Universal Joints and Driveshafts

H. Chr. Seherr-Thoss · F. Schmelz · E. Aucktor

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Analysis, Design, Applications

Second, enlarged edition with 267 Figures and 72 Tables

Translated by J. A. Tipper and S. J. Hill

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Preface to the second English edition

An important date in the history of automotive engineering was celebrated at the start of the 1980s: 50 years of front-wheel drives in production vehicles. This bicentennial event aroused interest in the development, theory and future of driveshafts and joints. The authors originally presented all the available knowledge on constant-velocity and universal-joint driveshafts in German as long ago as 1988, followed by English in 1992 and Chinese in 1997.

More than ten years have passed since then, in which time technology has also made major progress in the field of driveshafts. Driveshaft design and manufacturing process has kept pace with the constantly growing demands of the various users. More powerful engines with higher torques, new fields of application with increased stresses, e.g. off-road and heavy goods vehicles or rolling mills, improved materials, new production processes and advanced experimental and test methods have imposed completely new requirements on the driveshaft as a mechanical component.

GKN Driveline has made a major contribution to the further development of the driveshaft and will maintain this effort in the future. At our Research & Product Development Centres, GKN engineers have defined basic knowledge and conceived product improvements for the benefit of the automotive, agricultural, and machinery industry of mechanically engineered products for the world. In close cooperation with its customers GKN has created low-noise, vibration and maintenance-free driveshafts.

The cumulative knowledge acquired has been compiled in this second edition book that has been updated to reflect the latest state of the art. It is intended to serve both as a textbook and a work of reference for all driveline engineers, designers and students who are in some way involved with constant-velocity and universal-joint driveshafts.

Redditch, England
July 2005

Arthur Connelly
Chief Executive Officer
GKN Automotive Driveline Driveshafts

Preface to the second German edition

1989 saw the start of a new era of the driveline components and driveshafts, driven by changing customer demands. More vehicles had front wheel drive and transverse engines, which led to considerable changes in design and manufacture, and traditional methods of production were revisited. The results of this research and development went into production in 1994–96:

- for Hooke's jointed driveshafts, weight savings were achieved through new forgings, noise reduction through better balancing and greater durability through improved lubrication.
- for constant velocity joints, one can talk about a "New Generation", employing something more like roller bearing technology, but where the diverging factors are dealt with. The revised Chapters 4 and 5 re-examine the movement patterns and stresses in these joints.

The increased demands for strength and precision led to even intricate shapes being forged or pressed. These processes produce finished parts with tolerances of 0.025 mm. 40 million parts were forged in 1998. These processes were also reviewed.

Finally, advances were made in combined Hooke's and constant velocity jointed driveshafts.

A leading part in these developments was played by the GKN Group in Birmingham and Lohmar, which has supported the creation of this book since 1982. As a leading manufacturer they supply 600,000 Hooke's jointed driveshafts and 500,000 constant velocity driveshafts a year. I would like to thank Reinhold Schoeberl for his project-management and typography to carry out the excellent execution of this book.

I would also like to thank my wife, Therese, for her elaboration of the indices. Moreover my thanks to these contributors:

Gerd Faulbecker (GWB Essen)
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Rainer Schaeferdiek
Karl-Ernst Strobel (Offenbach)

On 19th July 1989 our triumvirate lost Erich Aucktor. He made a valuable contribution to the development of driveline technology from 1937–1958, as an engine engineer, and from 1958–78 as a designer and inventor of constant velocity joints at Löhr & Bromkamp, Offenbach, where he worked in development, design and testing. It is thanks to him that this book has become a reference work for these engineering components.

Count Hans Christoph Seherr-Thoss

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Independent Tables

Tables of principal dimensions, torque capacities and miscellaneous data inside the Figures

Notation

Symbol	Unit	Meaning
1. Coordinate Systems		
$0, x, y, z$	mm	orthogonal, right-handed body system
$0, x', y', z'$	mm	spatial system, arising from the transformation with the rotation matrix $\boldsymbol{D}_\varphi = \begin{pmatrix} \sin\varphi & -\cos\varphi & 0 \\ \cos\varphi & \sin\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$
$0, x'', y'', z''$	mm	spatial system, arising from the transformation with the translation matrix $\boldsymbol{D}_\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{pmatrix}$
$0, r, \varphi$	mm, degrees	system of polar coordinates in Boussinesq half space
$\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{u}$		vectorial representation of a straight track
$\boldsymbol{a} = \begin{pmatrix} k \\ l \\ m \end{pmatrix}$		location vector and its components
$\boldsymbol{u} = \begin{pmatrix} n \\ p \\ q \end{pmatrix}$		direction vector and its components
Indices 1		body 1, driving unit
Indices 2		body 2, driven unit or intermediate body
Indices 3		body 3, driven unit for three bodies
$i = 1, 2, 3, \ldots n$		sequential numbering

Symbol	Unit	Meaning
2. Angles		
α	degrees	pressure angle
β	degrees	articulation angle
γ	degrees	skew angle of the track
δ	degrees	divergence of opening angle ($\delta = 2\varepsilon$), offset angle
ε	degrees	tilt or inclination angle of the track (Stuber pledge angle)
ϑ	degrees	complementary angle to α ($\alpha = 90° - \vartheta$)
τ	degrees	Hertzian auxiliary angle
$\cos\tau$	–	Hertzian coefficient
φ	degrees	angle of rotation through the joint body
χ	degrees	angle of intersection of the tracks
ψ	degrees	angle of the straight generators r
3. Rolling body data		
A	mm^2	contact area
$2a$	mm	major axis of contact ellipse
$2b$	mm	minor axis of contact ellipse
$2c$	mm	separation of cross axes of double Hooke's joints
c	mm	offset (displacement of generating centres)
d	mm	roller diameter
D	mm	trunnion diameter
D_m	mm	diameter of the pitch circle of the rolling bodies
k	N/mm^2	specific loading
l	mm	roller length (from catalogue)
l_w	mm	effective length of roller
r	mm	radius of curvature of rolling surfaces
R	m	effective joint radius
s_w	mm	joint plunge
ψ		reciprocal of the conformity in the track cross section
κ_Q		conformity in the track cross section
κ_L		conformity in the longitudinal section of the track
$\varrho = 1/r$	1/mm	curvature of the rolling surfaces
c_p	N/mm^2	coefficient of conformity
p_o	N/mm^2	Hertzian pressure
δ_o	mm	total elastic deformation at a contact point
δ_b	mm	plastic deformation
μ, ν		Hertzian elliptical coefficients
E	N/mm^2	Young's modulus ($2.08 \cdot 10^{11}$ for steel)
m		Poisson's ratio = 3/10
$\vartheta = \frac{4}{E}(1 - m^2)$	mm^2/N	abbreviation used by Hertz
z	–	number of balls

Symbol	Unit	Meaning
4. Forces		
P	N	equivalent dynamic compressive force
Q	N	Hertzian compressive force
F	N	radial component of equivalent compressive force P
A	N	axial component of equivalent compressive force P
Q_{total}	N	total radial force on a roller bearing used by Hertz/Stribeck
5. Moments		
M	Nm	moments in general
M_0	Nm	static moment
M_d	Nm	dynamic moment
M_N	Nm	nominal moment (from catalogue)
M_b	Nm	bending moment
M_B	Nm	design moment
6. Mathematical constants and coefficients		
ε		ratio of the front and rear axle loads $A_\mathrm{F}/A_\mathrm{R}$
ε_F		fraction of torque to the front axle
ε_R		fraction of torque to the rear axle
f_β		articulation coefficient
k_ω		equivalence factor for cyclic compressive forces
s		Stribeck's distribution factor = 5
s_0		static safety factor for oscillating bearings of Hooke's joints = 0.8 to 1.0
u		number of driveshafts
μ		friction coefficient of road
7. Other designations		
P_eff	kW	output power
V	kph	driving speed
ω	s^{-1}	angular velocity
m		effective number of transmitting elements
n	rpm	rotational speed (revolutions per minute)
π		plane of symmetry

Designations which have not been mentioned are explained in the text or shown in the figures.

Chronological Table

1352–54	Universal jointed driveshaft in the clock mechanism of Strasbourg Cathedral.
1550	*Geronimo Cardano's* gimbal suspension.
1663	*Robert Hooke's* universal joint. 1683 double Hooke's joint.
1824	Analysis of the motion of Hooke's joints with the aid of spherical trigonometry and differential calculus, and the calculation of the forces on the cross by *Jean Victor Poncelet*.
1841	Kinematic treatment of the Hooke's joint by *Robert Willis*.
1894	Calculation of surface stresses for crosses by *Carl Bach*.
1901/02	Patents for automotive joints by *Arthur Hardt* and *Robert Schwenke*.
1904	Series production of Hooke's joints and driveshafts by *Clarence Winfred Spicer*.
1908	First ball joint by *William A. Whitney*. Plunging + articulation separated.
1918	Special conditions for the uniform transmission of motion by *Maurice d'Ocagne*. 1930 geometrical evidence for the constant velocity characteristics of the Tracta joint.
1923	Fixed ball joint steered by generating centres widely separated from the joint mid-point, by *Carl William Weiss*. Licence granted to the Bendix Corp.
1926	*Pierre Fenaille's* "homokinetic" joint.
1927	Six-ball fixed joint with 45° articulation angle by *Alfred H. Rzeppa*. 1934 with offset steering of the balls. First joint with concentric meridian tracks.
1928	First Hooke's joint with needle bearings for the crosses by *Clarence Winfred Spicer*. Bipode joint by *Richard Bussien*.
1933	Ball joint with track-offset by *Bernard K. Stuber*.
1935	Tripode joint by *J. W. Kittredge*, 1937 by *Edmund B. Anderson*.
1938	Plunging ball joint according to the offset principle by *Robert Suczek*.
1946	Birfield-fixed joint with elliptical tracks, 1955 plunging joint, both by *William Cull*.
1951	Driveshaft with separated Hooke's joint and middle sections by Borg-Warner.
1953	Wide angle fixed joint ($\beta = 45°$) by *Kurt Schroeter*, 1971 by *H. Geisthoff*, *Heinrich Welschof* and *H. Grosse-Entrop*.
1959	AC fixed joint by *William Cull* for British Motor Corp., produced by Hardy-Spicer.
1960	Löbro-fixed joint with semicircular tracks by *Erich Aucktor/Walter Willimek*. Tripode plunging joint, 1963 fixed joint, both by *Michel Orain*.

1961	Four-ball plunging joint with a pair of crossed tracks by *Henri Faure*. DANA-plunging joint by *Phil. J. Mazziotti*, *E. H. Sharp*, *Zech*.
1962	VL-plunging joint with crossed tracks, six balls and spheric cage by *Erich Aucktor*.
1965	DO-plunging joint by *Gaston Devos*, completed with parallel tracks and cage offset by Birfield. 1966 series for Renault R 4.
1970	GI-tripode plunging joint by Glaenzer-Spicer. UF-fixed joint by *Heinr. Welschof/Erich Aucktor*. Series production 1972.
1985	Cage-guided balls for plunging in the Triplan joint by *Michel Orain*.
1989	AAR-joint by Löbro in series production.

1 Universal Jointed Driveshafts for Transmitting Rotational Movements

The earliest information about joints came from Philon of Byzantium around 230 BC in his description of censers and inkpots with articulated suspension. In 1245 AD the French church architect, Villard de Honnecourt, sketched a small, spherical oven which was suspended on circular rings. Around 1500, Leonardo da Vinci drew a compass and a pail which were mounted in rings [1.1].

1.1 Early Reports on the First Joints

Swiveling gimbals were generally known in Europe through the report of the mathematician, doctor and philosopher, Geronimo Cardano. He also worked in the field of engineering and in 1550 he mentioned in his book "De subtilitate libri XXI" a sedan chair of Emperor Charles V "which was mounted in a gimbal" [1.2]. In 1557 he described a ring joint in "De armillarum instrumento". Pivots staggered by 90° connected the three rings to one another giving rise to three degrees of freedom (Fig. 1.1). This suspension and the joint formed from it were named the "cardan suspension" or the "cardan joint" after the author.

The need to transmit a rotary movement via an angled shaft arose as early as 1300 in the construction of clocktowers. Here, because of the architecture of the tower, the clockwork and the clockface did not always lie on the same axis so that the transmission of the rotation to the hands had to be displaced upwards, downwards or sideways. An example described in 1664 by the Jesuit, Caspar Schott, was the clock in Strasbourg Cathedral of 1354 [1.3]. He wrote that the inclined drive could best be executed through a cross with four pivots which connected two shafts with forks (fuscinula) fitted to their ends (Fig. 1.2). The universal joint was therefore known long before Schott. He took his description from the unpublished manuscript "Chronometria Mechanica Nova" by a certain Amicus who was no longer alive. If one analyses Amicus's joint, the close relationship between the gimbal and the universal joint can be seen clearly (Figs. 1.3a and b). The mathematics of the transmission of movement were however not clear to Schott, because he believed that one fork must rotate as quickly as the other.

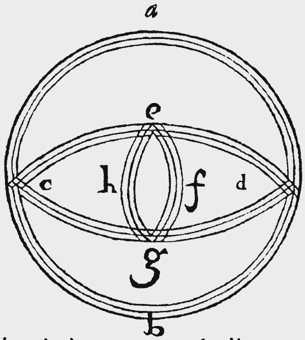
1.1.1 Hooke's Universal Joints

In 1663 the English physicist, Robert Hooke, built a piece of apparatus which incorporated an articulated transmission not quite in the form of Amicus's joint. In 1674 he described in his "Animadversions" [1.4] the helioscope of the Danzig astronomer,

CAPVT VII.

De Armillarum inſtrumento.

COnſtat ex circulis tribus inſtrumentum armillis ſimile, quorum ſuperiores ſunt duplicati, & polis ſecundus primo fixis infigitur. Vt ſit A B circulus primus, cui infixi ſint ad rectos polorum CD



poli interioris, & vterque duplicatus, vel vt media pars circumagi poſſit ſub eiſdem polis, quia inferior; vel quia ex dimidio ſtabilis præter polos, ex dimidio mobilis. Tertius autem in medio ſecundi, ita vt circumagi poſſint poli eius ex E in C, & C in G, & G in D. Et rurſus polis quaſi nullis ex E in F, & F in G, & G in H. Et vt lateat prorſus coniunctio, adeò vt annexus alter alteri videatur. Tale inſtrumentum vidi apud virum Maximiliani Cæſaris, Mathematicum Medicum & Philoſophum inſignem, Ioannem Sagerum Giſenhaigen Vratiſleuienſem, quanquam neque ipſe docuerit, quomodo inſertus eſſet; neque ego interrogauerim. Ergo fieri poteſt, vt circulus inferior D E C G circumuertatur, ſuperiore immobili: atque ita poli ferentur per E C G D: ſed tunc neceſſaria erit cauitas infra circulum ſecundum, per quam feratur. Sed ſi circulus E C G D integer ſit, fieri non poteſt vt ferantur poli niſi cum circulo, cui infixi ſint: hic autem eſt pars circuli prædicti media, aut etiam inferior. Erunt ergo tres modi. Inferior autem circulus, cum in ſeipſo reuoluitur, poterit manentibus quidem polis circumduci à lateribus, fixo manente medio: nam ſi medius transferatur cui polus infixus eſt, exibit polus circumductus circulum E C G D ſecundum latitudinem: aut transferetur polus per cauitatem. At tunc non erit circulus F G H E ſolidus. Cum ergo voluerimus circulos ambos eſſe ſolidos, relinquentur duo modi tantum, vt pars media circuli ſecundi, cui infixi ſint circuli, circumuoluatur; & extremæ inferioris partes, ſeu latera: aut vt pars inferior ſecundi circuli intruſa ſuperiori, & in qua ſint poli fixi edem modo ſub ſuperiore circumducatur, atque eo modo totus circulus E F G H circumagatur per E D G C. Ipſe verò circulus E D G C in ſeipſo vt prius manente fixa parte media, in qua ſunt poli infixi, circumducatur lateribur ſuis. Commune autem eſt ambobus, vt poli ſint infixi vtriſque circulis ſecundo & tertio, & quòd latera inferioris manente medio circumagantur. In ſecundo tantum differunt; cum vel media pars manentibus lateribus, vel inferior ſuperiore fixa circumduci poſſit.

Fig. 1.1. Ring joint of Hieronymus Cardanus 1557. In his "Mediolanensis philosophi ac medici", p. 163, he wrote the following about it: "I saw such a device at the house of Emperor Maximilian's adviser, Joannem Sagerum (Johann Sager) from Gisenhaigen (Gießenhagen, Geißenhain), an important mathematician, doctor and philosopher in the Pressburg diocese. He did not explain how he had arrived at this, on the other hand I did not ask him about this." This is proof that the suspension was already known before Cardanus and was only called the "cardan suspension" after this description [1.2]. Translated from the Latin by Theodor Straub, Ingolstadt. Photograph: Deutsches Museum, Munich

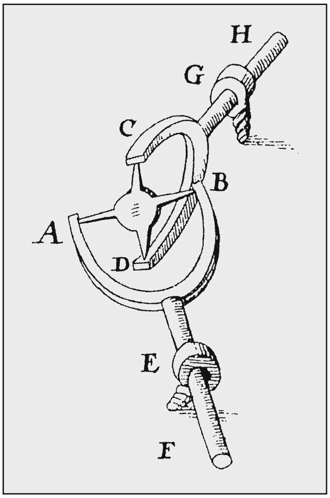


Fig. 1.2. Universal joint of Amicus (16th century). *ABCD* is a cross, the opposing arms *AB* of which are fitted into holes on the ends of a fork *ABF* (fuscinula). The other pair of arms *CD* is received in the same way by the fork *CDH* and the forks themselves are mounted in fixed rings *G* and *E* [1.3, 1.7]. Photograph: Deutsches Museum, Munich

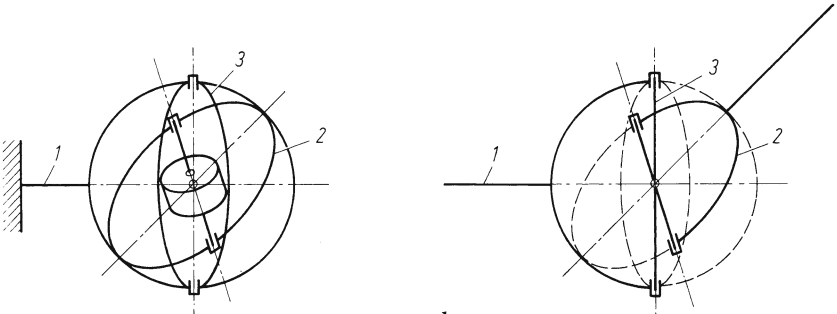


Fig. 1.3 a, b. Relationship between gimbal and universal joint. **a** swiveling gimble, the so-called "cardan suspension" (16th century). *1* attachment, *2* revolving ring, *3* pivot ring; **b** universal joint of Amicus (17th century), extended to make the cardan suspension. *1* drive fork, *2* driven fork, *3* cross

Johannes Hevelius, which comprised a universal joint similar to that of Amicus (Fig. 1.4). In 1676 he spoke of a "joynt" and a "universal joynt" because it is capable of many kinds of movements [1.5–1.7].

Hooke was fully conversant with the mathematics of the time and was also skilled in practical kinematics. In contrast to Schott he knew that the universal joint does not transmit the rotary movement evenly. Although Hooke did not publish any theory about this we must assume that he knew of the principle of the non-uniformity. He applied his joint for the first time to a machine with which he graduated the faces of sun dials (Fig. 1.5).

Cardano and Hooke can therefore be considered as having prepared the way for universal joints and driveshafts. The specialised terms "cardan joint" in Continental Europe and "Hooke's joint" in Anglo-Saxon based languages still remind us of the two early scholars.

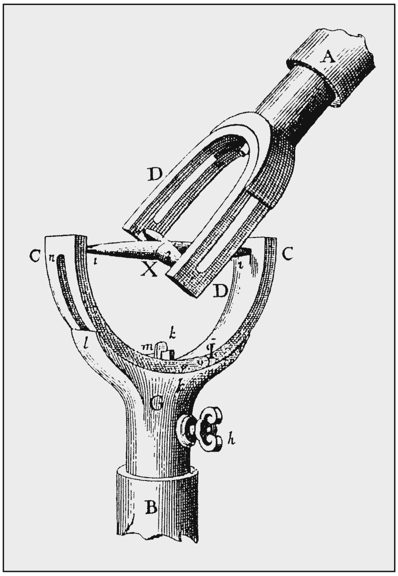


Fig. 1.4. Universal joint from the helioscope by Johannes Hevelius [1.5, 1.7]. Photograph: Deutsches Museum, Munich

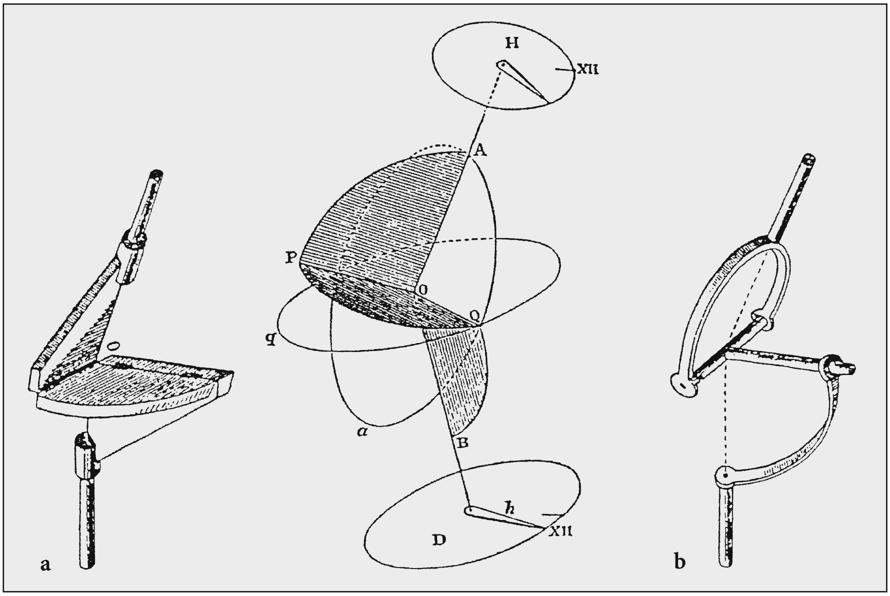


Fig. 1.5 a, b. Principle of the apparatus for graduating the faces of sun dials, according to Robert Hooke 1674 [1.11]. **a** Double hinge of the dividing apparatus in a shaft system, as in **b**; **b** relationship between the universal joint and this element. Photograph: Deutsches Museum, Munich

1.2 Theory of the Transmission of Rotational Movements by Hooke's Joints

Gaspard Monge established the fundamental principles of his Descriptive Geometry in 1794 at the École Polytechnique for the study of machine parts in Paris. The most important advances then came in the 19th century from the mathematician and engineer officer Jean-Victor Poncelet, who had taught applied mathematics and engineering since 1824 in Metz at the Ecole d'Application, at a time when mechanical, civiland industrial engineering were coming decisively to the fore. As the creator of projective geometry in 1822 he perceived the spatial relationships of machine parts so well that he was also able to derive the movement of the Hooke's joint [1.8, 1.9]. It was used a great deal at that time in the windmills of Holland to drive Archimedes screws for pumping water.

1.2.1 The Non-Uniformity of Hooke's Joints According to Poncelet

In 1824 Poncelet proved, with the aid of spherical trigonometry, that the rotational movement of Hooke's joints is non-uniform (Fig. 1.6a and b).

Let the plane, given by the two shafts CL and CM, be the horizontal plane. The starting position of the cross axis *AA'* is perpendicular to it[1]. The points of reference *ABA'B'* of the cross and the yokes move along circles on the surface of the sphere *K* with the radius *AC* = *BC* = *A'C* = *B'C*. The circular surface *EBE'B'* stands perpendicular to the shaft *CL and the circular surface DAD'A'* to the shaft *CM*. The angle of inclination *DCE* of these surfaces toone another is the angle of articulation β of the two shafts *CL* and *CM*.

The relationships between the movements were derived by Poncelet from the spherical triangle *ABE* in Fig. 1.6b. If the axis *CM* has rotated about the arc $EA = \varphi_1$

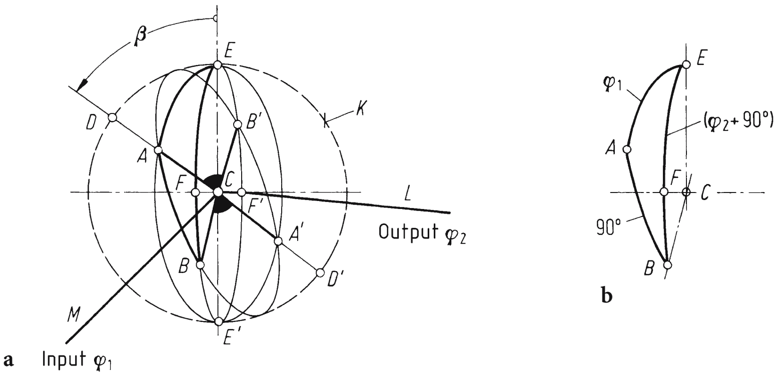


Fig. 1.6 a, b. Proof of the non-uniform rotational movement of a Hooke's joint by J. V. Poncelet 1824. **a** Original figure [1.8, 1.9]; **b** spherical triangle from **a**

[1] Following the VDI 2722 directive of 1978 this is called "orthogonal". If AA′ lies in the plane of the shaft then the starting position is "in-phase" [2.14].

while the axis CL turns about the arc $FB = \varphi_2$ then according to the cosine theorem [1.10, Sect. 3.3.12, p. 92]

$$\cos 90° = \cos \varphi_1 \cos(90 + \varphi_2) + \sin \varphi_1 \sin(90 + \varphi_2) \cos \beta.$$

Since $\cos 90° = 0$ it follows that

$$0 = \cos \varphi_1 (-\sin \varphi_2) + \sin \varphi_1 \cos \varphi_2 \cos \beta.$$

After dividing by $\cos \varphi_1 \sin \varphi_2$ Poncelet obtained

$$\tan \varphi_2 = \cos \beta \tan \varphi_1 \tag{1.1a}$$

or

$$\varphi_2 = \arctan(\cos \beta \tan \varphi_1). \tag{1.1b}$$

For the in-phase starting position, with $\varphi_1 + 90°$ and $\varphi_2 + 90°$ (1.1a). The following applies

$$\tan(\varphi_2 + 90°) = \cos \beta \tan(\varphi_1 + 90°) \Rightarrow \cot \varphi_2 = \cos \beta \cot \varphi_1$$

or

$$\frac{1}{\tan \varphi_2} = \cos \beta \frac{1}{\tan \varphi_1} \Rightarrow \tan \varphi_2 = \frac{\tan \varphi_1}{\cos \beta}. \tag{1.1c}$$

The *first derivative* of (1.1b) with respect to time gives the angular velocity

$$\frac{d\varphi_2}{dt} = \frac{1}{1 + \tan^2 \varphi_1 \cos^2 \beta} \frac{\cos \beta}{\cos^2 \varphi_1} \frac{d\varphi_1}{dt}$$

$$= \frac{\cos \beta}{\cos^2 \varphi_1 + \sin^2 \varphi_1 (1 - \sin^2 \beta)} \frac{d\varphi_1}{dt}$$

$$\frac{d\varphi_2}{dt} = \frac{\cos \beta}{1 - \sin^2 \varphi_1 \sin^2 \beta} \frac{d\varphi_1}{dt} \qquad \text{[1.10, Sect. 4.3.3.13, p. 107]}$$

or because $d\varphi_2/dt = \omega_2$ and $d\varphi_1/dt = \omega_1$

$$\frac{\omega_2}{\omega_1} = \frac{\cos \beta}{1 - \sin^2 \beta \sin^2 \varphi_1}. \tag{1.2}$$

Equations (1.1a–c) and (1.2) form the basis for calculating the angular difference shown in Fig. 1.7a:

$$\Delta\varphi = \varphi_2 - \varphi_1$$

and the ratio of angular velocities ω_2/ω_1 shown in Fig. 1.7b.

For the two boundary conditions, (1.2) gives

$$\varphi_2 = 0°, \ \varphi_1 = 90° \Rightarrow \omega_2 = \omega_{min} = \omega_1 \cos \beta,$$

$$\varphi_2 = 90°, \ \varphi_1 = 180° \Rightarrow \omega_2 = \omega_{max} = \omega_1/\cos \beta.$$

Figure 1.7a shows that an articulation of 45° gives rise to a lead and lag $\Delta\varphi$ about ±10°, and very unpleasant vibrations ensue. The reduction of these vibrations has been very important generally for the mechanical engineering and the motor

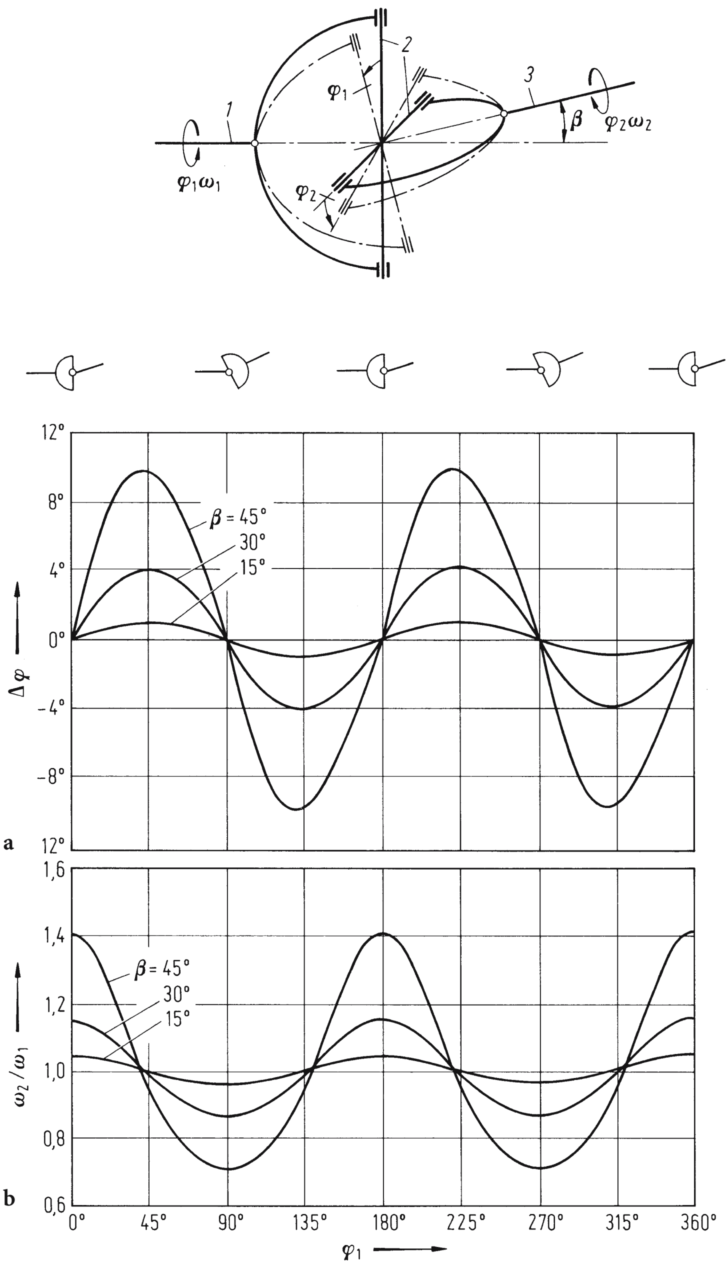


Fig. 1.7 a, b. Angular difference and angular velocities of Hooke's joints for articulation angles of 15°, 30° and 45°. Single Hooke's joint (schematically) [1.36]. *1* Input shaft; *2* intermediate member (cross); *3* output shaft. ω_1, ω_2 angular velocities of the input and output shafts, β angle of articulation, φ_1, φ_2 angle of rotation of the input and output shafts. **a** Angular difference $\Delta\varphi$ (cardan error); **b** angular velocities. Graph of the angular velocity ω_2 for articulation angles of 15°, 30° and 45°

vehicle industries: for the development of front wheel drive cars it has been imperative. $\Delta\varphi$ is also called the angular error or "cardan error".

The *second derivative* of (1.1b)

$$\frac{d\varphi_2}{dt} = \frac{d\varphi_1}{dt}\frac{\cos\beta}{1-\sin^2\beta\sin^2\varphi_1} = \frac{k'}{1-k^2\sin^2\varphi_1}$$

with respect to time gives the angular acceleration

$$\alpha_2 = \frac{d^2\varphi_2}{dt^2} = \frac{d\varphi_1}{dt}k'\frac{0(1-k^2\sin^2\varphi_1)-1(-k^2 2\sin\varphi_1\cos\varphi_1)}{(1-k^2\sin^2\varphi_1)^2}\frac{d\varphi_1}{dt}$$
$$= \left(\frac{d\varphi_1}{dt}\right)^2 \frac{k'k^2\,2\sin\varphi_1\cos\varphi_1}{(1-k^2\sin^2\varphi_1)^2} = \omega_1^2\frac{\cos\beta\sin^2\beta\sin 2\varphi_1}{(1-\sin^2\beta\sin^2\varphi_1)^2}\,. \quad (1.3)$$

This angular acceleration α_2 will be required in Sect. 4.2.7, for calculating the resulting torque, and for the maximum values of speed and articulation $n\beta$.

1.2.2 The Double Hooke's Joint to Avoid Non-uniformity

In 1683 Robert Hooke had the idea of eliminating the non-uniformity in the rotational movement of the single universal joint by connecting a second joint (Fig. 1.8). In this arrangement of two Hooke's joints the yokes of the intermediate shaft which rotates at ω_2 lie in-phase, i.e. as shown in Figs. 1.8, 1.9a and b in the same plane, as Robert Willis put forward in 1841 [1.11].

For joint 2φ_2 is the driving angle and φ_3 is the driven angle. Therefore from (1.1c) $\tan\varphi_3 = \tan\varphi_2/\cos\beta_2$. Using (1.1c) one obtains

$$\tan\varphi_3 = \frac{\cos\beta_1\tan\varphi_1}{\cos\beta_2}\,.$$

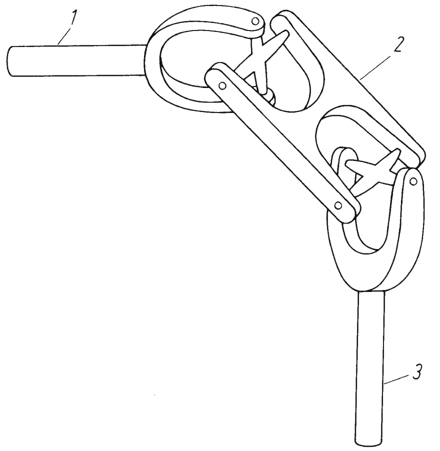


Fig. 1.8. Robert Hooke's double universal joint of 1683 [1.6]. *1* Input, *2* intermediate part, *3* output

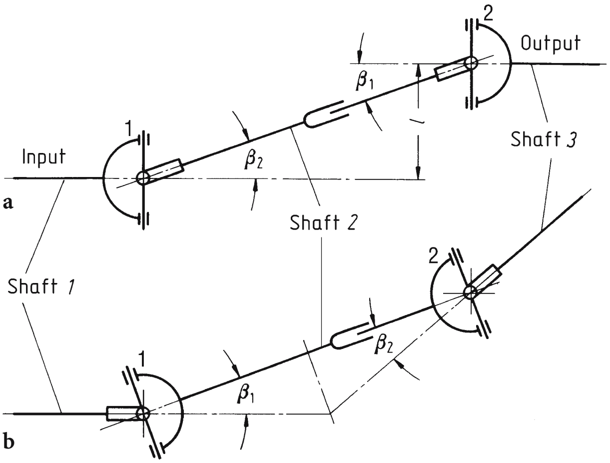


Fig. 1.9 a, b. Universal joints with in-phase yokes and $\beta_1 = \beta_2$ as a precondition for constant velocity $\omega_3 = \omega_1$ [1.37]. **a** Z-configuration, **b** W-configuration

From this uniformity of the driving and driven angles then follows for the condition $\beta_2 + \pm \beta_1$, because $\cos(\pm \beta)$ gives the same positive value in both cases. Therefore $\tan \varphi_{3\,=\,\tan} \varphi_1$ both in the Z-configuration in Fig. 1.9a and in the W-configuration in Fig. 1.9b, and also in the double Hooke's joint in Fig. 1.8. It follows that if $\varphi_3 = \varphi_1$

$$d\varphi_3/dt = d\varphi_1/dt \quad \text{or} \quad \omega_3 = \omega_1 = \text{const.} \tag{1.4}$$

Driveshafts in the Z-configuration are well suited for transmitting uniform angular velocities $\omega_3 = \omega_1$ to parallel axes if the intermediate shaft 2 consists of two prismatic parts which can slide relative to one another, e.g. splined shafts. The intermediate shaft 2 then allows the distance l to be altered so that the driven shaft 3 which revolves with $\omega_3 = \omega_1$ can move in any way required, which happens for example with the table movements of machine tools.

Arthur Hardt suggested in 1901 a double Hooke's joint in the W-configuration for the steering axle of cars (DRP 136605), "...in order to make possible a greater wheel lock...". He correctly believed that a single Hooke's joint is insufficient to accommodate on the one hand variations in the height of the drive relative to the wheel, and on the other hand sharp steering, up to 45°, with respect to the axle. For this reason he divided the steer angle over two Hooke's joints, the centre of which was on the axis of rotation of the steering knuckle, "... as symmetrical as possible to this ... in order to achieve a uniform transmission ...". Hardt saw here the possibility of turning the inner wheel twice as much as with a single Hooke's joint.

If, however, the yokes of the intermediate shaft *2* which revolves at ω_3 are out of phase by 90° then the non-uniformity in the case of shaft *3* increases to

$$\omega_{\max} = \frac{\omega_{\min}}{\cos\beta_1 \cos\beta_2},$$

which can be deduced in a similar way to (1.4).

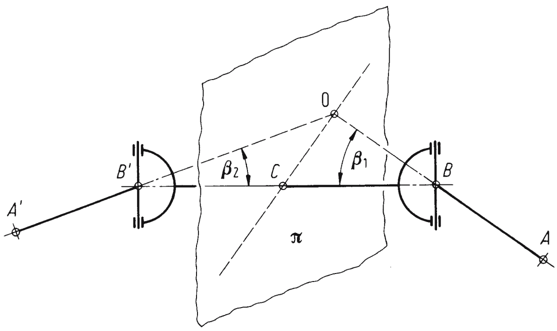


Fig. 1.10. Showing the constant velocity conditions for double Hooke's joints, according to d'Ocagne 1930 [1.14]

1.2.3 D'Ocagne's Extension of the Conditions for Constant Velocity

Is the condition $\beta_2 = \pm\, \beta_1$ sufficient for constant velocity? In 1841 Robert Willis deduced that the double Hooke's joint possesses constant velocity properties, even when the input and output shafts are neither parallel nor intersecting [1.11]. On these spatially oriented shafts constant velocity can only be achieved if the drive and driven axes are fixed. The yokes of the intermediate shaft *2* must be oppositely phased and disposed about the articulation angle φ according to the equation $\tan \varphi/2 = \tan \beta_2/\tan \beta_1$. In the case of planar axes $\beta_2 = \pm\, \beta_1$ can only be attained if the input and output shafts intersect one another. This was shown by Maurice d'Ocagne in 1918 [1.12]. According to his theory a double Hooke's joint only transmits in a uniform manner if, in accordance with Fig. 1.10, the following two conditions are fulfilled:

- the axes of the input and output shafts meet at the point *O*,
- the two Hooke's joints are arranged symmetrically to a plane π which goes through points *O* and *C*.

These conditions are not so simple to fulfil in practice. Without special devices it is not possible to guarantee that the axes will always intersect and that the angle β_1 and β_2 are always the same. Figure 1.11a shows a modern design of double Hooke's joint without centring. There are however joint designs (Fig. 1.11b) in which the conditions are approximated (quasi-homokinetic joint) and some (Fig. 1.11c) where they are strictly fulfilled (homokinetic joint)[2]. In 1971 Florea Duditza dealt with these three cases with the methods of kinematics [1.13].

1.2.4 Simplification of the Double Hooke's Joint

The quasi-homokinetic and also the strictly homokinetic double Hooke's joint as shown in Fig. 1.11b and c are complicated engineering systems made up of many dif-

[2] The term "homokinetic" (homo = same, kine = to move) was originated by two Frenchman Charles Nugue and Andre Planiol in the 1920s.

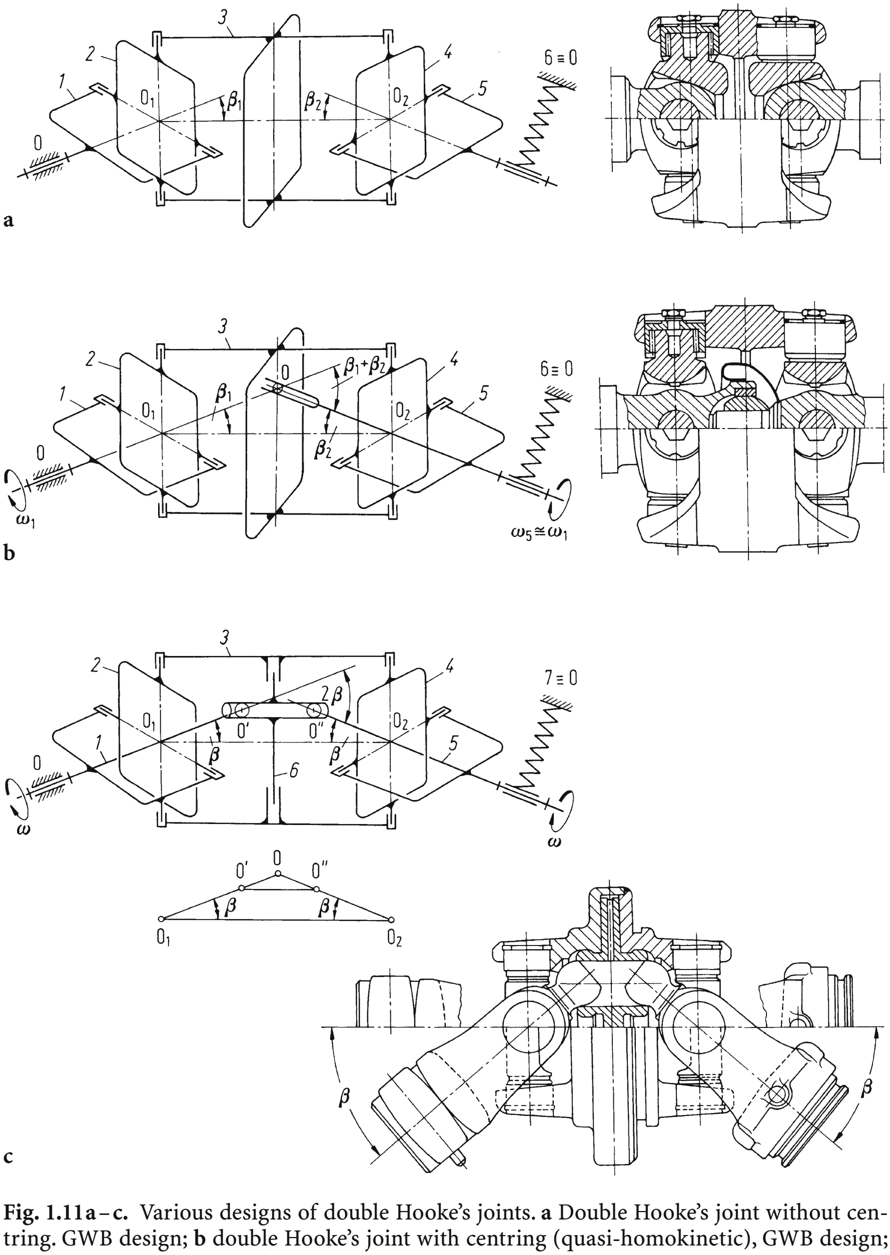


Fig. 1.11 a–c. Various designs of double Hooke's joints. **a** Double Hooke's joint without centring. GWB design; **b** double Hooke's joint with centring (quasi-homokinetic), GWB design; **c** double Hooke's joint with steereing attachment (strictly homokinetic) according to Paul Herchenbach (German patent 2802 572/1978), made by J. Walterscheid GmbH [2.14]

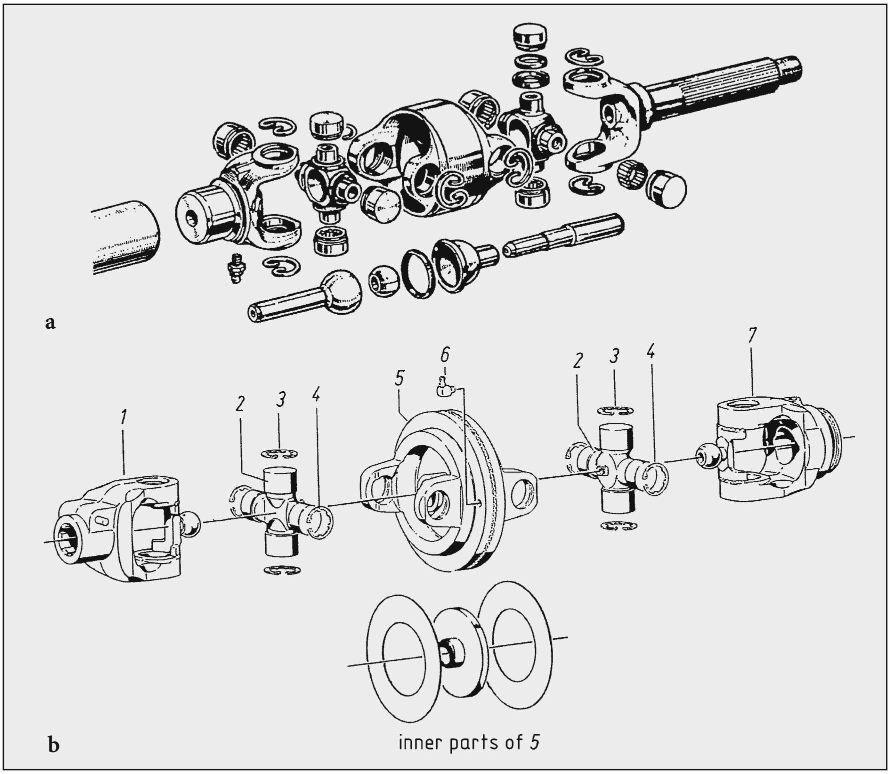


Fig. 1.12 a, b. Exploded views of double Hooke's joints. **a** Citroën design with needle bearings and centring, 1934. Quasi-homokinetic, $\beta = 40°$; **b** Walterscheid design by Hubert Geisthoff, Heinrich Welschof and Paul Herchenbach 1966–78 (German patent 1302735, 2802572). Fully-homokinetic, $\beta = 80°$. *1* ball stud yoke; *2* unit pack; *3* circlip; *4* circlip for double yoke; *5* double yoke; *6* right angled grease nipple; *7* inboard yoke/guide hub

ferent elements (Fig. 1.12a and b). The task of finding simpler and cheaper solutions with constant velocity properties has occupied inventors since the second decade of this century. The tracta joint of Pierre Fenaille 1926 has had the best success [1.14, 1.15].

1.2.4.1 Fenaille's Tracta Joint

The Tracta joint works on the principle of the double tongue and groove joint (Fig. 1.13a–c). It comprises only four individual parts: the two forks F and F' and the two sliding pieces T and M (centring spheres) which interlock. These two sliding pieces, guided by spherical grooves, have their centre points C always the same symmetrical distance from the centre of the joint O. Constant velocity is thus ensured; independently of the angle of articulation the plane of symmetry π passes at $\beta/2$ through O. In 1930 Maurice d'Ocagne proved the constant velocity properties of the Tracta joint before the Academie des Sciences in Paris (Fig. 1.13b) [1.14].