

Fundamentals of Airplane Flight Mechanics

David G. Hull

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With 125 Figures and 25 Tables

 Springer

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Dedicated to
Angelo Miele
who instilled in me his love for flight mechanics.

Preface

Flight mechanics is the application of Newton's laws ($F=ma$ and $M=I\alpha$) to the study of vehicle trajectories (performance), stability, and aerodynamic control. There are two basic problems in airplane flight mechanics: (1) given an airplane what are its performance, stability, and control characteristics? and (2) given performance, stability, and control characteristics, what is the airplane? The latter is called airplane sizing and is based on the definition of a standard mission profile. For commercial airplanes including business jets, the mission legs are take-off, climb, cruise, descent, and landing. For a military airplane additional legs are the supersonic dash, fuel for air combat, and specific excess power. This text is concerned with the first problem, but its organization is motivated by the structure of the second problem. Trajectory analysis is used to derive formulas and/or algorithms for computing the distance, time, and fuel along each mission leg. In the sizing process, all airplanes are required to be statically stable. While dynamic stability is not required in the sizing process, the linearized equations of motion are used in the design of automatic flight control systems.

This text is primarily concerned with analytical solutions of airplane flight mechanics problems. Its design is based on the precepts that there is only one semester available for the teaching of airplane flight mechanics and that it is important to cover both trajectory analysis and stability and control in this course. To include the fundamentals of both topics, the text is limited mainly to flight in a vertical plane. This is not very restrictive because, with the exception of turns, the basic trajectory segments of both mission profiles and the stability calculations are in the vertical plane. At the University of Texas at Austin, this course is preceded by courses on low-speed aerodynamics and linear system theory. It is followed by a course on automatic control.

The trajectory analysis portion of this text is patterned after Miele's flight mechanics text in terms of the nomenclature and the equations of motion approach. The aerodynamics prediction algorithms have been taken from an early version of the NASA-developed business jet sizing code called the General Aviation Synthesis Program or GASP. An important part of trajectory analysis is trajectory optimization. Ordinarily, trajectory optimization is a complicated affair involving optimal control theory (calculus of variations) and/or the use of numerical optimization techniques. However, for the standard mission legs, the optimization problems are quite simple in nature. Their solution can be obtained through the use of basic calculus.

The nomenclature of the stability and control part of the text is based on the writings of Roskam. Aerodynamic prediction follows that of the USAF Stability and Control Datcom. It is important to be able to list relatively simple formulas for predicting aerodynamic quantities and to be able to carry out these calculations throughout performance, stability, and control. Hence, it is assumed that the airplanes have straight, tapered, swept wing planforms.

Flight mechanics is a discipline. As such, it has equations of motion, acceptable approximations, and solution techniques for the approximate equations of motion. Once an analytical solution has been obtained, it is important to calculate some numbers to compare the answer with the assumptions used to derive it and to acquaint students with the sizes of the numbers. The Subsonic Business Jet (SBJ) defined in App. A is used for these calculations.

The text is divided into two parts: trajectory analysis and stability and control. To study trajectories, the force equations ($F=ma$) are uncoupled from the moment equations ($M=I\alpha$) by assuming that the airplane is not rotating and that control surface deflections do not change lift and drag. The resulting equations are referred to as the 3DOF model, and their investigation is called trajectory analysis. To study stability and control, both $F=ma$ and $M=I\alpha$ are needed, and the resulting equations are referred to as the 6DOF model. An overview of airplane flight mechanics is presented in Chap. 1.

Part I: Trajectory Analysis. This part begins in Chap. 2 with the derivation of the 3DOF equations of motion for flight in a vertical plane over a flat earth and their discussion for nonsteady flight and quasi-steady flight. Next in Chap. 3, the atmosphere (standard and exponential) is discussed, and an algorithm is presented for computing lift and drag of a subsonic airplane. The engines are assumed to be given, and the thrust and specific fuel consumption are discussed for a subsonic turbojet and turbofan. Next, the quasi-steady flight problems of cruise and climb are analyzed in Chap. 4 for an arbitrary airplane and in Chap. 5 for an ideal subsonic airplane. In Chap. 6, an algorithm is presented for calculating the aerodynamics of high-lift devices, and the nonsteady flight problems of take-off and landing are discussed. Finally, the nonsteady flight problems of energy climbs, specific excess power, energy-maneuverability, and horizontal turns are studied in Chap. 7.

Part II: Stability and Control. This part of the text contains static stability and control and dynamic stability and control. It is begun in Chap. 8 with the 6DOF model in wind axes. Following the discussion of the equations of motion, formulas are presented for calculating the aerodynamics of

a subsonic airplane including the lift, the pitching moment, and the drag. Chap. 9 deals with static stability and control. Trim conditions and static stability are investigated for steady cruise, climb, and descent along with the effects of center of gravity position. A simple control system is analyzed to introduce the concepts of hinge moment, stick force, stick force gradient, and handling qualities. Trim tabs and the effect of free elevator on stability are discussed. Next, trim conditions are determined for a nonsteady pull-up, and lateral-directional stability and control are discussed briefly. In Chap. 10, the 6DOF equations of motion are developed first in regular body axes and second in stability axes for use in the investigation of dynamic stability and control. In Chap. 11, the equations of motion are linearized about a steady reference path, and the stability and response of an airplane to a control or gust input is considered. Finally, the effect of center of gravity position is examined, and dynamic lateral-direction stability and control is discussed descriptively.

There are three appendices. App. A gives the geometric characteristics of a subsonic business jet, and results for aerodynamic calculations are listed, including both static and dynamic stability and control results. In App. B, the relationship between linearized aerodynamics (stability derivatives) and the aerodynamics of Chap. 8 is established. Finally, App. C reviews the elements of linear system theory which are needed for dynamic stability and control studies.

While a number of students has worked on this text, the author is particularly indebted to David E. Salguero. His work on converting GASP into an educational tool called BIZJET has formed the basis of a lot of this text.

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Austin, Texas

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Chapter 1

Introduction to Airplane Flight Mechanics

Airplane flight mechanics can be divided into five broad areas: trajectory analysis (performance), stability and control, aircraft sizing, simulation, and flight testing. Only the theoretical aspects of trajectory analysis and stability and control are covered in this text. Aircraft sizing and simulation are essentially numerical in nature. Airplane sizing involves an iterative process, and simulation involves the numerical integration of a set of differential equations. They are discussed in this chapter to show how they fit into the overall scheme of things. Flight testing is the experimental part of flight mechanics. It is not discussed here except to say that good theory makes good experiments.

The central theme of this text is the following: Given the three-view drawing with dimensions of a subsonic, jet-powered airplane and the engine data, determine its performance, stability, and control characteristics. To do this, formulas for calculating the aerodynamics are developed.

Most of the material in this text is limited to flight in a vertical plane because the mission profiles for which airplanes are designed are primarily in the vertical plane. This chapter begins with a review of the parts of the airframe and the engines. Then, the derivation of the equations governing the motion of an airplane is discussed. Finally, the major areas of aircraft flight mechanics are described.

1.1 Airframe Anatomy

To begin the introduction, it is useful to review the parts of an airframe and discuss their purposes. Fig. 1.1 is a three-view drawing of a Boeing 727. The body or fuselage of the airplane holds the crew, passengers, and freight. It is carried aloft by the lift provided by the wing and propelled by the thrust produced by jet engines housed in nacelles. This airplane has two body-mounted engines and a body centerline engine whose inlet air comes through an S-duct beginning at the front of the vertical tail. The fuel is carried in tanks located in the wing.

Since a jet transport is designed for efficient high-speed cruise, it is unable to take-off and land from standard-length runways without some configuration change. This is provided partly by leading edge slats and partly by trailing edge flaps. Both devices are used for take-off, with a low trailing edge flap deflection. On landing, a high trailing edge flap deflection is used to increase lift and drag, and brakes, reverse thrust, and speed brakes (spoilers) are used to further reduce landing distance.

A major issue in aircraft design is static stability. An airplane is said to be inherently aerodynamically statically stable if, following a disturbance from a steady flight condition, forces and/or moments develop which tend to reduce the disturbance. Shown in Fig. 1.2 is the body axes system whose origin is at the center of gravity and whose x_b , y_b , and z_b axes are called the roll axis, the pitch axis, and the yaw axis. Static stability about the yaw axis (directional stability) is provided by the vertical stabilizer, whereas the horizontal stabilizer makes the airplane statically stable about the pitch axis (longitudinal stability). Static stability about the roll axis (lateral stability) is provided mainly by wing dihedral which can be seen in the front view in Fig. 1.1.

Also shown in Figs. 1.1 and 1.2 are the control surfaces which are intended to control the rotation rates about the body axes (roll rate P , pitch rate Q , and yaw rate R) by controlling the moments about these axes (roll moment L , pitch moment M , and yaw moment N). The convention for positive moments and rotation rates is to grab an axis with the thumb pointing toward the origin and rotate counterclockwise looking down the axis toward the origin. From the pilot's point of view, a positive moment or rate is roll right, pitch up, and yaw right.

The deflection of a control surface changes the curvature of a wing or tail surface, changes its lift, and changes its moment about the

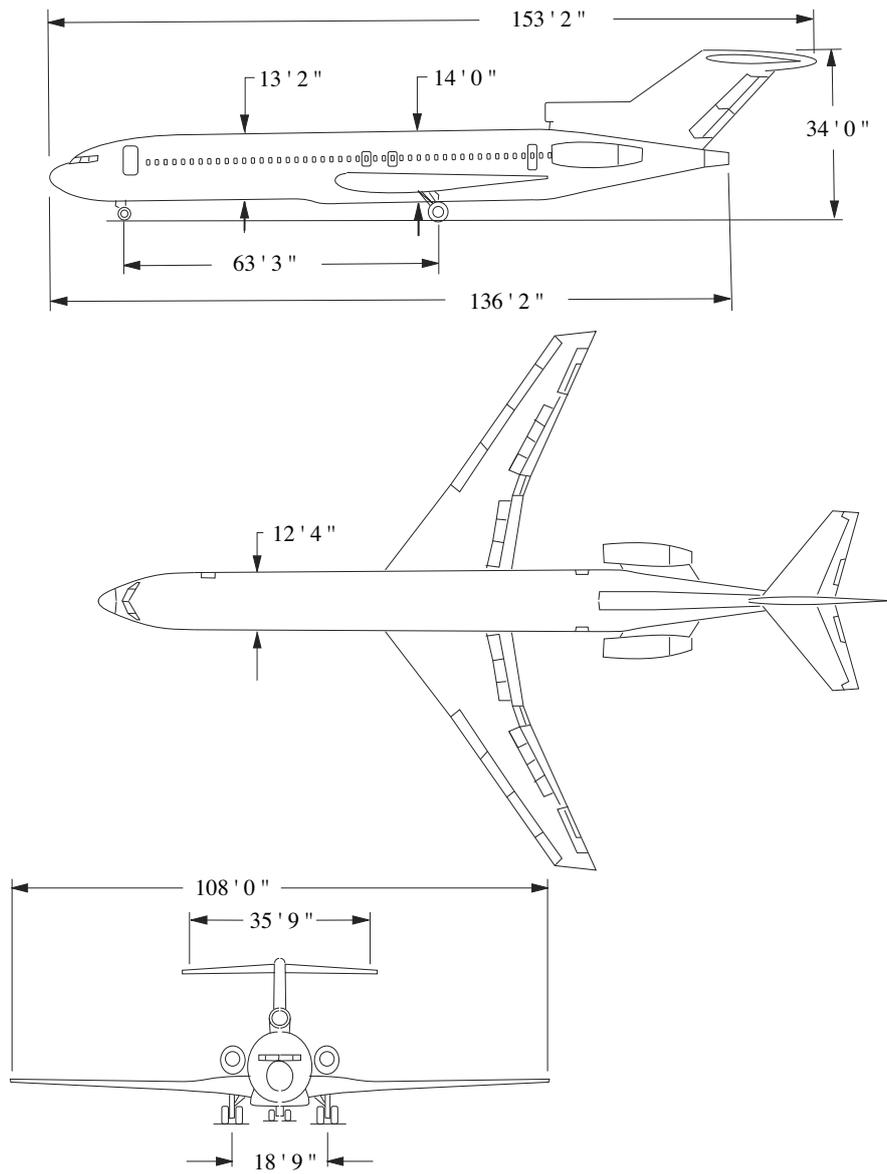


Figure 1.1: Three-View Drawing of a Boeing 727

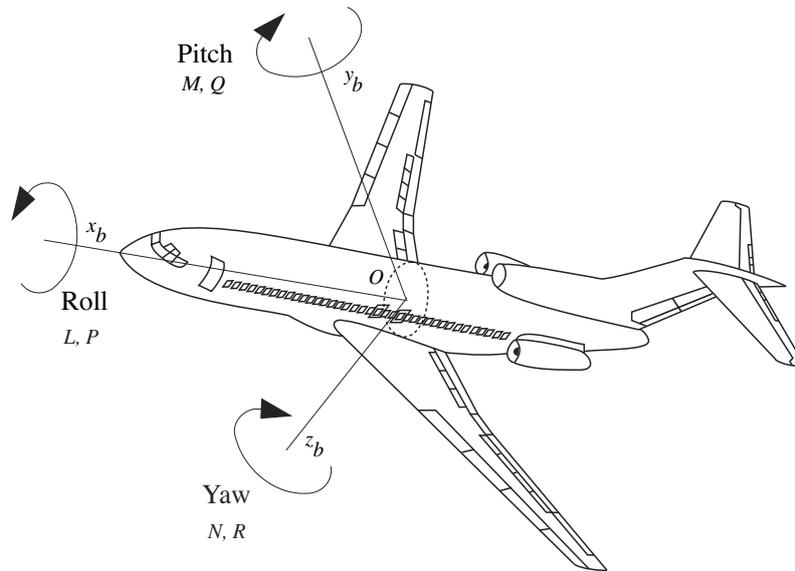


Figure 1.2: Body Axes, Moments, Rates, and Controls

corresponding body axis. Hence, the ailerons (one deflected upward and one deflected downward) control the roll rate; the elevator controls the pitch rate; and the rudder controls the yaw rate. Unlike pitching motion, rolling and yawing motions are not pure. In deflecting the ailerons to roll the airplane, the down-going aileron has more drag than the up-going aileron which causes the airplane to yaw. Similarly, in deflecting the rudder to yaw the airplane, a rolling motion is also produced. Cures for these problems include differentially deflected ailerons and coordinating aileron and rudder deflections. Spoilers are also used to control roll rate by decreasing the lift and increasing the drag on the wing into the turn. Here, a yaw rate is developed into the turn. Spoilers are not used near the ground for roll control because the decreased lift causes the airplane to descend.

The F-16 (lightweight fighter) is statically unstable in pitch at subsonic speeds but becomes statically stable at supersonic speeds because of the change in aerodynamics from subsonic to supersonic speeds. The airplane was designed this way to make the horizontal tail as small as possible and, hence, to make the airplane as light as possible. At

subsonic speeds, pitch stability is provided by the automatic flight control system. A rate gyro senses a pitch rate, and if the pitch rate is not commanded by the pilot, the elevator is deflected automatically to zero the pitch rate. All of this happens so rapidly (at the speed of electrons) that the pilot is unaware of these rotations.

1.2 Engine Anatomy

In this section, the various parts of jet engines are discussed. There are two types of jet engines in wide use: the turbojet and the turbofan.

A schematic of a *turbojet* is shown in Fig. 1.3. Air entering the engine passes through the diffuser which slows the air to a desired speed for entering the compressor. The compressor increases the pressure of the air and slows it down more. In the combustion chamber (burner), fuel is added to the air, and the mixture is ignited and burned. Next, the high temperature stream passes through the turbine which extracts enough energy from the stream to run the compressor. Finally, the nozzle increases the speed of the stream before it exits the engine.

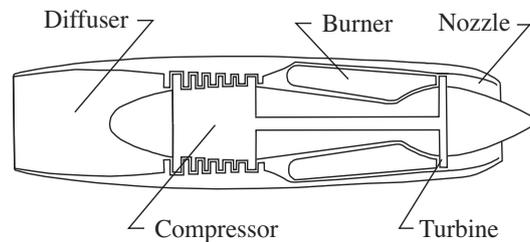


Figure 1.3: Schematic of a Turbojet Engine

The engine cycle is a sequence of assumptions describing how the flow behaves as it passes through the various parts of the engine. Given the engine cycle, it is possible to calculate the thrust (lb) and the fuel flow rate (lb/hr) of the engine. Then, the specific fuel consumption (1/hr) is the ratio of the fuel flow rate to the thrust.

A schematic of a *turbofan* is shown in Fig. 1.4. The turbofan is essentially a turbojet which drives a fan located after the diffuser and before the compressor. The entering air stream is split into a primary

part which passes through the turbojet and a secondary part which goes around the turbojet. The split is defined by the bypass ratio, which is the ratio of the air mass flow rate around the turbojet to the air mass flow rate through the turbojet. Usually, the fan is connected to its own turbine by a shaft, and the compressor is connected to its turbine by a hollow shaft which rotates around the fan shaft.

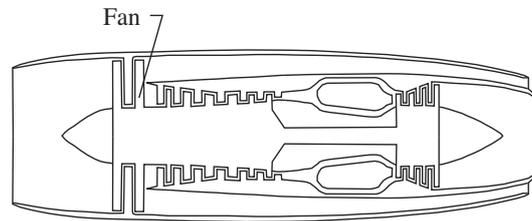


Figure 1.4: Schematic of a Turbofan Engine

1.3 Equations of Motion

In this text, the term flight mechanics refers to the analysis of airplane motion using Newton's laws. While most aircraft structures are flexible to some extent, the airplane is assumed here to be a rigid body. When fuel is being consumed, the airplane is a variable-mass rigid body.

Newton's laws are valid when written relative to an inertial reference frame, that is, a reference frame which is not accelerating or rotating. If the equations of motion are derived relative to an inertial reference frame and if approximations characteristic of airplane motion are introduced into these equations, the resulting equations are those for flight over a nonrotating flat earth. Hence, for airplane motion, the earth is an approximate inertial reference frame, and this model is called the flat earth model. The use of this physical model leads to a small error in most analyses.

A general derivation of the equations of motion involves the use of a material system involving both solid and fluid particles. The end result is a set of equations giving the motion of the solid part of the airplane subject to aerodynamic, propulsive and gravitational forces. To simplify the derivation of the equations of motion, the correct equations

for the forces are assumed to be known. Then, the equations describing the motion of the solid part of the airplane are derived.

The airplane is assumed to have a right-left plane of symmetry with the forces acting at the center of gravity and the moments acting about the center of gravity. Actually, the forces acting on an airplane in flight are due to distributed surface forces and body forces. The surface forces come from the air moving over the airplane and through the propulsion system, while the body forces are due to gravitational effects. Any distributed force (see Fig. 1.5) can be replaced by a concentrated force acting along a specific line of action. Then, to have all forces acting through the same point, the concentrated force can be replaced by the same force acting at the point of interest plus a moment about that point to offset the effect of moving the force. The point usually chosen for this purpose is the center of mass, or equivalently for airplanes the center of gravity, because the equations of motion are the simplest.

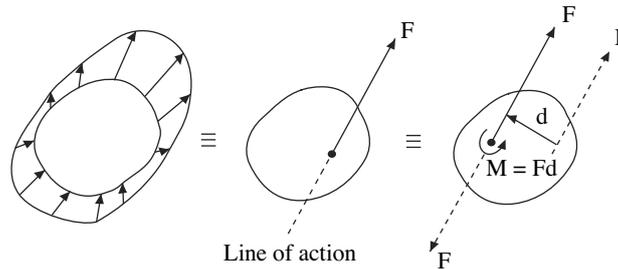


Figure 1.5: Distributed Versus Concentrated Forces

The equations governing the translational and rotational motion of an airplane are the following:

- a. Kinematic equations giving the translational position and rotational position relative to the earth reference frame.
- b. Dynamic equations relating forces to translational acceleration and moments to rotational acceleration.
- c. Equations defining the variable-mass characteristics of the airplane (center of gravity, mass and moments of inertia) versus time.
- d. Equations giving the positions of control surfaces and other movable parts of the airplane (landing gear, flaps, wing sweep, etc.) versus time.

These equations are referred to as the six degree of freedom (6DOF) equations of motion. The use of these equations depends on the particular area of flight mechanics being investigated.

1.4 Trajectory Analysis

Most trajectory analysis problems involve small aircraft rotation rates and are studied through the use of the three degree of freedom (3DOF) equations of motion, that is, the translational equations. These equations are uncoupled from the rotational equations by assuming negligible rotation rates and neglecting the effect of control surface deflections on aerodynamic forces. For example, consider an airplane in cruise. To maintain a given speed an elevator deflection is required to make the pitching moment zero. This elevator deflection contributes to the lift and the drag of the airplane. By neglecting the contribution of the elevator deflection to the lift and drag (untrimmed aerodynamics), the translational and rotational equations uncouple. Another approach, called trimmed aerodynamics, is to compute the control surface angles required for zero aerodynamic moments and eliminate them from the aerodynamic forces. For example, in cruise the elevator angle for zero aerodynamic pitching moment can be derived and eliminated from the drag and the lift. In this way, the extra aerodynamic force due to control surface deflection can be taken into account.

Trajectory analysis takes one of two forms. First, given an aircraft, find its performance characteristics, that is, maximum speed, ceiling, range, etc. Second, given certain performance characteristics, what is the airplane which produces them. The latter is called aircraft sizing, and the missions used to size commercial and military aircraft are presented here to motivate the discussion of trajectory analysis. The mission or flight profile for sizing a commercial aircraft (including business jets) is shown in Fig. 1.6. It is composed of take-off, climb, cruise, descent, and landing segments, where the descent segment is replaced by an extended cruise because the fuel consumed is approximately the same. In each segment, the distance traveled, the time elapsed, and the fuel consumed must be computed to determine the corresponding quantities for the whole mission. The development of formulas or algorithms for computing these performance quantities is the charge of trajectory analysis. The military mission (Fig. 1.7) adds three performance com-

putations: a constant-altitude acceleration (supersonic dash), constant-altitude turns, and specific excess power (P_S). The low-altitude dash gives the airplane the ability to approach the target within the radar ground clutter, and the speed of the approach gives the airplane the ability to avoid detection until it nears the target. The number of turns is specified to ensure that the airplane has enough fuel for air combat in the neighborhood of the target. Specific excess power is a measure of the ability of the airplane to change its energy, and it is used to ensure that the aircraft being designed has superior maneuver capabilities relative to enemy aircraft protecting the target. Note that, with the exception of the turns, each segment takes place in a plane perpendicular to the surface of the earth (vertical plane). The turns take place in a horizontal plane.

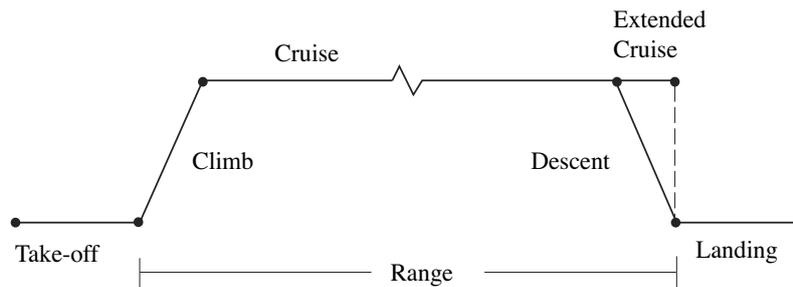


Figure 1.6: Mission for Commercial Aircraft Sizing

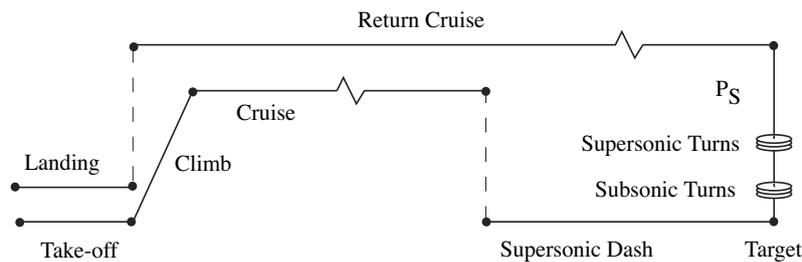


Figure 1.7: Mission for Military Aircraft Sizing

These design missions are the basis for the arrangement of the trajectory analysis portion of this text. In Chap. 2, the equations of motion for flight in a vertical plane over a flat earth are derived, and

their solution is discussed. Chap. 3 contains the modeling of the atmosphere, aerodynamics, and propulsion. Both the standard atmosphere and the exponential atmosphere are discussed. An algorithm for predicting the drag polar of a subsonic airplane from a three-view drawing is presented, as is the parabolic drag polar. Engine data is assumed to be available and is presented for a subsonic turbojet and turbofan. Approximate analytical expressions are obtained for thrust and specific fuel consumption.

The mission legs characterized by quasi-steady flight (climb, cruise, and descent) are analyzed in Chap. 4. Algorithms for computing distance, time, and fuel are presented for arbitrary aerodynamics and propulsion, and numerical results are developed for a subsonic business jet. In Chap. 5 approximate analytical results are derived by assuming an ideal subsonic airplane: parabolic drag polar with constant coefficients, thrust independent of velocity, and specific fuel consumption independent of velocity and power setting. The approximate analytical results are intended to be used to check the numerical results and for making quick predictions of performance.

Next, the mission legs characterized by accelerated flight are investigated. Take-off and landing are considered in Chap. 6. Specific excess power, P_S , and constant altitude turns are analyzed in Chap. 7. However, the supersonic dash is not considered because it involves flight through the transonic region.

In general, the airplane is a controllable dynamical system. Hence, the differential equations which govern its motion contain more variables than equations. The extra variables are called control variables. It is possible to solve the equations of motion by specifying the control histories or by specifying some flight condition, say constant altitude and constant velocity, and solving for the controls. On the other hand, because the controls are free to be chosen, it is possible to find the control histories which optimize some index of performance (for example, maximum distance in cruise). Trajectory optimization problems such as these are handled by a mathematical theory known as Calculus of Variations or Optimal Control Theory. While the theory is beyond the scope of this text, many aircraft trajectory optimization problems can be formulated as simple optimization problems whose theory can be derived by simple reasoning.

1.5 Stability and Control

Stability and control studies are concerned with motion of the center of gravity (cg) relative to the ground and motion of the airplane about the cg. Hence, stability and control studies involve the use of the six degree of freedom equations of motion. These studies are divided into two major categories: (a) static stability and control and (b) dynamic stability and control. Because of the nature of the solution process, each of the categories is subdivided into longitudinal motion (pitching motion) and lateral-directional motion (combined rolling and yawing motion). While trajectory analyses are performed in terms of force coefficients with control surface deflections either neglected (untrimmed drag polar) or eliminated (trimmed drag polar), stability and control analyses are in terms of the orientation angles (angle of attack and sideslip angle) and the control surface deflections.

The six degree of freedom model for flight in a vertical plane is presented in Chap. 8. First, the equations of motion are derived in the wind axes system. Second, formulas for calculating subsonic aerodynamics are developed for an airplane with a straight, tapered, swept wing. The aerodynamics associated with lift and pitching moment are shown to be linear in the angle of attack, the elevator angle, the pitch rate, and the angle of attack rate. The aerodynamics associated with drag is shown to be quadratic in angle of attack. Each coefficient in these relationships is a function of Mach number.

Chap. 9 is concerned with static stability and control. Static stability and control for quasi-steady flight is concerned primarily with four topics: trim conditions, static stability, center of gravity effects, and control force and handling qualities. The trim conditions are the orientation angles and control surface deflections required for a particular flight condition. Given a disturbance from a steady flight condition, static stability investigates the tendency of the airplane to reduce the disturbance. This is done by looking at the signs of the forces and moments. Fore and aft limits are imposed on allowable cg locations by maximum allowable control surface deflections and by stability considerations, the aft cg limit being known as the neutral point because it indicates neutral stability. Handling qualities studies are concerned with pilot-related quantities such as control force and how control force changes with flight speed. These quantities are derived from aerodynamic moments about control surface hinge lines. Trim tabs have been introduced to allow the

pilot to zero out the control forces associated with a particular flight condition. However, if after trimming the stick force the pilot flies hands-off, the stability characteristics of the airplane are reduced.

To investigate static stability and control for accelerated flight, use is made of a pull-up. Of interest is the elevator angle required to make an n -g turn or pull-up. There is a cg position where the elevator angle per g goes to zero, making the airplane too easy to maneuver. This cg position is called the maneuver point. There is another maneuver point associated with the stick force required to make an n -g pull-up.

While dynamic stability and control studies can be conducted using wind axes, it is the convention to use body axes. Hence, in Chap. 10, the equations of motion are derived in the body axes. The aerodynamics need for body axes is the same as that used in wind axes. A particular set of body axes is called stability axes. The equations of motion are also developed for stability axes.

Dynamic stability and control is concerned with the motion of an airplane following a disturbance such as a wind gust (which changes the speed, the angle of attack and/or the sideslip angle) or a control input. While these studies can and are performed using detailed computer simulations, it is difficult to determine cause and effect. As a consequence, it is desirable to develop an approximate analytical approach. This is done in Chap. 11 by starting with the airplane in a quasi-steady flight condition (given altitude, Mach number, weight, power setting) and introducing a small disturbance. By assuming that the changes in the variables are small, the equations of motion can be linearized about the steady flight condition. This process leads to a system of linear, ordinary differential equations with constant coefficients. As is known from linear system theory, the response of an airplane to a disturbance is the sum of a number of motions called modes. While it is not necessary for each mode to be stable, it is necessary to know for each mode the stability characteristics and response characteristics. A mode can be unstable providing its response characteristics are such that the pilot can easily control the airplane. On the other hand, even if a mode is stable, its response characteristics must be such that the airplane handles well (handling qualities). The design problem is to ensure that an aircraft has desirable stability and response characteristics throughout the flight envelope and for all allowable cg positions. During this part of the design process, it may no longer be possible to modify the configuration, and automatic control solutions may have to be used.

App. A contains the geometric and aerodynamic data used in the text to compute performance, stability and control characteristics of a subsonic business jet called the SBJ throughout the text. App. B gives the relationship between the stability derivatives of Chap. 11 and the aerodynamics of Chap. 8. Finally, App. C contains a review of linear system theory for first-order systems and second-order systems.

1.6 Aircraft Sizing

While aircraft sizing is not covered in this text, it is useful to discuss the process to see where performance and static stability fit into the picture.

Consider the case of sizing a subsonic business jet to have a given range at a given cruise altitude. Furthermore, the aircraft must take-off and land on runways of given length and have a certain maximum rate of climb at the cruise altitude. The first step in the design process is to perform conceptual design. Here, the basic configuration is selected, which essentially means that a three-view drawing of the airplane can be sketched (no dimensions). The next step is to size the engines and the wing so that the mission can be performed. To size an engine, the performance of an actual engine is scaled up or down. See Fig. 1.8 for a flow chart of the sizing process. The end result of the sizing process is a three-view drawing of an airplane with dimensions.

The sizing process is iterative and begins by guessing the take-off gross weight, the engine size (maximum sea level static thrust), and the wing size (wing planform area). Next, the geometry of the airplane is determined by assuming that the center of gravity is located at the wing aerodynamic center, so that the airplane is statically stable. On the first iteration, statistical formulas are used to locate the horizontal and vertical tails. After the first iteration, component weights are available, and statistical formulas are used to place the tails. Once the geometry is known, the aerodynamics (drag polar) is estimated.

The next step is to fly the airplane through the mission. If the take-off distance is too large, the maximum thrust is increased, and the mission is restarted. Once take-off can be accomplished, the maximum rate of climb at the cruise altitude is determined. If it is less than the required value, the maximum thrust is increased, and the mission is restarted. The last constraint is landing distance. If the landing

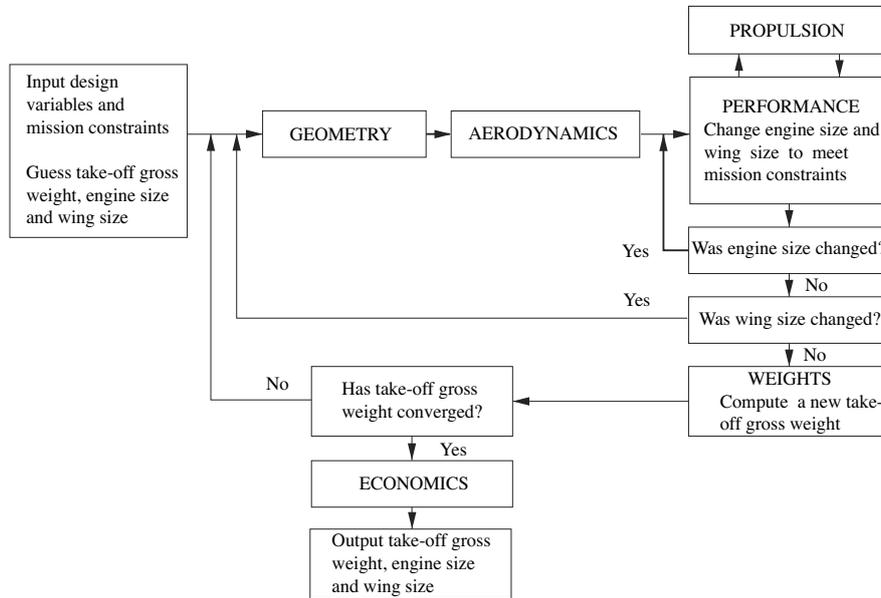


Figure 1.8: Aircraft Sizing Flowchart

distance is too large, the wing planform area is changed, and the mission is restarted. Here, however, the geometry and the aerodynamics must be recomputed.

Once the airplane can be flown through the entire mission, the amount of fuel required is known. Next, the fuel is allocated to wing, tip, and fuselage tanks, and statistical weights formulas are used to estimate the weight of each component and, hence, the take-off gross weight. If the computed take-off gross weight is not close enough to the guessed take-off gross weight, the entire process is repeated with the computed take-off gross weight as the guessed take-off gross weight. Once convergence has been achieved, the flyaway cost and the operating cost can be estimated.

1.7 Simulation

Simulations come in all sizes, but they are essentially computer programs that integrate the equations of motion. They are used to evaluate the

flight characteristics of a vehicle. In addition to being run as computer programs, they can be used with working cockpits to allow pilots to evaluate handling qualities.

A major effort of an aerospace company is the creation of a high-fidelity 6DOF simulation for each of the vehicles it is developing. The simulation is modular in nature in that the aerodynamics function or subroutine is maintained by aerodynamicists, and so on.

Some performance problems, such as the spin, have so much interaction between the force and moment equations that they may have to be analyzed with six degree of freedom codes. These codes would essentially be simulations.

Chapter 2

3DOF Equations of Motion

An airplane operates near the surface of the earth which moves about the sun. Suppose that the equations of motion ($F = ma$ and $M = I\alpha$) are derived for an accurate inertial reference frame and that approximations characteristic of airplane flight (altitude and speed) are introduced into these equations. What results is a set of equations which can be obtained by assuming that the earth is flat, nonrotating, and an approximate inertial reference frame, that is, the flat earth model.

The equations of motion are composed of translational (force) equations ($F = ma$) and rotational (moment) equations ($M = I\alpha$) and are called the six degree of freedom (6DOF) equations of motion. For trajectory analysis (performance), the translational equations are uncoupled from the rotational equations by assuming that the airplane rotational rates are small and that control surface deflections do not affect forces. The translational equations are referred to as the three degree of freedom (3DOF) equations of motion.

As discussed in Chap. 1, two important legs of the commercial and military airplane missions are the climb and the cruise which occur in a vertical plane (a plane perpendicular to the surface of the earth). The purpose of this chapter is to derive the 3DOF equations of motion for flight in a vertical plane over a flat earth. First, the physical model is defined; several reference frames are defined; and the angular positions and rates of these frames relative to each other are determined. Then, the kinematic, dynamic, and weight equations are derived and discussed for nonsteady and quasi-steady flight. Next, the equations of motion for flight over a spherical earth are examined to find out how good the flat

earth model really is. Finally, motivated by such problems as flight in a headwind, flight in the downwash of a tanker, and flight through a downburst, the equations of motion for flight in a moving atmosphere are derived.

2.1 Assumptions and Coordinate Systems

In deriving the equations of motion for the nonsteady flight of an airplane in a vertical plane over a flat earth, the following physical model is assumed:

- a. The earth is flat, nonrotating, and an approximate inertial reference frame. The acceleration of gravity is constant and perpendicular to the surface of the earth. This is known as the *flat earth model*.
- b. The atmosphere is at rest relative to the earth, and atmospheric properties are functions of altitude only.
- c. The airplane is a conventional jet airplane with fixed engines, an aft tail, and a right-left *plane of symmetry*. It is modeled as a variable-mass particle.
- d. The forces acting on an airplane in symmetric flight (no sideslip) are the thrust, the aerodynamic force, and the weight. They act at the center of gravity of the airplane, and the thrust and the aerodynamic force lie in the plane of symmetry.

The derivation of the equations of motion is clarified by defining a number of coordinate systems. For each coordinate system that moves with the airplane, the x and z axes are in the plane of symmetry of the airplane, and the y axis is such that the system is right handed. The x axis is in the direction of motion, while the z axis points earthward if the aircraft is in an upright orientation. Then, the y axis points out the right wing (relative to the pilot). The four coordinate systems used here are the following (see Fig. 2.1):

- a. The *ground axes system* $Exyz$ is fixed to the surface of the earth at mean sea level, and the xz plane is the vertical plane. It is an approximate inertial reference frame.

- b. The *local horizon axes system* $Ox_h y_h z_h$ moves with the airplane (O is the airplane center of gravity), but its axes remain parallel to the ground axes.
- c. The *wind axes system* $Ox_w y_w z_w$ moves with the airplane, and the x_w axis is coincident with the velocity vector.
- d. The *body axes system* $Ox_b y_b z_b$ is fixed to the airplane.

These coordinate systems and their orientations are the convention in flight mechanics (see, for example, Ref. Mi1).

The coordinate systems for flight in a vertical plane are shown in Fig. 2.1, where the airplane is located at an altitude h above mean sea level. In the figure, \mathbf{V} denotes the velocity of the airplane relative to the

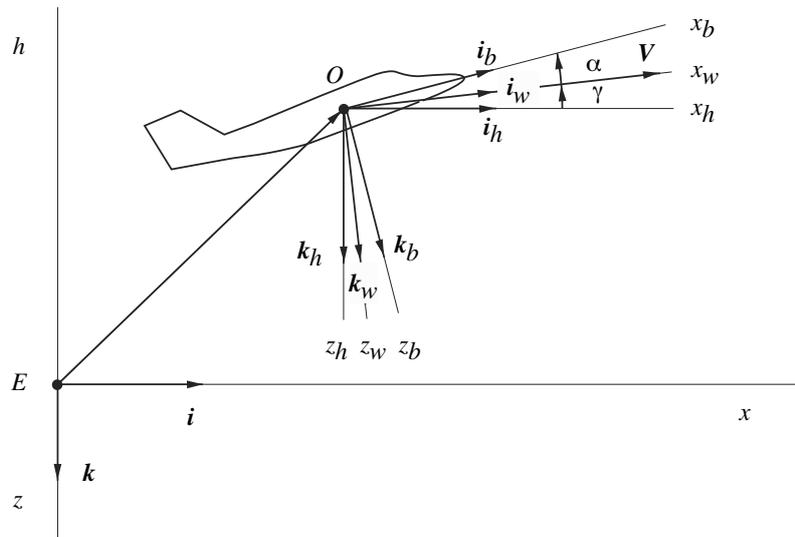


Figure 2.1: Coordinate Systems for Flight in a Vertical Plane

air; however, since the atmosphere is at rest relative to the ground, \mathbf{V} is also the velocity of the airplane relative to the ground. Note that the wind axes are orientated relative to the local horizon axes by the *flight path angle* γ , and the body axes are orientated relative to the wind axes by the *angle of attack* α .