

International Steam Tables

Second edition

Wolfgang Wagner · Hans-Joachim Kretzschmar

International Steam Tables

Properties of Water and Steam
Based on the Industrial Formulation IAPWS-IF97

Tables, Algorithms, Diagrams,
and CD-ROM Electronic Steam Tables

All of the equations of IAPWS-IF97 including a complete set
of supplementary backward equations for fast calculations
of heat cycles, boilers, and steam turbines

Second edition

Dr. Wolfgang Wagner, Professor Emeritus
Ruhr-University Bochum
Faculty of Mechanical Engineering
Chair of Thermodynamics
Universitaetsstr. 150
44780 Bochum
Germany
Wagner@thermo.rub.de

Dr. Hans-Joachim Kretzschmar, Professor
Zittau/Goerlitz University of Applied Sciences
Faculty of Mechanical Engineering
Department of Technical Thermodynamics
Theodor-Koerner-Allee 16
02763 Zittau
Germany
hj.kretzschmar@hs-zigr.de

Originally published as a German-English edition

ISBN 978-3-540-21419-9

e-ISBN 978-3-540-74234-0

DOI 10.1007/978-3-540-74234-0

Library of Congress Control Number: 2007940182

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Cover Design: WMXDesign GmbH, Heidelberg

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

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Preface to the Second Edition

The international research regarding the thermophysical properties of water and steam has been coordinated by the International Association for the Properties of Water and Steam (IAPWS). IAPWS is responsible for the international standards for thermophysical properties. These standards and recommendations are given in the form of releases, guidelines, and advisory notes. One of the most important standards in this sense is the formulation for the thermodynamic properties of water and steam for industrial use.

In 1997, IAPWS adopted the “IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam” for industrial use, called IAPWS-IF97 for short. The formulation IAPWS-IF97 replaced the previous industrial formulation IFC-67 published in 1967.

After the adoption of IAPWS-IF97 in 1997, further so-called backward equations were developed. These studies were coordinated by the IAPWS Task Group on Supplementary Backward Equations for IAPWS-IF97 chaired by one of the authors of this book (H.-J. K.). The final form of these equations is based on contributions by

J. R. Cooper	K. Knobloch	I. Stöcker
A. Dittmann	H.-J. Kretzschmar	R. Span
D. G. Friend	R. Mareš	W. Wagner
J. S. Gallagher	K. Miyagawa	I. Weber
A. H. Harvey	N. Okita	

In addition to these scientists, many other IAPWS colleagues, particularly the members of the working group “Industrial Calculations” (chairman up to 2001: B. Rukes, chairman from 2001 to 2003: K. Miyagawa, and chairman from 2004 onwards: Bill Parry) from 2002 onwards renamed in “Industrial Requirements and Solutions”, and the working group “Thermophysical Properties of Water and Steam” (chairman up to 2000: J. R. Cooper, chairman from 2000 to 2005: D. G. Friend, and chairman from 2005 onwards: H.-J. Kretzschmar), have contributed to the entire success of this IAPWS project; we appreciate their contribution very much. We are particularly grateful to the chairman of the evaluation task group, K. Miyagawa, for his exceptional efforts in testing these backward equations to ensure that they fulfill all requirements and checking the drafts of the several supplementary releases.

In 1998, Springer-Verlag published the book “Properties of Water and Steam” authored by W. Wagner and A. Kruse. This book described the industrial formulation IAPWS-IF97 as it was adopted by IAPWS in 1997. This new book is considered to be the second edition of the book published in 1998, although it has a different title and authorship and is only in English and no longer bilingual English/German. This second edition describes the industrial formulation in its current form, thus including all of the new so-called backward equations adopted by IAPWS between 2001 and 2005.

In addition to IAPWS-IF97, the industrial standard for the *thermodynamic* properties of water

and steam, the most recent equations for the *transport* properties dynamic viscosity and thermal conductivity are also presented. Moreover, equations for the surface tension, dielectric constant, and refractive index are given.

In contrast to the first edition, this second edition contains a number of extensions and new parts, namely:

- Incorporation of all “supplementary” backward equations.
- Inclusion of the uncertainty of the specific enthalpy into the uncertainty values of IAPWS-IF97 for the most important properties.
- Formulas to calculate all partial derivatives of the eight most important thermodynamic properties.
- Additional properties in the steam tables.
- Incorporation of the new basic equation for the high-temperature region (1073.15 K to 2273.15 K) with pressures up to 50 MPa (previously up to 10 MPa).
- Pressure-temperature diagrams with isolines of all properties contained in the steam tables and further properties.
- A compact disc (CD) providing the interactive program “IAPWS-IF97 Electronic Steam Tables” for the calculation of all properties (contained in the book) dependent on freely selectable pressures and temperatures in the single-phase region and on pressure or temperature along the saturated-vapour and saturated-liquid lines. Those properties for which it is reasonable can also be calculated within the two-phase region for given values of pressure or temperature and vapour fraction.

We are very grateful to Dr. K. Knobloch who developed the supplementary backward equations in her dissertation. We would like to thank Mr. M. Kunick for calculating and formatting the tables as Microsoft Excel sheets for Part B. We are very grateful to Dr. I. Stöcker, Dr. K. Knobloch, Ms. M. Weidner, and Mr. S. Buchholz for their help in producing all of the pressure-temperature diagrams in Part C of the book. Our warmest thanks are dedicated to Dr. U. Overhoff for his assistance in preparing the “IAPWS-IF97 Electronic Steam Tables” on the CD in Part D and for several checkups, and to Dr. I. Stöcker for her help in producing the large size Mollier h - s and T - s diagrams, which are included as attachments to the book. We thank Mr. R. Preusche, Mr. M. Markward, and Mr. B. Salomo for reprogramming all of the equations presented in the book. We would also like to thank Mrs. B. Esch for typing the text of the manuscript and Mrs. R. Gölzenleuchter for producing all of the figures. Our thanks go to Dr. O. Kunz for his help in creating the electronic printing version of Part A of the manuscript. Finally, we are grateful to Dr. E. W. Lemmon and Mrs. R. Smith for carefully reading the manuscript and for a number of suggestions on improving the English style.

One of us (H.-J. Kretschmar) is particularly grateful to the Saxon State Ministry for Science and Art for the financial support of the development of the supplementary backward equations at the Zittau/Görlitz University of Applied Sciences from 2001 to 2003.

Bochum and Zittau, November 2007

W. Wagner
H.-J. Kretschmar

Preface to the First Edition

In 1997, the International Association for the Properties of Water and Steam (IAPWS) adopted a new formulation for the thermodynamic properties of water and steam for industrial use. This formulation is called “IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam” and “IAPWS Industrial Formulation 1997” or “IAPWS-IF97” for short. The new formulation IAPWS-IF97 replaces the previous industrial formulation, IFC-67, that has formed the basis for power-plant calculations and other industrial applications since the late 1960’s. IAPWS-IF97 improves significantly both the accuracy and the speed of the calculation of thermodynamic properties. The differences from IFC-67 will require many users, particularly boiler and turbine manufacturers but also power-station companies and corresponding engineering offices, to modify design and application codes. In addition to these applications, IAPWS-IF97 is also of importance for energy-engineering applications in chemical industry and in other branches of industry. Therefore, this book presents the individual equations of IAPWS-IF97 for calculating the thermodynamic properties of water and steam for industrial use.

The IAPWS Industrial Formulation 1997 was developed in an international research project. This development was coordinated by the IAPWS Task Group “New Industrial Formulation” chaired by one of the authors of this book (W. W.). The final form of IAPWS-IF97 is based on contributions and equations by

J. R. Cooper	R. Mareš	Y. Takaishi
A. Dittmann	K. Oguchi	I. Tanishita
J. Kijima	H. Sato	J. Trübenbach
H.-J. Kretzschmar	I. Stöcker	W. Wagner
A. Kruse	O. Šifner	Th. Willkommen.

Besides these “developers” many other IAPWS colleagues, particularly the members of the two working groups “Industrial Calculations” and “Thermophysical Properties of Water and Steam”, contributed to the entire success of this comprehensive project; we appreciate their contribution very much. We are especially grateful to the chairmen of these two working groups, B. Rukes and J. R. Cooper. In addition, we would like to thank the members of the IAPWS Task Group “New Industrial Formulation - Evaluation” for testing IAPWS-IF97 regarding the fulfilment of requirements and checking the influence on real power-cycle calculations; concerning these important pieces of work we are particularly grateful to the chairman of this task group, K. Miyagawa, and his colleagues R. Spencer, R. B. McClintock, and H. W. Bradley for their exceptional efforts.

In addition to IAPWS-IF97, the industrial standard for the thermodynamic properties of water and steam, the most recent equations for the transport properties dynamic viscosity and thermal conductivity are also presented. Moreover, equations for the surface tension, static dielectric constant, and refractive index are given.

The text of this book is bilingual. Part A contains the description of the above mentioned equations for the thermophysical properties in English and Part B the corresponding description in German. Comprehensive tables of the most important thermophysical properties of water and steam are given in Part C in both languages.

The values in the tables of Part C were exclusively calculated from the corresponding equations summarized in Part A and Part B, respectively. These tables, which are mainly based on the new industrial formulation IAPWS-IF97, replace the tables “Properties of Water and Steam in SI-Units” prepared by E. Schmidt and edited by U. Grigull (Springer-Verlag Berlin Heidelberg New York, R. Oldenbourg München, Fourth, Enlarged Printing, 1989) which are based on the previous industrial formulation IFC-67.

We wish to express our warmest thanks to Mr. C. Bosen for his help in handling the computer programs for calculating the transport properties and for producing all the tables. We would also like to thank Mrs. A.-M. Sieg for typing the text of the manuscript. We are particularly grateful to the Deutsche Forschungsgemeinschaft for their financial support of that part of the development of IAPWS-IF97 which was carried out at the Ruhr-University Bochum.

Bochum, February 1998

W. Wagner
A. Kruse

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Nomenclature

Quantities

A	Function
a	Thermal diffusivity, $a = \lambda/(\rho c_p)$
a	Coefficient
B	Function
c_p	Specific isobaric heat capacity
$c_{p,m}^0$	Mean specific isobaric heat capacity in the ideal-gas state
c_v	Specific isochoric heat capacity
CTR	Computing-Time Ratio
f	Specific Helmholtz free energy, $f = u - Ts$
f^*	Fugacity
g	Specific Gibbs free energy, $g = h - Ts$
\bar{g}	\bar{g} -factor of Harris and Alder
h	Specific enthalpy
Δh_v	Specific enthalpy of vaporization, $\Delta h_v = h'' - h'$
I	Exponent
i	Serial number; Exponent
J	Exponent
j	Serial number; Exponent
k	Boltzmann's constant
M	Molar mass
N_A	Avogadro's number
n	Refractive index
n	Coefficient
Pr	Prandtl number, $Pr = \eta c_p \lambda^{-1}$
p	Pressure
R	Specific gas constant
R_m	Molar gas constant
s	Specific entropy
Δs_v	Specific entropy of vaporization, $\Delta s_v = s'' - s'$
T	Thermodynamic temperature ¹
t	Celsius temperature, $t/^{\circ}\text{C} = T/\text{K} - 273.15$
u	Specific internal energy
v	Specific volume
w	Speed of sound

¹ All temperature values given in this book are temperatures according to the International Temperature Scale of 1990 (ITS-90)

x	Vapour fraction
x	Arbitrary state variable
y	Arbitrary state variable
z	Compression factor, $z = pv/(RT)$
z	Arbitrary state variable
α	Mean molecular polarizability of the isolated water molecule
α_p	Relative pressure coefficient, $\alpha_p = p^{-1}(\partial p/\partial T)_v$
α_v	Isobaric cubic expansion coefficient, $\alpha_v = v^{-1}(\partial v/\partial T)_p$
β	Transformed pressure, Eq. (2.12a)
β_p	Isothermal stress coefficient, $\beta_p = -p^{-1}(\partial p/\partial v)_T$
γ	Dimensionless Gibbs free energy, $\gamma = g/(RT)$
Δ	Difference in any quantity
δ	Reduced density, $\delta = \rho/\rho^*$
δ_T	Isothermal throttling coefficient, $\delta_T = (\partial h/\partial p)_T$
ε	Dielectric constant (relative static dielectric constant or relative static permittivity)
ε_0	Permittivity of vacuum (electric constant)
η	Dynamic viscosity
η	Reduced enthalpy, $\eta = h/h^*$
θ	Reduced temperature, $\theta = T/T^*$
ϑ	Transformed temperature, Eq. (2.12b)
κ	Isentropic exponent, $\kappa = -vp^{-1}(\partial p/\partial v)_s$
κ_T	Isothermal compressibility, $\kappa_T = -v^{-1}(\partial v/\partial p)_T$
λ	Thermal conductivity
Λ	Reduced thermal conductivity, $\Lambda = \lambda/\lambda^*$
$\bar{\lambda}$	Wavelength of light
$\bar{\Lambda}$	Reduced wavelength of light, $\bar{\Lambda} = \bar{\lambda}/\bar{\lambda}^*$
μ	Joule-Thomson coefficient, $\mu = (\partial T/\partial p)_h$
μ	Dipole moment of the isolated water molecule
ν	Kinematic viscosity, $\nu = \eta\rho^{-1}$
π	Reduced pressure, $\pi = p/p^*$
ρ	Mass density
σ	Surface tension
σ	Reduced entropy, $\sigma = s/s^*$
τ	Inverse reduced temperature, $\tau = T^*/T$
ϕ	Dimensionless Helmholtz free energy, $\phi = f/(RT)$
Ψ	Reduced dynamic viscosity, $\Psi = \eta/\eta^*$
ω	Reduced volume, $\omega = v/v^*$

Superscripts

o	Ideal-gas part; ideal gas
r	Residual part
max	Maximum value of a quantity
min	Minimum value of a quantity

- * Reducing quantity
- ' Saturated-liquid state
- " Saturated-vapour state

Subscripts

- ad Adiabatic
- b Normal boiling point
- c Critical point
- h* At constant specific enthalpy
- ind Industrial equation for λ
- m State on the melting line
- m Mean value
- max Maximum value of a quantity
- p* At constant pressure
- perm Permissible
- RMS Root-mean-square value of a quantity, see below
- ρ At constant density
- s Saturation state
- s* At constant specific entropy
- sci Scientific equation for λ
- sub State on the sublimation line
- t Triple point
- T* At constant temperature
- v* At constant specific volume

Root-mean-square value:

$$(\Delta x)_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{n=1}^N (\Delta x_n)^2},$$

where Δx_n can be either absolute or percentage differences of the corresponding property x ; N is the number of Δx_n values (depending on the property, between 10 million and 100 million points are uniformly distributed over the respective range of validity)

Introduction

This book consists of five parts, Part A to Part E.

Part A presents the current internationally agreed upon equations for industrial calculations of the most relevant thermophysical properties of water and steam.

The current industrial standard for the *thermodynamic* properties, which replaced the former industrial standard IFC-67 [1], was adopted by the International Association for the Properties of Water and Steam (IAPWS) in 1997 under the name “IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam” or simply IAPWS-IF97 for short. All relevant numerical details about the entire set of equations of IAPWS-IF97 are given in Chap. 2.

As a comprehensive supplement of the first edition of this book, this second edition contains all of the so-called backward equations of IAPWS-IF97 developed after 1997 and adopted by IAPWS between 2001 and 2005. In addition to the uncertainty values given in IAPWS-IF97 for the properties specific volume, specific isobaric heat capacity, speed of sound, and saturation pressure, now uncertainty values for the specific enthalpy and differences in specific enthalpy are given as well. Moreover, formulas are presented in this new work to calculate all partial derivatives from the equations of IAPWS-IF97 formed by any three combinations of the properties pressure, temperature, and the specific properties volume, enthalpy, internal energy, entropy, Gibbs free energy, and Helmholtz free energy. For the high-temperature region (1073.15 K to 2273.15 K), the new basic equation that covers this temperature range for pressures up to 50 MPa (previously 10 MPa) is presented.

In addition to the equations for the thermodynamic properties of water and steam, Chap. 3 of Part A summarizes current equations for industrial use for the *transport* properties dynamic viscosity and thermal conductivity and also presents correlation equations for the surface tension, dielectric constant, and refractive index.

Part B contains the tables of the most important properties of water and steam, which were calculated from the corresponding equations of Chaps. 2 and 3 in Part A. In comparison with the first edition, additional tables with values of the properties compression factor, isochoric heat capacity, isobaric expansion coefficient, and isothermal compressibility are given. The table for the ideal-gas state was extended by including the properties isochoric heat capacity, isentropic exponent, and mean isobaric heat capacity between 0 °C and the given temperature t .

Part C of this book presents pressure-temperature diagrams with isolines of all the properties tabulated in Part B and of further properties such as the specific internal energy, Joule-Thomson coefficient, and a number of partial derivatives.

Part D contains a CD providing the interactive program “IAPWS-IF97 Electronic Steam Tables” to calculate all of the properties contained in the book dependent on pressure and temperature. In this way, users can calculate “personal” steam tables for arbitrary values of pressure and temperature, as well as properties in the two-phase region as a function of pressure or temperature together with vapour-fraction. With the addition of this possibility, the size of the printed steam tables (Part B) was reduced in comparison with the first edition of this book.

Part E contains the two wall charts, a Molier h - s diagram and a T - s diagram.

Part A

Equations for the Calculation of the Thermophysical Properties of Water and Steam

1 Reference Constants

This chapter summarizes all reference constants needed for evaluating the equations given in Chaps. 2 and 3.

The specific gas constant of ordinary water,

$$R = 0.461\,526 \text{ kJ kg}^{-1} \text{ K}^{-1}, \quad (1.1)$$

results from the recommended value of the molar gas constant [2],

$$R_m = 8.314\,51 \text{ kJ kmol}^{-1} \text{ K}^{-1}, \quad (1.2)$$

and from the molar mass of ordinary water,

$$M = 18.015\,257 \text{ kg kmol}^{-1}. \quad (1.3)$$

The value of the molar mass of ordinary water results from the molar mass of hydrogen, $M_H = 1.007\,975\,97 \text{ g mol}^{-1}$ (based on the molar mass of the isotopes ^1H and ^2H given in [3] and the isotopic concentration corresponding to the molar fraction of ^1H equal to 0.99985 and of ^2H equal to 0.00015 [4]), and the molar mass of oxygen, $M_O = 15.999\,304\,7 \text{ g mol}^{-1}$ (based on the molar mass of the isotopes ^{16}O , ^{17}O , and ^{18}O given in [3] and the isotopic concentrations corresponding to the molar fractions of ^{16}O , ^{17}O , and ^{18}O equal to 0.99762, 0.00038, and 0.002, respectively, considered to be characteristic for all natural occurrences of oxygen [4]).

The values of the critical parameters

$$T_c = 647.096 \text{ K}, \quad (1.4)$$

$$p_c = 22.064 \text{ MPa}, \text{ and} \quad (1.5)$$

$$\rho_c = 322 \text{ kg m}^{-3} \quad (1.6)$$

are from the corresponding IAPWS release [5]. The triple-point temperature is

$$T_t = 273.16 \text{ K} \quad (1.7)$$

according to the International Temperature Scale of 1990 (ITS-90) [6] and the triple-point pressure

$$p_t = 611.657 \text{ Pa} \quad (1.8)$$

was determined by Guildner et al. [7]. According to the scientific standard for the thermodynamic properties of ordinary water, the IAPWS-95 formulation [8, 9], the temperature of the normal boiling point (at a pressure of 0.101325 MPa (1 atm)) amounts to

$$T_b = 373.1243 \text{ K}. \quad (1.9)$$

2 IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam

At the IAPWS meeting in Erlangen, Germany in 1997, the “IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam” was adopted as the new international industrial standard for the thermodynamic properties of water and steam. This new industrial standard is also called “IAPWS Industrial Formulation 1997” or “IAPWS-IF97” for short. The IAPWS-IF97 formulation replaced the previous industrial standard IFC-67 [1]. In comparison with IFC-67, IAPWS-IF97 significantly improves both the accuracy and the calculation speed of thermodynamic properties.

This chapter presents all of the information about the individual equations of IAPWS-IF97 necessary for calculating the thermodynamic properties of water and steam. It also includes the supplementary backward equations developed after 1997 and adopted by IAPWS between 2001 and 2005, and also a new basic equation for the high-temperature region 1073.15 K to 2273.15 K for pressures up to 50 MPa (previously 10 MPa). In contrast to the first edition of this book, the backward equations are not presented region by region, but all backward equations dependent on the same input variables are summarized in the same section. Section 2.4 presents formulas to calculate all of the partial derivatives $(\partial z/\partial x)_y$, from the equations of IAPWS-IF97, where the variables x , y , and z can represent any of the thermodynamic properties: pressure p , temperature T , and the specific properties volume v , enthalpy h , internal energy u , entropy s , Gibbs free energy g , or Helmholtz free energy f . In addition to the uncertainties of the equations of IAPWS-IF97 in the properties specific volume, specific isobaric heat capacity, speed of sound, and saturation pressure, Sec. 2.5 also contains uncertainty statements on the specific enthalpy and differences in specific enthalpy. Moreover, illustrations show the achieved consistency between the basic equations along the region boundaries.

Information about the development of the IAPWS-IF97 equations and details about their quality and calculation speed in comparison with the previous industrial standard IFC-67 are given in the international publication on IAPWS-IF97 [10]. Details about the development of the supplementary backward equations can be found in the articles [11-14].

2.1 Characteristic Features of IAPWS-IF97

This section gives general information about the structure of the industrial formulation IAPWS-IF97, including the entire range of validity, and makes some general statements about the quality of IAPWS-IF97 concerning accuracy and consistency along the region boundaries. In addition, statements on the calculation speed are made not only when the basic equations are used for calculations of properties that are not dependent on pressure and temperature, but also when the supplementary backward equations are used.

2.1.1 Structure of IAPWS-IF97

The IAPWS Industrial Formulation 1997 consists of a set of equations for different regions which cover the following range of validity:

$$\begin{aligned} 273.15 \text{ K}^2 \leq T \leq 1073.15 \text{ K} & \quad 0 < p \leq 100 \text{ MPa} [15, 16] \\ 1073.15 \text{ K} < T \leq 2273.15 \text{ K} & \quad 0 < p \leq 50 \text{ MPa} [16]^3 \end{aligned}$$

Figure 2.1 shows the five regions which divide the entire range of validity of IAPWS-IF97; for the exact definition of the five regions see Sec. 2.2. Regions 1 and 2 are each covered by a fundamental equation for the specific Gibbs free energy $g(p, T)$, region 3 by a fundamental equation for the specific Helmholtz free energy $f(\rho, T)$, and region 4, the two-phase region (corresponding to the saturation curve in the p - T diagram), by a saturation-pressure equation $p_s(T)$. The high-temperature region 5 is also covered by a $g(p, T)$ equation. These five equations, shown in rectangular boxes in Fig. 2.1, form the so-called *basic equations*.

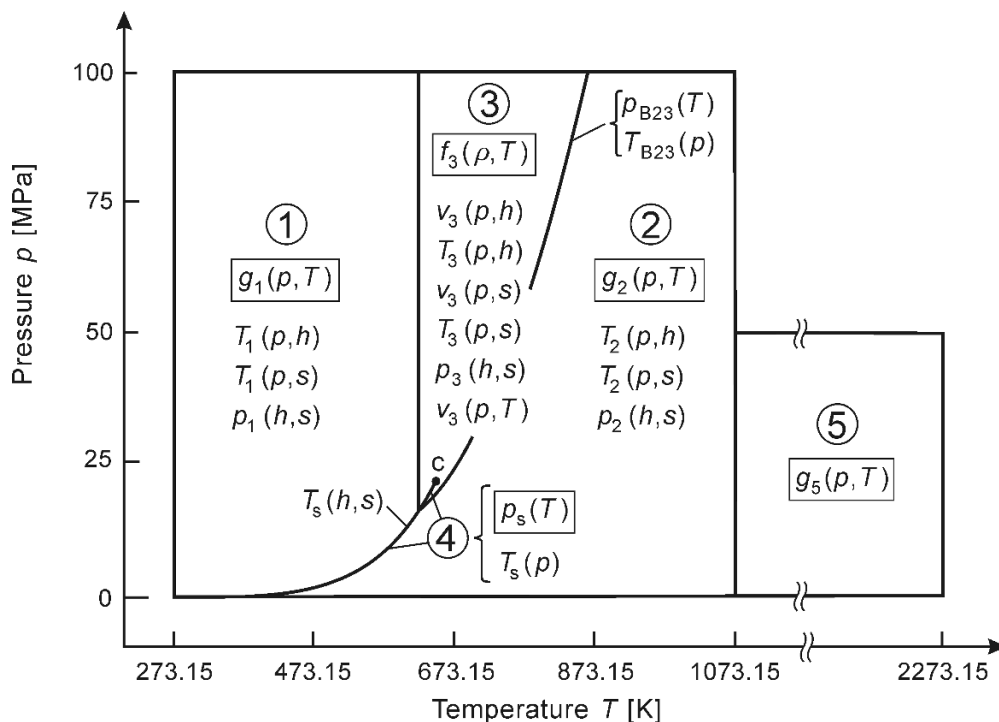


Fig. 2.1 Regions and equations of the industrial formulation IAPWS-IF97.

The industrial standard IAPWS-IF97 has been coupled to the scientific standard, the “IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General

² In order to remain consistent with the previous industrial formulation IFC-67 [1], the range of validity of IAPWS-IF97 in temperature starts at 273.15 K (0 °C) rather than at the triple-point temperature $T_t = 273.16 \text{ K}$ (0.01 °C). Thus, when being thermodynamically exact, states in the temperature range $273.15 \text{ K} \leq T \leq 273.16 \text{ K}$ and at pressures $p_{\text{sub}}(T) \leq p < p_m(T)$ are in the metastable region, where p_{sub} and p_m are the pressures along the sublimation and melting line [17], respectively.

³ The revision of the release of 1997 [15] only relates to the extension of region 5 up to pressures of 50 MPa (previously 10 MPa).

and Scientific Use” [8, 9], hereafter abbreviated to IAPWS-95. This coupling was achieved by fitting the basic equations of regions 1 to 3 and 5 to values of the specific volume v , specific enthalpy h , specific isobaric heat capacity c_p , and speed of sound w calculated from IAPWS-95 [9]. Accordingly, the basic equation for region 4, the saturation-pressure equation, was fitted to the values of the saturation pressure p_s calculated from IAPWS-95.

In addition to these basic equations, so-called *backward equations* are provided for all regions except for region 5, where the backward equations are only valid for pressures $p \geq p_s(273.15 \text{ K}) \approx 0.000611 \text{ MPa}$. These backward equations were developed in the following combinations of variables: For regions 1 and 2 as equations of the form $T(p, h)$, $T(p, s)$, and $p(h, s)$, for region 3 as equations of the form $v(p, h)$, $T(p, h)$, $v(p, s)$, $T(p, s)$, $p(h, s)$, and $v(p, T)$, for the entire region 4 as a saturation-temperature equation $T_s(p)$, and for the technically most important part of region 4 ($s \geq s''(623.15 \text{ K})$) as a saturation-temperature equation of the form $T_s(h, s)$. In Fig. 2.1, in addition to the (framed) basic equations, all of these types of backward equations are assigned to the corresponding region of IAPWS-IF97. The subscripts relate to the region for which the equation is valid.

These backward equations were developed in such a way that they are numerically very consistent with the corresponding basic equation. Thus, properties as functions of (p, h) , (p, s) , and (h, s) for regions 1 to 3, of (p) for the entire region 4, and of (h, s) for the technically most important part of region 4 can be calculated without any iteration. Due to the backward equation $v(p, T)$ for region 3, the specific volume can be calculated for this region without the necessity of its iteration from the basic equation $f_3(\rho, T)$. Consequently, properties such as $s(p, h)$ and $h(p, s)$ can be calculated directly from the corresponding backward equation or in combination with the corresponding basic equation, for example, $h(p, s)$ via the relation $h(p, T(p, s))$. As a result of this special concept of the industrial standard IAPWS-IF97, all important combinations of properties can be calculated extremely quickly; more details are given in the next section and in Sec. 2.3.

2.1.2 Quality of IAPWS-IF97

The achieved overall quality of the industrial formulation IAPWS-IF97 is characterized by the following general results in the light of the three criteria accuracy, consistency between basic equations along region boundaries and between backward equations along subregion boundaries, and calculation speed.

The accuracy of IAPWS-IF97 is illustrated by the fact that for its entire range of validity only 0.2% of the calculated v values, 6% of the c_p values, 2% of the w values, and none of the p_s values are outside the uncertainty of the corresponding IAPWS-95 values [9]. When carrying out the same test with the previous industrial standard IFC-67, between 47% (for v) and 80% (for p_s) of the IFC-67 values were outside the uncertainty of the corresponding IAPWS-95 values. Based on all comparisons made [10] it can be concluded that IAPWS-IF97 is more than one order of magnitude more accurate than IFC-67. The estimated uncertainties of IAPWS-IF97 in the properties v , c_p , w , p_s , h , and Δh over the entire range of validity are given in Sec. 2.5. In addition to the representation of the properties for the stable homogeneous regions and at saturation, the corresponding IAPWS-IF97 equations also yield reasonable values for both the

metastable superheated-liquid region and the metastable subcooled-vapour region close to the saturated-liquid line and the saturated-vapour line, respectively.

Compared with IFC-67, an additional important jump in quality was achieved by the fact that IAPWS-IF97 clearly meets the requirements regarding the consistencies along the region boundaries, see Fig. 2.1. IAPWS-IF97 is clearly within the permitted inconsistencies according to the so-called Prague values [18]. This is also true for the “difficult” boundary between regions 2 and 3, along which the consistency requirements for the specific isobaric heat capacity are also met by the basic equations of regions 2 and 3. For IAPWS-IF97, the maximum inconsistency in c_p at this boundary amounts to 0.35% whereas the corresponding IFC-67 inconsistency was greater than 6%. Details of the achieved consistencies along the boundaries between regions 1 and 3, regions 2 and 3, and regions 2 and 5 are given in Sec. 2.5.

The third and probably the greatest advantage is the very large improvement in the calculation speed compared with IFC-67. Even when using only the backward equations that existed in 1997 when IAPWS-IF97 was adopted (the equations $T(p,h)$ and $T(p,s)$ for regions 1 and 2) for the most important regions 1, 2, and 4, where the computing time is particularly relevant, the calculation-speed factor of IAPWS-IF97 in comparison with IFC-67 amounts to 5.1. This value was determined by taking into account the frequencies of use of the most relevant property functions in these regions based on a survey of the international power-cycle companies and related industries. This means that for these important regions IAPWS-IF97 is more than 5 times faster than IFC-67 as long as the individual equations are properly programmed. Details about the accuracy, consistency along region boundaries, and calculation speed (in comparison with IFC-67) are given in the comprehensive article on IAPWS-IF97 [10]. These calculation-speed factors were determined in 1996 based on a computer running 16-bit DOS with a processor 486DX/33 MHz [10]. When repeating these comparisons with a modern Pentium 4/3.0 GHz PC and the operating system Windows XP, then IAPWS-IF97 is not only 5.1 times faster than IFC-67 but 8.3 times [19].

A comparison of the calculation speed within IAPWS-IF97 with and without using the backward equations shows that these equations bring an enormous increase in the calculation speed. When using the backward equations $T(p,s)$ and $T(p,h)$ for regions 1 and 2 (these backward equations were developed along with the basic equations of IAPWS-IF97), the calculation of properties in these regions as functions of (p,s) and (p,h) is between 11 and 38 times faster than calculating these properties by iteration from the respective basic equation.

A further essential step towards even shorter computing times was made by the supplementary backward equations developed after 1997. When using these equations, the calculation of properties dependent on (h,s) in regions 1 and 2 is more than 35 times faster than iterating the basic equations. For region 3, the calculation speed is increased by a factor of more than 10 for calculating properties as functions of (p,h) , (p,s) , and (h,s) with the help of the respective backward equations. The calculation of properties as a function of (p,T) using the backward equations $v(p,T)$ in combination with the basic equation $f_3(\rho,T)$ is 17 times faster than determining these properties only by iteration from the basic equation. In the part of the two-phase region 4 that is important for designing steam turbines, the calculation of the saturation properties p_s , T_s , and the vapour fraction x as a function of (h,s) from backward equations is 14 times faster than the determination of these properties by iteration with the corresponding basic equations. Thus, the new backward equations allow a significant increase in the calculation speed.

2.2 Basic Equations of IAPWS-IF97

This section contains all of the details relevant for using the basic equations of IAPWS-IF97. Figure 2.2 shows the assignment of the five basic equations to the corresponding regions. The boundaries of the regions can be taken directly from Fig. 2.2 except for the boundary between regions 2 and 3; this boundary is defined by the so-called B23-equation given in Sec. 2.2.1.

The boundary $T = 623.15$ K belongs to regions 1 and 3, the boundary corresponding to the p_{B23} -line (the T_{B23} -line is exactly the same line, see Eqs. (2.1) and (2.2)) belongs to regions 2 and 3, and the boundary $T = 1073.15$ K belongs to regions 2 and 5. Thus, the properties along these boundaries could be calculated from equations $g_1(p, T)$ or $f_3(\rho, T)$ on the boundary $T = 623.15$ K, from equations $g_2(p, T)$ or $f_3(\rho, T)$ on the boundary $p_{B23}(T)$, and from equations $g_2(p, T)$ or $g_5(p, T)$ on the boundary $T = 1073.15$ K. In this way, on these boundaries one gets (slightly) different values from the g_1 and f_3 equations, from the g_2 and f_3 equations, and from the g_2 and g_5 equations. In order to avoid such ambiguities, the boundary $T = 623.15$ K is considered to belong to region 1, and the boundaries $p_{B23}(T)$ and $T = 1073.15$ K are considered to belong to region 2. Thus, the properties along these boundaries can be calculated unambiguously from the g_1 and g_2 equations, respectively.

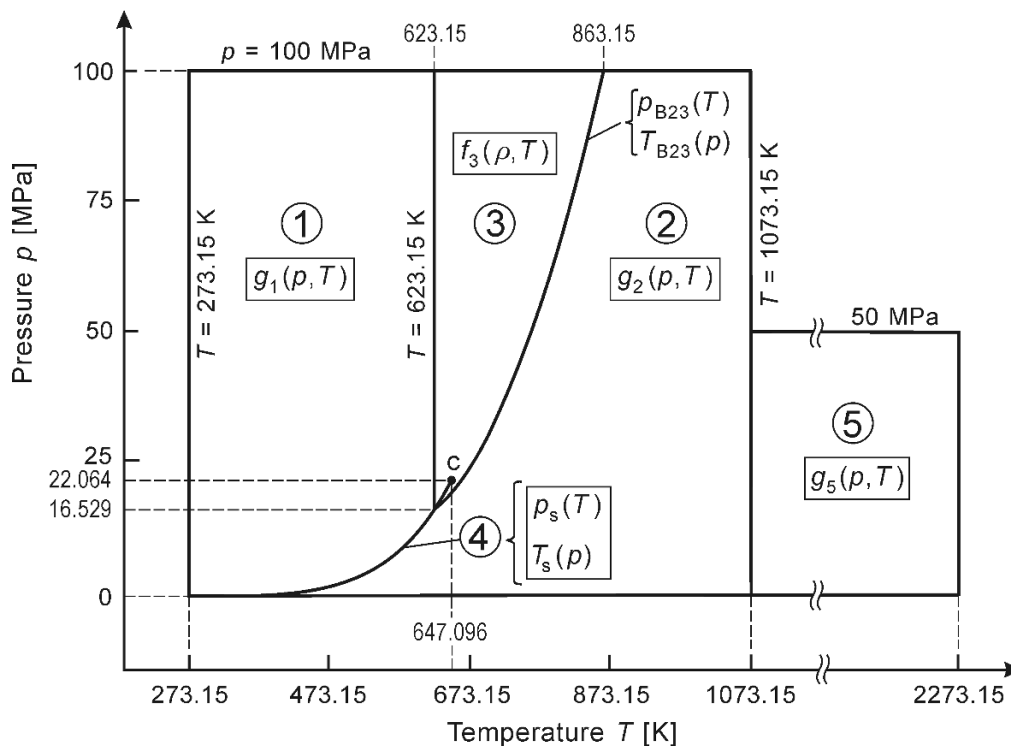


Fig. 2.2 The assignment of the basic equations to the five regions of IAPWS-IF97.

Although the saturation-temperature equation $T_s(p)$ is formally a backward equation, see Sec. 2.1.1 and [10, 15], it is nevertheless included in this section because it was derived from the same implicit quadratic equation for the saturation line, Eq. (2.12), as the saturation-pressure

equation $p_s(T)$, and is, in contrast to the “normal” backward equations given in Sec. 2.3, completely consistent with the $p_s(T)$ equation. Thus, from here onwards the saturation-temperature equation $T_s(p)$ is dealt with like a basic equation.

When using only the basic equations for the calculation of any thermodynamic property as a function of any of the most important combinations of input variables other than (p, T) , e.g. (p, h) , (p, s) , and (h, s) , due to the necessary iterations, the calculation is clearly slower than the calculation via the backward equations, but (within the iteration accuracy) consistent with all properties at the point fixed by the two input variables selected.

Uncertainty estimates of the most relevant properties, calculated from the IAPWS-IF97 basic equations, are summarized in Secs. 2.5.1 and 2.5.2. The inconsistencies between the corresponding basic equations along the boundaries between regions 1 and 3, regions 2 and 3, and regions 2 and 5 are given in Sec. 2.5.3.

Note. The user should be aware of these inconsistencies, in particular when calculating across and very near the region boundaries.

2.2.1 Auxiliary Equation for the Boundary between Regions 2 and 3

The boundary between regions 2 and 3, see Fig. 2.2, is defined by the following simple quadratic pressure-temperature relation (the B23-equation):

$$\frac{p_{B23}(T)}{p^*} = \pi(\theta) = n_1 + n_2\theta + n_3\theta^2, \quad (2.1)$$

where $\pi = p/p^*$ and $\theta = T/T^*$ with $p^* = 1$ MPa and $T^* = 1$ K. The coefficients n_1 to n_3 of Eq. (2.1) are listed in Table 2.1. Equation (2.1) roughly describes an isentropic line; the entropy values along this boundary line are between $s = 5.047$ kJ kg⁻¹ K⁻¹ and $s = 5.261$ kJ kg⁻¹ K⁻¹.

Alternatively, Eq. (2.1) can be expressed explicitly in temperature as

$$\frac{T_{B23}(p)}{T^*} = \theta(\pi) = n_4 + [(\pi - n_5) / n_3]^{0.5} \quad (2.2)$$

with θ and π as defined for Eq. (2.1) and the coefficients n_3 to n_5 listed in Table 2.1. Equations (2.1) and (2.2) cover the range from 623.15 K at 16.5292 MPa up to 863.15 K at 100 MPa.

Table 2.1 Coefficients of the equations $p_{B23}(p)$ and $T_{B23}(T)$, Eqs. (2.1) and (2.2)

i	n_i	i	n_i
1	$0.348\ 051\ 856\ 289\ 69 \times 10^3$	4	$0.572\ 544\ 598\ 627\ 46 \times 10^3$
2	$-0.116\ 718\ 598\ 799\ 75 \times 10^1$	5	$0.139\ 188\ 397\ 788\ 70 \times 10^2$
3	$0.101\ 929\ 700\ 393\ 26 \times 10^{-2}$		

Computer-Program Verification. Eqs. (2.1) and (2.2) must meet the following T - p point:
 $T = 0.623\ 150\ 000 \times 10^3$ K, $p = 0.165\ 291\ 642\ 5 \times 10^2$ MPa.

2.2.2 Basic Equation for Region 1

This section contains all details relevant for using the basic equation for region 1 of IAPWS-IF97, see Fig. 2.2. Uncertainty estimates of the most relevant properties calculated from IAPWS-IF97 can be found in Sec. 2.5.

The basic equation for this region is a fundamental equation for the specific Gibbs free energy g . This equation is expressed in dimensionless form, $\gamma = g/(RT)$, and reads

$$\frac{g_1(p, T)}{RT} = \gamma(\pi, \tau) = \sum_{i=1}^{34} n_i (7.1 - \pi)^{I_i} (\tau - 1.222)^{J_i}, \tag{2.3}$$

where $\pi = p/p^*$ and $\tau = T^*/T$ with $p^* = 16.53$ MPa and $T^* = 1386$ K; $R = 0.461526$ kJ kg⁻¹ K⁻¹ according to Eq. (1.1). The coefficients n_i and exponents I_i and J_i of Eq. (2.3) are listed in Table 2.2.

All thermodynamic properties can be derived from Eq. (2.3) by using the appropriate combinations of the dimensionless Gibbs free energy γ and its derivatives. The relations of the relevant thermodynamic properties to γ and its derivatives are summarized in Table 2.3. Moreover, with the information given in Sec. 2.4, particularly with the formulas of Sec. 2.4.1, all partial derivatives formed by the properties p, T, v, u, h, s, g , and f can be easily calculated. All required derivatives of the equation for the dimensionless Gibbs free energy γ , Eq. (2.3), are explicitly given in Table 2.4.

Since the 5th International Conference on the Properties of Steam in London in 1956, the specific internal energy and the specific entropy of the saturated liquid at the triple point have been set equal to zero:

$$u'_t = 0; \quad s'_t = 0. \tag{2.4}$$

In order to meet this condition at the temperature and pressure of the triple point, see Eqs. (1.7) and (1.8), the coefficients n_3 and n_4 in Eq. (2.3) have been adjusted accordingly, which results in a specific enthalpy of the saturated liquid at the triple point given by

$$h'_t = 0.000611783 \text{ kJ kg}^{-1}. \tag{2.5}$$

Table 2.2 Coefficients and exponents of the basic equation $g_1(p, T)$ in its dimensionless form, Eq. (2.3)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	-2	0.146 329 712 131 67	18	2	3	-0.441 418 453 308 46 × 10 ⁻⁵
2	0	-1	-0.845 481 871 691 14	19	2	17	-0.726 949 962 975 94 × 10 ⁻¹⁵
3	0	0	-0.375 636 036 720 40 × 10 ¹	20	3	-4	-0.316 796 448 450 54 × 10 ⁻⁴
4	0	1	0.338 551 691 683 85 × 10 ¹	21	3	0	-0.282 707 979 853 12 × 10 ⁻⁵
5	0	2	-0.957 919 633 878 72	22	3	6	-0.852 051 281 201 03 × 10 ⁻⁹
6	0	3	0.157 720 385 132 28	23	4	-5	-0.224 252 819 080 00 × 10 ⁻⁵
7	0	4	-0.166 164 171 995 01 × 10 ⁻¹	24	4	-2	-0.651 712 228 956 01 × 10 ⁻⁶
8	0	5	0.812 146 299 835 68 × 10 ⁻³	25	4	10	-0.143 417 299 379 24 × 10 ⁻¹²
9	1	-9	0.283 190 801 238 04 × 10 ⁻³	26	5	-8	-0.405 169 968 601 17 × 10 ⁻⁶
10	1	-7	-0.607 063 015 658 74 × 10 ⁻³	27	8	-11	-0.127 343 017 416 41 × 10 ⁻⁸
11	1	-1	-0.189 900 682 184 19 × 10 ⁻¹	28	8	-6	-0.174 248 712 306 34 × 10 ⁻⁹
12	1	0	-0.325 297 487 705 05 × 10 ⁻¹	29	21	-29	-0.687 621 312 955 31 × 10 ⁻¹⁸

Continued on next page.

Table 2.2 – Continued

i	I_i	J_i	n_i	i	I_i	J_i	n_i
13	1	1	$-0.218\ 417\ 171\ 754\ 14 \times 10^{-1}$	30	23	-31	$0.144\ 783\ 078\ 285\ 21 \times 10^{-19}$
14	1	3	$-0.528\ 383\ 579\ 699\ 30 \times 10^{-4}$	31	29	-38	$0.263\ 357\ 816\ 627\ 95 \times 10^{-22}$
15	2	-3	$-0.471\ 843\ 210\ 732\ 67 \times 10^{-3}$	32	30	-39	$-0.119\ 476\ 226\ 400\ 71 \times 10^{-22}$
16	2	0	$-0.300\ 017\ 807\ 930\ 26 \times 10^{-3}$	33	31	-40	$0.182\ 280\ 945\ 814\ 04 \times 10^{-23}$
17	2	1	$0.476\ 613\ 939\ 069\ 87 \times 10^{-4}$	34	32	-41	$-0.935\ 370\ 872\ 924\ 58 \times 10^{-25}$

Table 2.3 Relations of thermodynamic properties to the dimensionless Gibbs free energy γ and its derivatives when using Eq. (2.3)

Property	Relation
Specific volume $v = (\partial g / \partial p)_T$	$v(\pi, \tau) \frac{p}{RT} = \pi \gamma_\pi$
Specific enthalpy $h = g - T(\partial g / \partial T)_p$	$\frac{h(\pi, \tau)}{RT} = \tau \gamma_\tau$
Specific internal energy $u = g - T(\partial g / \partial T)_p - p(\partial g / \partial p)_T$	$\frac{u(\pi, \tau)}{RT} = \tau \gamma_\tau - \pi \gamma_\pi$
Specific entropy $s = -(\partial g / \partial T)_p$	$\frac{s(\pi, \tau)}{R} = \tau \gamma_\tau - \gamma$
Specific isobaric heat capacity $c_p = (\partial h / \partial T)_p$	$\frac{c_p(\pi, \tau)}{R} = -\tau^2 \gamma_{\tau\tau}$
Specific isochoric heat capacity $c_v = (\partial u / \partial T)_v$	$\frac{c_v(\pi, \tau)}{R} = -\tau^2 \gamma_{\tau\tau} + \frac{(\gamma_\pi - \tau \gamma_{\pi\tau})^2}{\gamma_{\pi\pi}}$
Speed of sound $w = v(-(\partial p / \partial v)_s)^{0.5}$	$\frac{w^2(\pi, \tau)}{RT} = \frac{\gamma_\pi^2}{\frac{(\gamma_\pi - \tau \gamma_{\pi\tau})^2}{\tau^2 \gamma_{\tau\tau}} - \gamma_{\pi\pi}}$
Isobaric cubic expansion coefficient $\alpha_v = v^{-1}(\partial v / \partial T)_p$	$\alpha_v(\pi, \tau) T = 1 - \frac{\tau \gamma_{\pi\tau}}{\gamma_\pi}$
Isothermal compressibility $\kappa_T = -v^{-1}(\partial v / \partial p)_T$	$\kappa_T(\pi, \tau) p = -\frac{\pi \gamma_{\pi\pi}}{\gamma_\pi}$
$\gamma_\pi = \left(\frac{\partial \gamma}{\partial \pi} \right)_\tau, \quad \gamma_{\pi\pi} = \left(\frac{\partial^2 \gamma}{\partial \pi^2} \right)_\tau, \quad \gamma_\tau = \left(\frac{\partial \gamma}{\partial \tau} \right)_\pi, \quad \gamma_{\tau\tau} = \left(\frac{\partial^2 \gamma}{\partial \tau^2} \right)_\pi, \quad \gamma_{\pi\tau} = \left(\frac{\partial^2 \gamma}{\partial \pi \partial \tau} \right)$	

Table 2.4 The dimensionless Gibbs free energy γ Eq. (2.3), and its derivatives

$$\begin{aligned}
\gamma &= \sum_{i=1}^{34} n_i (7.1 - \pi)^{I_i} (\tau - 1.222)^{J_i} & \gamma_\tau &= \sum_{i=1}^{34} n_i (7.1 - \pi)^{I_i} J_i (\tau - 1.222)^{J_i - 1} \\
\gamma_\pi &= \sum_{i=1}^{34} -n_i I_i (7.1 - \pi)^{I_i - 1} (\tau - 1.222)^{J_i} & \gamma_{\tau\tau} &= \sum_{i=1}^{34} n_i (7.1 - \pi)^{I_i} J_i (J_i - 1) (\tau - 1.222)^{J_i - 2} \\
\gamma_{\pi\pi} &= \sum_{i=1}^{34} n_i I_i (I_i - 1) (7.1 - \pi)^{I_i - 2} (\tau - 1.222)^{J_i} & \gamma_{\pi\tau} &= \sum_{i=1}^{34} -n_i I_i (7.1 - \pi)^{I_i - 1} J_i (\tau - 1.222)^{J_i - 1}
\end{aligned}$$

$$\gamma_\pi = \left(\frac{\partial \gamma}{\partial \pi} \right)_\tau, \quad \gamma_{\pi\pi} = \left(\frac{\partial^2 \gamma}{\partial \pi^2} \right)_\tau, \quad \gamma_\tau = \left(\frac{\partial \gamma}{\partial \tau} \right)_\pi, \quad \gamma_{\tau\tau} = \left(\frac{\partial^2 \gamma}{\partial \tau^2} \right)_\pi, \quad \gamma_{\pi\tau} = \left(\frac{\partial^2 \gamma}{\partial \pi \partial \tau} \right)$$

Range of Validity. Equation (2.3) covers region 1 of IAPWS-IF97 defined by the following range of temperature and pressure, see Fig. 2.2:

$$273.15 \text{ K} \leq T \leq 623.15 \text{ K} \quad p_s(T) \leq p \leq 100 \text{ MPa} .$$

In addition to the properties in the stable single-phase liquid region, Eq. (2.3) also yields reasonable values in the metastable superheated-liquid region close to the saturated-liquid line.

Computer-Program Verification. To assist the user in computer-program verification of Eq. (2.3), Table 2.5 contains test values of the most relevant properties.

Table 2.5 Thermodynamic property values calculated from the basic equation $g_1(p, T)$, Eq. (2.3), for selected temperatures and pressures ^a

Property	$T = 300 \text{ K}$ $p = 3 \text{ MPa}$	$T = 300 \text{ K}$ $p = 80 \text{ MPa}$	$T = 500 \text{ K}$ $p = 3 \text{ MPa}$
$v [\text{m}^3 \text{ kg}^{-1}]$	$0.100\ 215\ 168 \times 10^{-2}$	$0.971\ 180\ 894 \times 10^{-3}$	$0.120\ 241\ 800 \times 10^{-2}$
$h [\text{kJ kg}^{-1}]$	$0.115\ 331\ 273 \times 10^3$	$0.184\ 142\ 828 \times 10^3$	$0.975\ 542\ 239 \times 10^3$
$u [\text{kJ kg}^{-1}]$	$0.112\ 324\ 818 \times 10^3$	$0.106\ 448\ 356 \times 10^3$	$0.971\ 934\ 985 \times 10^3$
$s [\text{kJ kg}^{-1} \text{ K}^{-1}]$	0.392 294 792	0.368 563 852	$0.258\ 041\ 912 \times 10^1$
$c_p [\text{kJ kg}^{-1} \text{ K}^{-1}]$	$0.417\ 301\ 218 \times 10^1$	$0.401\ 008\ 987 \times 10^1$	$0.465\ 580\ 682 \times 10^1$
$c_v [\text{kJ kg}^{-1} \text{ K}^{-1}]$	$0.412\ 120\ 160 \times 10^1$	$0.391\ 736\ 606 \times 10^1$	$0.322\ 139\ 223 \times 10^1$
$w [\text{m s}^{-1}]$	$0.150\ 773\ 921 \times 10^4$	$0.163\ 469\ 054 \times 10^4$	$0.124\ 071\ 337 \times 10^4$
$\alpha_v [\text{K}^{-1}]$	$0.277\ 354\ 533 \times 10^{-3}$	$0.344\ 095\ 843 \times 10^{-3}$	$0.164\ 118\ 128 \times 10^{-2}$
$\kappa_T [\text{MPa}^{-1}]$	$0.446\ 382\ 123 \times 10^{-3}$	$0.372\ 039\ 437 \times 10^{-3}$	$0.112\ 892\ 188 \times 10^{-2}$

^a Programmed functions should be verified using 8 byte real values for all variables.

2.2.3 Basic Equation and Supplementary Equation for Region 2

This section contains all details relevant for using the basic equation of region 2 of IAPWS-IF97, see Fig. 2.2. The B23-equation for defining the boundary between regions 2 and 3 is given in Sec. 2.2.1. Uncertainty estimates of the most relevant properties calculated from IAPWS-IF97 can be found in Sec. 2.5.

2.2.3.1 Basic Equation

The basic equation for this region is a fundamental equation for the specific Gibbs free energy g . This equation is expressed in dimensionless form, $\gamma = g/(RT)$, and is separated into two parts, an ideal-gas part γ^0 and a residual part γ^r , so that it reads

$$\frac{g_2(p, T)}{RT} = \gamma(\pi, \tau) = \gamma^0(\pi, \tau) + \gamma^r(\pi, \tau) , \quad (2.6)$$

where $\pi = p/p^*$ and $\tau = T^*/T$ with $R = 0.461\,526\text{ kJ kg}^{-1}\text{ K}^{-1}$ given by Eq. (1.1), and γ^0 and γ^r according to Eqs. (2.7) and (2.8).

The equation for the dimensionless ideal-gas part γ^0 of the basic equation $g_2(p, T)$ reads

$$\gamma^0(\pi, \tau) = \ln \pi + \sum_{i=1}^9 n_i^0 \tau^{J_i^0} , \quad (2.7)$$

where $\pi = p/p^*$ and $\tau = T^*/T$ with $p^* = 1\text{ MPa}$ and $T^* = 540\text{ K}$. The coefficients n_1^0 and n_2^0 were adjusted in such a way that the values for the specific internal energy and specific entropy, calculated from Eq. (2.6), correspond to Eq. (2.4). Table 2.6 contains the coefficients n_i^0 and exponents J_i^0 of Eq. (2.7).

Table 2.6 Coefficients and exponents of the ideal-gas part γ^0 , Eq. (2.7)

i	J_i^0	n_i^0	i	J_i^0	n_i^0
1	0	$-0.969\,276\,865\,002\,17 \times 10^1$ ^a	6	-2	$0.142\,408\,191\,714\,44 \times 10^1$
2	1	$0.100\,866\,559\,680\,18 \times 10^2$ ^a	7	-1	$-0.438\,395\,113\,194\,50 \times 10^1$
3	-5	$-0.560\,879\,112\,830\,20 \times 10^{-2}$	8	2	$-0.284\,086\,324\,607\,72$
4	-4	$0.714\,527\,380\,814\,55 \times 10^{-1}$	9	3	$0.212\,684\,637\,533\,07 \times 10^{-1}$
5	-3	$-0.407\,104\,982\,239\,28$			

^a If Eq. (2.7) is incorporated into Eq. (2.9), instead of the values for n_1^0 and n_2^0 given above, the following values for these two coefficients must be used: $n_1^0 = -0.969\,372\,683\,930\,49 \times 10^1$, $n_2^0 = 0.100\,872\,759\,700\,06 \times 10^2$.

The form of the dimensionless residual part γ^r of the basic equation $g_2(p, T)$ is as follows:

$$\gamma^r(\pi, \tau) = \sum_{i=1}^{43} n_i \pi^{I_i} (\tau - 0.5)^{J_i} , \quad (2.8)$$

where $\pi = p/p^*$ and $\tau = T^*/T$ with $p^* = 1\text{ MPa}$ and $T^* = 540\text{ K}$. The coefficients n_i and exponents I_i and J_i of Eq. (2.8) are listed in Table 2.7.

All thermodynamic properties can be derived from Eq.(2.6) by using the appropriate combinations of the ideal-gas part γ^0 , Eq.(2.7), and the residual part γ^r , Eq.(2.8), of the dimensionless Gibbs free energy and their derivatives. The relations of the relevant thermodynamic properties to γ^0 and γ^r and their derivatives are summarized in Table 2.8. Moreover, with the information given in Sec. 2.4, particularly with the formulas of Sec. 2.4.1, all partial derivatives formed by the properties p , T , v , u , h , s , g , and f can be very easily calculated. All required derivatives of the equations for γ^0 and γ^r are explicitly given in Table 2.9 and Table 2.10, respectively.

Table 2.7 Coefficients and exponents of the residual part γ^f , Eq. (2.8)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	1	0	$-0.177\ 317\ 424\ 732\ 13 \times 10^{-2}$	23	7	0	$-0.590\ 595\ 643\ 242\ 70 \times 10^{-17}$
2	1	1	$-0.178\ 348\ 622\ 923\ 58 \times 10^{-1}$	24	7	11	$-0.126\ 218\ 088\ 991\ 01 \times 10^{-5}$
3	1	2	$-0.459\ 960\ 136\ 963\ 65 \times 10^{-1}$	25	7	25	$-0.389\ 468\ 424\ 357\ 39 \times 10^{-1}$
4	1	3	$-0.575\ 812\ 590\ 834\ 32 \times 10^{-1}$	26	8	8	$0.112\ 562\ 113\ 604\ 59 \times 10^{-10}$
5	1	6	$-0.503\ 252\ 787\ 279\ 30 \times 10^{-1}$	27	8	36	$-0.823\ 113\ 408\ 979\ 98 \times 10^1$
6	2	1	$-0.330\ 326\ 416\ 702\ 03 \times 10^{-4}$	28	9	13	$0.198\ 097\ 128\ 020\ 88 \times 10^{-7}$
7	2	2	$-0.189\ 489\ 875\ 163\ 15 \times 10^{-3}$	29	10	4	$0.104\ 069\ 652\ 101\ 74 \times 10^{-18}$
8	2	4	$-0.393\ 927\ 772\ 433\ 55 \times 10^{-2}$	30	10	10	$-0.102\ 347\ 470\ 959\ 29 \times 10^{-12}$
9	2	7	$-0.437\ 972\ 956\ 505\ 73 \times 10^{-1}$	31	10	14	$-0.100\ 181\ 793\ 795\ 11 \times 10^{-8}$
10	2	36	$-0.266\ 745\ 479\ 140\ 87 \times 10^{-4}$	32	16	29	$-0.808\ 829\ 086\ 469\ 85 \times 10^{-10}$
11	3	0	$0.204\ 817\ 376\ 923\ 09 \times 10^{-7}$	33	16	50	$0.106\ 930\ 318\ 794\ 09$
12	3	1	$0.438\ 706\ 672\ 844\ 35 \times 10^{-6}$	34	18	57	$-0.336\ 622\ 505\ 741\ 71$
13	3	3	$-0.322\ 776\ 772\ 385\ 70 \times 10^{-4}$	35	20	20	$0.891\ 858\ 453\ 554\ 21 \times 10^{-24}$
14	3	6	$-0.150\ 339\ 245\ 421\ 48 \times 10^{-2}$	36	20	35	$0.306\ 293\ 168\ 762\ 32 \times 10^{-12}$
15	3	35	$-0.406\ 682\ 535\ 626\ 49 \times 10^{-1}$	37	20	48	$-0.420\ 024\ 676\ 982\ 08 \times 10^{-5}$
16	4	1	$-0.788\ 473\ 095\ 593\ 67 \times 10^{-9}$	38	21	21	$-0.590\ 560\ 296\ 856\ 39 \times 10^{-25}$
17	4	2	$0.127\ 907\ 178\ 522\ 85 \times 10^{-7}$	39	22	53	$0.378\ 269\ 476\ 134\ 57 \times 10^{-5}$
18	4	3	$0.482\ 253\ 727\ 185\ 07 \times 10^{-6}$	40	23	39	$-0.127\ 686\ 089\ 346\ 81 \times 10^{-14}$
19	5	7	$0.229\ 220\ 763\ 376\ 61 \times 10^{-5}$	41	24	26	$0.730\ 876\ 105\ 950\ 61 \times 10^{-28}$
20	6	3	$-0.167\ 147\ 664\ 510\ 61 \times 10^{-10}$	42	24	40	$0.554\ 147\ 153\ 507\ 78 \times 10^{-16}$
21	6	16	$-0.211\ 714\ 723\ 213\ 55 \times 10^{-2}$	43	24	58	$-0.943\ 697\ 072\ 412\ 10 \times 10^{-6}$
22	6	35	$-0.238\ 957\ 419\ 341\ 04 \times 10^2$				

Range of Validity. Equation (2.6) covers region 2 of IAPWS-IF97 defined by the following range of temperature and pressure, see Fig. 2.2:

$$\begin{aligned}
 273.15\ \text{K} &\leq T \leq 623.15\ \text{K} & 0 < p \leq p_s(T) \\
 623.15\ \text{K} &< T \leq 863.15\ \text{K} & 0 < p \leq p_{\text{B}23}(T) \\
 863.15\ \text{K} &< T \leq 1073.15\ \text{K} & 0 < p \leq 100\ \text{MPa},
 \end{aligned}$$

where $p_s(T)$ is calculated from Eq. (2.13) and $p_{\text{B}23}(T)$ from Eq. (2.1). In addition to the properties in the stable single-phase vapour region, Eq. (2.6) also yields reasonable values in the metastable-vapour region for pressures above 10 MPa. Equation (2.6) is not valid in the metastable-vapour region at pressures $p \leq 10$ MPa; for this part of the metastable-vapour region see Sec. 2.2.3.2.

Table 2.8 Relations of thermodynamic properties to the ideal-gas part γ^o and the residual part γ^r of the dimensionless Gibbs free energy and their derivatives when using Eq. (2.6) or Eq. (2.9)

Property	Relation
Specific volume $v = (\partial g / \partial p)_T$	$v(\pi, \tau) \frac{P}{RT} = \pi(\gamma_\pi^o + \gamma_\pi^r)$
Specific enthalpy $h = g - T(\partial g / \partial T)_p$	$\frac{h(\pi, \tau)}{RT} = \tau(\gamma_\tau^o + \gamma_\tau^r)$
Specific internal energy $u = g - T(\partial g / \partial T)_p - p(\partial g / \partial p)_T$	$\frac{u(\pi, \tau)}{RT} = \tau(\gamma_\tau^o + \gamma_\tau^r) - \pi(\gamma_\pi^o + \gamma_\pi^r)$
Specific entropy $s = -(\partial g / \partial T)_p$	$\frac{s(\pi, \tau)}{R} = \tau(\gamma_\tau^o + \gamma_\tau^r) - (\gamma^o + \gamma^r)$
Specific isobaric heat capacity $c_p = (\partial h / \partial T)_p$	$\frac{c_p(\pi, \tau)}{R} = -\tau^2(\gamma_{\tau\tau}^o + \gamma_{\tau\tau}^r)$
Specific isochoric heat capacity $c_v = (\partial u / \partial T)_v$	$\frac{c_v(\pi, \tau)}{R} = -\tau^2(\gamma_{\tau\tau}^o + \gamma_{\tau\tau}^r) - \frac{(1 + \pi\gamma_\pi^r - \tau\pi\gamma_{\pi\tau}^r)^2}{1 - \pi^2\gamma_{\pi\pi}^r}$
Speed of sound $w = v(-\partial p / \partial v_s)^{0.5}$	$\frac{w^2(\pi, \tau)}{RT} = \frac{1 + 2\pi\gamma_\pi^r + \pi^2\gamma_\pi^{r2}}{(1 - \pi^2\gamma_{\pi\pi}^r) + \frac{(1 + \pi\gamma_\pi^r - \tau\pi\gamma_{\pi\tau}^r)^2}{\tau^2(\gamma_{\tau\tau}^o + \gamma_{\tau\tau}^r)}}$
Isobaric cubic expansion coefficient $\alpha_v = v^{-1}(\partial v / \partial T)_p$	$\alpha_v(\pi, \tau)T = \frac{1 + \pi\gamma_\pi^r - \tau\pi\gamma_{\pi\tau}^r}{1 + \pi\gamma_\pi^r}$
Isothermal compressibility $\kappa_T = -v^{-1}(\partial v / \partial p)_T$	$\kappa_T(\pi, \tau)p = \frac{1 - \pi^2\gamma_{\pi\pi}^r}{1 + \pi\gamma_\pi^r}$
$\gamma_\pi^r = \left(\frac{\partial \gamma^r}{\partial \pi}\right)_\tau$, $\gamma_{\pi\pi}^r = \left(\frac{\partial^2 \gamma^r}{\partial \pi^2}\right)_\tau$, $\gamma_\tau^r = \left(\frac{\partial \gamma^r}{\partial \tau}\right)_\pi$, $\gamma_{\tau\tau}^r = \left(\frac{\partial^2 \gamma^r}{\partial \tau^2}\right)_\pi$, $\gamma_{\pi\tau}^r = \left(\frac{\partial^2 \gamma^r}{\partial \pi \partial \tau}\right)_\pi$, $\gamma_\tau^o = \left(\frac{\partial \gamma^o}{\partial \tau}\right)_\pi$, $\gamma_{\tau\tau}^o = \left(\frac{\partial^2 \gamma^o}{\partial \tau^2}\right)_\pi$	