Artist view of the Thirty Meter Telescope project (TMT) based on a three-mirror design (Courtesy of the TMT Observatory Corporation)

Artist view of the European Extremely Large Telescope project (E-ELT) based on a five-mirror design (Courtesy of the European Southern Observatory)
Gérard René Lemaitre

Astronomical Optics and Elasticity Theory

Active Optics Methods

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Cover image: General view of the giant reflective Schmidt LAMOST, in Xinglong Station, which started operations in 2008 (courtesy National Astronomical Observatories, Chinese Academy of Sciences)

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Foreword

It is an honor as well as a pleasure to write this Foreword for this major work – “Astronomical Optics and Elasticity Theory” – by my friend and colleague, Prof. Gérard Lemaître. My situation is unusual in that I have not even seen, still less read, the manuscript. My Foreword must therefore be based on my general knowledge of Lemaître’s work and our close contact over many years.

The initiator of “stress polishing” for astronomical purposes was the great optician Bernhard Schmidt, who proposed the technique for the manufacture of Schmidt plates for his newly invented Schmidt cameras, as reported in 1932. Surprisingly, there was no significant advance on the theoretical basis of the method until Lemaître started his work, reported in 1972. Since then, he has established himself as the acknowledged world expert on both the complete theory and its manifest practical applications. In my “Reflecting Telescope Optics II,” I give a brief account of his work on pages 23–27. He has not only performed fundamental work for the classical case of spherical aberration, but also for correction of astigmatism, coma and other aberration modes. This includes applications to Cassegrain secondary mirrors. In 1989, I suggested also the application to primary mirrors for which manufacturing tolerances can be relaxed by Active Optics control in the operating phase. In this application, stress polishing can be seen as the first fundamental manufacturing step in a complete system of Active Optics.

More recently, Lemaître has extended his techniques to systems of variable focal length (optical power), which have found important application in the optical train of the 4-telescope complex of the ESO VLT. The function of the interferometric mode VLTI is dependent on this application. The correction applies not only for the axis but also in the field of the interferometric image and is achieved by a system of Variable Curvature Mirrors.

Prof. Lemaître’s book, giving a full account of all these developments, is a most valuable addition to the literature in this important branch of optical manufacture. Since its major applications are in astronomical optics, it has also been my pleasure to support the publication of this work by Springer-Verlag in the same Astronomy and Astrophysics Library Series as my own books “Reflecting Telescope Optics I and II.” I am confident it will become a worthy and widely recognized standard work.

Rohrbach

R. N. Wilson
Astronomical Optics and Elasticity Theory is intended to serve both as a text and as a basic reference on "active optics methods." Mainly elaborated for astronomy, and following a conceptual idea originated by Bernhard Schmidt, the first developments of active optics began in the 1960s. These methods allow one to transform by a highly continuous process a spherical surface into the desired aspherical surface, as well as to correct tilt and decentering errors between telescope mirrors, to control the focal position by curvature variation, etc, so as to achieve diffraction-limited performance. The recent spectacular increase in telescope sizes, active image correction of telescope errors and atmospheric degradation, and the advent of detectors having nearly perfect quantum efficiencies has led to remarkable progress in observational astronomy, whose large telescopes now currently operate with active optics.

The first chapter concerns optical design and elasticity theory; I thought it useful to introduce these two topics by brief historical accounts. Most of the following chapters are dedicated to the generation of axisymmetric aspheric mirrors, as well as non-axisymmetric mirrors. Active optics methods are investigated for corrections of focus, and for aberrations of third and higher orders. Optical aberration modes that can be superposed by elastic flexure belong to a subfamily that I called Clebsch-Seidel modes. Such aberration correction modes are generated by multimode deformable mirrors. Depending on the adopted thickness class – constant or variable – various active mirror configurations are discussed using the so-called tulip, cycloid, vase, meniscus, and double-vase mirrors. Two chapters are dedicated to optical designs with the Schmidt concept; the first includes my 1985 high-order analysis of the axial wavefront reflected by a spherical mirror, the system resolving power for each option – with either a refractive, a reflective, or a diffractive corrector – and the optimal corrector shape for each design type; in the second, active optics aspherization methods of the corrector element are developed for cata
dioptric or all-reflective telescope types and for aspherized grating spectrographs. Another chapter on large mirror support systems treats the minimization of flexure against gravity and in situ active optics control on large telescopes. A short chapter concerns the flexure of thin lenses when bent by a uniform load; this is useful to produce stigmatic singlet lenses by active optics. Grazing incidence X-ray telescopes can also greatly benefit from the ripple-free active aspherization process for various two-mirror designs and particularly for a mirror pair strictly satisfying the sine
condition; a theory of weakly conical shells is proposed in a special chapter where the aspherization of the mirrors is obtained by pure extension (or contraction).

The book provides a foundation for finding a mirror thickness geometry and an associated load configuration which can generate one or several fixed surface optical modes – this in the most practicable conditions. Computational modeling, the third branch of science which bridges analytical theory and experimentation, is the ultimate method for accurately solving the deformations of a solid for any configuration of equilibrium-force sets. In the final design stage for an active optics mirror, finite element analysis of the three-dimensional deformations allows optimizing its thickness geometry to obtain the desired mirror figure. However, geometrical optimizations with such codes must require sufficient user knowledge in elasticity theory, and a preliminary analytic solution of the problem by a first approximation theory. This preliminary approach with the theory – the aim of this book – is all the more necessary since there are generally several alternatives for generating a given surface type – as, for instance, with the various solutions presented here for variable curvature mirrors.

The beautiful theory of axisymmetric shallow shells, elaborated by Erik Reissner in 1946, is one of the greatest analytic achievements in elasticity theory. In the axisymmetric flexure case, this theory is here used for the aspherization of fast f-ratio mirrors. In addition, a convergent iteration vector which acts towards the required flexure is implemented for determining the thickness distribution of meniscus-, vase-, and closed-form mirror shells. The method has proved sufficiently accurate that no significant corrections were found necessary from finite element analysis. Active optics aspherizations of primary and secondary telescope mirrors were carried out by the Laboratoire d’Optique de l’Observatoire de Marseille (LOOM). The results of stress figuring or in-situ stressing of all the axisymmetric mirrors directly designed from Reissner’s theory – as for instance with the modified-Rumsey anastigmatic telescope presented here – show that the axial wavefront correction errors are within conventional diffraction limited criteria.

I am grateful to M. Ferrari for his contributions in the second chapter, to J. Caplan, S. Mazzanti and K. Dohlen for fruitful discussions on several points of the book, and also to P. Joulie and P. Lanzoni for their efficient preparation of the figures.

Marseille, October 2008

G. R. Lemaitre


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Notations

**Optics Notation**

- $x, y, z$: rectangular coordinates
- $\rho, \theta, z$: cylindrical coordinates
- $\lambda$: wavelength of a monochromatic propagation of light
- $\nu$: frequency of a monochromatic propagation of light
- $D$: clear aperture diameter
- $r_m$: semi-radius of clear aperture
- $f, f'$: focal length of a system in the object and image space respectively
- $\Omega$: focal ratio, or f-ratio
- $n, n', N$: index of refraction of a medium
- $i, i'$: conjugate incidence and emerging angles
- $R$: radius of curvature of axisymmetric optical surface
- $c$: curvature of an axisymmetric optical surface (the velocity of light in vacuum is also denoted $c$)
- $c_x, c_y$: principal curvatures of an optical surface
- $C_p$: Petzval curvature
- $\kappa$: conic constant of conicoid optical surface
- $u, u'$: conjugate aperture angles
- $\eta, \eta'$: conjugate ray heights
- $\bar{\eta}$: normalized ray height
- $\varphi$: field angle
- $\varphi_{\text{max}}$: maximum field angle
- $z, z'$: usual object and image conjugate distances
- $\xi, \xi'$: Newton object and image conjugate distances
- $M$: transverse magnification
- $K$: optical power
- $H$: Lagrange invariant
- $E$: étendue invariant
- $T$: telephoto effect
- $W_{[4]}$: aberration wavefront function of third order theory
- $\rho, \theta, \bar{\eta}$: normalized radius, azimuth angle and image height of a wavefront aberration function
Notations

$S_I$ to $S_V$ Seidel’s five coefficients of third order theory
$z_{n,m}(\rho, \theta)$ cylindrical coordinate representation of a wavefront mode or an optical surface. Also simply denoted $z_{nm}$
$Z$, $Z_{Opt}$ representation of a wavefront mode or an optical surface in active optics coaddition law
$\omega$ angular frequency of a wave propagation
$k$ wave number
$S$ Strehl intensity ratio

Elasticity Notation

$x$, $y$, $z$ rectangular coordinates
$\rho$, $\theta$, $z$ cylindrical coordinates
$t$ thickness of a plate or a shell
$T$ dimensionless thickness
$q$ intensity of a uniform load
$F$ intensity of a force at a single point
$\mu$ weight per unit volume
$g$ intensity of the gravity field
$I$ moment of inertia of a beam about a perpendicular axis
$I_p$ polar moment of inertia of a beam about its axis
$E$ Young’s elasticity modulus in uniaxial tension and compression
$\nu$ Poisson’s ratio
$G$ shear modulus of elasticity, sometimes called torsion modulus
$K$ isotropic modulus of elasticity in 3-D tension and compression
$D$ flexural rigidity
$R$ radius of curvature generated by axisymmetric flexure
$R_x$, $R_y$ principal radii of curvature generated by flexure of a surface
$\varepsilon_{xx}$, $\varepsilon_{yy}$, $\varepsilon_{zz}$ normal strain components in rectangular coordinates
$\varepsilon_{xy}$, $\varepsilon_{yx}$, $\varepsilon_{yz}$, $\varepsilon_{zy}$ shear strain components in rectangular coordinates
$\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{zz}$, $\sigma_{xy}$, $\sigma_{yx}$ normal stress components in rectangular coordinates
$\sigma_{yz}$, $\sigma_{zx}$, $\sigma_{xy}$ shear stress components in rectangular coordinates
$u$, $v$, $w$ components of the displacement vector in rectangular coordinates (sometimes denoted $u_x$, $u_y$, $u_z$, respectively)
$M_x$, $M_y$ bending moments per unit length of sections of a plate perpendicular to $x$ and $y$ axes, respectively
$M_{xy}$ twisting moment per unit length of section of a plate perpendicular to $x$ axis
$Q_x$, $Q_y$ shearing forces per unit length of sections of a plate perpendicular to $x$ and $y$ axes, respectively
$V_x$, $V_y$ net shearing forces per unit length of sections of a plate perpendicular to $x$ and $y$ axes, respectively
$N_x$, $N_y$ normal forces per unit length of sections of a plate in direction of $x$ and $y$ axes, respectively
normal strain components in cylindrical coordinates

shear strain components in cylindrical coordinates

normal stress components in cylindrical coordinates

shear stress components in cylindrical coordinates

components of the displacement vector in cylindrical coordinates (sometimes denoted \( u_r, u_t, u_z \), respectively)

radial and tangential bending moments per unit length of sections of a plate perpendicular to \( r \) and \( t \) axes, respectively

twisting moment per unit length of section of a plate perpendicular to \( r \) axis

shearing forces per unit length of sections of a plate perpendicular to \( r \) and \( t \) axes, respectively

net shearing forces per unit length of section of a plate perpendicular to \( r \)-axis

normal forces per unit length of sections of a plate in direction of \( r \) and \( t \) axes respectively, or for a shallow shell, in normal direction to curvilinear sections

equivalent representations of flexure with respect to \( z \)-axis used in active optics coaddition law

dimensionless flexure with respect to \( z \)-axis

dimensionless flexure with respect to \( z \)-axis used in cylindrical shells and weakly conical shells
Chapter 1
Introduction to Optics and Elasticity

1.1 Optics and Telescopes – Historical Introduction

The idea of using a mirror that provides optical rays all converging to a single point originated in the Hellenistic period of Greece more than two millennia ago. Because of the straightforward geometrical properties of conics as meridional sections of stigmatic mirrors, catoptrics was constituted long before dioptrics. Nevertheless, and surprisingly, the first telescopes built were not reflectors but refractors.

In Greece it was long known that some problems of a geometric nature were not soluble by straightedge and compass. A legend of the Classic period of Greece states that the gods were unhappy because geometry was not sufficiently studied. The oracle of Delos, which was consulted before major decisions, said ~430 BC that recovery of the gods’ clemency would require solving three problems: the angle trisection, the cube duplication, and the circle squaring. The first two problems were rapidly solved, but the third has baffled mathematicians for 2,300 years until F. Lindemann (1882) demonstrated that $\pi$ is transcendental, and thus showing that the construction of the circle length by purely geometrical means is insoluble.

1.1.1 The Greek Mathematicians and Conics

- **Menaechmus** (∼375–325 BC) who lived in Macedonia and Greece and was a tutor of Alexander the Great, formalized the notion of conics and found many of their geometrical properties. He solved the famous problem of *cube duplication* by determining $\sqrt[3]{2}$ from the intersection of two conics: a parabola and a hyperbola – with the Cartesian formalism $y = x^2$ and $xy = 2$, respectively (Fig. 1.1). He suspected that the solution was impossible with the classic method of straightedge and compass since a few decades earlier Hippocrates of Chios (471–410 BC, Athens) has been able to solve the not less difficult problem of the *angle trisection* by the less restrictive method of “declination” – *inclinatio* in Latin and υγμαλίς (neusis) in Greek – which is now called the *marked straightedge method*.

This method consists of revolving a straight line of a given length through a fixed point until it intersects two fixed straight lines (cf. Arnaudiès & Delezoide [6]).
In practice one uses a straightedge on which the given length is copied in two marks by revolving and sliding it at the fixed point. In Fig. 1.1, the angle to trisect is $\alpha OA$, a length $MN = 2 \times OA$ is reported on the straightedge that pivots in O, and M, N must be on the straight lines $x', y'$ that are parallel to $x, y$ through A. This method is equivalent to the two-conic intersection method.

- **Aristaeus the Elder** (~365–300 BC, Greece) wrote five books, now lost, entitled *Solid Loci* and concerning the conic sections. This is known from commentaries by Pappus (290–350 AD, Alexandria), one of the last Greek mathematicians.

- **Euclid** (325–265 BC, Alexandria) who is the classical reference in the founding of geometry with his 13 books, well known as *Elements*, also wrote several other works including *Optics*, *Conics*, and *Surface Loci*. *Optics* solely concerns perspective. According to Pappus, Euclid’s *Conics*, now lost, was mostly a compilation whereas Aristaeus’s book gave a more thorough discussion of the discoveries and properties of conics.

- **Apollonius of Perga** (262–~190 BC, born in Perga, now known as Murtana, on the south coast of Turkey) lived in Ephesus and Alexandria. It is to him that we owe the names ellipse, hyperbola, and parabola. He wrote a treatise in eight books, seven of which still exist, entitled *Conics*, which contains about four hundred propositions. He differentiated the various conic types by the angle of the intersecting plane with respect to the cone angle. Pappus states that “Apollonius, having completed Euclid’s four books of conics and added four others, handed down eight books on conics”. He also refers to Euclid’s last work on conics that is included in the *Treasury of Analysis* under *Surface Loci* but states that Apollonius’s *Conics* had become the classical reference on those questions. He gives some indications of the contents of six other works by Apollonius: *Cutting of a ratio, Cutting an area, Inclinations* (presently lost), *On determinate sections, Plane loci, On verging constructions and Tangencies*, but none of them seem to mention any optical property of a conic.

- **Diocles** (~240–~180 BC, lived in Arcadia and Karystos near Athens) published *On burning mirrors*. It is known from comments by Eutocius (480–540 AD) that
1.1 Optics and Telescopes – Historical Introduction

he solved the problem of angle trisection by inventing a method based on cissoid curves which thus differs from Hippocrates of Chios’s method. None of his writings were known in the West before 1920, but more recently (~1970) a complete Arabic translation of his work was found in the Astan Quds Library of Mashhad, Iran. The first translation, by Toomer [162], was published in 1976. It appears that Diocles discovered the property that a parabola can be defined as the locus of points satisfying a constant distance ratio equal to unity from a given point (focus) and a directrix line. One of his other studies leads to the following fact.

→ The fundamental optical property of stigmatism of a parallel beam reflected by a paraboloid was known by Diocles in ~200 BC.

• Anthemius of Tralles (~474 AD–before 558), who lived in Constantinople, is credited with discovering the relation for ellipses $MF+MF’ = \text{constant}$ and for the classical method of drawing them with a string. He found the catoptric stigmatism between the two foci of an ellipsoid, a property probably unknown by Diocles.

1.1.2 The Persian Mathematicians and Mirrors

After the end of the Alexandrine superiority in 641, a scientific renaissance took place in Persia – mainly in Baghdad – where most of the main Greek works were translated into Arabic during the following centuries (Rouse Ball [9]). Some advances in algebra had already been assimilated by the Persians along with mathematical developments previously achieved in India, such as decimal numbering and the important invention by Bramaguptas in 629 of the “0” symbol for zero. In 820 Al-Khwarizmi published Al-jabr wa’l Muqabala – from which the word algebra is derived – where one of the positive roots of some second degree equations is solved (although symbolic notation did not yet exist). In around 900, Persians geometers were aware of Archimedes’s failure to construct the regular heptagon and of the equally fruitless attempts by subsequent Greek geometers; they provided the first constructions of the heptagon around 970. Among their numerous contributions, we will limit our comments to the main works of Alhazen in optics.

• Al-Haytham, known in the West as Alhazen (965–1021), published in 1008 the Treatise on Optics: Kitab ul Manzir, containing seven books. In it he gives a detailed description of the human eye, explaining the function of each part. Here also Alhazen is the first to mention the camera obscura, some of which he built, noticing that the image is inverted. He gives the first explanation of atmospheric refraction. He investigates lenses as well as spherical and paraboloidal mirrors, and is aware of the spherical aberration and of the stigmatic property of a paraboloid already demonstrated by Diocles.

An important question which had been introduced by Ptolemy (85–165 AD, Alexandria) in his famous Almagest is known as Alhazen’s Problem: given a spherical mirror of center O, a point source A and another given point B, how can one geometrically construct the intersection point R at the mirror surface where the ray
AR is reflected towards B? Except for the trivial arrangements where the center O is on AB or on the median plane of AB, the general solution is impossible with the straightedge and compass method. Considering the AOB plane, the solution can be derived from the intersection of the circular section of the mirror with one of the homofocal ellipses of foci A and B, by selecting the ellipse which is tangent to this circle – there are generally two of them – by imposing two conditions: the equality of the ordinates and the equality of slopes. This leads to solving fourth degree equations where the solution is with a double root for the tangency. It seems that Alhazen solved this problem with the marked straightedge [6]. Huygens later gave a solution using the intersection of conics. A construction of point R by the intersection of a circle and a hyperbola is displayed in Fig. 1.2.

Al-Haytham notices that if a point source is at infinity, the image given by a spherical mirror of radius $R$ is located at a distance equal to or a little larger than $R/2$ from the mirror: this distance is the focal length. In some examples where the source point is at finite distance, he also gives the location of the image point after reflection on a concave mirror: this is the conjugate distance.

→ Although lacking symbolic notation, the Persians knew how to calculate the conjugate distance in $\sim1000$. This may be considered as the prelude to Gaussian optics.

Fig. 1.2 Al-Haytham’s Problem: Given two points A, B and a spherical mirror of center O, how can one find the point R where the ray AR is reflected through point B? In the AOB plane where the mirror intersection is the circle $\mathcal{C}$, one constructs the circles of diameter OA, OB and lines $\Delta_A, \Delta_B$ that intersect these diameters in $A', B'$. A hyperbola $\mathcal{H}$, and only one, can be constructed passing by $O, A', B'$ with its center $\Omega$ at the middle of $A'B'$ and its asymptotes in the direction of the bisections of lines OA and OB. Among the four points on $\mathcal{C}$ and $\mathcal{H}$, the figure shows the two solutions as points R or $R'$ for a convex or a concave mirror, respectively (Arnaudiès & Delozoide [6])
Al-Haytham engaged in remarkable technological developments, for instance by constructing mirrors of steel and probably of steel-silver alloy and of pure silver, but like others was unable to obtain accurate spheres. He also gave comments on the development of turning lathes.

It is mainly from Spain under Arab domination and not directly from Persia, that Persian scientific writings, rendered into Latin by Adelard, Gherard and many other translators, were introduced into Europe during several centuries after 1150, thus including the Greek heritage also. During this period most of the old European universities were created. This favored the assimilation of these heritages and gave rise to important developments which were materialized by the Renaissance.

### 1.1.3 End of European Renaissance and Birth of Telescopes

The emergence of small blown objects in glass originates in Phoenicia, Syria, and Egypt around \( \sim 200 \) BC and much earlier for non-blown objects. The magnification effects of transparent materials was known in Antiquity. The blown glass technique had passed through the ages via the Romans and settled in Venice before the first millennium. The first lenses – *lense* was the Latin name given to a biconvex disk – used as spectacles appear in Italy before 1300 for correcting presbyopia because hand-lenses were unappropriate for writing; this was followed up around 1450 by divergent spectacles for correcting myopia. During the European Renaissance (1400–1600), the crystal- and glasswork of Murano – a small island near Venice – was flourishing. From \( \sim 1300 \), the remarkable developments of Murano’s furnaces and the skillfulness of glass blowers allowed the manufacturing of bottles, drinking glasses, chandeliers, polychromatic vases, and ornaments, etc. Around 1550, it had become easy in Murano to procure positive or negative lenses of small optical powers for correcting usual defects of the human eye. In this context, by empirically separating a negative lens from a positive lens, Digges in *Pantometrie* (1571) and Della Porta in *Magia Naturalis* (1589) noticed that an object at some distance is seen enlarged; this device is generally considered as the primitive ancestor of the telescope and may be called an “enlarging monocular” (sometimes improperly called *spyglass*).

Historics on the enlarging monocular are commented in noticeable works by Danjon and Couder [44] and King [85] where the development of early telescope is also described. The construction of enlarging monoculars between \( \sim 1608 \) and 1609 in Holland, is mainly the result of technological advances in fine glass manufacturing and lens polishing in Italy.

Because of technological difficulties encountered in making accurate and efficient metal mirrors, the first telescopes were not reflectors; the refractor telescopes emerged first from transformations of the enlarging monocular by Galileo Galilei. Accounts on the development of telescopes are in Riekher [132] and Wilson [170]. The major milestones in these developments are resumed hereafter.
1.1.4 Refractive Telescopes

- **Galileo Galilei** (1564–1642) heard from France, in 1609, that Lippershey in Holland had constructed a sort of “enlarging monocular.” This device, made of a single lens of positive power at the first end of a tube and of a negative lens on a sliding tube, was in fact a chance arrangement of eyeglass lenses available on the market which thus may only magnify distant objects by two or three times. It must be considered as a poor half part of our ancient opera glasses, and was totally useless for astronomical observations. Over a few months’s time Galileo fully understood its principle and transformed it into a “telescope” by constructing three of them known as telescopes No. 1, 2 and 3. In 1610, with telescope No. 3 he discovered Jupiter’s satellites, Venus’ phases, and the Sun’s rotation (observations of Sunspots by the naked eye and natural camera obscuras were reported in China since 28 BC and later in Persia).

From the rustic enlarging monocular, Galileo discovered the basic optical features for obtaining two-lens systems with large magnifications, i.e. a telescope – an invention which he must be credited with –: with a plano-convex lens as his objective and one of various divergent lenses as an eyepiece, he derived afocal systems of large magnifications, i.e. large beam compressions. His second difficult task, and not the least, was to build accurate lenses able to provide such high magnifications (Fig. 1.3).

It is remarkable that all his objective lenses are close to a plano-convex shape which come up from currently available equiconvex lenses that, in order to obtain larger focal lengths, he probably re-figured by himself; this is relevant to his objective lens of telescope No. 3 – the only surviving piece of this telescope – which shows two concentric shapes on the same side: a flat or quasi-flat central zone defining the clear aperture surrounded by a useless convex surface. Galileo obtained magnifications somewhat higher than 20 requiring deep divergent lenses down to

![Fig. 1.3 Galileo’s first refracting telescopes. (Up) No. 1: length 980 mm, magnification 21, clear aperture ~16 mm, f/61. (Down) No. 2: length 1,360 mm, magnification 14, clear aperture ~26 mm, f/51. (Institute and Museum of History of Sciences, Florence)]
47 mm focal length which he figured by himself because eyeglass makers did not make strong enough negative lenses for correcting such a huge myopia. Although his afocal design gave a view of objects at infinity, he naturally moved the diverging lens towards the objective by a slight amount in order to vision the sky at the eye’s punctum proximum distance. His mother found it lucrative to sell lenses to persons who asked for them. Interferometric analyses of some Galileo telescope optics conserved at the Science Museum in Florence show that the emerging wavefronts were “diffraction limited” at a single wavelength; Galileo would not have grasped the nuance of such a compliment, however.

Galileo published his astronomical discoveries with Telescope No. 3 – with a first lens focal length \( f_1 = 1,650 \text{ mm} \), \( f_1/D = \sim 50 \) – in Sidereus Nuncius (1610), where he states having used a magnification up to 30 and recognizes that the objective lens could be replaced by a concave mirror.\(^2\)

- **Johannes Kepler** (1571–1630) introduced the term “focus” in his work of 1604, Ad Vitellionem Paralipomena. In optics, he was the first to establish the conjugate distance relation for a given focal length. He noticed that the human eye works with an inverted image on the retina. In Dioptrice (1611), Kepler discussed the theory of telescope and enounced the rule giving the magnification as the ratio of the focal lengths of the two lenses. He described a refractor with a positive eyepiece but he never used one. The first positive eyepieces were used by C. Scheiner and later by F. Fontana in 1646; however this is generally known as Kepler’s eyepiece.

- **Willebrord Snell** (1580–1626) discovered the sine law of refraction in 1621 from experiment. He died in 1626 without publishing his discovery. It was first published by Descartes in his Dioptrique (1637) without reference to Snell who communicated it privately to several people including Descartes (cf. Born and Wolf [17]).

- **René Descartes** (1595–1650) thoroughly elaborated the general theory of stigmatic curves based on analytic geometry – that he created for this purpose – and simultaneously introduced the standard symbolic writing which we are familiar with. It was known from Diocles (cf. Toomer [162]) for the paraboloid, and probably by Pappus for the ellipsoid and hyperboloid, that only conicoid mirrors provide a perfect reflected image of an axial source point.

  In La Géométrie (1637), Descartes [45] introduces the complete theory of perfect axial imagery by aspherical surfaces that cancel the spherical aberration. It contains the equations of stigmatic surfaces of mirrors or lenses for a finite or infinite distance conjugate. For mirrors, the meridian sections of stigmatic conicoids appear as degenerated second degree curves. For lenses, the analytic geometry allowed

\(^2\) A few years later, N. Peiresc observing the Moon with a Galileo refractor begun drawing a map of it with the help of P. Gassendi and of a distinguished engraver C. Mellan; at mid work he discovered the Moon libration – oscillations of \( \sim 8^\circ \) and \( 6^\circ \) in longitude and latitude – and then completed his task with three maps done. He was one of the few scientists to defend Galileo against the Vatican Inquisition which condemned all published works on heliocentrism – among them De Revolutionibus Orbium Coelestium by N. Copernicus (1543) – and, in 1600, condemned the Copernician G. Bruno to the stake.
Descartes’ ovals:
Consider a given source point F in a medium of refractive index unity, and its conjugate G in medium n. A refracting surface – diopter – of stigmatic shape satisfies
\[ FC + n \cdot CG = \text{constant}. \]
The locus of C points is drawn, with a constant tension of the string ECKCG, along the marked straight edge FE as it pivots around F (La Géométrie, 1637 [45]) (cf. Chap. 9)

him to derive the stigmatic ovoids (cf. Chap. 9), which meridian sections, namely Descartes’s ovals, are fourth degree curves.

Using the formalism of Greek geometrical methods, Descartes gave a famous construction of the ovals with the marked straight edge and a string (cf. Arnaudiès and Delozoide [6]) which provides, through a refractive surface, the stigmatism of axial conjugates at finite distance (Fig. 1.4). All possible shapes of stigmatic lenses, designed with one spherical surface which is centered on the object or image, are displayed in La Dioptrique [45], which is part of Discours de la Méthode.

No further advance was made in Descartes’ theory of stigmatic surfaces until Petzval (1843) and Seidel (1856) established the complete theory with field aberrations, more than two centuries later.

- Christiaan Huygens (1629–1695), who recognized the importance of atmospheric seeing, built a 5.7 cm aperture refractor of 4 m focal length in 1655 (singlet objective lens at f/70) with which he discovered Titan. Refractors then increased further in size with Hevelius, Cassini, and others. In 1686, Constantin Huygens built several refractors so-called “aerials” – the tubes were open to the air – reaching 22-cm aperture for a focal length of 70 m (objective lens at f/300). Another example is the ∼f/500 singlet objective at the Marseille Observatory (Caplan [24]), apparently used around 1700.

Throughout the period 1609–1740, single lens objectives evolved towards slower f-ratios, which still did not require any asphericity correction, but suffered hugely from chromatic aberrations and mainly from axial chromatism.

The axial chromatism provides a first order variation of the focal length with the wavelength. Further slow down of the f-ratio was not the right way to minimize its angular size: with such huge focal lengths, the human eye could not see any image at all by lack of sensitivity or integration time.

Lead oxide glass, known in Antiquity, was reinvented in English glass factories around 1620. A standard production process was set up in 1675 by Ravenscroft. This material, so-called crystal of England or light flint (LF) glass, offered the white brightness of (quartz-) crystal and was easy to elaborate from closed crucibles. Its refractive index at the yellow helium line was \( n_d = 1.58 \) instead of 1.52 for the crown (K) or borosilicate (BK) glasses.
• **Chester Moor Hall** invented in 1728 the *achromatic objectives* – corrected from axial chromatism – by combining two lenses together: a negative flint lens and a positive crown lens. First, experimenting with flint and crown prisms, he carefully measured both their mean deviation angles and color dispersion angles. Then he determined the ratio of the prism angles of a matching prism pair that minimizes the resulting color dispersion which, thus, provided an achromatic deviation. Next, considering a lens pair, Hall stated that if at any given axial height the local prism angle ratio is the same, the chromatism correction will be achieved. Denoting $K_1$ and $K_2$ the optical power (cf. Sect. 1.4.3) of each lens in glass of respective dispersive power $\delta n_1/(n_1-1)$ and $\delta n_2/(n_2-1)$, this means that Hall discovered the *achromatism condition* $K_1 \delta n_1/(n_1-1) + K_2 \delta n_2/(n_2-1) = 0$. After designing a lens-pair, in 1733, Hall sub-contracted the optical figuring of the two lenses which, when assembled as a 3.5-cm aperture telescope, revealed results in accordance to his theory.

His results were well understood by Peter Dollond (renowned instrument and lens maker; his son John later succeeded in obtaining a Dollond patent for doublet lens achromats which was Hall’s results! [85]), and proved that the dispersive power of a glass $\delta n/(n_d-1)$ completely differs between a flint and a crown. This brings to evidence Newton’s error who, by supposing that the dispersive power was linearly the same for all glasses, hastily concluded that achromatization was impossible. Essays by L. Euler in 1742 and by S. Klingenstierna some years later confirmed this error. In establishing his theory of primary chromatism correction, Hall made possible the major advance in the development of refractors.

• **Alexis Clairaut** (1713–1765) elaborated the *theory of achromatic doublet lenses* in the period 1756–1762. He more accurately repeated the refractive index measures of crown and flint by Hall and P. Dollond, and concluded that a doublet-lens *never* could be exactly matched for obtaining achromatism because chromatic residuals will remain (these residuals were later called *secondary spectrum*). In a first memoir to the Royal Academy of Sciences [32], Clairaut discussed achromats with a crown first element. In this case, considering that the chromatic aberration of the crown positive lens must be set exactly opposite to that of the flint negative lens, he showed that continuous pairings are possible (with more or less spherical aberration). The assembled lenses provide a net positive power with the same focal length at two different wavelengths.

In the second memoir he investigated various shape achromats and discovered the second solution class with a negative flint lens as the first element. Investigating the two classes, and by varying the mean curvature of the lenses – *cambrure* in French – he derived relationships for achromats with nulled spherical aberration. Denoting $c_1$, $c_2$ the surface curvatures of the first lens and $c_3$, $c_4$ those of the second lens, Clairaut introduced an equal curvature for both internal surfaces, $c_2 = c_3$. Among the infinite number of solutions, this particular solution is known as Clairaut’s *equal curvature condition* of minimizing the number of surfacing tools (which later allowed cementing the lenses for a higher throughput). It falls that this particular solution $c_2 = c_3$ is not far from the other particular solution with four
differing curvatures which, in addition to the spherical aberration correction, gives the correction of the (off-axis) coma.

In the third memoir of 1762 Clairaut investigated the field imagery and noticed that the focused images asymmetrically aberrated (coma and astigmatism) and did not remain in a plane (field curvature). In a figure, he displays an off-axis blur image that he derived from trigonometrical ray trace. Finally, he derived the two simultaneous algebraic equations for non-cemented achromatic objectives corrected from both spherical aberration and coma. This is the Clairaut aplanatism conditions in the third-order aberration theory which was usually solved algebraically by Clairaut and soon after by J. D’Alembert. An equivalent graphical solution was found much later by A.E. Conrady: considering a \((c_2, c_3)\) Cartesian plane and a positive crown lens as the first element, Clairaut’s conditions are represented by a two-branch hyperbola \((c_3 - b)^2/B^2 - (c_2 + a)^2/A^2 = 1\) for zero spherical aberration and a straight line for zero coma. From the two solutions corresponding to the two intersection points, only the solution with curvatures \(c_3 = 0.987c_2\) both negative is useful for aplanatic objectives; the second and freakish solution is with two meniscus lenses of curvatures \(c_3 = 2.520c_2\) both positive for a Schott glass BK7-F2.

**Fig. 1.5** Doublet-lens objectives achromatized for an object at infinity in the spectral range \([\lambda_C = 486; \lambda_F = 656\text{ nm}], the blue and red hydrogen lines, and \(\lambda_d = 587\text{ nm}, the yellow helium line. Optimizations with Kerber’s condition of focus defined from ray height at \(\sqrt{3}/2\) on the first surface, the entrance pupil. Effective focal length \(f' = 1\). Focal-ratio f/16. The curvatures are exaggerated on the drawings. **Left: Clairaut’s algebraic conditions**, like the later ones by A.E. Conrady, produce a graph of this sort which shows Sphe \(3 = 0\) as a hyperbola and Coma \(3 = 0\) as a straight line, for thin air-spaced doublets (cf. for instance Szulc [152]). The intersection points give two aplanats for crown first; with BK7-F2 from Schott, these are with \(c_2 = -2.813, c_3 = -2.777\) and \(c_2 = 2.631, c_3 = 6.636\). There are also two other aplanat solution classes for flint first. **Right: Clairaut-Mossotti aplanat** also called Cemented aplanat. Glass materials developed later, such as Schott SF5 and others, avoid half of the reflected light by cementing the two elements, \(c_2 = c_3\); however this is only possible for small aperture objectives. In addition they provide a reduced secondary spectrum. With BK7-SF5 objectives and a field diameter of 1 degree, both aplanat solution classes provide the same resolution of \(2\) arcsec; this becomes two times better with CaF2-KZFSN4.
A similar representation with other two solutions can be obtained with a negative flint lens as the first element.

A detailed historical account on Clairaut’s optics work and the latter geometric representation of his algebraic conditions for aplanatic objectives is given by J.A. Church.3

• **Joseph Fraunhofer** (1787–1826) investigated the diffraction of light by gratings, published his theory of diffraction in 1823, and laid down the basis of spectroscopy. In 1752, the first spectral lines were observed by T. Melvill in the spectra of flames into which metals or salts have been introduced. Each chemical element is associated with a set of spectral lines. Fraunhofer studied the absorption lines of the Sun’s spectra, originally observed by W. Wollaston (1802), and determined the wavelengths of the brightest lines of hydrogen, helium, oxygen, sodium, magnesium, iron, and calcium. The yellow helium d-line at 587.561 nm, and the blue Hβ and red Hα hydrogen lines, i.e. the F- and C-lines at 486.132 and 656.272 nm, have been mainly used to characterize the refractive index $n_d$ and the reciprocal dispersive power $v_d = (n_d - 1)/(n_F - n_C)$ – sometimes called V-number – of optical materials. Fraunhofer’s works on spectral lines made possible a major advance in the achromatization accuracy of doublet lenses.

Then without apparently using the algebraic results of Clairaut which includes the exact correction of coma – at least in theory – as reformulated by d’Alembert in 1764 and 1767, Fraunhofer designed achromatic doublet lenses by successive iterations of trigonometric ray traces. This allowed him to build excellent objectives with a reduced coma although this latter aberration was not exactly nulled.4

• **Ottaviano Mossotti** (1791–1863), professor of geodesics at the University of Pisa, elaborated the *theory of cemented aplanatic doublet lenses* in the period 1853–1859. Let us denote $n = n_d$ the refractive index of a glass, $v = v_d$ its reciprocal dispersive power and $K$ the power of a lens of this glass, so that two lenses of different glass can be characterized by $(K_1, v_1)$ and $(K_2, v_2)$. In Gaussian optics, the *Hall achromatic condition* $K_1/v_1 + K_2/v_2 = 0$, entails that the resulting focal length is exactly the

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3 Clairaut’s optical results continue to be misinterpreted in modern literature. For example, H.C. King’s *The History of the Telescope* [85] is very wide off the mark on p.157, where we read: “Clairaut managed to reduce astigmatism and [field] curvature to within reasonable limits, but he could do nothing for coma and considered it an irremediable evil of two-lens combinations...” which is totally false since he established the algebraic conditions for thin-lens achromatic aplanats; furthermore Clairaut and D’Alembert realized correctly that astigmatism and field curvature are not too serious in the narrow fields of view of telescopes.

4 Concerning some astronomical results obtained with Fraunhofer objectives, we must mention the Königsberg heliometer as the famous instrument with which Bessel discovered and measured the first stellar parallax, that of 61 Cygni. Referring to this instrument after publishing his aberration theory (1856), Seidel noticed that the coma was corrected [partly] and wrote that “this objective perfectly satisfies the Fraunhofer condition” but there was no such existing Fraunhofer condition since he used iterations of trigonometric ray traces.

This inappropriate claim by Seidel has often been repeated, thus introducing much confusion in the past and present literature. This condition is in fact the Seidel sum $C_H = 0$ (implicitly included in Clairaut’s algebraic formulation) and could have only been approximatively satisfied in Fraunhofer objectives.