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Dynamic and Transient Infinite Elements

Theory and Geophysical, Geotechnical and Geoenvironmental Applications

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Preamble

Effective and efficient modelling of infinite media is important to the production of accurate and useful solutions for many scientific and engineering problems involving infinite domains, such as earthquake wave propagation within the upper crust of the Earth in the fields of geophysics and seismology, dynamic structure–foundation interaction in the fields of geotechnical, civil and dam engineering, transient pore-fluid flow, heat transfer and mass transport within the interior of the Earth in the fields of geoscience and geoenvironmental engineering, to name only a few. Such an effective and efficient modelling provides useful analytical and numerical tools for simulating, both accurately and efficiently, the effect of the far field of a system on the near field of the system so that computational resources can be concentrated on the simulation aspects of multiple processes, multiple scales, complicated geological and geometrical conditions for the near field of the system. Towards this end, dynamic and transient infinite elements have been developed during the past few decades.

This monograph aims to provide a state-of-the-art report on the theory and application of dynamic and transient infinite elements for simulating the far fields of infinite domains involved in many scientific and engineering problems, based on the author's own work during the last two decades. For this purpose, while the theoretical aspects of either dynamic infinite elements or transient infinite elements are systematically presented, the related application examples are immediately followed to illustrate the usefulness and applicability of these infinite elements for simulating a wide range of dynamic and transient problems involving infinite domains. To broaden the readership of this monograph, common mathematical notations are used to derive the formulations of both dynamic and transient infinite elements. This enables this monograph to be used either as a useful textbook for postgraduate students or as a valuable reference book for computational geoscientists, geotechnical engineers, civil engineers and applied mathematicians. In addition, each chapter is written independently of the remainder of the monograph so that readers may read the chapters of interest separately.

In this monograph, the coupled computational method of finite elements and dynamic infinite elements is used to solve wave propagation problems in infinite domains. For a given wave propagation problem, the near field of the problem is simulated using finite elements so that complicated geometries and complex material properties can be considered in the coupled computational method. The far field of the problem is simulated using dynamic infinite elements so that waves can be propagated from the near field to the far field without causing spurious reflection and refraction at the interface between finite elements and dynamic infinite elements in the coupled computational model. By taking advantages of both finite elements and dynamic infinite elements, the coupled computational method of finite elements and dynamic infinite elements provides a powerful simulation tool for dealing with a wide range of practical problems, such as the distributions of free-field motion during earthquakes, the seismic responses of dam–reservoir water–sediment–foundation systems and the dynamic analyses of civil structure–foundation interactions. To simulate transient pore-fluid flow, heat transfer and mass transport problems in infinite domains, the coupled computational method of finite elements and transient infinite elements is also presented. As an application example, this coupled method has been used to investigate the effects of several key factors on contaminant transport processes in fractured porous media of infinite domains. The related theoretical developments and application results are briefly described as follows: (1) Owing to the characteristics of propagating waves from the near field to the far field of a system, the wave propagation function of a dynamic infinite element plays a key role in the formulation of the element. Since the wave propagation function is explicitly dependent on frequency, the coupled computational method of finite elements and dynamic infinite elements can be directly used to solve linear wave propagation problems in the frequency domain, while it can be only used to deal with nonlinear wave propagation problems in the hybrid frequency–time domain. (2) For a two-dimensional dynamic infinite element, the corresponding wave propagation function has two independent wavenumbers so that it can be used to simulate explicitly both P-wave and SV-wave propagation in the far field of a system. Similarly, for a three-dimensional dynamic infinite element, the corresponding wave propagation function has three independent wavenumbers so that it is capable of simulating simultaneously P-wave, SV-wave and R-wave propagation in the far field of a system. (3) The coupled computational model of finite elements and dynamic infinite elements can be used to solve both wave scattering and wave radiation problems in infinite domains. When dealing with wave scattering problems, a wave input procedure, which can be easily applied to the coupled computational model of finite elements and dynamic infinite elements, is presented to transform an incident wave into equivalent nodal loads at a wave input boundary located within the coupled computational model. (4) For the application of dynamic infinite elements to dam engineering problems, the coupled computational method of two-dimensional finite elements and dynamic infinite elements has been used to simulate the dynamic responses of both a gravity dam–water–sediment–foundation system and an embankment dam–water–sediment–foundation system. For a gravity dam, the related numerical results have indicated that the reservoir bottom sediment has a remarkable effect on the dynamic response of the dam, while in the case of an embankment dam, the corresponding results have demonstrated that both the type and the location of impervious members within the dam have significant influences on the dynamic response of the embankment dam. (5) As an application example of simulating wave scattering problems in the fields of geophysics and seismology, the coupled computational method of two-dimensional finite elements and dynamic infinite elements has been used to investigate the effects of canyon topographical and geological conditions on the distributions of free-field motion during earthquakes. The related numerical results have demonstrated that both topographical and geological conditions have significant influences on seismic acceleration distributions along the surface of a canyon, implying that structures located on softer soils may be subjected to stronger seismic loads than those located on stiffer rocks. (6) The coupled computational model of three-dimensional finite elements and dynamic infinite elements has been used to solve dynamic framed structure–raft foundation–underlying medium interaction problems in the field of civil engineering. The related numerical results have demonstrated that since the radiation damping of an underlying medium plays a predominant role in determining the total damping of the underlying medium, the dynamic response of a three-dimensional framed structure on a layered medium is much stronger than that on a homogeneous medium, as a result of wave reflection and refraction within the soft layer. (7) To construct transient infinite elements for simulating transient pore-fluid flow and heat transfer problems in fluid-saturated porous media of infinite domains, the hydraulic head distribution and heat transfer functions are used to derive the formulations of the transient infinite elements. Since these functions are explicitly dependent on time, the coupled computational method of finite elements and transient infinite elements can be straightforwardly employed to solve transient pore-fluid flow and heat transfer problems in the time domain. (8) Based on the mass transport function concept, the formulations of transient infinite elements are derived for simulating the far fields of mass transport problems in fractured porous media of infinite domains. With the use of the double porosity continuum approach, the porous block and fissured network in a fractured porous medium can be treated as an equivalent medium consisting of two overlapping continua. This enables the coupled computational method of finite elements and transient infinite elements to be used for investigating the effects of various key factors on contaminant transport processes in fractured porous media of infinite domains. The related numerical results have demonstrated that the leakage between the porous block and the fissure network, the porosity ratio of the fissured network to the porous block, pore-fluid advection and solute dispersion have significant effects on contaminant concentration distributions in fractured porous media of infinite domains.

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Contents

Nomenclature

The following symbols are commonly used with the attached definitions, unless otherwise specified in the monograph.

A	area of a finite element
C	contaminant concentration
$\{C\}$	contaminant concentration vector
C_1	contaminant concentration in the porous block
C_2	contaminant concentration in the fissured network
c_p	specific heat of pore-fluid
D	dispersion coefficient
g	acceleration due to gravity
h	hydraulic head
H	reference length
K	hydraulic conductivity
L	length of a problem domain
M	mapping function
N	shape function
$[N]$	shape function matrix
p	pressure
P	nodal force
$\{P\}$	nodal force vector
P_0	concentrated force
P_λ	nodal force on the wave input boundary
S	boundary length of a finite element
T	temperature
$\{T\}$	temperature vector
t	temporal variable
u	displacement in the x direction
v	displacement in the y direction
V	volume of a finite element
w	displacement in the z direction
x, y, z	spatial coordinates in a global coordinate system

λ	thermal conductivity
λ_{e0}	reference thermal conductivity in the horizontal direction
ϕ	porosity
ψ	vector potential function
ρ	density
ν	Poisson's ratio
β	stress increase factor
σ	normal stress
τ	shear stress
ω	circular frequency
ξ, η, ζ	spatial coordinates in a local coordinate system
θ	wave incident angle
η_d	hysteretic damping coefficient
χ	transmissive coefficient between the porous block and the fissured network in a fractured porous medium

Subscripts

f	pertaining to pore-fluid
0	pertaining to reference quantities
P	pertaining to P-wave
SV	pertaining to SV-wave

Superscripts

e	pertaining to quantities in a finite element level
$*$	pertaining to dimensionless quantities
s	pertaining to solid matrix
T	pertaining to the transpose of a matrix

Chapter 1
Introduction

Effective and efficient modelling of infinite media is important for the production of accurate and useful solutions for many scientific and engineering problems involving infinite domains (Bettess 1977, 1980; Chow and Smith 1981; Medina and Taylor 1983; Zhang and Zhao 1987; Zhao et al. 1989; Zhao and Valliappan 1993a, b, c, d; Astley 1996, 1998; Yang et al. 1996; Yang and Huang 2001; Yun et al. 2000, 2007; Wang et al. 2006). Some typical examples involving infinite domains are as follows: (1) earthquake wave propagation within the upper crust of the Earth in the fields of geophysics and seismology; (2) dynamic structure–foundation interaction in the fields of geotechnical, civil and dam engineering; and (3) transient pore-fluid flow, heat transfer and mass transport within the interior of the Earth in the fields of geoscience and geoenvironmental engineering. Although the solid Earth is viewed as a bounded domain at the terrestrial scale, it can be treated as an unbounded domain at the human scale. For instance, in the case of predicting possible property damages caused by an earthquake, only a limited region around the epicentre is of interest because the earthquake wave energy is significantly reduced as the distance from the epicentre is increased. Compared with the region of interest around an epicentre, which is called *the near field or the interior domain of a system*, the outside region, referred to as *the far field or the exterior domain of the system*, is large enough to be treated as an infinite domain, from the mathematical point of view. Similarly, the sizes of engineering structures such as civil buildings, dams, embankments, retaining walls and nuclear reactors are very small, compared with those of their foundations. Since only the response of a structure and its surrounding foundation is of interest, from the structural design point of view, computational resources should be concentrated on the analysis of the structure and the near field of the foundation.

In terms of simulating the near fields of systems involving infinite domains, the finite element method provides a very powerful tool in the sense that complicated geometries and complex material distributions can be effectively and efficiently considered in a finite element model (Zienkiewicz 1997; Rao 1989). In particular, the numerical adequacy and convergence properties of a finite element model were extensively studied, so that many numerical simulation criteria have been established. For example, when a finite element is used to simulate wave propagation problems, there is a mesh size requirement criterion available, which states that in

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order to ensure the numerical adequacy and convergence of a finite element model, the size of a linear finite element should be less than one-eighth of the wavelength to be simulated, whereas the size of a quadratic finite element should be less than one-fourth of the wave length to be simulated in the finite element model. Similarly, when a finite element is used to simulate transient mass transport problems, the size of the element should satisfy the Courant number, so that the numerical adequacy and convergence of a finite element model can be ensured (Zienkiewicz 1977; Zhao et al. 1994). For these reasons, the finite element mesh of the near field can be designed on the basis of the related mesh criteria available, without a need to conduct a mesh refinement study.

Since the finite element method can be used to simulate problems of finite domains, it is necessary to develop useful numerical techniques for simulating the far fields of problems when they are of infinite domains. Towards this end, static, dynamic and transient infinite elements have been developed for simulating the far fields of many scientific and engineering problems involving infinite domains during the past few decades. *Static infinite elements refer to the time-independent infinite elements* suitable for simulating the far fields of static problems. *Dynamic infinite elements refer to the frequency-dependent infinite elements* suitable for simulating the far fields of dynamic and wave propagation problems, while *transient infinite elements refer to the time-dependent infinite elements* suitable for simulating the far fields of transient pore-fluid flow, heat transfer and mass transport problems. On the other hand, for most scientific and engineering problems involving infinite domains, the near field of a problem can be appropriately determined so that the nonlinear behaviour of the problem can be simulated by finite elements. As a result, for the sake of developing dynamic and transient infinite elements, linear dynamic elasticity (in certain cases including linear visco-elasticity) is used to represent the mechanical behaviour of the far field, while linearized ground water flow and diffusive mass transport equations are used to approximately represent the behaviours of the corresponding far fields.

For the numerical simulation of an infinite domain, a primitive and very simple method, in which the infinite domain was approximately truncated as a large-enough finite domain, was widely used at the early stage of the finite element analysis. The major disadvantages in using this primitive method are as follows: (1) the simulation of a large-enough domain leads to a significant increase in computational resources; (2) the boundary conditions of a problem at infinity cannot be rigorously satisfied. For instance, stresses and displacements attenuating zero at infinity for a static problem and the wave radiation condition in the far field for a dynamic problem have to be violated in the numerical analysis; (3) stretching a fixed number of finite elements to model a vast domain can result in a severe loss of solution accuracy for static problems, while it results in spurious solutions for dynamic problems because the element size requirement for appropriately simulating dynamic problems cannot be satisfied in the numerical simulation; (4) for transient heat transfer and mass transport problems, the use of artificially truncated boundaries can cause unexpected numerical reflections back into the near field, where the solutions are usually of great interest to the analyst, of a system.

To overcome the above-mentioned disadvantages, infinite elements have been developed to simulate, both effectively and efficiently, the physical and mechanical effects of the far field of a system on the near field of the system. In this respect, Ungless (1973) presented the static infinite element concept for simulating infinite domains of static problems. This concept attracted considerable research on the development and application of static infinite elements during both the 1970s and the 1980s (Bettess 1977, 1980, 1992; Beer and Meek 1981; Booker and Small 1981; Zhao et al. 1986). In the early 1980s, Chow and Smith (1981) extended the static infinite element concept to the simulation of infinite domains for dynamic problems. Owing to the wave propagation characteristics associated with dynamic problems, a large amount of research has been contributed to the development and application of dynamic infinite elements for simulating the far-field effects of infinite domains since the 1980s (Medina and Taylor 1983; Zhang and Zhao 1987; Zhao et al. 1989; Zhao and Valliappan 1993a, b, c, d; Astley 1996, 1998; Yang et al. 1996; Yang and Huang 2001; Yun et al. 2000, 2007; Wang et al. 2006). As most of the research conducted in the development of dynamic infinite elements is associated with steady-state wave propagation problems in the frequency domain, it remains desirable to directly develop dynamic infinite elements in the time domain for simulating elastic wave propagation problems involving infinite domains in the future. On the other hand, for dealing with the numerical simulation of infinite domains associated with transient pore-fluid flow, heat transfer and mass transport problems, Zhao and Valliappan (1993e, f, 1994a) presented (time-dependent) transient infinite elements in the time domain. Since the proposed transient infinite elements are time dependent, they have been successfully used, with a combination of the conventional finite element method, to solve a wide range of transient pore-fluid flow, heat transfer and mass transport problems in fluid-saturated porous media of infinite domains (Zhao et al. 1994b, c; Khalili et al. 1999a, b; Lai et al. 2002; Zhang et al. 2007).

The prediction of an earthquake and related property damages has been a hot research topic in the fields of geology, geophysics and seismology. Although earthquakes cannot be predicted using the present day's knowledge of geoscientists, modern advances in computational simulation methods provide some useful tools suitable for investigating the detailed dynamic processes and mechanisms associated with an earthquake. From the computational simulation point of view, an earthquake may involve the following two important stages: an inception stage and an occurrence stage. At an inception stage, the deformation rate of crustal materials (i.e. about a few centimetres per year) is so slow that the geological system related to the inception of an earthquake can be treated as a quasi-static system, indicating that the whole geological system can be simulated using the coupled computational method of finite elements and static infinite elements. However, at the occurrence stage of an earthquake, the resulting earthquake wave propagates at a speed of a few thousand kilometres per second within the crust of the Earth, so that the geological system related to the occurrence of an earthquake must be treated as a dynamic system. In this situation, the whole geological system needs to be simulated using the coupled computational method of finite elements and dynamic infinite elements. To demonstrate the potential application of dynamic infinite elements in the fields of geophysics and seismology, the coupled computational method of two-dimensional finite elements and dynamic infinite elements is used to investigate the effects of canyon topographical and geological conditions on the distributions of free-field motion during earthquakes. This is studied in the fourth chapter of this monograph.

As extensive studies on the dynamic response of concrete gravity and embankment dams due to earthquake loadings have demonstrated, the dynamic response of either a concrete gravity dam or an embankment dam is mainly affected by the following factors: (1) the interaction between the dam and the impounded reservoir water (Chopra 1968; Chakarbarti and Chopra 1974; Liam-Finn et al. 1977); (2) the compressibility of the impounded water (Chopra and Gupta 1982); (3) the interaction between the dam and the foundation rock (Liam-Finn et al. 1977; Liam-Finn and Varoglu 1972a, b,1975); (4) the materials at the reservoir bottom (Hall and Copra 1982; Fenves and Chopra 1983, 1984, 1985; Lotfi et al. 1987; Medina et al. 1990. Based on a substructure method, Chopra and his colleagues considered the above factors and made some interesting conclusions on the dynamic response of concrete gravity dams due to earthquake loadings (Chopra 1968; Chakarbarti and Chopra 1974; Hall and Copra 1982; Fenves and Chopra 1983, 1984, 1985). Owing to the limitations of the substructure method, the reservoir bottom material was assumed to have zero thickness. However, in certain circumstances such as concrete gravity and embankment dams built in the Yellow River valley, China, not only materials at a reservoir bottom have considerable thicknesses, but sediments at the reservoir bottom are also comprised of very soft clay materials. Although some basic studies have been carried out to investigate how reservoir bottom sediments affect the dynamic response of concrete gravity dams (Medina et al. 1990; Zhao 1994), further studies are needed to investigate the detailed dynamic mechanisms associated with the effects of reservoir bottom sediments on the dynamic response of concrete gravity and embankments dams. In view of this fact, the coupled computational method of finite and dynamic infinite elements is used for investigating the effects of reservoir bottom sediments on the dynamic response of concrete gravity and embankment dams. Since the coupled computational model keeps all advantages of the conventional finite element method, complicated geometrical, physical and mechanical properties of a dam–water–foundation system, including the reservoir bottom sediment effect, can be straightforwardly considered in the corresponding numerical simulations.

Transient pore-fluid flow, heat transfer and mass transport in fluid-saturated porous media of infinite domains are important phenomena in many scientific and engineering fields. For example, in the field of exploration geoscience, pore-fluid flow, heat transfer and mass transport from the interior of the Earth to the surface of the Earth are three important physical processes to control ore body formation and mineralization within the upper crust of the Earth. Owing to the increasing demand for natural minerals and the possible exhaustion of existing mineral resources in the foreseeable future, there has been an ever-increasing interest in the study of key controlling processes associated with ore body formation and mineralization within the upper crust of the Earth (Phillips 1991; Yeh and Tripathi 1991; Nield and Bejan 1992; Steefel and Lasaga 1994; Raffensperger and Garven 1995; Schafer et al.

1998a, b; Xu et al. 1999; Schaubs and Zhao 2002; Ord et al. 2002; Gow et al. 2002; Zhao et al. 1997–2008). In the field of environmental engineering, carbon dioxide gas sequestration in the deep Earth is becoming a potential way to reduce the greenhouse effect. Even in our daily lives, pore-fluid flow and contaminant transport through fluid-saturated porous soils can be encountered almost everywhere. This means that transient infinite elements can be used to solve a wide range of scientific and engineering problems encountered in nature. To illustrate how transient infinite elements are used to solve contaminant transport problems in the field of geoenvironmental engineering, the coupled computational method of finite elements and transient infinite elements is used for investigating the effects of various key factors on contaminant transport processes in fractured porous media of infinite domains.

The arrangements of the forthcoming parts of this monograph are as follows. In Chap. 2, the formulations of two-dimensional dynamic infinite elements are presented in detail. To use the coupled computational method of two-dimensional finite elements and dynamic infinite elements for wave scattering problems in infinite media, a wave input procedure is also presented in this chapter. In Chap. 3, the coupled computational method of two-dimensional finite elements and dynamic infinite elements is used to solve dynamic dam–water–sediment–foundation interaction problems in the fields of geotechnical and dam engineering. Both a concrete gravity dam and an embankment dam are considered and some interesting results are presented. In Chap. 4, the coupled computational method of two-dimensional finite elements and dynamic infinite elements is used to simulate the spatial distribution of free-field motion during an earthquake, which is a fundamental scientific problem in the fields of geophysics and seismology. The effects of different topographical and geological conditions on the spatial distributions of free-field motion during earthquakes have been investigated. The detailed formulations associated with three-dimensional dynamic infinite elements are presented in Chap. 5. Through a combination of three-dimensional finite elements and dynamic infinite elements, two benchmark problems have been used to verify the correctness and usefulness of the proposed three-dimensional dynamic infinite elements for simulating wave radiation problems in three-dimensional infinite media. Based on the related formulations presented in Chap. 5, the coupled computational method of three-dimensional finite elements and dynamic infinite elements is used in Chap. 6 to simulate dynamic structure–foundation interaction problems in the fields of civil and structural engineering. For the purpose of understanding the dynamic mechanisms of a structure–foundation interaction problem, a fundamental problem, namely the vibration of a rigid plate foundation on a visco-elastic half-space, is considered before the dynamic response of a three-dimensional framed structure–raft foundation–underlying medium system is simulated by the coupled computational method of three-dimensional finite elements and dynamic infinite elements. In Chap. 7, the detailed formulations of transient infinite elements are presented for simulating pore-fluid flow and heat transfer problems in fluid-saturated porous media of infinite domains, because such problems can be found in a broad range of scientific and engineering fields. Two different approaches are employed to derive the property matrices of these transient infinite elements. The detailed formulations associated with transient infinite elements for simulating mass transport problems are presented in Chap. 8, when the coupled computational method of finite elements and transient infinite elements is used to simulate contaminant transport problems in fractured porous media of infinite domains. On the basis of the double porosity concept, a fractured porous medium can be treated as an equivalent medium consisting of two overlapping continua, namely a porous continuum and a fissured continuum. Finally, some conclusions are given at the end of the monograph.

Chapter 2
Theory of Two-Dimensional Dynamic Infinite Elements for Simulating Wave Propagation Problems in Infinite Media

Numerical simulation of wave propagation problems in infinite media has attracted significant attention in many scientific and engineering fields such as geophysics, seismology, civil engineering and earthquake engineering. From a wave motion point of view, structural vibration problems can be divided into two categories. One is a wave radiation problem, or an interior domain problem, in which wave energy is produced within the near field and then propagated into the far field of the problem in various wave forms. Typical examples of this category are foundation vibration problems as a result of trains passing on railways, machine vibration problems on the foundations of buildings, impacting vibration problems on the ground surface of an airport during airplanes landing, to name just a few. The other is a wave scattering problem, or an exterior domain problem, in which wave energy is produced in the far field and propagated into the near field of the problem. An earthquake source or an explosion in the far field is a typical example of this category. Since the formulations of dynamic infinite elements for simulating wave radiation problems are essentially the same as those for simulating wave scattering problems, the focus of this chapter is on the formulation and derivation of dynamic infinite elements for simulating wave scattering problems. In this regard, a wave input method needs to be developed for simulating incoming waves from the far field of an infinite medium.

For the theoretical analysis of wave scattering problems in the fields of seismology and geophysics, extensive work has been carried out over the past years. Aki and Larner (1970) proposed a practical method using both the discrete wavenumber representation for a wave field and the related Rayleigh assumption. In their method, a scattered wave field is expressed as the superposition of plane waves, which have unknown complex amplitudes and propagate in various directions. The total motion of the system is obtained through integration over the horizontal wavenumber. Under the assumption of horizontal periodicity of irregularity, the resulting integral can be replaced by an infinite series. Truncation of this series and application of interface conditions of continuity for both stress and displacement lead to a set of linear equations for the unknown complex amplitudes. This method has found many applications in the fields of geophysics and seismology (Bard 1982; Geli et al. 1988). Trifunac (1973) and Wong and Trifunac (1974) presented analytical solutions for SH-wave scattering problems around semi-circular and semi-elliptical valleys

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using a Hankel function expansion. By means of boundary integral equations, Wong (1982) improved the discrete wavenumber method (Aki and Larner 1970) and presented a general inverse method to solve the P-wave, SV-wave and Rayleigh wave scattering problems for alluvial valleys of both semi-circular and semi-elliptical shapes. Lee (1984) and Eshraghi and Dravinski (1989a, b) used a wave function expansion method to solve wave scattering problems around either hemi-spherical valleys or dipping layers, respectively. Sanchez-Sesma (1983) applied the boundary integration method to the scattering of elastic waves around axisymmetric irregularities. Kawase (1988) suggested a discrete wavenumber boundary element method, in which the conventional boundary element method is used with Green's functions of discrete wavenumbers, for dealing with wave scattering problems. Khair et al. (1989) introduced the hybrid method of finite and boundary elements for solving three-dimensional scattering problems of plane P-waves and SV-waves around cylindrical valleys. Obviously, all methods mentioned above are mainly suitable for dealing with linear, isotropic and homogeneous materials as a result of using half-space elastic wave theory to describe earthquake excitations in these methods.

The finite element method (Zienkiewicz 1977) is one of the most powerful numerical methods for solving complex and complicated problems in both scientific and engineering fields (Zhao et al. 1994, 1995, 1997, 1998). However, for the finite element simulation of wave scattering problems in infinite media, the following two issues have to be considered. The first is the infinite extension of a problem domain, while the second is the incidence of an earthquake wave from the far field of a system. To simulate infinite media both effectively and efficiently, Ungless (1973) and Bettess (1977, 1980) presented a static infinite element method for dealing with static problems. Chow and Smith (1981), Medina and Taylor (1983) and Zhao et al. (1989) extended the static infinite element method to the solution of wave radiation problems in infinite media. Using a combination of finite and infinite elements, a whole system can be divided into a near field and a far field. The near field is simulated using finite elements, while the far field is simulated using dynamic infinite elements. For the seismic analysis of a structure, the main concern is usually about the dynamic response of the structure, so that only a small region of the infinite medium needs to be treated as the near field of the system. This can result in a significant reduction in the total number of finite elements that are used to simulate the computational domain of the system. Since dynamic infinite elements are capable of simulating wave propagation within themselves, unwanted wave reflection phenomena at the interface between a finite element and an infinite element can be avoided.

Early work on earthquake input procedures for the finite element method was carried out by Reimer et al. (1974), with particular attention to the finite element analysis of arch dams. Due to the difficulties of this problem, they suggested a massless finite element model, which is called the *massless foundation earthquake input model*. In this model, only a limited massless foundation is simulated using finite elements, and the acceleration of an earthquake is applied to the whole finite element model. This procedure has some obvious discrepancies from physical reality, as it cannot simulate wave propagation effects in the foundation of an arch dam. To overcome these discrepancies, Clough et al. (1985) proposed a sophisticated earthquake input method, called the *free field input method*, for the finite element analysis of arch dams subjected to earthquakes. Although both the spatial amplitude and the spatial phase differences of an earthquake can be considered in the free field input method, it is difficult to apply this method to practical problems in the field of earthquake engineering, because few earthquakes have been recorded along the surfaces of natural canyons. Another wave input method used in the hybrid model of finite and boundary elements is called the *standard wave input method*, which is established on the basis of the half-space elastic wave theory. Furthermore, Zhao (1987) presented a wave propagation input method on the basis of using finite and dynamic infinite element coupled models and considering wave propagation characteristics in elastic solid media. As this method has been successfully used to solve SH-wave scattering problems (Zhang and Zhao 1988), it is worth extending this method to the solution of P-wave and SV-wave scattering problems because of wave mode conversions in these situations.

To take advantage of the coupled computational model of finite and dynamic infinite elements for simulating natural foundations, a numerical model for dealing with wave scattering problems in infinite media is presented in this chapter. With consideration of P-wave and SV-wave reflection characteristics on a fixed boundary, the harmonic P-wave and SV-wave propagating from the far field of a system are transformed into nodal dynamic forces on the wave input boundary, where scattering waves from canyons or structures can be transmitted back into the far field of the system through dynamic infinite elements. The major advantage of using the proposed model is that, by choosing a horizontal boundary in the underlying rock, full-space elastic wave theory can be used to describe earthquake excitations in the coupled computational model of finite and dynamic infinite elements. As a result, the proposed model is capable of simulating wave propagation and scattering mechanisms within the region of interest, which is located above the wave input boundary, under any geometrical and geological conditions. In addition, the proposed numerical model is clear in the physical concept and easy to be included into the existing finite element computer code. The related numerical results from solving P-wave and SV-wave scattering problems in a half-plane and a semi-circular canyon have been obtained using the proposed model.

2.1 Formulation of Two-Dimensional Dynamic Infinite Elements and Wave Input Method

To derive the formulations of two-dimensional dynamic infinite elements, it is natural to consider wave motion equations in a half-plane. Since an earthquake wave can be decomposed into the sum of several harmonic waves, it is necessary to investigate the propagation behaviours of harmonic waves in a half-plane. Since material damping occurring in the soil/rock involves a frictional loss of energy, it can be considered as hysteretic damping, which is independent of frequency, by means of the correspondence principle (Wolf 1985). Using this principle, the damped solution of a system can be obtained from the elastic one by replacing the elastic constants with the corresponding complex ones. Under the assumption that the dynamic system is subjected to a harmonic wave loading and that the medium of the system exhibits hysteretic damping, the corresponding governing wave equations of the system can be expressed as follows:

$$G^*\nabla^2 u + \left(\lambda^* + G^*\right)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y}\right) + f_x = \rho\frac{\partial^2 u}{\partial t^2}, \tag{2.1}$$

$$G^*\nabla^2 v + \left(\lambda^* + G^*\right)\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}\right) + f_y = \rho\frac{\partial^2 v}{\partial t^2}, \tag{2.2}$$

$$G^* = (1 + i\eta_d)\, G, \quad \lambda^* = (1 + i\eta_d)\, \lambda, \tag{2.3}$$

where G is the shear modulus; λ is the Lamé constant; η_d is the hysteretic damping coefficient of the medium; u and v are displacements in the x and y directions; f_x and f_y are body forces in the x and y directions, respectively; ρ is the density of the medium; ∇^2 is the second-order two-dimensional Laplace operator.

Using the Galerkin weighted residual procedure and neglecting body forces in Eqs. (2.1) and (2.2), the discretized wave equation of the system can be derived as

$$-\,\omega^2\,[M]\,\{\Delta\} + (1 + i\eta_d)\,[K]\,\{\Delta\} = \{F_0\}\,, \tag{2.4}$$

where $\{\Delta\}$ is the unknown nodal displacement vector; ω is the circular frequency of the harmonic wave; $[M]$ and $[K]$ are the global mass and stiffness matrices of the system respectively; and $\{F_0\}$ is the amplitude vector of the applied harmonic load. $[M]$, $[K]$ and $\{F_0\}$ can be assembled from the following element submatrices and subvectors:

$$[M]^e = \iint_A [N]^T \rho\, [N]\, dA, \tag{2.5}$$

$$[K]^e = \iint_A [B]^T \left[D^*\right] [B]\, dA, \tag{2.6}$$

$$\{F_0\}^e = \int_S [N]^T \left\{\overline{X}_0\right\} dS + [N]^T \left\{\overline{P}_0\right\}, \tag{2.7}$$

where A and S are the area and boundary length of the element; $\{\bar{X}_0\}$ is the amplitude vector of element boundary traction; $\{\bar{P}_0\}$ is the amplitude vector of concentrated loads acting on the element; $[D^*]$ is the constitutive matrix of the element material; and $[B]$ and $[N]$ are the strain matrix and shape function matrix of the element. It needs to be pointed out that Eqs. (2.5), (2.6) and (2.7) are equally valid for both finite and dynamic infinite elements. Since the derivation of two-dimensional finite element formulation is well known (Zienkiewicz 1977; Rao 1989), only the formulation of two-dimensional dynamic infinite elements is derived below.

2.1.1 Formulation of Two-Dimensional Dynamic Infinite Elements

For dealing with wave propagation problems in infinite media of geometrical irregularities and geological complexities, the use of a coupled computational model of finite and dynamic infinite elements is very effective (Zhao 1987; Zhang and Zhao 1987; Zhao et al. 1989, 1991, 1992; Zhao and Valliappan 1993a, b, c, d, e, f). Considering a dynamic infinite element shown in Fig. 2.1, the corresponding coordinate mapping can be expressed as follows:

$$x = \sum_{i=1}^{5} M_i x_i, \tag{2.8}$$

$$y = \sum_{i=1}^{5} M_i y_i, \tag{2.9}$$

where M_i ($i = 1, 2, \ldots, 5$) is the following mapping function of the dynamic infinite element:

$$M_1 = \frac{1}{2}(\xi - 1)(\eta - 1), \tag{2.10}$$

$$M_2 = 0, \tag{2.11}$$

$$M_3 = -\frac{1}{2}(\xi - 1)(\eta + 1), \tag{2.12}$$

$$M_4 = \frac{1}{2}\xi(\eta + 1), \tag{2.13}$$

$$M_5 = -\frac{1}{2}\xi(\eta - 1). \tag{2.14}$$

The displacement field within this dynamic infinite element can be expressed as follows:

$$u = \sum_{i=1}^{3} N_i u_i, \tag{2.15}$$

$$v = \sum_{i=1}^{3} N_i v_i, \tag{2.16}$$

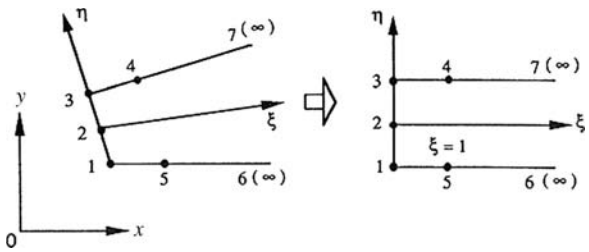

Fig. 2.1 Two-dimensional dynamic infinite element: nodes 1, 2 and 3 are the end nodes to be connected with a finite element; nodes 4 and 5 are the middle nodes with $\xi = 1$; nodes 6 and 7 are at infinity

where N_i ($i = 1$, 2 and 3) is the following displacement shape function of the dynamic infinite element:

$$N_1 = P(\xi)\frac{\eta(\eta - 1)}{2}, \tag{2.17}$$

$$N_2 = -P(\xi)(\eta - 1)(\eta + 1), \tag{2.18}$$

$$N_3 = P(\xi)\frac{\eta(\eta + 1)}{2}, \tag{2.19}$$

where $P(\xi)$ is the wave propagation function of the dynamic infinite element. From a harmonic wave propagation point of view, $P(\xi)$ can be expressed in the following form:

$$P(\xi) = \exp\left[-(\alpha + i\beta)\xi\right], \tag{2.20}$$

where α and β are the displacement–amplitude decay factor and nominal wavenumber of the dynamic infinite element in the local coordinate system. Physically, $\exp(-\alpha\xi)$ expresses the behaviour of displacement amplitude attenuation within the dynamic infinite element as a result of wave energy dissipation; while $\exp(-i\beta\xi)$ expresses the behaviour of phase delays as a result of wave propagation in the local coordinate system.

Equations (2.15) and (2.16) can be written in the matrix form as follows:

$$\begin{Bmatrix} u \\ v \end{Bmatrix}^e == [N]\{\Delta\}^e, \tag{2.21}$$

where $[N]$ is the shape function matrix of the dynamic infinite element; $\{\Delta\}^e$ is the nodal displacement vector of the element. They are of the following forms:

$$[N] = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix},$$

$$\{\Delta\}^e = \left\{u_1 \ v_1 \ u_2 \ v_2 \ u_3 \ v_3\right\}^T. \tag{2.23}$$

Using the above definitions, the strain matrix of the dynamic infinite element can be expressed as follows:

$$\{\varepsilon\}^e = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial N_1}{\partial x} & 0 & \dfrac{\partial N_2}{\partial x} & 0 & \dfrac{\partial N_3}{\partial x} & 0 \\ 0 & \dfrac{\partial N_1}{\partial y} & 0 & \dfrac{\partial N_2}{\partial y} & 0 & \dfrac{\partial N_3}{\partial y} \\ \dfrac{\partial N_1}{\partial y} & \dfrac{\partial N_1}{\partial x} & \dfrac{\partial N_2}{\partial y} & \dfrac{\partial N_2}{\partial x} & \dfrac{\partial N_3}{\partial y} & \dfrac{\partial N_3}{\partial x} \end{bmatrix} = [B]\{\Delta\}^e, \tag{2.24}$$

where $[B]$ is the strain matrix of the dynamic infinite element; $\{\varepsilon\}^e$ is the strain vector of the element. The strain matrix of the dynamic infinite element can be further expressed as

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} \end{bmatrix}. \tag{2.25}$$

To evaluate the strain matrix of the dynamic infinite element, it is necessary to calculate the first derivatives of the displacement shape functions with respect to the local ξ and η coordinates as follows:

$$\frac{\partial N_i}{\partial \xi} = \frac{\partial N_i}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial N_i}{\partial y}\frac{\partial y}{\partial \xi} \qquad (i = 1, 2, 3), \tag{2.26}$$

$$\frac{\partial N_i}{\partial \eta} = \frac{\partial N_i}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial N_i}{\partial y}\frac{\partial y}{\partial \eta} \qquad (i = 1, 2, 3). \tag{2.27}$$

Equations (2.26) and (2.27) can be readily expressed in the following matrix form:

$$\begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} = [J] \begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} \qquad (i = 1, 2, 3), \tag{2.28}$$

where the matrix $[J]$, called the Jacobian matrix, is given by the following equation:

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}. \tag{2.29}$$

Substituting Eqs. (2.8) and (2.9) into Eq. (2.29) yields the final expression for the Jacobian matrix as follows:

$$[J] = \begin{bmatrix} \sum_{i=1}^{5} \left(\frac{\partial M_i}{\partial \xi} x_i \right) & \sum_{i=1}^{5} \left(\frac{\partial M_i}{\partial \xi} y_i \right) \\ \sum_{i=1}^{5} \left(\frac{\partial M_i}{\partial \eta} x_i \right) & \sum_{i=1}^{5} \left(\frac{\partial M_i}{\partial \eta} y_i \right) \end{bmatrix}. \tag{2.30}$$

Thus, the first derivatives of the displacement shape functions with respect to the global x and y coordinates can be expressed as follows: