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Editors

Scientific Computing in Electrical Engineering

With 231 Figures, 112 in Color, and 33 Tables

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Editors

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Preface

The sixth international conference on Scientific Computing in Electrical Engineering (SCEE) was another event in the SCEE series, aiming to bring together scientists from universities and industry with the goal of intensive discussions about modeling and simulation of electronic circuits and electromagnetic fields. It was held in Sinaia, Romania, from 17^{th} to 22^{nd} September 2006 and it was endorsed by Philips Research Laboratories, Eindhoven (*http://www.philips.nl*), Infineon Technologies from Munich (*http://www.infineon.com*), ST Microelectronics (*http://www.st.com*), Computer Simulation Technology (*http://www.cst.com*), IEEE Romania Section (*http://www.ieee.ro*), Romanian Ministry of Education and Research by the CEEX program (*http://www.mct-excelenta.ro*).

The history of SCEE begun in 1997, as a national German meeting held in Darmstadt and then in Berlin (1998), both under the auspices of the DMV (Deutscher Mathematiker Verein). In 2000, the first truly international workshop was organized in Warnemünde by the University of Rostock, Germany (*http://www.scee-2000.uni-rostock.de/*). In 2002, the 4th SCEE conference was jointly organized by the Eindhoven University of Technology (TU/e) and Philips Research Laboratories Eindhoven, The Netherlands (*http://www.win.tue.nl/scee2002/*). In 2004, the 5th SCEE conference took place in Capo D'Orlando, Italy, organized by Universita di Catania and Consorzio Catania Ricerche (*http://www.dmi.unict.it/scee2004/*). A SCEE Summer School on Computational methods for microelectronics was organized in 2005 as a follow up of the SCEE04 conference (*http://unict.it/sceeschools*). The 6th conference was organized by "Universitatea Politehnica din Bucuresti (UPB), Centrul de Inginerie Electrica Asistata de Calculator (CIEAC) - Laboratorul de Metode Numerice (LMN)" in Sinaia, Romania (*http://www.scee06.org/*).

As on all previous occasions, the conference was supported both from the industrial sector and academia, thus being guaranteed the relevance of work to practical situations and challenging open problems.

One of the main aims of the SCEE events is to strengthen the interaction between electrical or electronic engineers and the mathematics community. This aim is also illustrated by the SCEE logo which has some lines that might be interpreted as field lines or wave fronts and part of a bracket which stands for mathematical bracket but also symbolizes the idea of connecting together several communities mathematicians and engineers, university and industry. This logo was designed by Ramona Weyde-Ferch for SCEE 2000.

The conference provided an excellent opportunity to the European Community for project meetings (*www.chameleon-rf.org, www.comson.org*) or to discuss new research projects in the EU seventh research program FP7.

The conference topics were: **Computational Electromagnetics** (Modelling and parameter extraction, Discretization and Solution Methods, Applications :Antennas, Microwave, Interconnects and on-chip passive structures), **Circuit Simulation** and Design (Reduced Order Modeling, Numerical Integration Techniques, TCAD/EDA tools and techniques, Applications: Radio Frequency, Power Electronics,Optical Networks), **Coupled Problems** (Field-circuit coupled problems, Multi-physics (coupling, Coupling with electrical, thermal and mechanical problems, Application: Co-Simulation, Electromagnetic Compatibility, Bio-engineering), **Mathematical and Computational Methods** (Inverse Problems, Optimization, Multi-Scale Schemes, Solutions methods for large linear systems, Differential-Algebraic Equations, Grid Computing, Grid Computing).

The Program Committee consisted of:

- Prof. A. M. Anile - Universita di Catania, Italy
- Dr. A. Bossavit - Ecole Superiore delectricite Gif sur Yvette, France
- Assoc. Prof. Dr. G. Ciuprina - Univ. Politehnica din Bucuresti, Romania
- Dr. U. Feldmann - Infineon Technologies AG, Germany
- Prof. Dr. M. Günther - Bergische Universitat Wuppertal, Germany
- Prof. Dr. D. Ioan - Univ. Politehnica din Bucuresti, Romania
- Prof. Dr. U. Langer - Johannes Kepler Univ., Austria
- Dr. E. J. W. ter Maten - Philips Research, The Netherlands
- Prof. Dr. U. van Rienen - Univ. Rostock, Germany
- Prof. Dr. W. H. A. Schilders - Philips Research, Eindhoven Univ. of Technology, The Netherlands
- Prof. Dr. T. Weiland - Technische Univ. Darmstadt, Germany

The Program Committee selected invited speakers from industry and academia for each of the four topics. Thus, SCEE 2006 was honoured by the presence of the following **invited speakers**:

- Prof. Athanasios C. Antoulas, (Rice University - Electrical and Computer Engineering Dpt. ECE, Houston, Texas - USA): “Approximation of large-scale dynamical systems: An overview and some new results”;
- Dr. Janne Roos, (Helsinki University of Technology, Circuit Theory Lab -APLAC - Finland): “Overview of Circuit-Simulation Activities at TKK CTL”;
- Prof. Luis Miguel Silveira, (Technical University of Lisbon (IST), School of Engineering, Department of Electrical and Computer Engineering, INESC-ID, Lisbon - Portugal): “Outstanding Challenges in Model Order Reduction”;
- Dr. Francois Henrotte, (RWTH - Aachen University - Institut fur Elektrische Maschinen, Germany): “The energy viewpoint in computational electromagnetics”;
- Dr. Irina Munteanu, (CST - Germany): “RF & Microwave Simulation with the Finite Integration Technique - From component to system design”;
- Dr. Herbert De Gersem, (Technical University Darmstadt, Computational Electromagnetics Lab. - TEMF - Germany): “Transient field-circuit coupled models with switching elements for the simulation of electric energy transducers”;
- Dr. Andrea Marmiroli, (STMicroelectronics, - Italy): “Technology and Device modelling in micro and nanoelectronics: current and future challenges”;
- Prof. Barbara Wohlmuth, (Stuttgart University - Institut fur Angewandte Analysis und Numerische Simulation IANS - Germany): “Advances in Mathematical and Computational Methods Applied in Electrical Engineering”;
- Prof. Piet Hemker, (Centre for Mathematics and Computer Science - CWI, Dpt. Modelling, Analysis and Simulation, Amsterdam, Univ. of Amsterdam, Dpt. of Mathematics, - The Netherlands): “Space mapping and defect correction for efficient optimization:.

Overall, there were about 100 contributions (40 oral presentations and 60 posters) including the talks of the Invited Speakers. As in previous editions, there were sessions dedicated to short oral introduction of poster, where each contributor was given two minutes to advertise his/her work.

It has always been the policy of these conferences to encourage participants from all countries, and this conference has been remarkably succesfull, there were about 90 participants from 14 countries. This confirmed that SCEE 2006 was a truly international event.

The papers appearing in this book represent a selection of papers presented at the conference. Each paper was carefully referreed by two or three referees chosen by the Program Committee. The Program Commitee supervised the reviewing iterative process, aiming to improve the published form of the articles.

The selected papers have been organized according to the scientific area. Therefore, there are four parts, respectively devoted to Coupled Problems, Circuit Simulation, Electromagnetism and General Mathematical Computational Methods.

We would like to thank the referees of the papers who have spent a lot of time in order to ensure a high quality scientific level of the papers in this book and also to their effort to help us in completing the reviewing process according to the time schedule.

The local organizing committee is greatly indebted to the financial support received from the sponsors and to all the people whose enthusiasm and hard work ensured the success of the conference. Special thanks go to Prof. Mihai Iordache, the Dean of the Electrical Engineering Faculty of the Politehnica University of Bucharest for his constant and precious support. Finally, we would like to thank Ph.D. students Diana Mihalache and Alexandra Stefanescu for the care they have shown in assembling all the information into this book.

Bucharest, *Gabriela Ciuprina*
March, 2007 *Daniel Ioan*

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Part I

Coupled Problems

Comparison of Model Reduction Methods with Applications to Circuit Simulation*

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Summary. We compare different model reduction methods applied to the dynamical system of a coupled transmission line: balanced truncation (BT), truncation by balancing one gramian (or PMTBR - poor man's truncated balanced reduction), positive real balanced truncation (PRBT) and its Hamiltonian implementation (PRBT-Ham), PRIMA, spectral zero method (SZM) and its Hamiltonian implementation (SZM-Ham), and finally, optimal $\mathcal{H}_2$. Their performance is analyzed in terms of several criteria such as: preservation of controllability, observability, stability and passivity, relative $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms, and the computational cost involved.

1 Introduction

This paper presents different reduction methods together with results obtained by applying each method on a dynamical system given by a coupled transmission line. In Sect. 2, a modified nodal analysis (MNA)-similar representation of the system is derived. The model reduction methods are grouped in two main categories, *gramian based* and *Krylov based*, discussed in sections 3 and 4 respectively. Sect. 3 outlines the theory behind gramian based reduction methods: BT, PMTBR and PRBT. Krylov based reduction methods PRIMA, SZM and optimal $\mathcal{H}_2$ are described in Sect. 4. In Sect. 5 we compare all methods in terms of: preservation of some important properties like controllability, observability, stability and passivity, the relative $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms and in terms of the computational cost. In Sect. 6, error systems resulting from different methods are compared. This allows us to identify frequency ranges where one particular method approximates the original system more accurately. Sect. 7 presents additional results obtained with the optimal $\mathcal{H}_2$ method. Finally, Sect. 8 summarizes our analysis and motivates further research.

2 State-space representation

The model reduction problem of transmission lines has been studied extensively, see for instance [8]. Our system consists of two transmission lines with inductive

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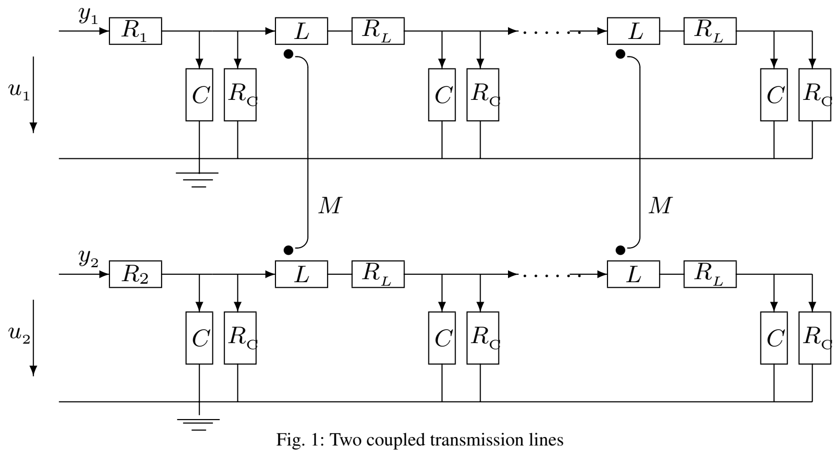

Fig. 1: Two coupled transmission lines

coupling as shown in Fig. 1. Each section consists of an inductor and its associated resistor, in series with a capacitor and its associated resistor. The first section has no inductor. All capacitor values C_i are equal. The same holds for the inductors L_i, the coupling inductors M_{ij}, the resistors associated with the capacitors R_{C_i}, the resistors associated with the inductors R_{L_i} and the input resistors, R_1 and R_2.

To simulate this circuit, the *state-space representation* of the system needs to be derived. Choosing the state variables as the currents through the inductors and the voltages across the capacitors, we obtain a system of order $n = 4N - 2$, where N is the number of sections of the circuit. The state-space representation in *modified nodal analysis (MNA)*-similar form is the following:

$$\left.\begin{aligned} \mathbb{C}\dot{\mathbf{x}}(t) &= \mathbf{G}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{L}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \end{aligned}\right\} \tag{1}$$

where $\mathbb{C} \in \mathbb{R}^{n\times n}$, $\mathbf{G} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times 2}$, $\mathbf{L} \in \mathbb{R}^{2\times n}$, $\mathbf{D} \in \mathbb{R}^{2\times 2}$ and $\mathbf{x}(t) \in \mathbb{R}^{n}$, $\mathbf{u}(t) \in \mathbb{R}^2$, $\mathbf{y}(t) \in \mathbb{R}^2$.

The problem will be studied under the following simplifying assumptions:

(1) the equations are in an MNA-similar form so that the resulting $\mathbb{C}$ matrix in (1) is nonsingular and positive definite (this means that all variables are state variables and none is redundant). In general, $\mathbb{C}$ resulting from circuit simulation is singular, due to additionally generated variables at the nodes between L_i and R_{L_i}.

(2) The transmission line has one input and one output, that is $u_2 = 0$ and only y_1 is observed, so that $\mathbf{u} = u_1$ and $\mathbf{y} = y_1$.

These assumptions are made to ease certain technical issues and allow a comparison of all reduction methods enumerated above; for instance, the optimal $\mathcal{H}_2$ method is currently available for single-input-single-output (SISO) systems only. None of these assumptions is essential for the validity of the results presented. Similar results for a system with MNA equations (where $\mathbb{C}$ is singular), using in part results from [5], will be reported in a future analysis.

For simplicity we will show the form of the equations by deriving them for $N = 3$ sections, namely for a circuit with 6 capacitors and 4 inductors, resulting in 10 states. In particular, the elements of the first line, from left to right will be

$$R_1,\ C_1,\ R_{C_1};\quad L_1,\ R_{L_1},\ C_2,\ R_{C_2};\quad L_2,\ R_{L_2},\ C_3,\ R_{C_3},$$

and those of the second line from left to right

$$R_2,\ C_4,\ R_{C_4};\quad L_3,\ R_{L_3},\ C_5,\ R_{C_5};\quad L_4,\ R_{L_4},\ C_6,\ R_{C_6}.$$

The state variables are:

$$\mathbf{x}_{C_1},\ \mathbf{x}_{L_1},\ \mathbf{x}_{C_2},\ \mathbf{x}_{L_2},\ \mathbf{x}_{C_3}, \mathbf{x}_{C_4},\ \mathbf{x}_{L_3},\ \mathbf{x}_{C_5},\ \mathbf{x}_{L_4},\ \mathbf{x}_{C_6},$$

and the state is chosen as:

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_C \\ \mathbf{x}_L \end{pmatrix},\ \ \mathbf{x}_C = \begin{pmatrix} \mathbf{x}_{C_1} \\ \mathbf{x}_{C_2} \\ \mathbf{x}_{C_3} \\ \mathbf{x}_{C_4} \\ \mathbf{x}_{C_5} \\ \mathbf{x}_{C_6} \end{pmatrix},\ \ \mathbf{x}_L = \begin{pmatrix} \mathbf{x}_{L_1} \\ \mathbf{x}_{L_3} \\ \mathbf{x}_{L_2} \\ \mathbf{x}_{L_4} \end{pmatrix}.$$

The associated system matrices are[2]:

$$\mathbb{C} = \begin{pmatrix} \tilde{\mathbb{C}} & 0 \\ 0 & \tilde{\mathbb{L}} \end{pmatrix},\ \mathbf{G} = \begin{pmatrix} -\mathbf{R}_C & \tilde{\mathbb{E}} \\ -\tilde{\mathbb{E}}^* & -\mathbf{R}_L \end{pmatrix},\ \mathbf{B} = \left(\tfrac{1}{R_1}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \right)^*,$$

$\mathbf{L} = -\mathbf{B}^*$ and $\mathbf{D} = \frac{1}{R_1}$, where:

$$\tilde{\mathbb{C}} = \mathrm{diag}(C_1, C_2, C_3, C_4, C_5, C_6),\ \tilde{\mathbb{L}} = \begin{pmatrix} L_1 & M_{13} & & \\ M_{13} & L_3 & & \\ & & L_2 & M_{24} \\ & & M_{24} & L_4 \end{pmatrix} \text{ and}$$

$$\tilde{\mathbb{E}} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix},\quad \begin{aligned} &\mathbf{R}_C = \mathrm{diag}\left(\tfrac{1}{R_1} + \tfrac{1}{R_{C_1}}, \tfrac{1}{R_{C_2}}, \tfrac{1}{R_{C_3}}, \tfrac{1}{R_2} + \tfrac{1}{R_{C_4}}, \tfrac{1}{R_{C_5}}, \tfrac{1}{R_{C_6}}\right) \\ &\mathbf{R}_L = \mathrm{diag}(R_{L_1}, R_{L_3}, R_{L_2}, R_{L_4}). \end{aligned}$$

The values of the elements used in the simulation are as follows: the input resistors are $R_1 = R_2 = 10\Omega$, the capacitors are $C_i = 5.4 \cdot 10^{-12} F$ and the associated resistors $R_{C_i} = 10^3 \Omega,\ (i = 1, \ldots, 6)$, the inductors are $L_i = 0.25 \cdot 10^{-9} H,\ (i = 1, \ldots, 4)$, the mutual inductors are $M_{ij} = 0.2 L_i\ (i = 1, 2,\ j = 3, 4)$ of that value. The associated resistors are zero $R_{L_i} = 0,\ (i = 1, \ldots, 4)$.

3 Gramian based methods

Gramian based methods involve diagonalization of gramians by congruence. These can either be the positive definite solutions to the Lyapunov equations (called *controllability* and *observability gramians*) or the positive definite solutions to algebraic Ricccati equations (called *positive real controllability* and *observability gramians*). The methods that we discuss are balanced truncation (BT) in Sect. 3.1 which

[2] For a matrix $\mathbf{M}$, $\mathbf{M}^*$ denotes transposition followed by complex conjugation if the matrix is complex.

performs simultaneous diagonalization of the controllability and the observability gramians, an equivalent of poor man's truncated balanced reduction (PMTBR) in Sect. 3.2 in which only one of the gramians is diagonalized and positive real balanced truncation (PRBT) in Sect. 3.3 in which positive definite solutions to the algebraic Ricatti equations are simultaneously diagonalized.

3.1 Balanced truncation (BT)

The idea behind balanced truncation is to simultaneously diagonalize the two infinite gramians, $\mathcal{P}$ and $\mathcal{Q}$ [1]. These are the solutions to the controllability and observability *Lyapunov equations* respectively, which are associated with the state space formuation (1). The mathematical model of the system may come in two representations: standard state-space and MNA-similar representation (or invertible descriptor form), respectively. We describe the application of model reduction methods for both cases of models.

Standard state-space representation

The standard state-space representation $(\mathbf{A}_{ss}, \mathbf{B}_{ss}, \mathbf{C}_{ss}, \mathbf{D}_{ss})$ is obtained from (1) by inverting the $\mathbb{C}$ matrix.

$$\mathbf{A}_{ss} = \mathbb{C}^{-1}\mathbf{G}, \mathbf{B}_{ss} = \mathbb{C}^{-1}\mathbf{B}, \mathbf{C}_{ss} = -\mathbf{B}^*, \mathbf{D}_{ss} = \mathbf{D}$$

The controllability and observability gramians are given by the symmetric positive definite solutions to the controllability and observability Lyapunov equations:

$$\mathbf{A}_{ss}\mathcal{P} + \mathcal{P}\mathbf{A}_{ss}^* + \mathbf{B}_{ss}\mathbf{B}_{ss}^* = 0 \tag{2}$$

$$\mathbf{A}_{ss}^*\mathcal{Q} + \mathcal{Q}\mathbf{A}_{ss} + \mathbf{C}_{ss}^*\mathbf{C}_{ss} = 0 \tag{3}$$

BT is performed in two steps. First, the balancing projection is computed (both gramians become equal and diagonal, with the Hankel singular values (HSVs) on the diagonal). Second, the states which are equally difficult to reach and to observe are truncated. This amounts to eliminating the states corresponding to the HSVs which are below a certain tolerance. Setting a tolerance for the reduced system a priori defines the number of states to be kept. The procedure is the following.

1. Compute the Cholesky factors of $\mathcal{P} = \mathbf{U}\mathbf{U}^*$ and $\mathcal{Q} = \mathbf{L}\mathbf{L}^*$
2. Compute the singular value decomposition of the product $\mathbf{U}^*\mathbf{L}$
$$\mathbf{U}^*\mathbf{L} = \mathbf{Z}\mathbf{S}\mathbf{Y}^* \tag{4}$$
The diagonal elements: $\mathbf{S} = \mathrm{diag}(\sigma_1, \dots \sigma_n)$, $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n$, where $\sigma_i = \sqrt{\lambda_i(\mathcal{P}\mathcal{Q})}$ are the *Hankel singular values* of the system. Choosing only the first k singular values and the first k columns of $\mathbf{Z}$ and $\mathbf{Y}$ gives the reduced system of order k after applying the projection $\mathbf{\Pi}$
3. $\mathbf{\Pi} = \mathbf{V}\mathbf{W}^*$ where $\mathbf{V} = \mathbf{U}\mathbf{Z}_k\mathbf{S}_k^{-\frac{1}{2}},\ \mathbf{V} \in \mathbb{R}^{n\times k},\ \ \mathbf{W} = \mathbf{L}\mathbf{Y}_k\mathbf{S}_k^{-\frac{1}{2}},\ \mathbf{W} \in \mathbb{R}^{n\times k}$
4. Compute the representation of the reduced system:
$$\tilde{\mathbf{A}}_{ss} = \mathbf{W}^*\mathbf{A}_{ss}\mathbf{V}, \tilde{\mathbf{B}}_{ss} = \mathbf{W}^*\mathbf{B}_{ss}, \tilde{\mathbf{C}}_{ss} = \mathbf{C}_{ss}\mathbf{V}, \tilde{\mathbf{D}}_{ss} = \mathbf{D}_{ss}$$

The corresponding diagonalized controllability and observability gramians are given by $\tilde{\mathcal{P}} = \mathbf{W}^*\mathcal{P}\mathbf{W} = \mathbf{S}_k,\ \tilde{\mathcal{Q}} = \mathbf{V}^*\mathcal{Q}\mathbf{V} = \mathbf{S}_k$ where $\mathbf{S}_k$ is the matrix containing the largest k HSV's on the diagonal.

Descriptor form representation

The MNA-similar representation is precisely (1). For simplicity, we rename the matrices in (1) to match the standard descriptor system representation:[3]

$$\mathbf{E}_{ds} = \mathbb{C}, \mathbf{A}_{ds} = \mathbf{G}, \mathbf{B}_{ds} = \mathbf{B}, \mathbf{C}_{ds} = \mathbf{L}, \mathbf{D}_{ds} = \mathbf{D}$$

The gramians are now the solutions to the following Lyapunov equations:

$$\mathbf{A}_{ds}\mathcal{P}\mathbf{E}_{ds}^* + \mathbf{E}_{ds}\mathcal{P}\mathbf{A}_{ds}^* + \mathbf{B}_{ds}\mathbf{B}_{ds}^* = 0 \tag{5}$$

$$\mathbf{A}_{ds}^*\hat{\mathcal{Q}}\mathbf{E}_{ds} + \mathbf{E}_{ds}^*\hat{\mathcal{Q}}\mathbf{A}_{ds} + \mathbf{C}_{ds}^*\mathbf{C}_{ds} = 0, \tag{6}$$

where $\mathcal{P}$ in (5) is precisely the solution of (2), while the original observability gramian corresponding to the solution of (3) is obtained by means of the congruence transformation

$$\mathcal{Q} = \mathbf{E}_{ds}^*\hat{\mathcal{Q}}\mathbf{E}_{ds}$$

The balancing and truncation procedures follow as described in Sect. 3.1, where (4) is replaced by:

$$\mathbf{U}^*\mathbf{E}_{ds}\mathbf{L} = \mathbf{Z}\mathbf{S}\mathbf{Y}^*$$

The system representation in the new basis now becomes:

$$\tilde{\mathbf{E}}_{ds} = \mathbf{W}^*\mathbf{E}_{ds}\mathbf{V} = \mathbf{I}_k, \tilde{\mathbf{A}}_{ds} = \mathbf{W}^*\mathbf{A}_{ds}\mathbf{V},$$

$$\tilde{\mathbf{B}}_{ds} = \mathbf{W}^*\mathbf{B}_{ds}, \tilde{\mathbf{C}}_{ds} = \mathbf{C}_{ds}\mathbf{V}, \tilde{\mathbf{D}}_{ds} = \mathbf{D}_{ds}.$$

Gramians $\mathcal{P}$ and $\mathcal{Q}$ are simultaneously diagonalized as mentioned in Sect. 3.1.

Solving the Lyapunov equation

There are many methods for solving the Lyapunov equation $\mathbf{A}\mathcal{P} + \mathcal{P}\mathbf{A}^* = \mathbf{Q}$ [1]. We will use the so-called *square-root method*, which directly computes $\mathbf{U}$ such that $\mathcal{P} = \mathbf{U}\mathbf{U}^*$. In MATLAB, this is implemented by `lyapchol`. Another important tool is the *sign function method*, which is discussed next.

The Lyapunov equation is a particular form of the Sylvester equation $\mathbf{A}\mathbf{X} + \mathbf{X}\mathbf{B} = \mathbf{C}$. To treat this generalized case, consider a matrix of the type

$$\mathbf{Z} = \begin{pmatrix} \mathbf{A} & -\mathbf{C} \\ \mathbf{0} & -\mathbf{B} \end{pmatrix},$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\Re(\lambda_i(\mathbf{A})) < 0$, $\mathbf{B} \in \mathbb{R}^{k\times k}$, $\Re(\lambda_i(\mathbf{B})) < 0$, and $\mathbf{C} \in \mathbb{R}^{n\times k}$. The sign function iteration $\mathbf{Z}_{n+1} = (\mathbf{Z}_n + \mathbf{Z}_n^{-1})/2$, $\mathbf{Z}_0 = \mathbf{Z}$ converges to

$$\lim_{j\to\infty} \mathbf{Z}_j = \begin{pmatrix} -\mathbf{I}_n & 2\mathbf{X} \\ \mathbf{0} & \mathbf{I}_k \end{pmatrix}$$

where $\mathbf{X}$ is the solution to the equation $\mathbf{A}\mathbf{X} + \mathbf{X}\mathbf{B} = \mathbf{C}$.

For the Lyapunov equation $\mathbf{A}\mathcal{P} + \mathcal{P}\mathbf{A}^* = \mathbf{Q}$, the starting matrix is

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[3] As mentioned earlier, our analysis of the system in descriptor form is restricted to the case in which matrix $\mathbf{E}_{ds} = \mathbb{C}$ is invertible.

$$\mathbf{Z} = \begin{pmatrix} \mathbf{A} & -\mathbf{Q} \\ \mathbf{0} & -\mathbf{A}^* \end{pmatrix}, \ \mathbf{A} \in \mathbb{R}^{n \times n}, \ \Re(\lambda_i(\mathbf{A})) < 0 \Rightarrow \mathbf{Z}_j = \begin{pmatrix} \mathbf{A}_j & -\mathbf{Q}_j \\ \mathbf{0} & -\mathbf{A}_j^* \end{pmatrix}$$

where the iterations can be written as follows

$$\mathbf{A}_{j+1} = \tfrac{1}{2}\left(\mathbf{A}_j + \mathbf{A}_j^{-1}\right), \ \ \mathbf{A}_0 = \mathbf{A}; \quad \mathbf{Q}_{j+1} = \tfrac{1}{2}\left(\mathbf{Q}_j + \mathbf{A}_j^{-1}\mathbf{Q}_j\mathbf{A}_j^{-*}\right), \ \ \mathbf{Q}_0 = \mathbf{Q}.$$

The limits of these iterations are $\mathbf{A}_\infty = -\mathbf{I}_n$ and $\mathbf{Q}_\infty = 2\mathcal{P}$ where $\mathcal{P}$ is the solution of the Lyapunov equation $\mathbf{A}\mathcal{P} + \mathcal{P}\mathbf{A}^* = \mathbf{Q}$.

Often, the constant term in the Lyapunov equation above is provided in factored form $\mathbf{Q} = \mathbf{R}\mathbf{R}^*$. As a consequence, it is possible to obtain the solution in factored form. In particular, the $(j+1)^{st}$ iterate in factored form is

$$\mathbf{Q}_{j+1} = \mathbf{R}_{j+1}\mathbf{R}_{j+1}^* \text{ where } \mathbf{R}_{j+1} = \tfrac{1}{\sqrt{2}}\left[\mathbf{R}_j, \mathbf{A}_j^{-1}\mathbf{R}_j\right] \Rightarrow \mathbf{Q}_\infty = \mathbf{R}_\infty\mathbf{R}_\infty^* = 2\mathcal{P}$$

$\mathbf{R}_\infty$ has infinitely many columns, although its rank cannot exceed n. This can be avoided by performing at each step a rank revealing RQ factorization $\mathbf{R}_j\mathbf{P}_j = \mathbf{T}_j\mathbf{U}_j$ with $\mathbf{P}_j$ the permutation matrix and $\mathbf{T}_j\mathbf{P}_j = \left[\Delta_j^*, \mathbf{0}\right]^*$. Δ_j is upper triangular and $\mathbf{U}_j\mathbf{U}_j^* = \mathbf{I}_j$. Thus, at the j^{th} step, $\mathbf{R}_j$ is replaced by Δ_j which has as many columns as the rank of $\mathbf{R}_j$. For accelerating convergence, the eigenvalues of $\mathbf{A}$ can be scaled [3]: at each step, $\mathbf{A}_j$ is replaced by $\frac{1}{\gamma_j}\mathbf{A}_j$ where the factors γ_j can be chosen as $\gamma_j = |det(\mathbf{A}_j)|^{\frac{1}{n}}$ in order to minimize the distance of the geometric mean of the eigenvalues of $\mathbf{A}_j$ from 1.

Convergence of the iteration which uses scaling is quadratic. The time required to compute the Cholesky factor by MATLAB's `lyapchol` function versus the iterative implementation of the sign function method in [3] is as follows: on a Pentium M at 1.3Ghz with 768MB RAM, `lyapchol` runs in 0.751s for a matrix $\mathbf{A}$ of dimension 242, while the implementation in [3] requires 5.423s and converges in $16 \approx \sqrt{242}$ steps. Even if, in theory, no scaling should also give quadratic convergence, in practice, due to numerical issues, convergence occurs after 20 steps.

3.2 Truncation by diagonalization of one gramian or poor man's truncated balanced reduction (PMTBR)

For the standard state-space representation, the procedure is the following [1].

1. Compute the gramian to be diagonalized (controllability gramian $\mathcal{P}$ in our case)
2. Compute the eigenvalue decomposition of $\mathcal{P} = \mathbf{V}\Sigma\mathbf{V}^*$
3. Choose the eigenvectors corresponding to the largest k eigenvalues to obtain the transformation $\mathbf{T} = \mathbf{V}_k^*$
4. The reduced system is

$$\tilde{\mathbf{A}}_{ss} = \mathbf{T}\mathbf{A}_{ss}\mathbf{T}^*, \ \tilde{\mathbf{B}}_{ss} = \mathbf{T}\mathbf{B}_{ss}, \ \tilde{\mathbf{C}}_{ss} = \mathbf{C}_{ss}\mathbf{T}^*, \ \tilde{\mathbf{D}}_{ss} = \mathbf{D}_{ss}$$

PMTBR is presented in [10] and uses numerical quadrature to approximate the gramian $\mathcal{P}$, without solving the Lyapunov equation. The algorithm used in our analysis, however, diagonalizes the exact solution $\mathcal{P}$ of the Lyapunov equation. As mentioned in Sect. 3.1, the solution to the Lyapunov equation can be computed either by using the *sign function method* or by using MATLAB's `lyapchol` function.

3.3 Positive real balanced truncation (PRBT)

Coupled transmission lines such as the one in Fig. 1 are passive systems, with *positive real* transfer functions (further information on passivity and positive realness is provided in [1]). We are therefore interested in reduced order models that are passive. In general, BT is not a passivity preserving method, since the resulting reduced system may have a non-positive real transfer function. PRBT, however, is a passivity preserving method. It yields reduced order models with positive real transfer functions by simultaneously diagonalizing the positive definite solutions $\mathcal{P}$ and $\mathcal{Q}$ of the controllability and observability algebraic *Riccati equations* respectively. This desirable result cannot be guaranteed with BT, where the solutions to the Lyapunov equations are diagonalized, rather than the solutions the Riccati equations. Riccati equations have a different form depending on whether the system is in standard state-space form or in descriptor form.

Historical note: this method was first introduced by Ober [6] and rediscovered by Phillips, Daniel and Silveira [9]. For an overview see also [1].

Standard state-space representation

The controllability and observability positive real Riccati equations are:

$$\mathbf{A}_{ss}\mathcal{P} + \mathcal{P}\mathbf{A}_{ss}^* + (\mathcal{P}\mathbf{C}_{ss}^* - \mathbf{B}_{ss})\Delta(\mathcal{P}\mathbf{C}_{ss}^* - \mathbf{B}_{ss})^* = 0 \tag{7}$$

$$\mathbf{A}_{ss}^*\mathcal{Q} + \mathcal{Q}\mathbf{A}_{ss} + (\mathcal{Q}\mathbf{B}_{ss} - \mathbf{C}_{ss}^*)\Delta(\mathcal{Q}\mathbf{B}_{ss} - \mathbf{C}_{ss}^*)^* = 0 \tag{8}$$

where $\Delta = (\mathbf{D}_{ss} + \mathbf{D}_{ss}^*)^{-1}$.
The procedure is the same as for BT (see Sect. 3.1), except that now balancing is performed on the minimal solutions of the Riccati equations. The diagonal elements of $\mathbf{S}$ in (4) are the *positive real singular values* of the system, which we denote by π_i: $\mathbf{S} = \mathrm{diag}(\pi_1, \dots \pi_n)$, where $\pi_1 \geq \pi_2 \geq \dots \geq \pi_n$.

Descriptor form representation

The corresponding algebraic Riccati equations in descriptor form are

$$\mathbf{A}_{ds}\mathcal{P}\mathbf{E}_{ds}^* + \mathbf{E}_{ds}\mathcal{P}\mathbf{A}_{ds}^* + (\mathbf{E}_{ds}\mathcal{P}\mathbf{C}_{ds}^* - \mathbf{B}_{ds})\Delta(\mathbf{E}_{ds}\mathcal{P}\mathbf{C}_{ds}^* - \mathbf{B}_{ds})^* = 0 \tag{9}$$

$$\mathbf{A}_{ds}^*\hat{\mathcal{Q}}\mathbf{E}_{ds} + \mathbf{E}_{ds}^*\hat{\mathcal{Q}}\mathbf{A}_{ds} + (\mathbf{E}_{ds}^*\hat{\mathcal{Q}}\mathbf{B}_{ds} - \mathbf{C}_{ds}^*)\Delta(\mathbf{E}_{ds}^*\hat{\mathcal{Q}}\mathbf{B}_{ds} - \mathbf{C}_{ds}^*)^* = 0 \tag{10}$$

where $\Delta = (\mathbf{D}_{ds} + \mathbf{D}_{ds}^*)^{-1}$. The observability gramian given by the solution of (8) is obtained via the congruence transformation $\mathcal{Q} = \mathbf{E}_{ds}^*\hat{\mathcal{Q}}\mathbf{E}_{ds}$.

Balancing and truncation are now performed on the solutions to (9) and (10) and the procedure follows as in 3.1.

Hamiltonian Riccati Balanced Truncation (PRBT-Ham)

Solutions to Riccati equations ((7),(8)) (or ((9),(10)) for MNA-similar form) can be obtained using the MATLAB function `care`. This can be applied to a system in usual state space form or in descriptor form. An alternative is to solve for $\mathcal{P}$ and $\hat{\mathcal{Q}}$ by means of the Hamiltonian eigenvalue problem [11]:

$$\begin{bmatrix} \mathbf{A}_{ds} - \mathbf{B}_{ds}\Delta\mathbf{C}_{ds} & -\mathbf{B}_{ds}\Delta\mathbf{B}_{ds}^* \\ \mathbf{C}_{ds}^*\Delta\mathbf{C}_{ds} & -\mathbf{A}_{ds}^* + \mathbf{C}_{ds}^*\Delta\mathbf{B}_{ds}^* \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \end{bmatrix} = \begin{bmatrix} \mathbf{E}_{ds} & \\ & \mathbf{E}_{ds}^* \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \end{bmatrix} \begin{bmatrix} \Lambda_- & \\ & \Lambda_+ \end{bmatrix} \tag{11}$$

where $\Delta = (\mathbf{D}_{ds} + \mathbf{D}_{ds}^*)^{-1}$, and Λ_-, Λ_+ are the Hamiltonian eigenvalues, with negative and positive real parts respectively (i.e. the *stable* and *antistable spectral zeros* of the system). We can partition $\mathbf{X}$ and $\mathbf{Y}$ according to the stable and antistable eigenvalues of the Hamiltonian into

$$\begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_- & \mathbf{X}_+ \\ \mathbf{Y}_- & \mathbf{Y}_+ \end{bmatrix}$$

The minimal solutions to (9) and (10) are given by:

$$\mathcal{P} = -\mathbf{X}_+(\mathbf{Y}_+)^{-1}\mathbf{E}_{ds}^{-*} \tag{12}$$

$$\hat{\mathcal{Q}} = -\mathbf{Y}_-(\mathbf{X}_-)^{-1}\mathbf{E}_{ds}^{-1} \tag{13}$$

and are the same as the ones resulting from the MATLAB `care` routine. The stabilizing solution (corresponding to the stable spectral zeros) is $\hat{\mathcal{Q}}$ while $\mathcal{P}$ is the antistabilizing solution (corresponding to the antistable spectral zeros). Both $\hat{\mathcal{Q}}$ and $\mathcal{P}$ are obtained form the same Hamiltonian eigenvalue computation (11). The original positive real observability gramian as solution to (8) is $\mathcal{Q} = \mathbf{E}_{ds}^*\hat{\mathcal{Q}}\mathbf{E}_{ds}$, so the positive real Hankel singular values are $\pi_i = \sqrt{\lambda_i(\mathcal{P}\mathcal{Q})}$, i.e. the diagonal elements of $\mathbf{X}_+(\mathbf{Y}_+)^{-1}\mathbf{Y}_-(\mathbf{X}_-)^{-1}$. We see that the positive real Hankel singular values can be computed without any inversion of $\mathbf{E}_{ds}$. The reduction procedure follows as in Sect. 3.1 using the computed (12) and (13).

If the system is the in usual state space form rather than in descriptor form, $\mathbf{E}_{ds}$ in (11) is simply replaced by $\mathbf{I}$. The resulting solutions $\mathcal{P}$ and $\hat{\mathcal{Q}}$ computed as (12) and (13) respectively, are precisely the positive real gramians solving (7) and (8). They are also the same as the solutions obtained with the MATLAB `care` routine in the usual state space form. The reduction procedure follows as in Sect. 3.3.

NOTE: The gramians used in balanced truncation, i.e. the solutions to the Lyapunov equations ((2), (3)) (and correspondingly ((5), (6)) for descriptor form) can be obtained using (11) with $\Delta = \mathbf{I}$, $\mathbf{C} = \mathbf{0}$ (for controllability) and $\mathbf{B} = \mathbf{0}$ (for observability).

4 Krylov based methods

Krylov based reduction methods exploit the use of Krylov subspace iterations to achieve system approximation by *moment matching* [1]. Three such methods are: PRIMA, the spectral zero method (SZM) and optimal $\mathcal{H}_2$. As outlined next, PRIMA matches k moments at zero by means of an *orthogonal projection*. SZM matches $2k$ moments of the original system, at k stable spectral zeros and their mirror images (the corresponding k antistable spectral zeros), by means of an *oblique projection*. Finally, using an oblique projection, the optimal $\mathcal{H}_2$ method matches $2k$ moments of the original system at the mirror images of the k poles of the reduced system (2 moments are matched at each pole). Hence an iteration is required.

4.1 PRIMA

For PRIMA, the moments of the transfer function $\mathbf{H}(s) = \mathbf{L}(s\mathbb{C} - \mathbf{G})^{-1}\mathbf{B} + \mathbf{D}$ are defined as the coefficients of the Taylor expansion of $\mathbf{H}(s)$ around $s_0 = 0$: $\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \ldots$, where

$$\mathbf{M}_0 = \mathbf{D} - \mathbf{L}\mathbf{G}^{-1}\mathbf{B} \;\text{ and }\; \mathbf{M}_k = (-1)^{(k+1)}\mathbf{L}(\mathbb{C}^{-1}\mathbf{G})^{-(k+1)}\mathbb{C}^{-1}\mathbf{B}, \;\text{for } k > 0.$$

PRIMA computes a k^{th} order reduced system by matching k moments of the original system. This is achieved by computing the orthogonal projection $\mathbf{\Pi} = \mathbf{X}_k\mathbf{X}_k^*$ such that $\mathbf{X}_k^*\mathbb{C}^{-1}\mathbf{G}\mathbf{X}_k = \mathbf{H}_k$ with $\mathbf{H}_k$ upper Hessenberg; the *column span* of $\mathbf{X}_k$ is the same as the *column span* of:

$$[\mathbb{C}^{-1}\mathbf{B},\ (\mathbb{C}^{-1}\mathbf{G})^{-1}\mathbb{C}^{-1}\mathbf{B},\ (\mathbb{C}^{-1}\mathbf{G})^{-2}\mathbb{C}^{-1}\mathbf{B},\ \ldots,\ (\mathbb{C}^{-1}\mathbf{G})^{-(k-1)}\mathbb{C}^{-1}\mathbf{B}].$$

The procedure is as follows [7].

1. Solve $\mathbf{G}\mathbf{R} = \mathbf{B}$ for $\mathbf{R}$.
2. $(\mathbf{X}_0, \mathbf{T})$ =QR($\mathbf{R}$); QR Factorization of $\mathbf{R}$
3. For $i = 1, 2, \ldots, k$
   Set $\mathbf{V} = \mathbb{C}\mathbf{X}_{i-1}$
   Solve $\mathbf{G}\mathbf{X}_i^{(0)} = \mathbf{V}$ for $\mathbf{X}_i^{(0)}$
   For $j = 1, 2, \ldots, i$
   $\mathbf{H} = \mathbf{X}_{i-j}^*\mathbf{X}_i^{(j-1)}$
   $\mathbf{X}_i^{(j)} = \mathbf{X}_i^{(j-1)} - \mathbf{X}_{i-j}\mathbf{H}$
   $(\mathbf{X}_i, \mathbf{T})$ = QR($\mathbf{X}_i^{(i)}$); QR Factorization of $\mathbf{X}_i^{(i)}$
4. Set $\mathbf{X} = [\mathbf{X}_0\ \ \mathbf{X}_1, \ldots, \mathbf{X}_{i-1}]$ and truncate $\mathbf{X}$ so that it has k columns only
5. Compute $\hat{\mathbb{C}} = \mathbf{X}^*\mathbb{C}\mathbf{X}$, $\hat{\mathbf{G}} = \mathbf{X}^*\mathbf{G}\mathbf{X}$, $\hat{\mathbf{B}} = \mathbf{X}^*\mathbf{B}$ and $\hat{\mathbf{L}} = \mathbf{L}\mathbf{X}$

4.2 Spectral zero method (SZM)

With PRIMA, system approximation was achieved by matching k moments of the transfer function at zero. In the general case, using the *rational Krylov* approach [1], reduced systems are obtained which match moments at preassigned *interpolation points* in the complex plane. SZM is a rational Krylov reduction method, in which the interpolation points are chosen as a subset of the spectral zeros of the original system [2], [11]. This selection guarantees the stability and passivity of the reduced system [2], [11]. The spectral zeros are given by Λ in (11). The real spectral zeros s_i come in pairs $(s_i, -s_i)$ while the complex spectral zeros come in quadruples of the form:

$$\begin{aligned} s_i \;&=\; \Re(s_i) + j\cdot\Im(s_i), \\ s_{i+1} &= \;\Re(s_i) - j\cdot\Im(s_i) \;=\; s_i^*, \\ s_{i+2} &= -\Re(s_i) + j\cdot\Im(s_i) \;=\; -s_i^*, \\ s_{i+3} &= -\Re(s_i) - j\cdot\Im(s_i) \;=\; -s_i, \end{aligned}$$

where without loss of generality, we assume $\Re(s_i) < 0$.

The usual procedure

The usual procedure for obtaining a k^{th} order reduced system with SZM is as follows.

1. Construct matrices $\mathbf{V}$ and $\mathbf{W}$ using $2k$ interpolation points:

$$\mathbf{V} = \left[\,(s_1\mathbf{E}_{ds} - \mathbf{A}_{ds})^{-1}\mathbf{B}_{ds},\, (s_2\mathbf{E}_{ds} - \mathbf{A}_{ds})^{-1}\mathbf{B}_{ds},\, \cdots,\ (s_k\mathbf{E}_{ds} - \mathbf{A}_{ds})^{-1}\mathbf{B}_{ds}\,\right]$$