cross-section of the molecular absorption are available in the short wavelength range.

The data of the first kind are tabulated as a dependence of the experimental cross-sections upon wavelength and in some cases upon temperature, i.e. $C_{a,i}(\lambda, T)$. Regretfully, the databases of mentioned cross-sections are not freely accessible nowadays. Therefore, during the concrete calculation we have been using the database collected from Sedunov et al. (1991) and Bass and Paur (1984) together with the data taken from the base of GOMETRAN computer code (Pozanov et al. 1995; Vasilyev et al. 1998) with the kind permission of its authors Vladimir Rozanov and Yuri Timofeyev. The cross-section of the molecular absorption of the specific gas (subscript "$i$" is omitted) is calculated for the data of the first kind as a simple linear interpolation over the look-up table:

$$\begin{aligned} C_a(\lambda, T) = &\Delta_1(\lambda, j)\Delta_1(T, k)C_a(\lambda_j, T_k) + \Delta_1(\lambda, j)\Delta_2(T, k)C_a(\lambda_j, T_{k+1}) \\ &+ \Delta_2(\lambda, j)\Delta_1(T, k)C_a(\lambda_{j+1}, T_k) + \Delta_2(\lambda, j)\Delta_2(T, k)C_a(\lambda_{j+1}, T_{k+1}), \end{aligned} \qquad (1.28)$$

where

$$\Delta_1(y, l) = \frac{y_{l+1} - y}{y_{l+1} - y_l}, \quad \Delta_2(y, l) = \frac{y - y_l}{y_{l+1} - y_l},$$

and numbers $j$ and $k$ are chosen over nodes of the table grid under the conditions $\lambda_j \le \lambda \le \lambda_{j+1}$, $T_k \le T \le T_{k+1}$. In the absence of the temperature dependence it is enough to set formally $\Delta_1(T, k) = 1$ and $\Delta_2(T, k) = 0$ in (1.28).

The data of the second kind describe the separate absorption lines of the gases (parameters of the fine structure). The theoretical aspects of the calculations using these data have been interpreted in detail, e.g. in the book by Penner (1959). For the concrete calculations, we have been using the database HITRAN-92 (Rothman et al. 1992). The volume coefficient of the molecular absorption according to the data of the second kind depends on the tempera-