New Trends in Macroeconomics

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Editors

in collaboration with Olivier Darné

New Trends in Macroeconomics

With 44 Figures
and 38 Tables

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Preface

That macroeconomic theory and macroeconometrics are, in the near future and more than ever, indispensable tools in the study of economics is no longer a very controversial statement. It is now generally agreed that economic theory, combined with historical, statistical and mathematical methods are necessary at the theoretical level, to formulate problems precisely, to draw conclusions from postulates and to gain insight into workings of complicated processes, and at the applied level, to measure variables, to estimate parameters and to organise the elaborate calculations involved in reaching empirical results. This book is an illustration of the Editors belief in this principle. It offers new insights in macroeconomic analysis. It deals with both theory and empirical results related to the dynamics within the structure of macroeconomic variables as well as between them. More precisely five axes are distinguished. There are theoretical and applied works with developments on (1) mechanisms of economic dynamics, (2) structures of macroeconomic variables, and (3) relationships between macroeconomic time series. The book also presents methodologies where (4) linear testing is improved and (5) new non-linear techniques are applied.
Turning to the individual contributions now.

Bénassy's chapter studies the propagation of macroeconomic shocks using a dynamic model with wage and price staggering. He finds evidence in favour of a persistent response of both output and inflation to monetary shocks.

Karagiannis, Palivos and Papageorgiou present an one-sector growth model where the technology is described by a Variable Elasticity of Substitution production function. It is shown that this model can exhibit unbounded endogenous growth despite the absence of exogenous technical change and the presence of non-reproducible factors, such as labour.

Stengos and Liang study the effect of financial development on growth using an additive Instrumental Variable-augmented Partially Linear Regression model. They conclude that financial development affects growth in a positive but non-linear way employing a Liquid Liabilities index and in an almost linear way when a Private Credit index is taken into account. Nevertheless, the effect becomes ambiguous in the case of a Commercial Central Bank index.

The transition from theoretical evidence to empirical testing is well done by Hendry. Hendry's chapter is focused in the gap that exists between macroeconomic theory models and applied econometric findings. He describes some of the sources of these gaps and suggests possible solutions.

In the Gogas and Serletis chapter the revenue-smoothing hypothesis is tested using annual data for the US over the period form 1934-1994. Although Mankiw (1987) and Poterba and Rotemberg (1990) works found evidence supporting the previous hypothesis in the US, the obtained results by Gogas and Serletis do not support the theory of optimal seigniorage.

The performance of structural VAR models to capture structures produced by two stochastic dynamic general equilibrium models is the main point of study in the Canova and Pires Pina's chapter. More specifically, their criticism is to a particular type of identification restrictions routinely used in applied work. To avoid eventual biases they propose an alternative identification technique.

Paya and Peel examine the Keynes-Einzig conjecture by using monthly data for six currencies against the US Dollar for the period 1921-1936. Empirical findings suggest that excess returns are predictable, and that deviations form covered interest parity (CIP) are large and systematic. Evidence of non-linear adjustment of CIP is also provided.

In a fractional cointegration framework given in Davidson's chapter, it is investigated the relationship between government popularity and economic performance in the UK. The tests reveal little or no evidence of a link between the political and economic cycles. This conclusion reinforces the idea that political cycles are generated by the internal dynamics of the opinion formation process.

As it has been underlined by Hendry, the gap between macroeconomic theory models and applied econometric findings arises because much of the observed macroeconomic data variability is due to various non-stationarities. These sources of non-stationarity, deriving from the technical progress, new legislation, institutional change, financial innovation and political factors, induce both evolution and structural breaks which change the distributional properties of the data.

Darné and Diebolt, in their chapter, propose a more technical approach to deal with non-stationarity in macroeconomic series. They give a selective survey on different non-stationarity tests and discuss some problems with these tests and some solutions and alternatives that have been suggested. They also present the relation between non-stationarity and some economic theory.

The importance of seasonality and non-stationarity for the forecasting accuracy is emerged in the Kouassi and Labys's chapter. This is illustrated in the context of structural time series models. The major result of their work is that the recognition of the presence of seasonal unit-roots can have important implications for forecasting and modelling.

The Kyrtsou and Volrow's chapter concludes this collective volume. The authors suggest the use of a new methodology, well known in physical sciences, for the identification of complex underlying dynamics in economic series. This method is the Recurrence Quantification Analysis. The empirical results of the chapter provide evidence for the existence of highly complex deterministic dynamics in the US macroeconomic and financial series. The possibility to obtain such features in real economic series, that we would not be able to find using only traditional linear techniques, makes the new world of non-linear complex dynamics very attractive. Further research on the impact of the application of these new tools to macroeconomic data is certainly needed.

From the mechanisms of propagation of macroeconomic shocks to growth and monetary theories, macroeconometrics and complex dynamics, we hope to provide a complete overview on the recent developments and ”New Trends in Macroeconomics”. It is now time to let the authors speak for themselves!

Strasbourg, France | *Claude Diebolt*
April 2005 | *Catherine Kyrtsou*

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The Propagation of Macroeconomic Shocks: A Dynamic Model with Contracts and Imperfect Competition

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Summary. In order to study rigorously the propagation of macroeconomic shocks, we construct a dynamic model with wage and price staggering, where wage and price contracts are set by fully maximizing agents in a framework of imperfect competition. We derive the optimal values for wage and price contracts and compute closed form solutions to the resulting dynamics. We show that wage and price contracts of reasonable durations can create persistence and a hump in the response of both output and inflation to monetary shocks.

Key words: Persistence, Staggered wages, Staggered prices, Imperfect competition.

JEL codes: E32, E52

1 Introduction

The purpose of this article is to construct a dynamic general equilibrium model including staggered wage and price contracts, as well as imperfect competition. We will study with it the issue of the propagation of macroeconomic shocks, and notably whether one can obtain a response to monetary shocks similar to that observed in reality. On the empirical side, a number of recent studies have shown that both output and inflation display a persistent response to monetary shocks. Moreover this response seems to be humpshaped, first increasing, then decreasing (see, for example, Cogley and Nason [9]; Christiano, Eichenbaum and Evans [8]). On the other hand RBC type models have often had problems creating such a response to monetary shocks. Recently wage and price contracts[1] have been introduced in that line of models, in order notably to make the corresponding economies more responsive to monetary shocks, and a debate has developed as to whether such modeling would allow to obtain a persistent and hump shaped response to these shocks. Surprisingly

[1] In line with the initial works by Gray [12], Fischer [11], Phelps and Taylor [16], Phelps [15], Taylor [17, 18] and Calvo [6].

the answers are widely divergent. For example, Chari, Kehoe and McGrattan [5] conclude that there will be no persistence with reasonable values of the parameters, while Collard and Ertz [10] obtain a hump-shaped and persistent response with one or two years wage contracts[2]. The objective of this article is to investigate the matter analytically, which seems particularly useful in view of the conflicting answers indicated above. For that purpose we shall build a rigorous dynamic stochastic general equilibrium model with *both* price and wage contracts[3], solve it analytically and express the dynamics of output and inflation as a function of the fundamental underlying parameters. The reason why we study wage and price contracts together, and not in isolation, is that this appears to be instrumental in obtaining a hump-shaped response in both output and inflation, as we shall see below. We shall see that a persistent and humpshaped response of both output and inflation to monetary shocks can be obtained with reasonable parameters.

2 The Model

2.1 Markets and Agents

The economy studied is a monetary economy with markets for goods, at the (average) price P_t and markets for labor, at the (average) wage W_t. The goods and labor markets function under a system of imperfectly competitive labor contracts, which will be detailed below. There are firms and households. Let us begin with the production side. The output index Y_t is an aggregate of a continuum of output types, indexed by $i \in [0,1]$:

$$Log Y_t = \int_0^1 Log Y_{it} di \tag{1}$$

Each index Y_{it} is itself the aggregate of another infinity of output types indexed by k:

$$Y_{it} = \left(\int_0^1 Y_{ikt}^{\theta} dk \right)^{1/\theta} \tag{2}$$

One should think of the index i as representing sectors, while the index k refers to firms in these sectors. Quite naturally the substitutability is higher within sectors than across sectors. The representative firm has a Cobb-Douglas technology[4]:

$$Y_{ikt} = Z_t N_{ikt}^{\alpha} \tag{3}$$

[2] Other contributions along the same lines are found, for example, in Ambler, Guay and Phaneuf [1], Andersen [2], Ascari [3], Jeanne [13] and Yun [19].

[3] Microfounded dynamic models with one rigidity and analytical solutions are found in Jeanne [13] for price contracts, and in Ascari [3] and Bénassy [4, 5] for wage contracts. Andersen [2] compares the two types of contracts.

[4] Although capital could be introduced explicitly (cf. for example Bénassy [4]), we omit it here because it complicates substantially the exposition, and does not add much to the dynamics because of the low actual depreciation rate.

The representative household (we omit the index k at this stage, since the situation of all households in a sector i is fully symmetrical) works N_{it}, consumes C_{it} and ends the period with a quantity of money M_{it}. He maximizes the expected value of his discounted utility, with the following intertemporal utility:

$$U = \sum_t \beta^t \left[Log C_{it} + \omega Log \frac{M_{it}}{P_t} - V(N_{it}) \right] \tag{4}$$

where V is a convex function. At the beginning of period t there is a stochastic multiplicative monetary shock as in Lucas (1972): money holdings carried from the previous period M_{it-1} are multiplied by the same factor μ_t for all agents, so that the representative household starts period t with money holdings $\mu_t M_{it-1}$. His budget constraint in t is thus:

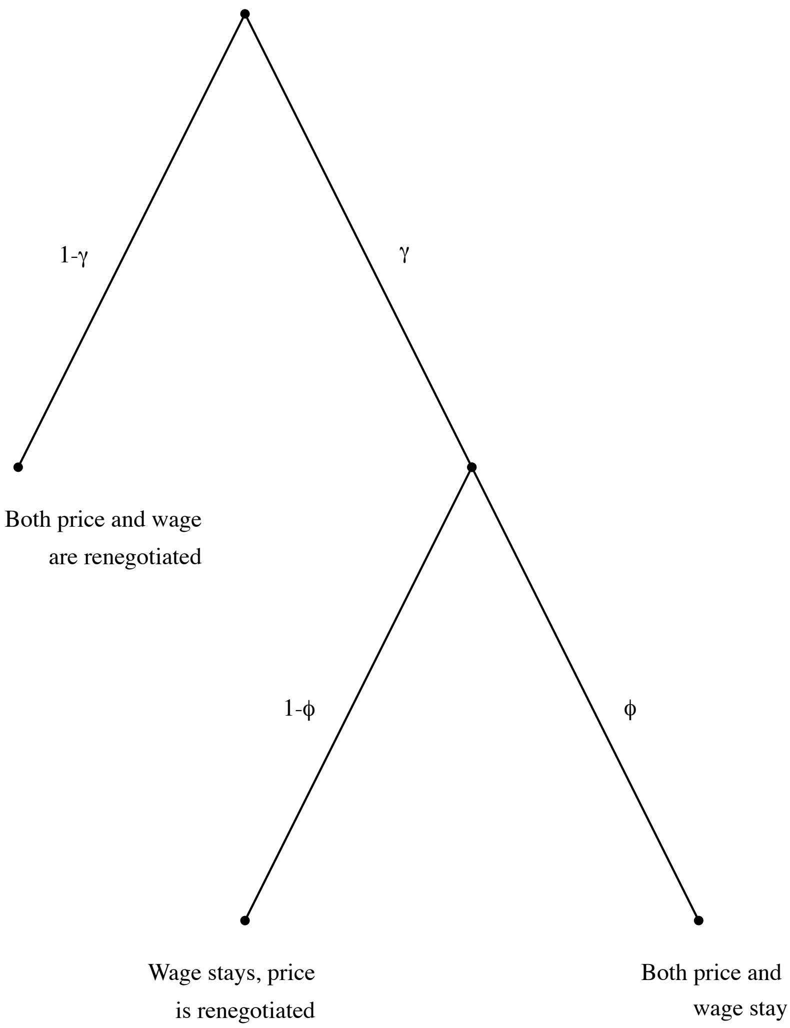
$$C_{it} + \frac{M_{it}}{P_t} = \frac{W_{it}}{P_t} N_{it} + \frac{\mu_t M_{it-1}}{P_t} + \Pi_{it} \tag{5}$$

where Π_{it} is the profits in sector i, which are distributed to the households who work in this sector.

2.2 Wage and Price Contracts

We will now describe the wage and price contracts, which are taken from Bénassy [4, 5], themselves inspired by Calvo [6]. Consider first the wage contracts. As in Calvo [6], in each period there is a random draw for all wage contracts, after which any particular contract continues unchanged (with probability γ), or is terminated (with probability $1-\gamma$). In this last case a new contract wage is decided in each firm by the households working in that firm, on the basis of all information currently available. In period s a wage contract is negotiated for each period $t \geq s$. This new contract is denoted X_{st}. The difference with Calvo [6] is that he assumed X_{st} to be the same for all $t \geq s$, whereas we assume that the X_{st} can be different whatever $t \geq s$. Price contracts are modelled in a similar manner. If wages are not renegotiated, then the price mechanism is completely symmetrical to the wage mechanism: price contracts continue with probability ϕ, and are terminated with probability $1-\phi$. If, however, wages in one particular firm are renegotiated, then prices in this firm are also automatically renegotiated. These possibilities, and their probabilities, are summarized in Figure 1. The basic idea underlying this formalization is that, if a firm is faced with a change in its cost structure because of wage renegotiation, then it will always want to change its price, which seems a reasonable assumption. We thus see that, taking into account both possibilities, the probability for a price contract to continue unchanged is $\gamma\phi$, and the probability to be renegotiated $1-\gamma\phi$. We denote by Q_{st} the price contract signed in period s to be in effect in period $t \geq s$.

Fig. 1



We still have to specify more precisely how wage and price renegotiations are related across firms and sectors. We shall assume that all firms in the same sector i renegotiate their wages or prices at exactly the same time, which means that the random draws are actually organized sector by sector. These random draws are independent across sectors.

3 The Walrasian Regime

We shall now compute as a benchmark the Walrasian equilibrium of this economy. In that case there is a unique price P_t and wage W_t, which clear the goods and labor markets. The real wage is equal to the marginal productivity of labor:

$$\frac{W_t}{P_t} = \alpha \frac{Y_t}{N_t} \tag{6}$$

The households maximize the expected value of their utility (4) subject to the budget constraints (5). The Lagrangean of this maximization program is:

$$\sum \beta^t \left[LogC_t + \omega Log \frac{M_t}{P_t} - V(N_t) \right]$$

$$+ \sum \beta^t \lambda_t \left[\frac{W_t N_t}{P_t} + \frac{\mu_t M_{t-1}}{P_t} + \Pi_t - C_t - \frac{M_t}{P_t} \right] \tag{7}$$

and the first order conditions:

$$\lambda_t = \frac{1}{C_t} \tag{8}$$

$$\frac{\lambda_t}{P_t} = \frac{\omega}{M_t} + \beta E_t \left(\frac{\lambda_{t+1} \mu_{t+1}}{P_{t+1}} \right) \tag{9}$$

$$V'(N_t) = \frac{\lambda_t W_t}{P_t} \tag{10}$$

Using (8) and the fact that $\mu_{t+1} = M_{t+1}/M_t$, equation (9) is rewritten:

$$\frac{M_t}{P_t C_t} = \omega + \beta E_t \left(\frac{M_{t+1}}{P_{t+1} C_{t+1}} \right) \tag{11}$$

which solves as:

$$\frac{M_t}{P_t C_t} = \frac{\omega}{1-\beta} \tag{12}$$

Combining (6), (8) and (10), we see that Walrasian employment is constant and equal to N, given by:

$$NV'(N) = \alpha \tag{13}$$

In what follows we shall work with the following disutility for labor:

$$V(N_t) = \xi \frac{N_t^\nu}{\nu} \tag{14}$$

in which case equation (13) yields:

$$N = \left(\frac{\alpha}{\xi} \right)^{1/\nu} \tag{15}$$

and the Walrasian wage W_t^* and price P_t^* are equal to:

$$W_t^* = \frac{\alpha(1-\beta)}{\omega}\left(\frac{\xi}{\alpha}\right)^{1/\nu} M_t \tag{16}$$

$$P_t^* = \frac{1-\beta}{\omega}\left(\frac{\xi}{\alpha}\right)^{\alpha/\nu} \frac{M_t}{Z_t} \tag{17}$$

4 The Demand for Goods and Labor

We shall now study our model under wage and price contracts. It is assumed that households, possibly through trade-unions, decide on the level of wages, and supply the amount of labor demanded by firms at these wages. Similarly firms set prices and supply the amount of goods demanded. An important element for the determination of wage and price contracts is of course the demand for goods and labor, so we begin with that.

4.1 The Demand for Goods

At any time there may be a multiplicity of prices. This variety of prices can be due to two causes: first, there may be staggered prices, and thus there are different prices because price contracts have been signed at different points in time. Secondly, even if prices are fully flexible in each period, the workers in different firms may have different wage contracts, so that prices will differ even if all other economic conditions are the same. Consider first the firms producing final output. They competitively maximize profits, i.e. they solve the following program:

$$\text{Max } P_t Y_t - \int_0^1 P_{it} Y_{it} di \qquad \text{s.t.} \qquad \int_0^1 Log Y_{it} di = Log Y_t$$

whose solution is:

$$Y_{it} = \frac{P_t Y_t}{P_{it}} \tag{18}$$

$$Log P_t = \int_0^1 Log P_{it} di \tag{19}$$

Now firms in a sector i will similarly maximize profits, i.e. they solve:

$$\text{Max } P_{it} Y_{it} - \int_0^1 P_{ikt} Y_{ikt} dk \qquad \text{s.t.} \qquad \left(\int_0^1 Y_{ikt}^{\theta} dk\right)^{1/\theta} = Y_{it}$$

whose solution is:

$$Y_{ikt} = Y_{it}\left(\frac{P_{ikt}}{P_{it}}\right)^{-1/(1-\theta)} \tag{20}$$

$$P_{it} = \left(\int_0^1 P_{ikt}^{-\theta/(1-\theta)} dk \right)^{-(1-\theta)/\theta} \tag{21}$$

Putting together equations (18) and (20) we obtain the expression of the demand for goods:

$$Y_{ikt} = \frac{P_t Y_t}{P_{it}} \left(\frac{P_{ikt}}{P_{it}} \right)^{-1/(1-\theta)} \tag{22}$$

An important thing to remember for what follows is that, in view of equation (18), all sectors have exactly the same value of sales:

$$P_{it} Y_{it} = P_t Y_t \qquad \forall i \tag{23}$$

This will also imply that all households, whatever the sector they work in, have the same income, and consequently the same consumption and money holdings.

4.2 The Demand for Labor

Since firms supply the quantity of goods demanded (22), the demand for labor is simply obtained by combining (3) and (22), which yields:

$$N_{ikt} = \left(\frac{P_t Y_t}{Z_t P_{it}} \right)^{1/\alpha} \left(\frac{P_{ikt}}{P_{it}} \right)^{-1/\alpha(1-\theta)} \tag{24}$$

5 Price Contracts

We now turn to the derivation of optimal price contracts. They are characterized through the following proposition:

Proposition 1: *Assume that in period τ the wage in firm (i,k) is $W_{ik\tau}$. Then the price contract $Q_{ik\tau t}$ signed in period τ for a period $t \geq \tau$ is given by:*

$$\left(\frac{Q_{ik\tau t}}{Q_{i\tau t}} \right)^{(1-\alpha\theta)/\alpha(1-\theta)} = \frac{W_{ik\tau}}{\alpha\theta} \left(\frac{1-\beta}{\omega} \right)^{(1-\alpha)/\alpha} \left(\frac{1}{Q_{i\tau t}} \right)^{1/\alpha} \Omega_{\tau t} \tag{25}$$

with:

$$Q_{i\tau t} = \left(\int_0^1 Q_{ik\tau t}^{-\theta/(1-\theta)} dk \right)^{-(1-\theta)/\theta} \tag{26}$$

$$\Omega_{\tau t} = E_\tau \left[M_t^{(1-\alpha)/\alpha} Z_t^{-1/\alpha} \right] \tag{27}$$

Proof: Appendix 1.

Now we shall see below, within the proof of proposition 3 (Appendix 2), that all firms in the same sector i will actually have the same wage, and therefore, in view of (25), the same price. We shall now derive the value of this common price contract through the following proposition:

Proposition 2: *Assume that in period τ the common wage in sector i is $W_{i\tau}$. Then the price contracts $Q_{i\tau t}$ signed in period τ for period $t \geq \tau$ by all firms in sector i are given by:*

$$Q_{i\tau t}^{1/\alpha} = \frac{1}{\alpha\theta}\left(\frac{1-\beta}{\omega}\right)^{(1-\alpha)/\alpha} W_{i\tau}\Omega_{\tau t} \tag{28}$$

where $\Omega_{\tau t}$ has been defined in formula (27).

Proof: Replace in formula (25) $W_{ik\tau}$ by $W_{i\tau}$ and $Q_{ik\tau t}$ by $Q_{i\tau t}$ Q.E.D.

6 Wage Contracts

We shall now compute the wage contracts signed in a period s for a period $t \geq s$. As will appear in the proof of proposition 3 (Appendix 2), these contracts will be the same for all firms and sectors, and we shall accordingly denote them as X_{st}. Before moving to a precise proposition, let us define some probabilities. If the wage contract X_{st} is still in effect at time t, it will be associated with prices which may have been set in any period τ, $s \leq \tau \leq t$. In view of the "survival rate" ϕ of price contracts, the probabilities $\pi_{\tau t}$ that the price was set in period τ are computed as:

$$\pi_{\tau t} = \phi^{t-s} \quad \tau = s \qquad \pi_{\tau t} = (1-\phi)\,\phi^{t-\tau} \quad s < \tau \leq t \tag{29}$$

Proposition 3: *The wage contract X_{st} signed in s for period $t \geq s$ is given by:*

$$X_{st}^{\nu} = \frac{\xi}{\alpha^2\theta^2}\left[\frac{\alpha\theta\,(1-\beta)}{\omega}\right]^{\nu} \sum_{s\leq\tau\leq t} \pi_{\tau t} E_s\left[\left(\frac{M_t}{Z_t}\right)^{\nu/\alpha}\left(\frac{1}{\Omega_{\tau t}}\right)^{\nu}\right] \tag{30}$$

where $\Omega_{\tau t}$ is given by equation (27) *and the probabilities $\pi_{\tau t}$ by equation* (29).

Proof: Appendix 2.

7 Macroeconomic Dynamics

We shall now compute the dynamics of the system under the following traditional processes for money and technology shocks[5]:

[5]Lowercase letters denote the logarithms of the corresponding uppercase letters.

$$m_t - m_{t-1} = \frac{u_t}{1-\rho L} \tag{31}$$

$$z_t = \frac{\omega_t}{1-\varphi L} \tag{32}$$

7.1 The Dynamics of Output and Inflation

We shall first characterize the dynamic evolution of output and inflation through the following proposition:

Proposition 4: *Under the monetary and technology processes* (31) *and* (32) *the dynamic evolutions of output and inflation are given by:*

$$y_t = z_t - \frac{\gamma\phi\varepsilon_t}{1-\gamma\phi\varphi L} + \frac{\alpha\gamma u_t}{(1-\gamma L)(1-\gamma\rho L)} + \frac{(1-\alpha)\gamma\phi u_t}{(1-\gamma\phi L)(1-\gamma\phi\rho L)} \tag{33}$$

$$\pi_t = p_t - p_{t-1} = \frac{u_t}{1-\rho L} - \frac{(1-\gamma\phi)(1-L)\varepsilon_t}{(1-\varphi L)(1-\gamma\phi\varphi L)}$$
$$- \frac{\alpha\gamma(1-L)u_t}{(1-\gamma L)(1-\gamma\rho L)} - \frac{(1-\alpha)\gamma\phi(1-L)u_t}{(1-\gamma\phi L)(1-\gamma\phi\rho L)} \tag{34}$$

Proof: Appendix 3.

With an explicit expression for the dynamics of output and inflation, we can potentially compute any measure of persistence. With five autoregressive roots for output and seven for inflation (formulas 33 and 34), a numerical discussion would quickly become very clumsy, so we shall rather discuss the issue of whether the response of output and inflation to monetary shocks displays a hump, since this has been the object of controversy and is easily assessed from our formulas.

7.2 Output Dynamics and the Hump

Let us start with the output dynamics in response to a monetary shock, and see under which conditions we shall obtain a humpshaped response. From formula (33) the first period impact of a money shock on output is:

$$\alpha\gamma + (1-\alpha)\gamma\phi \tag{35}$$

and the second period one:

$$\alpha\gamma(\gamma+\gamma\rho) + (1-\alpha)\gamma\phi(\gamma\phi+\gamma\phi\rho) \tag{36}$$

So there is a hump if:

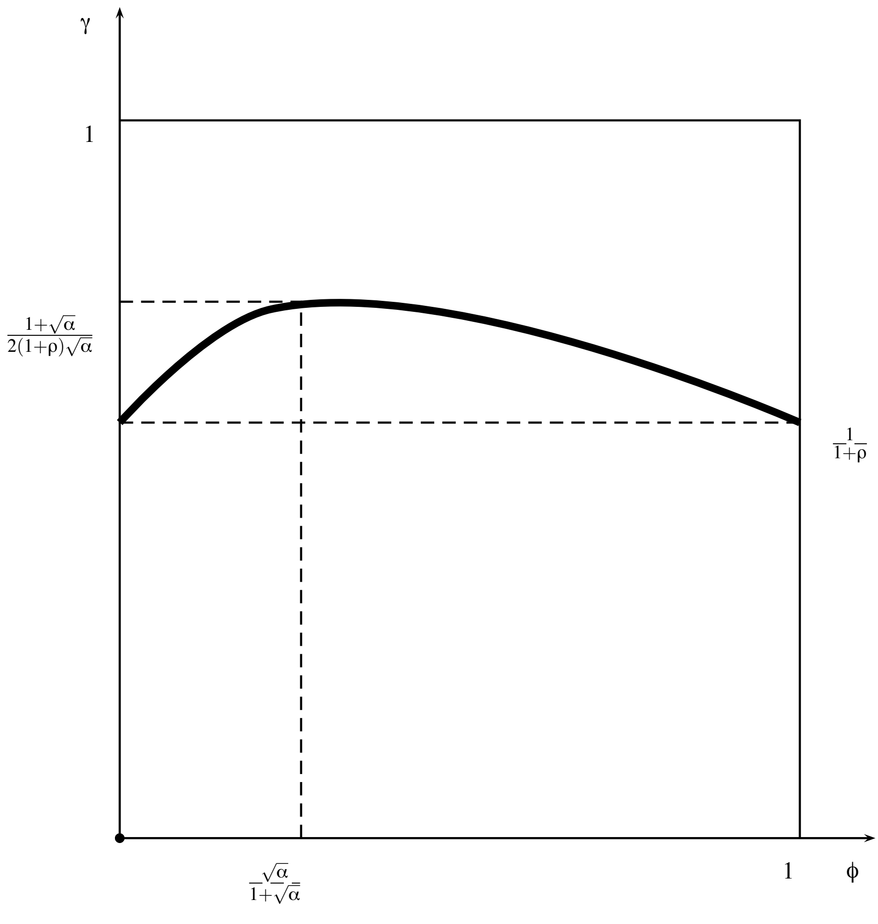
$$\alpha\gamma + (1-\alpha)\gamma\phi < \alpha\gamma^2(1+\rho) + (1-\alpha)\gamma^2\phi^2(1+\rho) \tag{37}$$

which can be rewritten as:

$$\gamma(1+\rho) > \frac{\alpha + (1-\alpha)\phi}{\alpha + (1-\alpha)\phi^2} \tag{38}$$

The corresponding region in (ϕ, γ) space is shown in Figure 2 as the set above the heavy line.

Fig. 2



For given α the right hand side of formula (38) is maximal for $\phi = \sqrt{\alpha}/\left(1+\sqrt{\alpha}\right)$ and then takes the value $\left(1+\sqrt{\alpha}\right)/2\sqrt{\alpha}$. This means that if:

$$\gamma > \frac{1+\sqrt{\alpha}}{2(1+\rho)\sqrt{\alpha}} \tag{39}$$

then there will be a hump in output no matter what the degree of rigidity of prices. To get a numerical idea, we can consider the traditional values $\alpha = 2/3$, $\rho = 1/2$. Then we find that, if wage contracts are at least three quarters long on average, there will always be a hump in the response of output to monetary shocks.

7.3 Inflation Dynamics and the Hump

From formula (34) the first period impact on inflation is:

$$1-\alpha\gamma-(1-\alpha)\gamma\phi \tag{40}$$

and the second period one:

$$\rho-\alpha\gamma(\gamma+\gamma\rho-1)-(1-\alpha)\gamma\phi(\gamma\phi+\gamma\phi\rho-1) \tag{41}$$

The condition for a hump in inflation is thus:

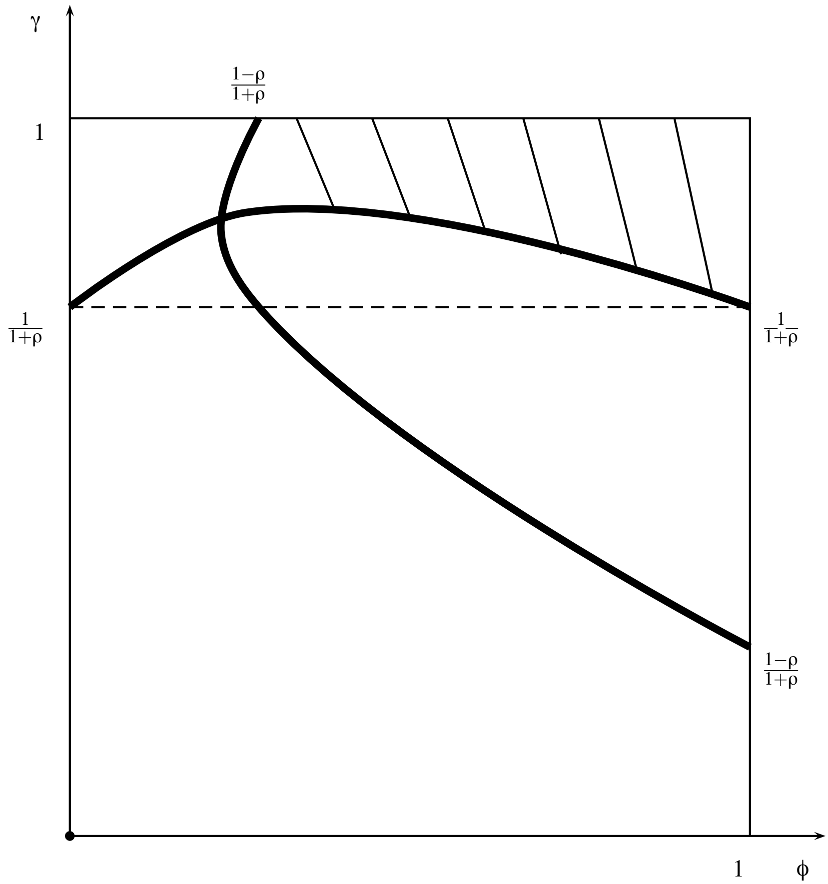
$$1-\rho<\alpha\gamma(2-\gamma-\gamma\rho)+(1-\alpha)\gamma\phi(2-\gamma\phi-\gamma\phi\rho) \tag{42}$$

The corresponding locus is represented in (ϕ,γ) space in Figure 3, together with the corresponding locus for the hump in output. The relevant region is to the right of the heavy line.

7.4 The Double Hump

The region with stripes in Figure 3 corresponds to the combinations of the (ϕ,γ) parameters such that the response of both output and inflation to monetary shocks displays a hump. To get a practical idea about this region, with the traditional values $\alpha = 2/3$, $\rho = 1/2$, if $\gamma > 3/4$ and $\phi > 1/3$, we will have a double hump. This corresponds to an average duration of contracts of 3 quarters for wages, and 1 month for prices, some very reasonable values indeed!

Fig. 3



8 Conclusions

We constructed in this article a dynamic stochastic general equilibrium model where both wages and prices are staggered and set in a rigorous framework of monopolistic competition. We used a framework akin to that in Calvo (1983), so that the average duration of wage and price contracts can take any value between zero and infinity. We first derived the optimal prices and wages, then computed the resulting macroeconomic dynamics and obtained closed form solutions for the evolution of output and inflation. These formulas showed that it was possible to obtain a persistent response to monetary shocks. We investigated notably under which conditions a hump shaped response of output and inflation could be obtained, and we found that this would obtain for very reasonable durations.

Appendix 1

Proof of Proposition 1

Firm (i,k) maximizes its discounted expected real profits weighted by the marginal utility of goods (i.e. multiplied by $1/C_t$ since utility is logarithmic in consumption). We shall consider here only the terms corresponding to the price contracts signed at time τ and still in effect at time t. Since price contracts have a probability $\gamma\phi$ to survive each period, the contract signed in τ has a probability $\gamma^{t-\tau}\phi^{t-\tau}$ to be still in effect in period t, and the firm will thus maximize the following expected profit:

$$E_\tau \sum_{t=\tau}^{\infty} (\beta\gamma\phi)^{t-\tau} \frac{1}{P_t C_t} \left(P_{ikt} Y_{ikt} - W_{ik\tau} N_{ikt}\right) \tag{43}$$

subject to equation (22) giving the demand for goods :

$$Y_{ikt} = \frac{P_t Y_t}{P_{it}} \left(\frac{P_{ikt}}{P_{it}}\right)^{-1/(1-\theta)} \tag{44}$$

Note that in formula (43) we put $W_{ik\tau}$ as the relevant wage for all periods $t \geq \tau$. Indeed we consider only the price contracts that will still be in effect in period t. But, as we indicated above, if the wage changes, then the prices are automatically renegotiated, so that all price contracts that will remain must be based on the current wage $W_{ik\tau}$. Note also that, in the above formulas (43) and (44), we have to replace P_{ikt} by $Q_{ik\tau t}$ and P_{it} by $Q_{i\tau t}$ since these are the relevant prices for our maximization. Firms indexed by (i,k) maximize (43) subject to (44). Let us insert the value of Y_{ikt} (equation 44) into (43). Taking into account $Y_{ikt} = Z_t N_{ikt}^{\alpha}$, $C_t = Y_t$, $P_{ikt} = Q_{ik\tau t}$ and $P_{it} = Q_{i\tau t}$, the part of the maximand concerning $Q_{ik\tau t}$ is written, omitting irrelevant constant terms:

$$\left(\frac{Q_{ik\tau t}}{Q_{i\tau t}}\right)^{-\theta/(1-\theta)} - W_{ik\tau} \left(\frac{Q_{ik\tau t}}{Q_{i\tau t}}\right)^{-1/\alpha(1-\theta)} \left(\frac{1}{Q_{i\tau t}}\right)^{1/\alpha} E_\tau \left[\frac{1}{P_t Y_t}\left(\frac{P_t Y_t}{Z_t}\right)^{1/\alpha}\right] \tag{45}$$

The first order condition in $Q_{ik\tau t}$ is:

$$\left(\frac{Q_{ik\tau t}}{Q_{i\tau t}}\right)^{-1/(1-\theta)} = \frac{W_{ik\tau}}{\alpha\theta} \left(\frac{Q_{ik\tau t}}{Q_{i\tau t}}\right)^{-1/\alpha(1-\theta)-1} \left(\frac{1}{Q_{i\tau t}}\right)^{1/\alpha} E_\tau \left[\frac{1}{P_t Y_t}\left(\frac{P_t Y_t}{Z_t}\right)^{1/\alpha}\right] \tag{46}$$

In view of equation (12) this is rewritten:

$$\left(\frac{Q_{ik\tau t}}{Q_{i\tau t}}\right)^{(1-\alpha\theta)/\alpha(1-\theta)} = \frac{W_{ik\tau}}{\alpha\theta}\left(\frac{1-\beta}{\omega}\right)^{(1-\alpha)/\alpha}\left(\frac{1}{Q_{i\tau t}}\right)^{1/\alpha} E_\tau\left[\frac{1}{M_t}\left(\frac{M_t}{Z_t}\right)^{1/\alpha}\right] \tag{47}$$

which is equation (25).

Appendix 2

Proof of Proposition 3

Household (i,k) (i.e. a household working in firm k in sector i) maximizes his discounted expected utility. We will consider here only the terms corresponding to the wage contracts signed at time s and still in effect at time t, which we will denote as X_{ikst}. Since wage contracts have a probability γ to survive each period, the wage contract signed in s has a probability γ^{t-s} to be still in effect in period t, and the household (i,k) will thus maximize the following expected utility:

$$E_s \sum_{t\geq s} \beta^{t-s}\gamma^{t-s}\left[LogC_{ikt} + \omega Log\frac{M_{ikt}}{P_t} - \frac{\xi N_{ikt}^{\nu}}{\nu}\right] \tag{48}$$

subject to the budget constraints in each period:

$$C_{ikt} + \frac{M_{ikt}}{P_t} = \frac{X_{ikst}}{P_t}N_{ikt} + \frac{\mu_t M_{ikt-1}}{P_t} + \Pi_{it} \tag{49}$$

and the equations giving the demand for labor (24):

$$N_{ikt} = \left(\frac{P_t Y_t}{Z_t P_{it}}\right)^{1/\alpha}\left(\frac{P_{ikt}}{P_{it}}\right)^{-1/\alpha(1-\theta)} \tag{50}$$

We see that N_{ikt} depends on P_{ikt}, the price effective in period t , which itself depends on the period τ when it has been set. This period τ can be any period between s and t. So we index employment by τ as well, denoting it $N_{ik\tau t}$, and formula (50) is rewritten, for $s \leq \tau \leq t$:

$$N_{ik\tau t} = \left(\frac{P_t Y_t}{Z_t Q_{i\tau t}}\right)^{1/\alpha}\left(\frac{Q_{ik\tau t}}{Q_{i\tau t}}\right)^{-1/\alpha(1-\theta)} \tag{51}$$

Now the price $Q_{ik\tau t}$ set in period τ, $s \leq \tau \leq t$, is given by formula (25), where the relevant wage is X_{ikst}:

$$\left(\frac{Q_{ik\tau t}}{Q_{i\tau t}}\right)^{(1-\alpha\theta)/\alpha(1-\theta)} = \frac{X_{ikst}}{\alpha\theta}\left(\frac{1-\beta}{\omega}\right)^{(1-\alpha)/\alpha}\left(\frac{1}{Q_{i\tau t}}\right)^{1/\alpha}\Omega_{\tau t} \tag{52}$$

where $\Omega_{\tau t}$ is defined in equation (27). The employment corresponding to a price set in period τ is thus, combining equations (51) and (52):