Piezoelectric Transducers and Applications

Antonio Arnau Vives (Ed.)

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Second Edition

Springer

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ISBN: 978-3-540-77507-2 e-ISBN: 978-3-540-77508-9

DOI: 10.1007/978-3-540-77508-9

Library of Congress Control Number: 2008922498

Cover design: eStudio Calamar Girona, Spain

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

Foreword

Since the publication of the first edition, the richness of the study of piezoelectric transducers has resulted in a large number of studies dealing both with new understandings underlying the principles, with new technological advances in its applications and indeed with developing new areas of utility for these transducers. The motivations driving the publication of that first edition as described in its foreword (which follows) continues with increased validity. The value of a second edition to include these new developments has been prepared. During the interim, the contributors and their students have not only continued, but increased their mutual interactions resulting in an amazing energy and synergy which is revealed in this edition.

One of the most valuable aids to those beginning to investigate a new area of study is a source which will guide them from beginning principles, through detailed implementation and applications. Even for seasoned investigators, it is useful to have a reasonably detailed discussion of closely related topics in a single volume to which one can refer. This is often difficult for many emerging areas of studies because they are so multidisciplinary. The subject matter of the principles, techniques and applications of piezoelectric transducers certainly fits into this category. The host of emerging new uses of piezoelectric devices that are being commercialized as well as the growing number of potential applications ensures that this field will encompass more and more disciplines with passing time. It is extremely fortunate and timely that this volume becomes available to the student at this time.

Piezoelectricity is a classical discipline traced to the original work of Jacques and Pierre Curie around 1880. This phenomenon describes the relations between mechanical strains on a solid and its resulting electrical behavior resulting from changes in the electric polarization. One can create an electrical output from a solid resulting from mechanical strains, or can create a mechanical distortion resulting from the application of an electrical perturbation. In the former case, the unit acts as a receiver of mechanical variations, converting it into electrical output, as in the case of a microphone. In the latter case, the unit can act as a transmitter converting the electrical signal into a mechanical wave. The piezoelectric units can be used both in narrow-band or resonant modes, and under broad-band regimes for detection and imaging applications. One of the remarkable properties of these devices is the ability to use them in a viscous medium, such as a liquid. When excited sinusoidally, these devices can generate waves in the immersing medium. Typically, as a result of the physical size of these devices, the waves are in the ultrasonic regime. From this classical discipline, an astounding number of applications are developing. From its use as a frequency generating standard in the earlier part of the 20th century, additional uses have seen these devices used as highly sensitive mass balances for use both in vacuum deposition and in electrochemical applications, as well as chemical specific sensors, as Doppler devices for fluid velocity measurements and for ultrasonic imagery. There are many other emerging applications in the bio-sciences for example. The number of applications is astounding.

It is clear that the discipline is inter-disciplinary. The authors of the contents of this book are a select group who has all been challenged by the intellectual diversity of the field. To successfully pass on such diverse information, intellectual competence is only a beginning. A devotion to, and love of clear communication is also required. These authors are members of the PETRA organization, (Piezoelectric Transducers and Applications) sponsored by the European Union, devoted to the collection and dissemination of knowledge and skills in the piezoelectric arts to students among the participating universities in Europe and Latin America. I have personally observed many of the authors interacting with students and have been very impressed by their care and mentoring. Contributions from such dedicated and seasoned teachers are now available to the student in this volume. This book fills a real need for a unified source for information on piezoelectric devices, ranging from broadband applications to resonant applications and will serve both experienced researchers and beginning students well.

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Preface

Following the execution of the project PETRA-I, co-financed by the European Union in the framework of the ALFA Program (America Latina Formación Académica), as coordinator of the PETRA Network (PiezoElectric TRansducers and their Applications), I edited the first edition of this book. Now, four years later, I am submitting the manuscript of this revised and enlarged 2nd edition.

This edition has been, in fact, an unexpected result of the project PETRA-II, also co-financed by the European Union in the framework of the ALFA Program. Effectively, halfway through the execution of this project, Springer-Verlag informed me of the good reception that the book had received and asked me if we had thought about a new edition. This very good news meant that the work during the previous four years had been worth it. The initial idea of collecting in one single volume a set of "tutorials" covering topics spread on different disciplines and linked by the use of piezoelectric devices was therefore useful. The interdisciplinary character of the discipline was made clear and the "tutorial" based format could be useful as a guide for doctoral degree students and even researchers going into this complex and multidisciplinary issue. Now we have the opportunity of improving that first approach but without losing what we think are the keys of its success: the "tutorial" style and the multidisciplinary character of the contents.

The new edition covers, in 18 chapters and two appendices, different aspects of piezoelectric devices and their applications, as well as fundamental topics of related disciplines. The contents were selected according to the different areas of research of the partners of the PETRA Network; therefore, this book does not intend to be an encyclopaedia on piezoelectric transducers and their applications, which would be completely impossible in a single-volume work. Three different parts, although not explicitly separated, can be distinguished in the book: one part corresponds to general concepts on piezoelectric devices and to the fundamentals of related topics (Chapters 1,2, 7-12 and the two appendices), another part deals with piezoelectric sensors and related applications (Chapters 1,3,5,12-14) and the other part focuses on ultrasonic transducers and systems and related applications (Chapters 4,6, 15-18).

Basic concepts of piezoelectricity are presented in Chap. 1 along with an introduction into the field of microgravimetric sensors; appendices A and B, at the end of the book, include fundamental concepts of electrostatics and physical properties of crystals which complement this initial introduction. Chapter 2 offers an overview of acoustic sensors, their basic principles of operation, the different types and their potential applications. Recent new excitation principles for bulk acoustic wave sensors such as lateral field or magnetic excitations have been added in this edition, as well as the topic of micromachined resonators such as cantilevers (MEMS) based on silicon technologies which are attracting current interest.

Chapters 3, 5, 13 and 14 delve more deeply into resonant sensors, especially bulk acoustic wave thickness shear mode resonators and their applications as quartz crystal microbalance sensors, their fundamentals and models (Chap. 3), electronic interfaces and associated problems (Chap. 5), the problems associated with the analysis and interpretation of experimental data (Chap. 14) and complementary techniques used with QCM (Chap. 13). In this 2nd edition, a thorough revision of these chapters with the addition of some important topics has been made. Sub-chapters dealing with the gravimetric and non-gravimetric regimes in QCM applications and the important aspect of kinetic analysis in acoustic wave sensor-based chemical applications have been added to Chap. 3. A comprehensive review of the different electronic interfaces for QCM sensors has been included in Chap. 5; in particular the topic of oscillators for in-liquid QCM applications is deeply treated in this edition, as well as the new interface systems based on lock-in techniques. Techniques based on impedance analysis, or adapted impedance analyzers, and decay method techniques have also been updated and interfaces for fast QCM applications, such as ac-electrogravimetry (Chap. 13), have also been included. The problem of compatibility between QCM and electrochemical set-ups is treated in Chap. 13. Chapter 14 has been thoroughly revised and a completely new section with case studies has been added to complement the complex aspect of data analysis and interpretation in real experiments. The section devoted to “other effects”, which complicates even more the interpretation of results, has been extended with the inclusion of the roughness effect.

As the case studies section in Chap. 14 makes clear, acoustic wave sensors are involved in applications such as biosensors, electrochemistry and polymer properties’ characterization, which require a minimum background to deal with. This background is intended to be given in Chaps. 7-12. Thus, Chap. 7 introduces the concept of viscoelasticity and describes in depth the physical properties of polymers. A very important aspect in resonant sensor applications is the shear parameter determination that has been added as a new subchapter in this tutorial.

Chapter 8 introduces the fundamentals of electrochemistry; in relation to the first edition, the section on “What is an electrode reaction?” has been extended with more explanation on the process of electron transfer and a corresponding schematic figure. The section on “Rates of electrode reactions” now includes a paragraph and figure describing the important role of the interfacial region and the definition of Faraday’s law. Additionally, the section on electrochemical techniques has been significantly enlarged with respect to steady-state, pulse and impedance techniques. The final section shows the range of possible applications of electrochemistry.

Chapter 9 provides an overview of chemical sensors, which is of great interest for establishing the differences between chemical sensors based on piezoelectric transducers and those based on other techniques such as electrochemical, optical, calorimetric, conductimetric (added in this version) and magnetic techniques, with the aim of facilitating the interpretation of the different data. Chapter 10 treats the specific topic of biosensors from a biological point of view; this treatment is specifically useful to understand the mechanism of biological recognition and its potential use for the development of biosensors and especially for piezoelectric biosensors, which is a field of much current interest. A new chapter (Chap. 12) has been added which introduces the fundamentals of piezoelectric immunosensors giving the basic schemes of biosensor functioning, immunoassay formats, and the principle of competitive immunoassay. The different steps involved in the production and immobilization of immunoreagents are treated in detail in this chapter which finishes with a real example of characterization of a piezoelectric immunosensor.

The processes involved in a piezoelectric immunosensor make clear the necessity of the resonator sensor surface modification. This topic is treated in depth in Chap. 11 which provides a guide to the important subject of modification of piezoelectric surfaces in piezoelectric transducers for sensor applications. Some additional examples have been added in this new version.

Chapters 4, 6, 15-18 deal with ultrasonic systems and applications. Chapter 4 introduces the basic aspects and the different models of piezoelectric transducers for broadband ultrasonic applications; electronic interfaces used in broadband configurations are introduced in Chap. 6; implementations of ultrasonic schemes and electronic interfaces for non-destructive testing industrial applications are detailed and analysed in Chap. 16, and some applications of ultrasound in chemistry and in medicine are treated in Chaps. 15 (Sonoelectrochemistry), 17 (Medical imaging) and 18 (Ultrasound hyperthermia). In this edition new topics have been added in the previous chapters. In Chap. 4 three sub-chapters have been added dealing with the transfers functions and time responses at emission and reception of the transducer, the acoustic impedance matching and the electrical matching and tuning. Chap. 6 includes a new sub-chapter dealing with the analysis of electrical responses in pulse-driven piezoelectric transducers by means of linear approaches has been added, including the inductive tuning case. In Chap. 16 two new sections have been added dealing with the electronic sequential scanning of ultrasound beams for fast operation in non-destructive testing applications. Chap. 17 is a new chapter added to deal with the application of ultrasound systems for medical imaging and tissue characterization; a basic introduction to the ultrasound properties of biological materials with different ultrasonic imaging modes is followed by a comprehensive review of the different techniques used for medical imaging. Chapter 18 includes a concise introduction to the clinical procedure and biological basis of hyperthermia therapy. In this second edition key information concerning the technical perspective of this treatment has been added: the ultrasound field measurement by the mechanically scanning method is described. In this section, it is explained how a 3D representation of the space domain response of the transducer can be obtained by using a hydrophone. In another section, the way ultrasound produces temperature increases in tissues in described. A description of the components of a general hyperthermia ultrasound system has also been included. Superficial and deep heat systems are also depicted. Finally, the ways in which ultrasound hyperthermia systems are characterized are treated, such as in the preparation and measuring of the properties of a tissue mimicking material (phantom) for use in ultrasonic hyperthermia.

Finally, Chap. 15 deals with the application of ultrasound in electrochemistry. In this edition the sections on basic consequences of ultrasound and on the experimental arrangements have been extended. In the former case, more discussion of the formation of cavitation bubbles and their collapse is included. In the latter case, the horn probes are discussed in more detail. Some more applications are referred to in particular nanomaterials (new sub-section).

The present volume is therefore a revised and enlarged version of the first edition which would not have been possible without the effort and dedication of all my colleagues, who contributed with the different chapters, to all of whom I will always be in debt. I would like to take advantage of this new opportunity to thank them again for giving me their confidence as coordinator of the PETRA group. My thanks also go to Springer for undertaking this new edition.

New challenges are waiting for us in the near future. I hope we will be able to face them enthusiastically and with excitement. The future is a challenge that we pose to our thoughts and makes sense of our lives.

Antonio Arnau Vives
November 2007

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1 Fundamentals of Piezoelectricity

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1.1 Introduction

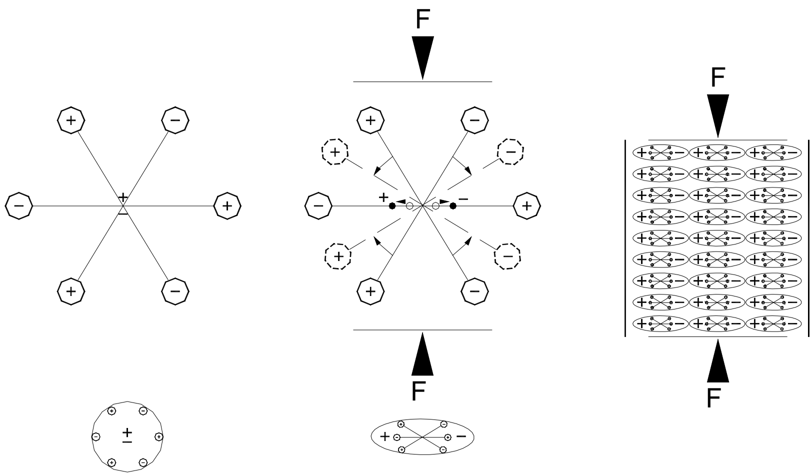
The topic of the following chapter is relatively difficult and includes different areas of knowledge. The piezoelectric phenomenon is a complex one and covers concepts of electronics as well as most of the areas of classical physics such as: mechanics, elasticity and strength of materials, thermodynamics, acoustics, wave's propagation, optics, electrostatics, fluids dynamics, circuit theory, crystallography etc. Probably, only a few disciplines of engineering and science need to be so familiar to so many fields of physics. Current bibliography on this subject is vast though dispersed in research publications, and few of the books on this topic are usually compilations of the authors' research works. Therefore, they are not thought for didactic purposes and are difficult to understand, even for postgraduates. The objective of this chapter is to help understand the studies and research on piezoelectric sensors and transducers, and their applications. Considering the multidisciplinary nature, this tutorial's readers can belong to very different disciplines. They can even lack the necessary basic knowledge to understand the concepts of this chapter. This is why the chapter starts providing an overview of the piezoelectric phenomenon, doing consciously initial simplifications, so that the main concepts, which will be progressively introduced, prevail over the accessories. The issues covered in this chapter must be understood without the help of additional texts, which are typically included as references and are necessary to study in depth specific topics. Finally, the quartz crystal is introduced as a micro-gravimetric sensor to present the reader an application of the piezoelectric phenomenon, which will be dealt with along the following chapters.

A.A. Vives (ed.), *Piezoelectric Transducers and Applications*, doi: 10.1007/978-3-540-77508-9_1,

1.2 The Piezoelectric Effect

The word *Piezoelectricity* comes from Greek and means "electricity by pressure" (*Piezo* means pressure in Greek). This name was proposed by Hankel [1] in 1881 to name the phenomenon discovered a year before by the Pierre and Jacques Curie brothers [2]. They observed that positive and negative charges appeared on several parts of the crystal surfaces when comprising the crystal in different directions, previously analysed according to its symmetry.

Figure 1.1a shows a simple molecular model; it explains the generating of an electric charge as the result of a force exerted on the material. Before subjecting the material to some external stress, the gravity centres of the negative and positive charges of each molecule coincide. Therefore, the external effects of the negative and positive charges are reciprocally cancelled. As a result, an electrically neutral molecule appears. When exerting some pressure on the material, its internal reticular structure can be deformed, causing the separation of the positive and negative gravity centres of the molecules and generating little dipoles (Fig. 1.1b). The facing poles inside the material are mutually cancelled and a distribution of a linked charge appears in the material's surfaces (Fig. 1.1c). That is to say, the material is polarized. This polarization generates an electric field and can be used to transform the mechanical energy used in the material's deformation into electrical energy.



a **b** **c**

Fig. 1.1. Simple molecular model for explaining the piezoelectric effect: **a** unperturbed molecule; **b** molecule subjected to an external force, and ***c*** polarizing effect on the material surfaces

Figure 1.2a shows the piezoelectric material on which a pressure is applied. Two metal plates used as electrodes are deposited on the surfaces where the linked charges of opposite sign appear. Let us suppose that those electrodes are externally short circuited through a wire to which a galvanometer has been connected. When exerting some pressure on the piezoelectric material, a linked charge density appears on the surfaces of the crystal in contact with the electrodes. This polarization generates an electric field which causes the flow of the free charges existing in the conductor. Depending on their sign, the free charges will move towards the ends where the linked charge generated by the crystal's polarization is of opposite sign. This flow of free charges will remain until the free charge neutralizes the polarization effect (Fig. 1.2a). When the pressure on the crystal stops, the polarization will disappear, and the flow of free charges will be reversed, coming back to the initial standstill condition (Fig. 1.2b). This process would be displayed in the galvanometer, which would have marked two opposite sign current peaks. If a resistance is connected instead of a short-circuiting, and a variable pressure is applied, a current would flow through the resistance, and the mechanical energy would be transformed into electrical energy.

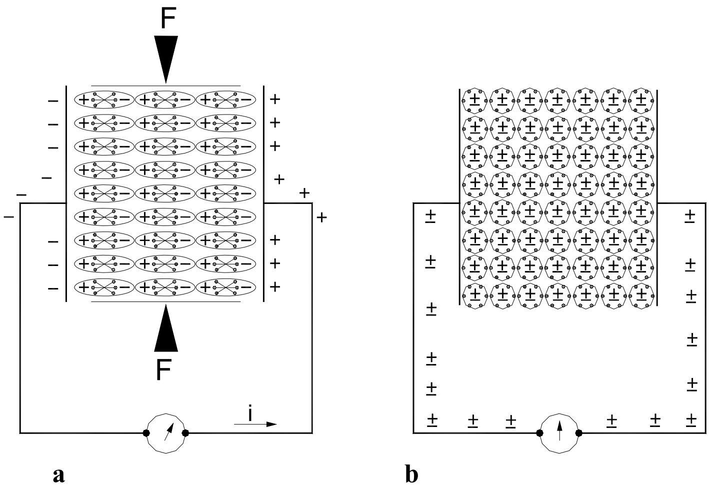


Fig. 1.2. Piezoelectric phenomenon: **a** neutralizing current flowing through the short-circuiting established on a piezoelectric material subjected to an external force; **b** absence of current through the short-circuited material in an unperturbed state

The Curie brothers verified, the year after their discovery, the existence of the reverse process, predicted by Lippmann (1881) [3]. That is, if one arbitrarily names *direct piezoelectric effect*, to the generation of an electric charge, and hence of an electric field, in certain materials and under certain laws due to a stress, there would also exist a *reverse piezoelectric effect* by which the application of an electric field, under similar circumstances, would cause deformation in those materials.

In this sense, a mechanical deformation would be produced in a piezoelectric material when a voltage is applied between the electrodes of the piezoelectric material, as shown in Fig.1.2. This strain could be used, for example, to displace a coupled mechanical load, transforming the electrical energy into mechanical energy.

1.3 Mathematical Formulation of the Piezoelectric Effect. A First Approach

In a first approach, the experiments performed by the Curie brothers demonstrated that the surface density of the generated linked charge was proportional to the pressure exerted, and would disappear with it. This relationship can be formulated in a simple way as follows:

$$P_p = d\,T \tag{1.1}$$

where P_p is the piezoelectric polarization vector, whose magnitude is equal to the linked charge surface density by piezoelectric effect in the considered surface, d is the piezoelectric strain coefficient and T is the stress to which the piezoelectric material is subjected.

The Curie brothers verified the reverse piezoelectric effect and demonstrated that the ratio between the strain produced and the magnitude of the applied electric field in the reverse effect, was equal to the ratio between the produced polarization and the magnitude of the applied stress in the direct effect. Consistently, the reverse piezoelectric effect can be formulated in a simple way, as a first approach, as follows:

$$S_p = d\,E \tag{1.2}$$

where S_p is the strain produced by the piezoelectric effect and E is the magnitude of the applied electric field.

The direct and reverse piezoelectric effects can be alternatively formulated, considering the elastic properties of the material, as follows:

$$P_p = d\,T = d\,c\,S = e\,S \tag{1.3}$$

$$T_p = c\,S_p = c\,d\,E = e\,E \tag{1.4}$$

where c is the elastic constant, which relates the stress generated by the application of a strain ($T = c\,S$), s is the compliance coefficient which relates the deformation produced by the application of a stress ($S = s\,T$), and e is the piezoelectric stress constant. (Note that the polarizations, stresses, and strains caused by the piezoelectric effect have been specified with the p subscript, while those externally applied do not have subscript. Although unnecessary, it will be advantageous later on.

1.4 Piezoelectric Contribution to Elastic Constants

The piezoelectric phenomenon causes an increase of the material's stiffness. To understand this effect, let us suppose that the piezoelectric material is subjected to a strain S. This strain will have two effects. On the one hand, it will generate an elastic stress T_e which will be proportional to the mechanical strain $T_e = c\,S$; on the other hand, it will generate a piezoelectric polarization $P_p = e\,S$ according to Eq. (1.3). This polarization will create an internal electric field in the material E_p given by (see Appendix A):

$$E_p = \frac{P_p}{\varepsilon} = \frac{e\,S}{\varepsilon} \tag{1.5}$$

where ε is the dielectric constant of the material.

This electric field, of piezoelectric origin, produces a force against the deformation of the material's electric structure, creating a stress $T_p = e\,E_p$. This stress, as well as that of elastic origin, is against the material's deformation. Consistently, the stress generated as a consequence of the strain S will be:

$$T = T_e + T_p = c\,S + \frac{e^2}{\varepsilon}S = \left(c + \frac{e^2}{\varepsilon}\right)S = \bar{c}\,S \tag{1.6}$$

Therefore, the constant $\bar{c}$ is the piezoelectrically stiffened constant, which includes the increase in the value of the elastic constant due to the piezoelectric effect. This coefficient will appear later on.

1.5 Piezoelectric Contribution to Dielectric Constants

When an external electric field E is applied between two electrodes where a material of dielectric constant ε exists, an electric displacement is created towards those electrodes, generating a surface charge density $\sigma = \sigma_o + \sigma_p$

which magnitude is $D = \varepsilon E$ [1]. If that material is piezoelectric, the electric field E produces a strain given by: $S_p = d\,E$. This strain of piezoelectric origin increases the surface charge density due to the material's polarization in an amount given by: $P_p = e\,S_p = e\,d\,E$ (Fig. 1.3). Because the electric field is maintained constant, the piezoelectric polarization increases the electric displacement of free charges towards the electrodes in the same magnitude ($\sigma_p = P_p$). Therefore, the total electrical displacement is:

$$D = \varepsilon E + P_p = \varepsilon E + e d\, E = \bar{\varepsilon} E \qquad (1.7)$$

where $\bar{\varepsilon}$ is the effective dielectric constant which includes the piezoelectric contribution.

1.6 The Electric Displacement and the Internal Stress

As shown in the previous paragraph, the electric displacement produced when an electric field E is applied to a piezoelectric and dielectric material is:

$$D = \varepsilon E + P_p = \varepsilon E + e S_p \qquad (1.8)$$

Under the same circumstances we want to obtain the internal stress in the material. The reasoning is the following: the application of an electric field on a piezoelectric material causes a deformation in the material's structure given by: $S_p = d\,E$. This strain produces an elastic stress whose magnitude is $T_e = c\,S_p$. On the other hand, the electric field E exerts a force on the material's internal structure generating a stress given by: $T_p = e\,E$. This stress is, definitely, the one that produces the strain and is of opposite sign to the elastic stress which tends to recover the original structure. Therefore, the internal stress that the material experiences will be the resultant of both. That is:

$$T = c S_p - e E \qquad (1.9)$$

[1] The free charge density which appears on the electrodes, will be the sum of the charge density which appears in vacuum plus the one that appears induced by the dielectric effect, i.e.:

$$\sigma_o + \sigma_d = \varepsilon_o E + \chi E = (\varepsilon_o + \chi) E = \varepsilon E$$

where ε_o is the vacuum dielectric permittivity and χ is the dielectric susceptibility of the material.