Pressure Vessel Design

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With 275 Figures and 4 Tables

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Preface

1 Industrial Sectors Interested in Pressure Vessels

Pressure vessels are probably the most widespread “machines” within the different industrial sectors. In fact, there is no factory without pressure vessels, steam boilers, tanks, autoclaves, collectors, heat exchangers, pipes, etc. More specifically, pressure vessels represent fundamental components in sectors of enormous industrial importance, such as the nuclear, oil, petrochemical, and chemical sectors. There are periodic international symposia on the problems related to the verification of pressure vessels.

For many years an ISO committee was dedicated to pressure vessels design. There is also a technical committee of the EU specifically assigned to this field. All the industrialized countries have a code relative to pressure vessels design. However, even when the code includes specific regulations to determine the thickness of the different components, typically not all issues facing the designer are discussed. Finally, it is worth noting that a few regulations cause some perplexity.

In Italy, a specific area of ISPESL regulations (VSR collection) is devoted to pressure vessels.

2 Current Know-How with Regard to Resistance Verification

A pressure vessel is not an easy machinery in terms of resistance verification. A layman can easily make the mistake of considering somewhat simple structural forms that are in fact quite difficult to analyze, especially if one would like to apply the most modern criteria of verification (elastoplasticity, self-limiting stresses, etc.).

Regardless of the enormous interest in the topic and numerous efforts, many problems have not been studied in-depth, and there is still no agreement among scholars and the institutions of the various countries that define regulations. In addition, economical reasons and technical progress constantly present new challenges in connection with new forms and solutions, the necessity to reduce thickness to a minimum, etc.

Finally, the growth of the nuclear sector has highlighted the necessity of an investigation beyond the simplistic analysis of stresses, and this also led to a systematic analysis of the impact of fatigue phenomena. These accomplishments notwithstanding, much still needs to be done if, from a practical standpoint, one wishes to move from general principles to operational guidelines of calculation criteria that are as simple as possible.

3 Current State of Technical Literature

With regard to Italy, when pressure vessels are treated, they are included in general textbooks about mechanical engineering. This leads to a somewhat generic and often outdated treatment of the subject with regard to modern verification criteria, and hence the outcome is of little practical interest. Outside of Italy, there are textbooks and various publications specifically on pressure vessels.

However, these publications have a number of shortcomings:

(a) Simply a guide to apply the code's rules correctly.
(b) Sound scholarly framework that often does not extend itself to the point of analyzing the practical cases, thus becoming of little use to the designer.
(c) Lack of interest in problems that may seem marginal but are in reality those causing many obstacles to the designer, specifically those that are not analyzed in detail and also happen not to be included in regulations and codes.
(d) Experimental emphasis that for the cases under study is of obvious help. However, because the number of cases is considerable, and a theoretical background is lacking, the designer is unable to use the available data by applying "similarity approaches."
(e) Lack of interest in verification methodology which is essential for sizing; the designer is faced with values for stresses that he or she does not know how to evaluate; the situation becomes even more complex when the verification methodology exists but does not correspond to the modern verification criteria.

4 General Characteristics of the Book

The book focuses on general problems as well as fundamental ones derived from the previous ones, and on problems that may be incorrectly considered of secondary importance but are in fact crucial in the design phase.

The basic approach is rigorously scientific with a complete theoretical development of the topics treated, but the analysis is always pushed so far as to offer concrete and precise calculation criteria that can be immediately applied to actual designs. This is accomplished through appropriate algorithms that lead to final equations or to characteristic parameters defined through mathematical equations. Given the complexity of many of these, representative graphs are shown.

In other cases experimental graphs are shown. Their limit of applicability is discussed, also by including a basic theoretical treatment to justify their specific behavior. The result of this is a textbook with a large number of equations and graphs, both fundamental for the actual design of pressure vessels.

The topics treated are grouped in ten chapters.

The first chapter describes how to achieve verification criteria, the second analyzes a few general problems, such as stresses of the membrane in revolution solids and edge effects. The third chapter deals with cylinders under pressure from the inside, while the fourth focuses on cylinders under pressure from the outside. The fifth chapter covers spheres, and the sixth is about all types of heads. Chapter seven discusses different components of particular shape as well as pipes, with special attention to flanges. The eighth chapter discusses the influence of holes, while the ninth is devoted to the influence of supports. Finally, chapter ten illustrates the fundamental criteria regarding fatigue analysis.

5 Original Contributions of the Author to the Solution of Various Problems

Besides the rather unique approach to the entire work, see Sect. 4 above, original contributions can be found in most chapters, thanks to the author's numerous publications on the topic and to studies performed ad hoc for this book.

Specifically, we would like to draw your attention to the following topics:

3.4 Allowable out of Roundness
3.5 Stiffened Cylinders
3.6 Partially Plastic Deformed Cylinders
3.7 Stresses due to Thickness Variation
4.2 and 4.3 Cylinders Under External Pressure (special emphasis on ovalization)
5.4 Partially Plastic Deformed Spheres
6.4 Flat Heads
7.4 Flanges
7.6 Expansion Compensators
8.3 Isolated Holes on Cylinders, Spheres, and Cones: Y and T Branches
8.4 Flat Head with Central Hole
8.5 Drilled Plates
9.2 Spherical Vessels Resting on a Parallel

Contents

Notation

A	Cross-sectional area
C	Form factor (heads)
D, d	Diameter
D_e, d_e	Outside diameter
D_i, d_i	Inside diameter
D_m, d_m	Average diameter
E	Young's modulus; modulus of elasticity
F	Force, load
f	Basic allowable stress
f_c	Allowable stress for piping
f_a	Allowable stress for fatigue
H	Height
I	Moment of inertia
$k_1 \ldots\ldots\ldots k_{10}$	Dimensionless factors
L, l	Length, width
M	Bending moment
N	Normal force
n	Number of waves (buckling); number of cycles
p	Pressure
p_c, p_{ce}, p_{cp}	Critical pressures
R, r	Radius
R_e, r_e	Outside radius
R_i, r_i	Inside radius
R_m, r_m	Average radius
s	Thickness
T	Shear force
t	Temperature
u	Ovalization
W	Section modulus
z	Weld joint efficiency; efficiency of ligaments
α	Thermal expansion coefficient
δ	Deviation from roundness
ε	Deformation (strain)

ε_{id}	Ideal deformation
λ	Thermal conductivity coefficient
μ	Poisson's ratio
ν	Safety factor
σ	Normal stress
$\sigma_I, \sigma_{II}, \sigma_{III}$	Principal stresses
σ_a	Longitudinal stress, axial stress
σ_m	Meridian stress
σ_r	Radial stress
σ_t	Hoop stress, circumferential stress
σ_{id}	Ideal stress
σ_s	Yield strength
σ_R	Rupture stress
τ	Shear stress
$\alpha, \vartheta, \varphi, \omega$	Angles

1

Preliminary Considerations

1.1 Mechanical Characteristics of Steel

If we exert a tensile load on a specimen made of mild carbon steel, and we transfer on the x-axis the values of the elongation per unit of length between the references (ε) (called strain) and on the y-axis the values of the stress (σ) that equals the load applied to the specimen divided by its original cross-sectional area, we obtain a diagram qualitatively similar to the one shown in Fig. 1.1.

We notice that there is proportionality between stress and strain in the first portion of the curve, i.e., the steel follows Hooke's law that constitutes the basis of classic calculation in the elastic field. In fact, the steel behaves in an elastic fashion, i.e., the deformations completely disappear after removal of the load, and the specimen returns to its original shape.

The angular coefficient E of the straight portion given by the relationship σ/ε is called modulus of elasticity, or Young's modulus. The point on the curve at the end of the linear section identifies a value of σ which is called proportional limit.

Steel behaves in an elastic fashion even beyond the proportional limit, as long as another characteristic point corresponding to stress called elastic limit is not exceeded. Note that the two points mentioned above are near, and the second one is not easy to determine. In practice, we typically equate the proportional limit to the elastic limit.

By increasing the load applied to the specimen, we reach a point on the curve corresponding to a stress σ, called upper yield strength, that represents the maximum value of σ taking place at the onset of the yielding phenomenon. In fact, after reaching the upper yield strength the load decreases, and we reach a relative minimum of the curve that identifies the stress called lower yield strength.

The yielding phenomenon is characterized by large deformations (when compared to those typical of the elasticity field) under practically constant load.

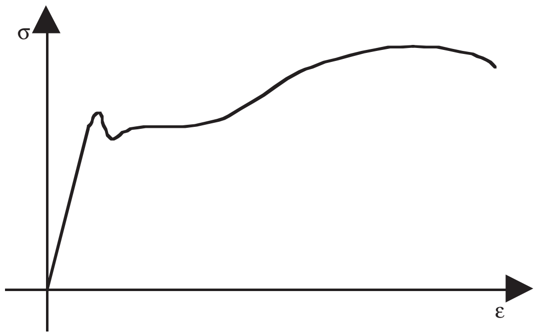

Fig. 1.1

This portion of the curve is then followed by a portion characterized by progressive increase in stress with large deformations. This is the well-known phenomenon of steel hardening, which persists until the stress reaches a maximum value called ultimate strength. After that σ decreases (again with regard to the original cross-sectional area of the specimen), and we reach rupture.

Conversely, if we consider the actual cross-sectional area of the specimen in the different stages, the highest value of σ is reached in correspondence with the rupture point. In fact, substantial elongations in correspondence with yielding and hardening areas happen together with a significant reduction of the cross-sectional area.

The lower value of the yield strength (simply known as “yield strength,” σ_s) and the maximum value of σ that precedes the rupture are the most significant parameters of the steel’s mechanical properties, and are therefore indicated in test certificates and represent the basis of resistance calculus.

The yield strength basically shows the condition under which the material starts yielding. At this point, the yielded fiber is not able to absorb growing stresses, and thus to contribute to the equilibrium of forces applied to the vessel. This is because we rule out the possibility that under safety conditions the deformations become so large that one is forced to consider the hardening phenomenon.

The fiber can be plastic deformed and, as we shall see, this has an important impact on the behavior of neighboring fibers, if we start from the assumption that they have not yet reached the yield strength. This leads to a different kind of calculation, somewhat different from the classic one based on the elasticity behavior of the entire component.

In view of the above considerations, one can replace the curve in Fig. 1.1 with that in Fig. 1.2, whereas in a first segment σ is proportional to ε (totally elastic behavior of material) followed by a segment parallel to the x-axis (perfectly plastic behavior of the material). Such simplification is most frequently used for resistance verification in the elastic–plastic field.

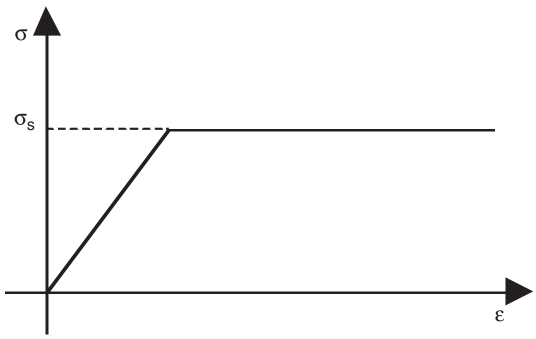

Fig. 1.2

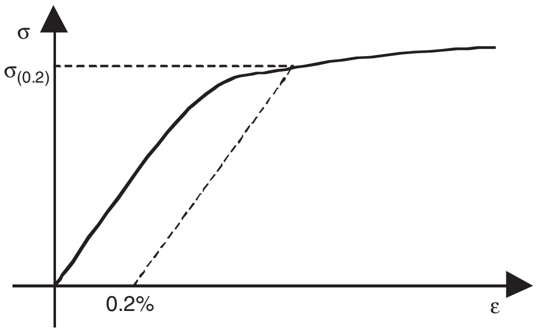

Fig. 1.3

The maximum value of σ in the tensile test is usually called rupture stress, although a better term would be unitary maximum load in the tensile test (σ_R). This is a characteristic parameter of the steel's behavior that cannot be ignored with regard to safety since it identifies its maximum level of resistance. It is therefore considered by adopting a relatively high factor of safety to ensure that the stresses present in the vessel are substantially distant from such value.

Steel does not always show a curve σ–ε similar to the one in Fig. 1.1; in the case of steel with a high content of carbon, for instance, the first segment of the curve has a shape similar to the one shown in Fig. 1.3. Moreover, this shape also characterizes steel used at high temperatures that exhibits a curve σ–ε, as in Fig. 1.1 at room temperature.

After the first linear segment the curve exhibits a substantial and progressive slope decrease, but the portion characterized by increasing deformations at basically constant stress is no longer present.

Since σ_s has not been found, we consider a conventional stress that substitutes for all practical purposes the classic yield strength with regard to calculus.

This is the stress that during the specimen's release that takes place according to Hooke's law causes a permanent deformation equal to 0.2% (Fig. 1.3). Therefore, it is indicated with the symbol $\sigma_{(0.2)}$.

Up to this point, we have discussed the steel's behavior at room temperature. It is, however, of the greatest importance to be aware of the influence of temperature on the mechanical characteristics of the material.

As we shall see, not only temperature but also time may have a strong influence, but right now we shall focus on the effects of temperature on the results of the classic tensile test.

The temperature also affects the values of resilience, of the elongation to rupture and of the area reduction. Limiting our focus to the values of $\sigma_{(0.2)}$ and σ_R, we notice that $\sigma_{(0.2)}$ decreases with the increase in temperature, while σ_R increases initially within a moderate range of temperatures and then decreases. In Fig. 1.4, we show the representative curves of carbon steel as a reference.

Moreover, the decrease of $\sigma_{(0.2)}$ has an important impact on the sizing of the vessel and, under certain conditions, on the selection of the steel to be used. In fact, the decrease of $\sigma_{(0.2)}$ can be more or less substantial for steel of different composition.

Knowing $\sigma_{(0.2)}$ and σ_R, however, is not always sufficient to identify the mechanical characteristics of steel under hot conditions, in order to calculate the allowable stress, as we shall see in Sect. 1.2.

When temperatures are typically below 300°C (570°F), the elongation of the specimen under tensile load does not increase over time, or it does so in a negligible fashion. When temperatures are higher than 300°C (570°F)

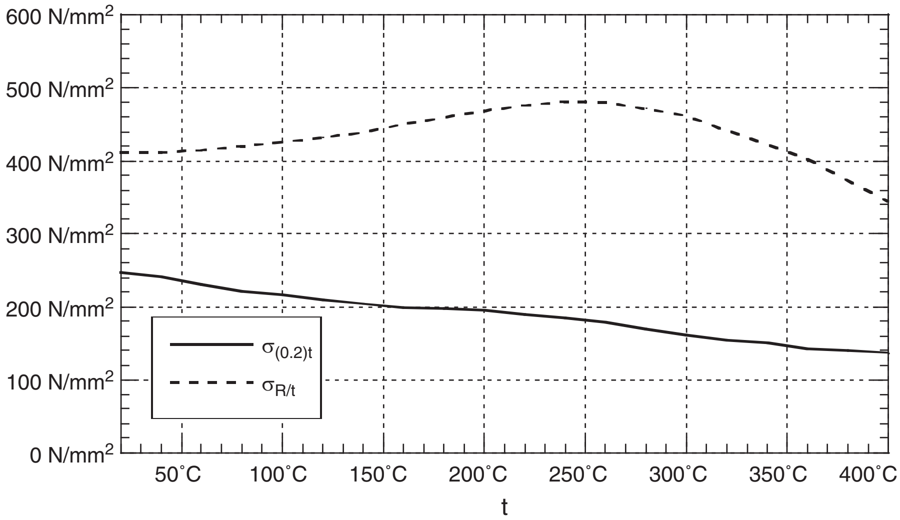

Fig. 1.4

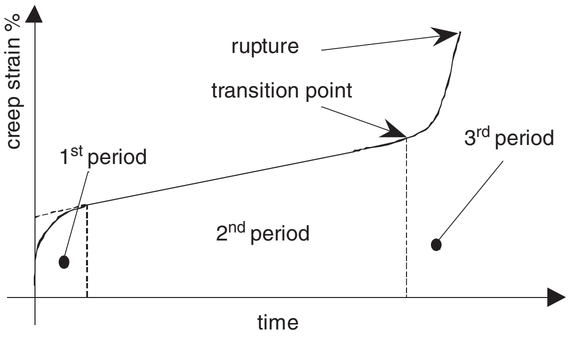

Fig. 1.5

the specimen is instead subject to an increased elongation over time; such elongation is also of variable entity as a function of temperature and stress applied to it.

Under certain temperature and load conditions the specimen can go into rupture over time. Such phenomenon, called creep, is clearly of great importance for the behavior of vessels over time for both safety and business reasons. If we examine the phenomenon in greater detail (see Fig. 1.5), we notice that the strain increases while the elongation's speed decreases over time until it reaches a minimum value.

This first portion of the curve identifies that which is called the first period. In the second period the elongation's speed remains practically constant; the representative point of the end of the second period is called point of transition. Finally, in the third period the elongation's speed and the values of the strain increase rapidly up to the rupture.

Amongst the three analyzed periods, the first and third are rather short, while the second takes most of the total time during which the specimen goes into rupture. As a reference point, please note that for a total time of about 100,000 h the first period represents at most a few hundred hours.

Finally, it is important to understand the great importance played by the point of transition that represents the beginning of the short period during which the specimen goes into rupture. The literature sometimes considers its reference time more important than the rupture time itself. In order to be able to take into account the creep in the sizing of components working under hot conditions, researchers and institutions initially developed different proposals that had in common the characteristic of forecasting tests of short duration, even though they differed with respect to the duration of the test and the evaluation of the results.

In Europe, the most popular proposal came from the German Metallurgic Union (DVM). Based on the name of this organization, the value of the stress derived from the test was called σ_{DVM} and was adopted for some time, and not only in Germany, to determine the allowable stress at high temperatures.

The stress σ_{DVM} is the one that between the 25th and the 35th hour causes an elongation's speed inside the specimen not higher than 0.001% h^{-1} and a permanent deformation after 45 h not greater than 0.2%.

The advantage of such tests is the short duration, but they were soon abandoned, mostly due to the extremely high variability of the values. In fact, the test occurs during the first period of the curve that may have a rather variable behavior from casting to casting of the same steel, even when the behavior of the curve during the second period is basically the same. The value of σ_{DVM} is a poor indicator of the performance of steel over long periods of time. Thus it became necessary to carry out tests over very long periods of time.

Nowadays, very many values of specimen made of steel of common usage that have reached the 100,000 h, and a relevant number of specimen that have been tested over even longer periods of time, are available. This allows us to determine with absolute certainty the behavior of steel of common usage up to 100,000 h of usage. Such is in fact the reference time that is typically taken into consideration for resistance verification purposes.

The value that is considered for resistance verification is the average of the rupture stress for a creep lasting 100,000 h. Such value is indicated with the symbol $\sigma_{R/100000/t}$ for a generic test temperature t. Note that this is an average value: these values are characterized by a certain amount of scattering. We admit that the variability may have a range around the average value considered for calculations equal to no more than 20% of the average value itself. If the minimum value is outside such range, we shall assume the minimum value multiplied by 1.25. If for a given steel and a given temperature we indicate the time on the x-axis and the values of the stress on the y-axis that cause the rupture of the specimen, which we call σ_R, in a doubly logarithmic diagram, we obtain curves that are qualitatively like the one shown in Fig. 1.6.

These curves look like broken lines with more or less evident knees. There are also instances – in most cases to be considered exceptional – where such knees are missing, and the broken line becomes a straight line. Therefore, the values of $\sigma_{R/100000/t}$ are derived from these curves for 100,000 h. As we said above, for steel of common usage there is no problem given the amount of values for the rupture stress per 100,000 h that are available. The problems arise when we deal with steel of new fabrication, or that has not yet undergone extensive tests; in such instances there is nothing else to do but to extrapolate on the basis of the values known for shorter periods of time. The extrapolation is possible with all the necessary cautionary tales, but it is sufficiently reliable only if it is not pushed too far. This is because of the presence of the knees, and also because even a small mistake in the determination of the values related to longer experimental time frames would result in substantial errors in the values obtained through extrapolation.

An acceptable extrapolation should not involve a timing ratio greater than 10; limiting such ratio to five leads to values that are sufficiently reliable.

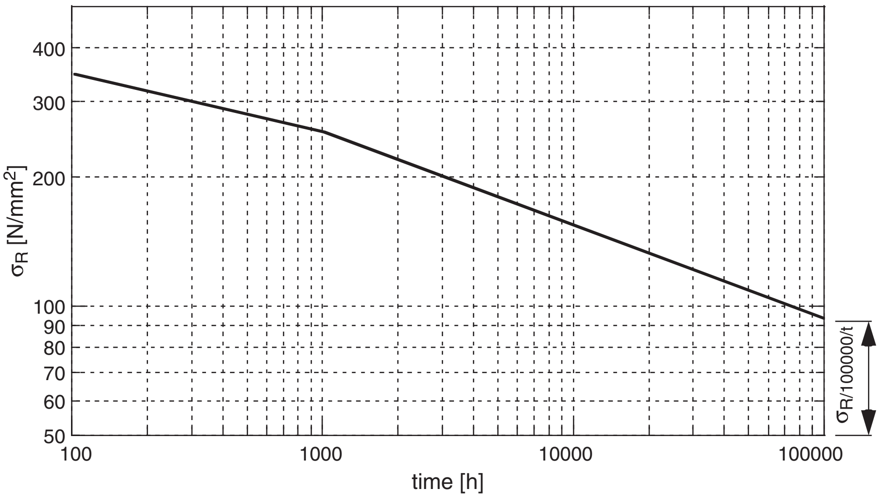

Fig. 1.6

In other words, to obtain reliable values for 100,000 h the longest test shall be at least of 20,000 h.

1.2 Allowable Stress

The following stresses are typically considered to determine the basic allowable stress of steel:

> σ_R = minimum value of the unitary maximum load during the tensile test (rupture stress) at room temperature.
> $\sigma_{(0.2)/t}$ = minimum value of the unitary load during the tensile test at temperature t with a permanent deformation equal to 0.2% of the initial length between references after removal of load.
> $\sigma_{R/100000/t}$ = average value of the unitary rupture stress for creep after 100,000 h at temperature t.

In the case of austenitic steel there is general agreement that instead of a permanent deformation of 0.2% we refer to a deformation of 1%. Note that the temperature t is the average wall temperature.

As discussed in Sect. 1.1, the meaning of these values is certainly clear to the reader; this not withstanding, further clarifications are due.

The rupture stress during the tensile test refers to room temperature. This may sound surprising since the resistance verification must be executed at design temperature t, to which the other two values above in fact refer. As pointed out in Sect. 1.1, the value of the rupture stress at moderate temperatures is greater than the one at room temperature. Within this temperature range, the adoption of this last value addresses basic safety criteria. The use of this value is in fact justified within the range of moderate temperatures: the goal here is to guarantee, through an adequate safety factor (as we shall see later on), that stresses acting upon the vessel do not cause a dangerous situation leading to rupture. Moreover, the value of σ_R is at times crucial, as far as allowable stress is concerned, when steel with high levels of yield strength are adopted. In this case if the design temperature is either room temperature or anyway moderate, the value of $\sigma_{(0.2)/t}$ is very close to σ_R. The determination of the allowable stress solely based on $\sigma_{(0.2)/t}$ may lead to a value that does not sufficiently protect against rupture.

Considering now the unitary load that causes a permanent deformation of 0.2% at release during the tensile test (we use the symbol $\sigma_{(0.2)/t}$ to remember that one should refer to the design temperature t), we pointed out in Sect. 1.1 that it practically replaces the yield strength when the steel does not exhibit the classic yielding phenomenon. If, on the other hand, this were the case, it would be easy to determine, due to the entity of the resulting deformations, that the value of $\sigma_{(0.2)}$ coincides with the lower value of the yield strength.

We shall use the symbol σ_s instead of $\sigma_{(0.2)/t}$ in the following chapters for simplicity purposes. The latter is in most cases crucial to determine the value of allowable stress. We will also use the term "yield strength", even though it is formally incorrect, to simplify the language. Finally, as far as the rupture stress per creep at 100,000 h is concerned, its importance is now evident, in view of what discussed in Sect. 1.1, if the design temperature is high. For such stress, given the dispersion of values present even for similar types of steel, one refers to the mean value of the range, generally assuming that the size of the range itself does not go beyond ±20%.

In order to obtain the allowable stress, the three characteristic stresses are associated to safety factors, the values of which lack a general consensus, and that have undergone numerous modifications over time. The general trend has been to reduce them, as the behavior of different kinds of steel became better understood, to require more stringent inspections and refine calculation methodologies. We therefore recommend the following criterion that will be applied throughout the book.

The basic allowable stress of the material that from now on we will call f is given by the smallest of the following values:

$$\frac{\sigma_R}{2.4}, \qquad \frac{\sigma_{(0.2)/t}}{1.5}, \text{ and } \quad \frac{\sigma_{R/100{,}000/t}}{1.5}.$$

From a conceptual point of view, the basic value is the one derived from $\sigma_{(0.2)/t}$. For this reason, we will always refer to the yield strength when we have to correlate the allowable stress with a value typical of the material's resistance. The other two values mentioned above occur in special instances, even though $\sigma_{R/100000/t}$ is found quite frequently.

We have already discussed elsewhere the reasons that suggest to take σ_R into consideration. $\sigma_{R/100000/t}$ is critical for f when its value is lower than the yield strength, since the safety factor has the same value of the one relative to $\sigma_{(0.2)/t}$. For instance, this happens for carbon steel at temperatures beyond 380–420°C (715–790°F), for low-alloy steel at temperatures higher than 470–500°C (880–930°F), and for austenitic steel at temperatures higher than 500°C (930°F).

Finally, note that we have defined the stress f as basic allowable stress of the material. This does not necessarily mean that it corresponds to the allowable stress during resistance verification of a specific piece in a specific position.

As we shall often return to this, in some cases it is acceptable that the ideal stress may reach the yield strength or even, in spite of being physically impossible, twice the yield strength (only from the point of view of calculation in the elastic field). We will discuss this issue in Sects. 1.4 and 1.5. The stress f, which in other cases actually corresponds to the maximum stress allowable for the piece, and at any rate to the maximum ideal stress of the membrane, represents a reference point, since it is present in all equations to compute the thickness of the various components. As a matter of fact, even when greater allowable values are assumed, they are correlated to the value of f.

1.3 Theories of Failure

This is a widely discussed topic in construction theory. In this book it is neither necessary nor relevant to examine all failure theories. We shall limit ourselves to consider those that directly relate to resistance verification of pressure vessels, and even for these we will highlight only those aspects that are required to understand what follows next.

Pressure vessels are characterized by the existence of stresses along three axis. First of all, due to pressure, there is a principal stress directed as the pressure itself and thus orthogonal to the wall of the vessel, while two additional principal stresses act on the plane orthogonal to the previous one.

In the case of cylindrical elements the first of such stresses is radial, the other two are directed, respectively, along the circumference and along the axis of the cylinder.

Similarly, in the case of spherical elements the first stress is radial, the other two are directed, respectively, along the meridian and the circumference orthogonal to the meridian, and they are obviously identical.

Different situations may occur with elements that are neither cylindrical nor spherical, e.g., in the case of a flat head the first of the three stresses mentioned above is orthogonal with respect to the head, and therefore along the same direction of the axis of the vessel, if the flat head is orthogonal to the latter. The other two have circumferential and radial direction.

In any case, we are faced with a state of stresses along three axis. This calls for a failure theory that allows one to correlate such state of stress with the resistance values of the material, derived from the tensile test that in turn is based on a single stress directed along the axis of the specimen.

The most generally accepted failure theories for ductile materials, such as steel used to build pressure vessels, are the well-known theory of maximum shear stress or Guest–Tresca, and the one known as distorsion energy theory or Huber–Hencky.

According to Guest–Tresca the level of danger is captured by the maximum shear stress, in other words all states having the same maximum shear stress are equivalent with respect to danger. The state of stress relative to the specimen being subjected to single tensile stress is represented in Mohr's plane by the only circle shown in Fig. 1.7. The maximum shear stress acting at 45° with respect to the only principal stress σ_{III} is equal to $\sigma_{III}/2$.

Therefore, if we associate a dangerous condition to the yielding of the material and we call σ_s the corresponding stress, the shear stress is given by

$$\tau_s = \frac{\sigma_s}{2}. \tag{1.1}$$

If the state of stress is along three axis, and we call σ_I, σ_{II}, and σ_{III} the three principal stresses, let us agree that the three increase in value from σ_I to σ_{III} (see Fig. 1.8).

The three maximum shear stresses on the three planes where they operate are hence given, respectively, by

$$\begin{aligned} \tau_{III,I} &= (\sigma_{III} - \sigma_I)/2; \\ \tau_{III,II} &= (\sigma_{III} - \sigma_{II})/2; \\ \tau_{II,I} &= (\sigma_{II} - \sigma_I)/2. \end{aligned} \tag{1.2}$$

With the above agreement the maximum value of the shear stress is given by $\tau_{III.I}$ and therefore the condition of danger is represented by the following

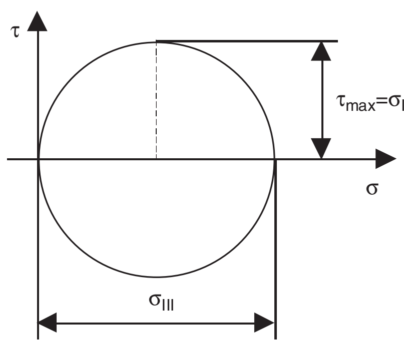

Fig. 1.7

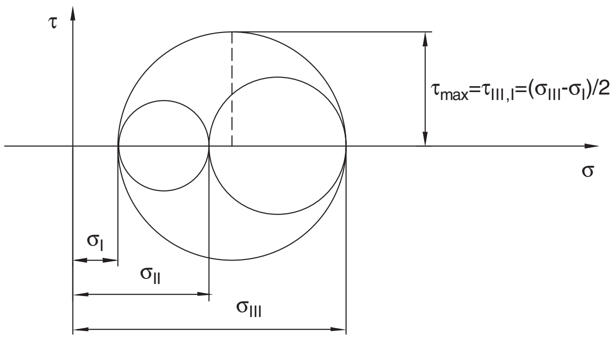

Fig. 1.8

expression derived from (1.1):

$$\tau_{III,I} = \frac{\sigma_{III} - \sigma_I}{2} = \tau_S = \frac{\sigma_S}{2}, \tag{1.3}$$

or

$$\sigma_{III} - \sigma_I = \sigma_S. \tag{1.4}$$

From a formal point of view (let us not forget that conceptually the stress at the basis of the theory is the shear one) a conventional stress (also called ideal stress or stress intensity) appears which is given by

$$\sigma_{id} = \sigma_{III} - \sigma_I. \tag{1.5}$$

Such stress, in the case of stress condition along more than one axis, takes over the same function that the only applied axial stress has inside the specimen, i.e., it can pinpoint the examined fiber's working condition with respect to danger.

In fact, when the ideal stress reaches the value of yield strength, in view of (1.4) and (1.5), according to the failure theory adopted here, we face a situation of danger. In practice, the ideal stress is therefore assumed to be the characteristic stress of the state of stress and to be limited to the allowable stress in order to obtain the sizing of the piece, as we shall see later on.

According to Huber and Hencky, the level of danger is captured by the distorsion energy, i.e., all conditions of stress that produce the same distorsion energy are equivalent to the condition of danger.

Defining again the three principal stresses as σ_I, σ_{II}, and σ_{III}, such energy is given by the following equation:

$$L = \frac{1}{6G}\left(\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2 - \sigma_I\sigma_{II} - \sigma_{II}\sigma_{III} - \sigma_{III}\sigma_I\right). \tag{1.6}$$

In the specific case of the specimen since $\sigma_I = \sigma_{II} = 0$ we have

$$L = \frac{1}{6G}\sigma_{III}^2. \tag{1.7}$$

The condition of danger characterized by $\sigma_{III} = \sigma_s$ corresponds to a distorsion energy equal to

$$L_s = \frac{1}{6G}\sigma_s^2. \tag{1.8}$$

For a state of stresses along more than one axis the condition of danger is therefore given by

$$L = L_s = \frac{1}{6G}\sigma_s^2. \tag{1.9}$$

From (1.6) and (1.9) we obtain

$$\sqrt{\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2 - \sigma_I\sigma_{II} - \sigma_{II}\sigma_{III} - \sigma_{III}\sigma_I} = \sigma_s. \tag{1.10}$$

Therefore, also in this case an ideal stress is determined

$$\sigma_{id} = \sqrt{\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2 - \sigma_I\sigma_{II} - \sigma_{II}\sigma_{III} - \sigma_{III}\sigma_I}. \tag{1.11}$$

This ideal stress relates to the stress state and allows one to identify the working condition of the fiber under scrutiny. As for the previously discussed theory, when σ_{id} reaches the value of the yield strength a dangerous condition takes place, according to (1.10) and (1.11). Similarly, the sizing of the piece is obtained by making sure that σ_{id} does not go beyond the allowable stress.

According to von Mises, the condition of danger depends on the value of the following conventional stress:

$$\tau_{id} = \sqrt{\tau_{III,I}^2 + \tau_{III,II}^2 + \tau_{II,I}^2}. \tag{1.12}$$

It must not go above a threshold that depends on the average value of the principal stresses given by

$$\sigma_m = \frac{\sigma_I + \sigma_{II} + \sigma_{III}}{3}. \tag{1.13}$$

If we rule out the dependency of the value of danger of τ_{id} on σ_m, we realize that von Mises' theory formally corresponds to Huber–Hencky's, in the sense that the ideal stress σ_{id} is the same as that in (1.11).

In fact, on the basis of (1.2) we obtain from (1.12):

$$\tau_{id} = \frac{1}{\sqrt{2}}\sqrt{\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2 - \sigma_I\sigma_{II} - \sigma_{II}\sigma_{III} - \sigma_{III}\sigma_I}. \tag{1.14}$$

In the case of the specimen

$$\tau_{id} = \frac{1}{\sqrt{2}}\sigma_{III}, \tag{1.15}$$

and the condition of danger is reached when $\sigma_{III} = \sigma_s$.

By replacing σ_{III} with σ_s in (1.15), and by comparing it with (1.14), we obtain (1.10). The ideal stress is represented by (1.11) in this case as well. Therefore, it is customary to talk about failure theory of Huber–Hencky–von Mises every time (1.11) is adopted, even though, as we have seen above, von Mises starts from assumptions that are completely different from a conceptual point of view.

Today this theory is the most generally accepted for resistance verification of pieces for which ductile materials are used, more than the Guest–Tresca theory. Note, though, that codes in the most important industrialized countries are based on the theory of Guest–Tresca.

The reason for this is because the theory of Guest–Tresca is more conservative than that of Huber–Hencky and easier to apply as well, as one can immediately realize by comparing the equations of σ_{id} in both cases.

We will generally refer to this theory of failure as well, without neglecting to refer to the theory of Huber–Hencky, however, every time it will be necessary or appropriate.

1.4 Plasticity Collaboration

In the sizing of pressure vessels the possibility of plastic collaboration of steel is widely exploited. This is a topic that, if dealt with in great depth, would result in a vast analysis that is in fact not justified considering what is of practical interest for resistance verification, a topic we will shortly introduce. Therefore, we will concentrate on reviewing the fundamental concepts by modeling the problems in a simple way, and by analyzing only those aspects of the phenomenon that find actual application in the analysis presented in the coming chapters.

The principle at the core of plastic collaboration consists of the possibility that less stressed fibers may contribute to the resistance of the piece by helping the most stressed ones. More precisely, the adoption of the criteria of plastic collaboration goes against the traditional concept of verification in the elastic field, which says that the condition of danger is reached when the most stressed fiber begins to show signs of yielding in the material. If the material is ductile it can withstand major deformations before rupturing. Therefore, the fact that the material yields in one area of the piece does not represent a condition of danger, if the nearby fibers are still far from yielding.

Let us consider a simplified stress–deformation curve, as is usually done in this cases, as in Fig. 1.9. The intuition is that the behavior of the steel is elastic–plastic, where the first section of the curve shows a perfectly elastic behavior, while the second section is parallel to the axis of deformations (i.e., a perfectly plastic behavior). In other words, we ignore the hardening, which actually has a favorable effect on resistance, and we assume that the material shows substantial yielding.

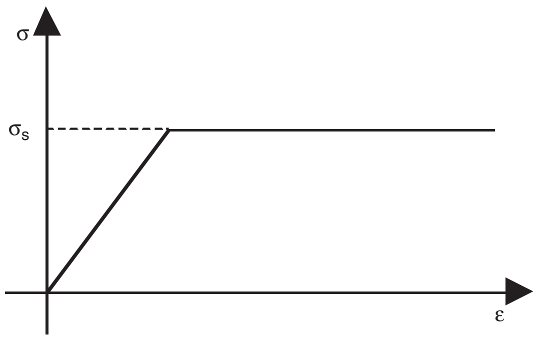

Fig. 1.9

Once yielding in a portion of the piece is reached, if we increase the external forces acting on the piece itself (in our case it is generally the pressure), the yielded fiber or fibers are unable to absorb another increase in stress. They undergo an increase in deformation, but the stress remains constant. At the same time, the nearby fibers that are still far from yielding are able to absorb increasing stresses. These are greater than those derived from calculation in the elastic field because of the greater deformation following the yielding of nearby fibers or, operating within an equilibrium framework, given the requirement to balance the increase in external forces for which the yielded fibers are no longer able to provide a contribution.

The condition of danger is reached when all fibers have exhausted the possibility to absorb an increase in stress; in other words, the condition of danger is represented by the plastic flow of the entire piece. At this point, while peaks of deformation are present given the constraint to the conditions of congruence, peaks of stress have disappeared since in every point the stress is equal to the yield strength. Following this simplified model, taking into account plastic collaboration corresponds in practice to ignoring the peaks of stress.

This issue is in fact more complex since there is vast case history, and every stress condition should be examined separately through a process known as stress analysis. This helps to identify the exact nature of the peak, in order to determine which criteria to apply to carry out verification. Such procedure can be found, e.g., in Sect. III of the ASME Code that deals with the verification of nuclear vessels, where the responsibility of the investigation of the stress condition is left to the designer. This can be done through traditional computational methodologies, if the theoretical modeling of the problem is possible, or through investigation criteria that are nowadays possible with the help of a computer and finite element analysis techniques.

The ASME code indicates at this point which criteria should be used to determine whether the values of stresses are compatible with the safety of the vessel, based on the nature of the stresses themselves, including general and local membrane stresses, those concerning bending, self-limiting ones, etc. We will focus on this topic in detail in Sect. 1.5 when we introduce the modern criteria of verification. We shall refer to them in the next chapters when we examine those cases that lend themselves to a theoretical analysis of the problem. Through the criteria illustrated in Sect. 1.5, we shall find confirmation to what stated in the beginning, i.e., the exploitation of plastic collaboration of the material according to various criteria that indeed depend on the different nature of peaks.

To understand the philosophy at the basis of the criteria in Sect. 1.5, it is therefore appropriate to try to model the most frequent situations. From this point of view a number of considerations can be made. The verification may involve a component for which an average stress not equal to zero or not localized is clearly identifiable. This is the case, for instance, of stresses in a cylinder subject to pressure from the inside without holes, or, if holes are present, in areas of the same but at such distance from the holes not to be influenced by them. In this case, as we shall see, the circumferential and the radial stress vary with the radius through the wall. Their difference represents the ideal stress according to Guest. This has a value that varies across the wall and shows a nonzero average value. With respect to the average value, the ideal stress shows both positive and negative peaks that are going to disappear if the cylinder becomes plastic, since the average stress, if ideally distributed over the entire thickness, is able to balance the acting pressure.

The methodology of plastic collaboration corresponds to neglecting these peaks by doing the calculations on the base of average values for the stresses. As we shall see, in the case at hand it is also possible to define a mathematical formulation of the distribution of stresses under total plastic conditions. This makes it possible to reason on such boundary situation by examining the resistance of the piece from a global viewpoint. Similar situations occur if there are drilling lines with holes that are so close to be considered non isolated, as we shall discuss in more detail later on. Even in this case the circumferential stress between holes, and in some instances the longitudinal stress as well, shows significant peaks in correspondence of the holes' edges. Adopting the criteria of plastic collaboration, these peaks are neglected by referring in the calculation to the average value of stresses that occur between holes.

A completely different situation occurs when stresses through the wall show a change in sign. In this case the average value may be even zero if there is pure bending. In reality this never takes place because of the simultaneous presence of stresses with a constant sign that overlap those caused by the bending. The average value is by the way very small with respect to the maximum values for stresses due to the clearly prevailing bending moment.

An example of this is a flat head or a vessel with a quadrangular section. It is obvious that even adopting plastic collaboration, one cannot here refer to the average value of stresses since it does not balance the external acting forces.

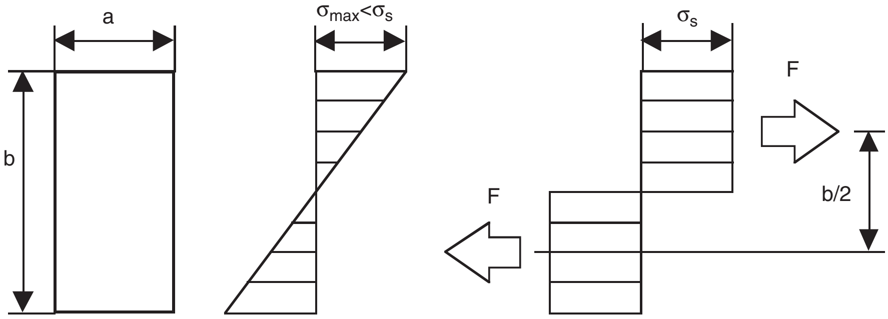

Fig. 1.10

Let us consider at this point how plastic collaboration occurs on a beam with a rectangular section under pure bending, as in Fig. 1.10.

In the elastic field stresses behave as shown on the left side of the figure. When increasing the bending moment up to the total plastic flow, the first fibers to yield are the external ones; subsequently, the yielding moves more and more towards the neutral axis, until the diagram becomes as shown on the right side of the figure.

At the limit of elastic behavior the moment is equal to

$$M_e = W\sigma_s = \frac{1}{6}ab^2\sigma_s, \tag{1.16}$$

where W is the section modulus. When the section is completely yielded, the corresponding moment is equal to

$$M_p = F\frac{b}{2} = a\frac{b}{2}\sigma_s\frac{b}{2} = \frac{1}{4}ab^2\sigma_s. \tag{1.17}$$

The coefficient of plastic collaboration is therefore given by the relationship between these two moments that represent the condition of danger, according to the calculation that takes plastic collaboration into account, and according to traditional computation practices in the elastic field, respectively. By indicating the coefficient of collaboration with ψ we have:

$$\psi = \frac{M_p}{M_e} = 1.5, \tag{1.18}$$

a well-known value in the literature. As far as stress peaks that occur in correspondence with isolated holes, the most appropriate way to approach the problem consists of examining the stress status around the edge of the hole in detail, and to apply the verification criteria discussed in Sect. 1.5.

If we are unable or do not want to perform such analysis, it is possible to adopt the criteria discussed in Chap. 8 that have been included with minor variations in the codes of the most industrialized countries for non-nuclear pressure vessels. As we shall see, even in this case plastic collaboration is factored in, by limiting the width of the area where the fibers collaborate with those subjected to most stress. In doing so, a limit is set to deformations in correspondence of peaks. In fact, an indiscriminate extension of the yielding to fibers far from reaching a peak, with regard to danger, would imply the onset of large deformations in correspondence with the peak itself, something clearly to be avoided.

It is also important to consider those peaks that are the consequence of the respect for congruence of deformations among pieces of different geometrical shape connected with each other. If they were ideally isolated, they would be characterized by different values of deformations in correspondence with the junction. Heads (whether flat, hemispherical or torospherical) connected to the ends of the cylinders represent a typical case.

In these cases, the stresses are, as we usually say, self-limiting, since they occur only out of necessity to respect congruence, and not to balance external forces. If we reach the yielding point and the subsequent deformations are relatively large, congruence is maintained without leading the material to rupture.

In fact, in these situations it would be more logical to carry out the analysis in terms of deformations. In practice this is done according to the laws of elasticity and peak stresses are obtained. We should also not forget the nature of such stresses, and we should not be surprised, if values for stress higher than the yield strength are introduced, which may seem absurd.

That is, the peak deformation takes on a value greater than that of the yield strength. The stress resulting from the calculation is greater than σ_s only from a formal point of view, as the product of deformation by the modulus of elasticity; in reality, it is obvious that the stress is equal to σ_s.

In these cases it is acceptable that the ideal stress computed according to the laws of elasticity may be even double that of the yield strength. Let us see now why this is the case. Let us look at Fig. 1.11 and assume that the deformation is greater than the yield point deformation in correspondence of the peak.

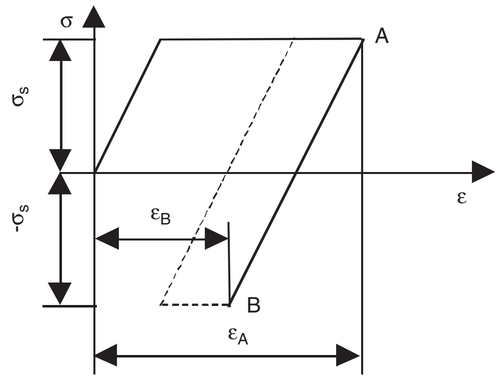

Fig. 1.11

The deformation is ε_A in correspondence of point A on the typical curve for perfectly elastic–plastic material. At release the material behaves elastically, and we also assume that the residual deformation is ε_B in correspondence of point B, and a residual stress equal to the negative yield strength.

If we next load the piece, we return to point A, and every subsequent cycle causes the stress to vary between $-\sigma_s$ and σ_s. In other words, except for the first cycle, for all the following ones the piece's behavior is elastic between point A and B, without further deformations since ε_A constitutes the deformation that takes congruence into account.

In order for this condition to occur, it is necessary and sufficient that the stress, computed according to the laws of elasticity, be double than the yield strength. Of course, if it happens to be lower the behavior of the material is completely analogous with the only difference that during release the negative yield strength is not reached. If, conversely, the stress should be more than double the yield strength, we would have the cycle that appears dashed in the figure. The material would therefore have an elastic–plastic behavior in those cycles following the first one, with the danger that incremental plastic deformations occur that the adopted model does not seem to justify, but that may actually happen in reality.

The calculation criterion introduced here is not limited to the junctions of pieces of different geometrical shape, but can be extended in general to all those situations where stresses originate from noncongruent deformations. Therefore, even the stresses due to thermal flux fall into this category, i.e., those stresses caused by variable thermal expansion through the wall of the piece. To conclude, it is noteworthy that the exploitation of plastic collaboration and the deriving lack of interest in peaks is possible and appropriate not only in relation with the type of steel used, but due to the absence of fatigue phenomena.

When an investigation about fatigue is required, the verification criteria are obviously different, and this is discussed in Sect. 1.5. Moreover, it is important to keep in mind that the verification criteria and the deriving equations for sizing discussed in the following chapters refer to work conditions that do not imply significant fatigue phenomena, due to the presence of a limited number of cycles.

If this were not the case, the validity of the equations to calculate the stresses that take peaks into account notwithstanding (thus ruling out the equations that consider the components of the vessel as membranes), the verification criteria must follow what discussed in Sect. 1.5 and in more detail in Chap. 10.

1.5 Verification Criteria

According to modern verification criteria, stresses can be divided into three categories: primary, secondary, and peak stresses. Primary stresses can then be divided into general membrane stresses, local membrane stresses, and primary bending stresses. In summary, there are the following types of stresses: general membrane, local membrane, primary bending, secondary, and peak.

1.5.1 General Membrane Stresses (σ_m)

They correspond to stresses derived from calculation when one considers the element under test as a membrane. More generally, they correspond to the average value of the stresses through the thickness of the vessel. In contrast to local membrane stresses, that we will discuss shortly, the fundamental characteristic of these stresses is that a potential yielding of the material does not cause a redistribution of the stresses, since the same stress is present in all the surrounding fibers.

A typical example of general membrane stresses is represented by the average values of the stresses acting in a cylinder without holes (or in an area that is not influenced by holes or by the junction with the heads). The same is true for the average values of the stresses acting on a sphere, or in the central area of a hemispheric or torospherical head.

1.5.2 Local Membrane Stresses (σ_{ml})

Here as well we are dealing with the average values of the stresses through the thickness in the analyzed section. In contrast with the previous ones, they involve a limited area of the component, and this means that the surrounding fibers are subject to membrane stress of lower value. A potential yielding of the material happens together with a redistribution of the stresses to the surrounding fibers that are still able to contribute to the local resistance of material, since they are not yielded.

Typical examples of stresses of this kind are the membrane stresses produced in the cylinder and in the dished heads in correspondence with their junction, or the membrane stresses that occur in the cylinder (or in the sphere), and in the nozzle welded on the same in correspondence of a hole.

Once again, these are the average values of the stresses. Stress peaks both in the junction cylinder-heads and in correspondence of the nozzles are generated that do not fall into this kind of stress category.

1.5.3 Primary Bending Stresses (σ_f)

These stresses belong to the category of primary stresses, such as the ones mentioned above, but they are characterized by the fact that their value is proportional to the distance of the fiber from the neutral axis of the section. As the previous ones, they derive from the balance conditions between internal stresses and external forces acting upon the vessel (pressure or mechanical loads). A typical example is represented by the stresses at the center of a flat head. The stresses produced by bending moments exerted on a vessel with a quadrangular section fall into this category, as well.

At this point we must discuss a very important concept in more detail. A bending moment may also generate a distribution of stresses that do not vary linearly across the thickness. An example can be seen at the corners of a vessel with a quadrangular section. Here, the primary bending stresses are those that balance the acting moment with linear variation through the thickness. With respect to these, the stresses that are actually present in the section, display positive or negative differences characterized by a resultant and a moment that are zero. These differential stresses do not balance the forces applied to the vessel, but only guarantee the congruence of the deformations. Therefore, they fall into the category of secondary stresses. The analogy with membrane stresses is evident: the latter ones represent the average value of the stresses produced by a normal load. The primary bending stresses represent the average behavior of the stresses caused by a bending moment.

The differential stresses versus the membrane ones (in the presence of a normal load), or with respect to the primary bending stresses (in the presence of a bending moment), are secondary stresses.

1.5.4 Secondary Stresses (σ_{sec})

Their fundamental characteristic is not to be involved in balancing the forces applied to the vessel, and to be for this reason self-limiting. Their only purpose is to guarantee the congruence of the deformations and, therefore, once the required deformations are produced (even though this happens through the yielding of the material) they do neither cause further deformations nor do they force the intervention of the surrounding fibers, as is the case instead for the local membrane stresses.

The stresses in correspondence of the junction between cylinder and heads belong to this category (not the membrane ones because in that case they are local membrane stresses); the stresses still not related to membranes in the cylinder or in the sphere and in the welded nipple in correspondence of a hole belong to this category, as well. In this last case local peaks due to the presence of sharp edges are excluded, as they belong to the next category.

The stresses due to thermal flux are secondary, as well. In fact, they are also self-limiting, since they are produced solely to reestablish the congruence of the deformations that differ in the various fibers because of the temperature gradient. Even though they originate elsewhere, some differences in stress levels also belong to this category (with respect to the average value that represents the general membrane stress); they occur in a cylinder or a sphere, especially in the case of great thickness, along the radius. These differences in stress do not contribute to balancing the pressure (as the balance is guaranteed by the general membrane stress), but only to ensure the congruence of the deformations.