

Torsion and Shear Stresses in Ships

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ISBN 978-3-642-14632-9

e-ISBN 978-3-642-14633-6

DOI 10.1007/978-3-642-14633-6

Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2010933503

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To my wife

*For her love, patience, encouragement
and support*

To my late parents

*For their continuous care and
encouragement*

To my students

*Whose enthusiasm and hard work have
encouraged me to prepare the course
material of this book.*

Preface

In the last few decades, much research work was conducted to improve ship structure analysis and design. Most of the efforts were directed to improve the strength of hull girder and to use the method of finite element analysis more efficiently and effectively. Because of the high degree of complexity of ship structures the interaction between hull girder strength and local strength require special attention. Any structural element of the ship hull girder is subjected to several types of stresses including the fabrication and residual stresses. The stresses induced by hull girder and local loadings include the primary stresses, secondary stresses and tertiary stresses. Local loading comprises tensile, compressive, lateral, shear and torsion loadings. This complex system of stresses could produce unacceptable deformations and high values of equivalent stresses. Most of the methods commonly used for ship structure analysis and design focus on the stresses induced by hull girder bending and shear as well as the stresses induced by local lateral loadings.

This book is intended to cover an area of ship structure analysis and design that has not been exhaustively covered by most published text books on ship structures.

Also, it addresses a very complex subject in the design of ship structure and presents it in a simple and suitable form for research students and practicing engineers.

In addition it presents the basic concepts of the methods and procedures required to calculate torsion and shear stresses in ship structures.

Moreover, it presents valuable analysis and design material on torsion and shear loading and stresses. The book therefore should be very useful for practicing naval architects and students of marine engineering and naval architecture. The book is enhanced with a set of some solved and unsolved problems.

Outline of the Book

The book is composed of three parts: *Part I* is devoted to Torsion stresses in ships; *Part II* is concerned with Shear stresses in ships, whereas *Part III* is specialized to modeling the aforementioned methodology as separate subjective modules managed by a main executive program.

Part I of this book introduces the basic elements of pure torsion of uniform thin-walled open sections are presented. The various cases of local torsion loading on beam elements are presented. The basic equations of torsion of thin-walled closed sections, multi-cell box-girders and the general case of combined thin-walled open and closed sections are given.

Torsion of a thin-walled variable section beam subjected to non-uniform torque for both cases of free warping and constrained warping are considered. Warping deformations and flexural warping stresses of thin-walled sections are also presented for different types of loading and end constraints. Warping deformations and stresses in deck structure of container ships are highlighted.

Solution of the torsion equation for an assumed two distributions of torsional loading of container ships traveling obliquely in a sea-way is presented. The method of calculation is based on using an idealized ship section for calculating the sectorial properties of the ship section (principal sectorial area diagram, sectorial static moment). The position of the shear centre, torsion constant J_t and the warping constant $J(\omega)$, the shear and flexural warping stresses are then calculated. The total stress in the deck plating of a container ship due to hull girder bending and torsional loading is discussed. A numerical example is given to clarify the calculation procedure.

Chapter 5 gives the basic concepts and calculation procedure of the sectorial properties of open sections. Chapter 6 gives a general solution of the torsion equation.

Part II of this book presents the basic principles and concepts of shear flow, shear stress, shear deformation and the application of these principles to ship structure. The shear lag effect in thin-walled structures resulting from the effect of shear stress on bending stress is presented.

Methods of calculating the distribution of shear flow and stresses over symmetrical and asymmetrical thin-walled open sections are given.

Shear centre for symmetrical and asymmetrical thin-walled sections is explained. The distribution of shear stresses over thin-walled single and multi-box girders is given. The methods of calculation are explained and supported by numerical examples.

Methods of calculation of the distribution of shear flow and stress over ship sections are addressed. The methods of calculation are based on the introduction of a simplified idealization of ship section using an effective thickness for the shell plating and the attached stiffeners. For ship sections having closed boxes, a correcting shear flow is introduced to eliminate any torsional distortions induced by the assumed shear flow distribution.

The method is used to calculate the shear flow distribution over ship sections of single and double deck cargo ships and oil tankers with one and twin longitudinal bulkheads. A method for calculating shear load carried by the side shell plating and longitudinal bulkheads is given. The importance of calculating the distribution of shear stresses over ship sections of the hull girder is emphasized so as to determine the maximum allowable shearing force for a given ship section.

A damage occurring in any part of the ship structure will cause redistribution of the shear and bending stresses over the remaining intact structural members. Some structural members will be over stressed and others may be lightly stressed. The shear stress distribution over ship sections experiencing local damages is examined so as to ensure adequate safety of the overloaded structural members.

Shear loading on ship hull girder is given together with shear force distribution for alternate hold loading in bulk carriers.

Bulk carriers experience unique problems which result from the particular structural configuration and loading of these ships (alternate hold loading system).

In bulk carriers, the longitudinal vertical shearing force is carried by the side shell plating, top wing tanks, and hopper tanks. The side shell, therefore, may carry a high proportion of the longitudinal vertical shearing force. Consequently, shear stresses in the side shell plating may reach unfavorable values. Shear buckling, or high values of combined stresses, may occur in some panels in the side shell plating. Adequate measures should be taken, therefore, to prevent the initiation of instability and high stresses. The effect of using alternate hold loading system on the magnitude and distribution of shear loading along ship length is presented.

In order to calculate the shear stress distribution over a typical ship section of a bulk carrier, the ship section is idealized by a simplified configuration so as to reduce the laborious calculations associated with shear flow distribution. The idealized structure should affect neither the magnitude nor the distribution of shear flow distribution around the top wing tanks, hopper tanks, and side shell. Procedures for calculating shear flow and stresses in bulk carriers are given in detail.

Part III of this book is specialized to modeling the aforementioned methodology as separate subjective modules managed by a main executive program namely *PROP*[®]. *FORTRAN*[®] is the programming language in which the modules are written. The program has been written from scratch by *Dr. K. A. Hafez*,

Department of Naval Architecture and Marine Engineering, Faculty of Engineering, Alexandria University, Egypt.

For any arrangement of rectangular cross sections, **PROP**[®] calculates their physical, sectional, sectorial properties, and the shear center of closed, opened, and combined cross sections. Also, the interested researcher may use the attached standard mathematical subroutines to manipulate the distribution of the sectional and/or sectorial properties together with their first few derivatives.

The information gathered in this program is expected to be sufficient for the first glance without going into more detailed discussion. However, for further details of the formulations and their associated computer programs, the interested researcher may consult the appropriate subjective chapters of this book, or he may refer to the listed references at the end of the book. All of the surveyed formulations and their associated computer program are computationally fast using the standard **IBM**[®] compatible computers, without any special requirements of the hardware configuration.

One of the main goals of the **PROP**[®] program is the easy possibility of addition to and deletion of functional modules as required. The program is not an optimization routine but still considered to belong to the preliminary structural design stage without any economical or optimization consideration.

Finally, this part includes three solved examples that surely help in tracing the algorithm of the **PROP**[®] program and understanding the way of input and output. Also, for the interested students and/or researchers a collection of non-solved problems is introduced.

Acknowledgments

I would like to thank *Dr. K. A. Hafez*, for receiving and correcting the presentation of this book and also for enhancing the value of the material presented herein by adding the source list and output of his computer program in *Chapter 12*. Also, I wish to thank all my graduate and undergraduate students who inspired me to write this book.

List of Symbols

A	Sectional area
B	Ship breadth
b	Flange width
C	Torsion rigidity
C_1	Warping constant
C_b	Block coefficient at summer load waterline
C_w	Warping constant
d	Web depth
dL	Elementary length
du	Linear deformation
dV	Elementary volume
E	Modulus of elasticity
e	Distance of shear center
e_y	Vertical distance of the shear center
f	A factor representing the degree of constrained against warping
G	Shearing modulus of elasticity
GJ	Torsional rigidity
I_h	Second moment of area of ship section about the y-axis
I_p	Polar moment of inertia
J	Torsion constant
L	Ship length
M	Bending moment
M_h	Horizontal bending moment
m	Intensity of torque load
M_{sw}	Still water bending moment
$M(\omega)$	Bimoment
p	Pressure load
q	Shear flow
R	Radius
r	Radial distance

S	Length parameter
$S(\omega)$	Sectorial static moment
T	Torque
T_e	Torsional moment at the end of the member
T_ω	Warping torque
T_s	Saint Venant torque
t	Thickness
t_f	Flange thickness
t_w	Web thickness
u	Linear displacement
β	Angle of deformation
γ	Shear angle
δ	Flexural warping coefficient
φ	Angle of twist
θ	Rate of twist
ν	Poisson's ratio
σ_1	Stress at the inner point of the flange
σ_2	Stress at the outer edge of the flange
σ_p	Stress at the attached plating
σ_ω	Flexural warping stress
τ	Shear stress
ω	Sectorial area

SI Units

International System of Units

This system can be divided into basic units and derived units as given in Tables 1 and 2.

Table 1 Basic units

Quantity	Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Thermodynamic temperature	Degree Kelvin	°K
Luminous intensity	Candela	cd

Table 2 Derived units

Quantity	Unit	Symbol
Force	Newton	$N = \text{kg m/s}^2$
Work, energy	Joule	$J = N \text{ m}$
Power	Watt	$W = J/s$
Stress, pressure	Pascal	$\text{Pa} = N/\text{m}^2$
Frequency	Hertz	$\text{Hz} = \text{s}^{-1}$
Acceleration	Meter per second squared	$g = \text{m/s}^2$
Area	Square meter	m^2
Volume	Cubic Meter	m^3
Density	Kilogram per cubic meter	$\rho = \text{kg/m}^3$
Velocity	Meter per second	$v = \text{m/s}$
Angular velocity	Radian per second	rad/s
Dynamic viscosity	Newton second per meter squared	$N \text{ s/m}^2$
Kinematic viscosity	Meter squared per second	m^2/s
Thermal conductivity	Watt per (meter degree Kelvin)	$W/(\text{m.deg.k})$

Table 3 Summary of the quantities commonly used in naval architecture

Quantity	SI unit
Length	0.3048 m
	1,842 m
	1,609 m
Area	0.0929 m ²
Volume	0.02832 m ³
Velocity	0.3048 m/s
	0.5144 m/s
Standard acceleration	9.8066 m/s ²
Mass	0.4536 kg
	1,016 kg
	1.016 tonne
Force	4.4482 N
Pressure, stress	6.8947 kN/m ²
	15.444 MN/m ²
Energy	1.3558 J
Power	745.7 W
Density (SW)	0.975 m ³ /tonne
	0.01 MN/m ³
Density (FW)	1.0 m ³ /tonne
	0.0098 MN/m ³
Modulus of elasticity (E)	20.9 GN
TPI (SW)	1.025A _w tonne/m
TPM	10 ⁴ A _w (N/m)
MCT'' (sw)	$\Delta\overline{GM}_L/L$ (MN m/m)
Displacement mass force (Δ)	9964 N
Displacement mass (Δ)	1.016 tonne
Wetted surface	$\Delta = \text{tonne}, L = \text{m}$

Table 4 Power conversion

Quantity	Common unit	SI unit
BHP	P _B	W
SHP	P _S	W
DHP	P _D	W
EHP	P _E	W

Table 5 Multiples and sub-multiples

Prefix	Factor	Symbol
Tera	10 ¹²	T
Giga	10 ⁹	G
Mega	10 ⁶	M
Kilo	10 ³	k
Milli	10 ⁻³	m
Micro	10 ⁻⁶	μ
Nano	10 ⁻⁹	n
Pico	10 ⁻¹²	p
Femto	10 ⁻¹⁵	f
Atto	10 ⁻¹⁸	a

Table 6 General units

Gravity acceleration: g	9.807 m/s ²
Water density (salt water): ρ_{sw}	1.025 tonne/m ³
Modulus of elasticity: E	20.9 MN/cm ²
Atmospheric pressure: p_{at}	10.14 kN/m ²
1.0 ton displacement	9964 N

Table 7 Shipbuilding units

(a) General	
Dimensions/distances	m
Primary spacing	m
Secondary spacing	mm
Area	m ²
Volume	m ³
Mass	kg
Velocity	m/s
Acceleration	m/s ²
(b) Hull girder properties	
Dimensions	m
Area	m ²
Section modulus	m ³
Inertia	m ⁴
Moment of area	m ³
Dimensions	mm
Area	cm ²
Thickness	mm
(c) Loads	
Pressure	kN/m ²
Loads	kN
Shear force	kN

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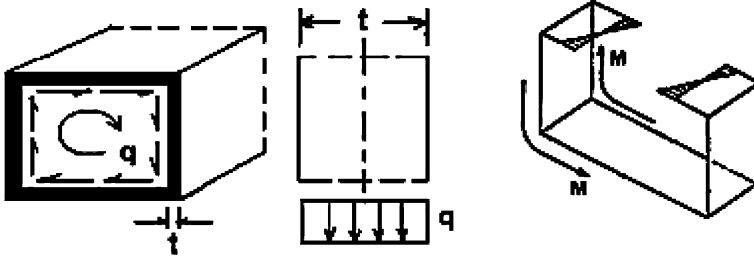
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Part I

Torsion Stresses in Ships



Chapter 1 introduces the basic elements of pure torsion of uniform thin-walled open sections. The basic equations of torsion of thin-walled closed sections are given together with the distribution of torsion shear flow and stress over the section as well as the angle and rate of twist. It also gives a comparison between open and closed thin-walled circular and box sections with regard to torsion stresses and rate of twist. Torsion of uniform thin-walled closed sections with attached open sections is also given. A list of the torsion and warping constants for some thin-walled open sections is given.

Chapter 2 introduces torsion of multi-cell box-girders. It covers torsion of uniform thin-walled two-cell box-girders, torsion stresses in a two identical cells box-girder and torsion of three-cell box-girder. It introduces the general case of torsion of uniform thin-walled multi-cell box-girders. The general case of combined thin-walled open and closed sections is presented. The special cases of combined open sections with one and two closed cell are presented.

Chapter 3 introduces torsion of a thin-walled variable section beam subjected to non-uniform torque. It is shown that plane sections no longer remain plane and torsion is associated with warping. Both free warping and constrained warping are considered. It covers also warping deformations and stresses in deck structure of container ships. Warping and flexural warping stresses of thin-walled sections are also considered for different types of loading and end constraints.

Chapter 4 introduces the definitions and differences between *St. Venant* torsion, warping torsion and the bi-moment. The main elements of torsion of open-decked ships are presented. Hull girder torsion loading and the various cases of local torsion loading on beam elements are considered.

Torsion of container ships traveling obliquely in a sea-way is highlighted. Solution of the torsion equation for assumed two distributions of torsional loading are presented. The method of calculation is based on a simplified idealization of the ship section, calculation of the sectorial properties of the ship section (principal sectorial area diagram, sectorial static moment), position of shear centre, torsion constant J_t and the warping constant $J(x)$. The calculation of the shear and flexural warping stresses are carried out for two cases of boundary conditions, free end, i.e. free warping and constrained end, i.e. constrained warping. The total stress in the deck plating of a container ship due to hull girder bending and torsional loading is presented. A numerical example is given for a container ship to illustrate and clarify the calculation procedure.

Chapter 5 presents the calculation procedure of the sectorial properties of open sections.

Chapter 6 presents a general procedure for the solution of the torsion equation.

Chapter 1

Torsion Stresses in Ships

1.1 Introduction

The torsion analysis is devoted to determining the stress distribution in twisted single-span or continuous numbers with solid, thin-walled open or closed cross sections.

A prismatic member resists a twisting moment in two ways.

1. By producing a circulatory shear flow in the cross section.
2. By inducing shear stresses resulting from the change in axial stresses.

The first is called *St. Venant* torsion and the second is called *warping* torsion or flexural twist. A flexural twist causes always some bending moments in a structure i.e., a pair or more of bending moments. These are called “*Bi-moment*”.

1.2 Torsion Loading of Beam Elements

1.2.1 Direct Torsion Loads

- Concentrated torsion load, see Fig. 1.1.
- Uniformly distributed torsion loading, see Fig. 1.2.
- Linearly distributed torsion loading, see Fig. 1.3.

1.2.2 Induced Torsion Load

Torsion loads may be induced by lateral forces acting at an offset distance from the shear center of the beam section. For the thin-walled channel section shown in

Fig. 1.1 Concentrated torque at the free end of the beam

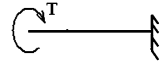


Fig. 1.2 Uniformly distributed torsion loading

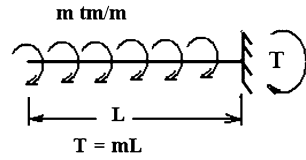


Fig. 1.3 Linearly distributed torsion loading

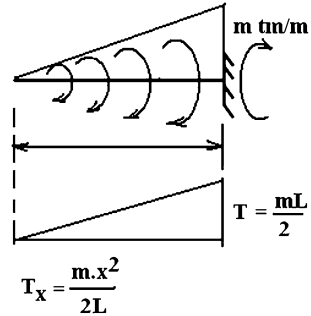


Fig. 1.4 Concentrated torque induced by the lateral force F

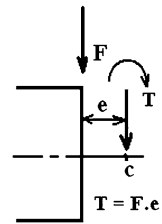


Fig. 1.4, the shear center of the section is located at a distance e from the web of the section. The induced concentrated torque is given by

$$T = F \cdot e$$

where F = lateral concentrated force; e = distance of shear center from the web of the section; T = induced concentrated torque.

1.3 Variation of Torque and Angle of Twist along Beam Length

1.3.1 Beams Subjected to Concentrated Torques

1.3.1.1 Concentrated Torque at End of Beam

The distribution of torque loading and the variation of angle of twist along a cantilever beam subjected to a concentrated torque at its free end are shown in Fig. 1.5.

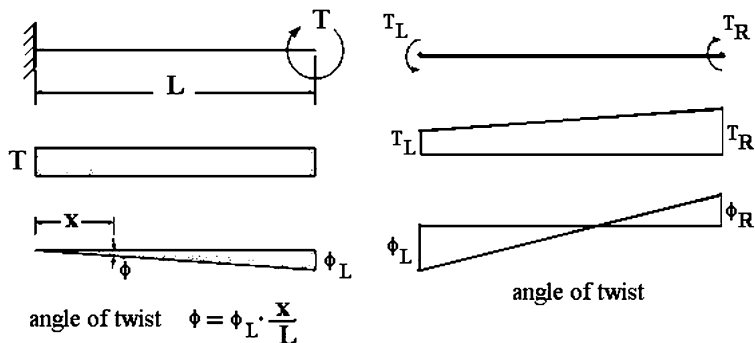


Fig. 1.5 Distribution of torsion loading and variation of angle of twist along a beam length subjected to end concentrated torque

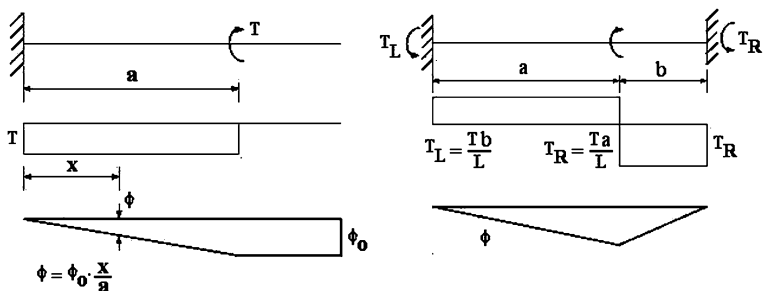


Fig. 1.6 Distribution of torsion loading and variation of angle of twist along a beam length subjected to a concentrated torque within beam length

1.3.1.2 Concentrated Torque within the Beam Length

The distribution of torsion loading and variation of angle of twist along a beam length subjected to a concentrated torque within the beam length is shown in Fig. 1.6.

1.3.2 Beams Subjected to Uniformly Distributed Torsion Loading

Figure 1.7 shows the distribution of torsional moment and the variation of angle of twist along a beam subjected to uniform torque along the beam length for different end conditions.

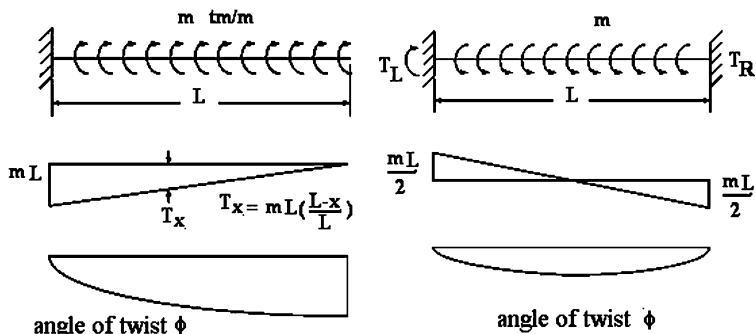


Fig. 1.7 Distribution of torsion loading and variation of angle of twist along a beam length subjected to uniform torque for different end conditions

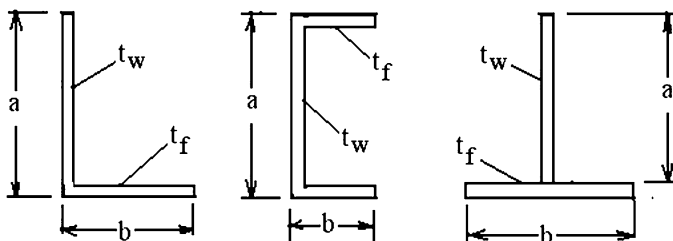


Fig. 1.8 Geometry and scantlings for three types thin-walled open sections

1.4 Torsion of Uniform Thin Walled Sections

1.4.1 Pure Torsion of Uniform Open Thin-Walled Girders

Pure Torsion is the case when a bar is twisted by couples applied at the ends and these ends are free to warp. Pure torsion of thin-walled sections with free ends produce only shear stresses which are the same for all sections.

Plane sections do not remain plane and warping takes place. However, the effect of warping on the calculation of shear stresses and angle of twist could be neglected for simplicity. The distribution of the shear stresses depends on the shape of the section.

For thin-walled open-sections, see Fig. 1.8, the angle of twist/unit length is given by

$$\theta = T/C \tag{1.1}$$

where T = Torque; G = Shearing modulus of elasticity; J = Torsion constant; C = Torsional rigidity of the section; C = G/J.

The torque is related to the angle of twist by the following equation.

$$T = G \cdot J \cdot \theta \tag{1.2}$$

1.4.1.1 Torsion Constant

The torsion constant J of an open thin-walled section is given by

$$J = \sum (s_i t_i^3 / 3)$$

where $s = a$ and b , see Fig. 1.8.

For the angle section shown in Fig. 1.8.

$$J = \sum (s_i t_i^3 / 3) = at_w^3 / 3 + bt_f^3 / 3$$

For the channel section

$$J = \sum (s_i t_i^3 / 3) = at_w^3 / 3 + 2bt_f^3 / 3$$

For the T section

$$J = \sum (s_i t_i^3 / 3) = at_w^3 / 3 + bt_f^3 / 3$$

The torsion and warping constants for three thin-walled open sections are given in Table 1.1.

1.4.1.2 Distribution of Torsion Shear Flow and Stress Over a Thin-Walled Open Section

For open sections, the shear flow and stress are linearly distributed over the wall thickness of the open section and is given by, see Fig. 1.9.

The shear flow is given by

$$q = T \cdot t^2 / J$$

The shear stress is given by

$$\tau = T \cdot t / J$$

The torsion constant

$$J = \sum (s_i t_i^3 / 3)$$

For the thin-walled rectangular section, see Fig. 1.9.

$$J = dt^3 / 3$$

Hence

$$\tau = 3T / dt^2$$

Table 1.1 Torsion and warping constants for some thin-walled open sections

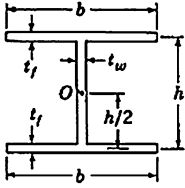
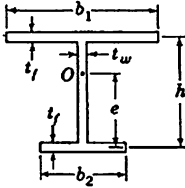
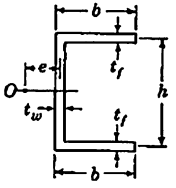
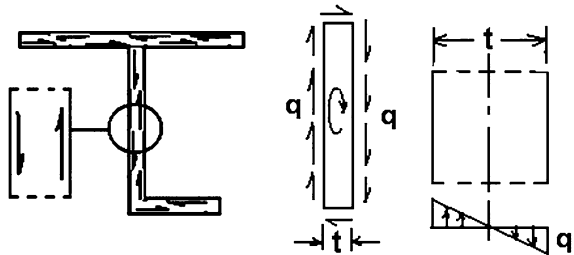
O = shear center	J = torsion constant	C_w = warping constant
	$J = \frac{2bt_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f h^2 b^3}{24}$	<p>If $t_f = t_w = t$:</p> $J = \frac{t^3}{3}(2b + h)$
	$e = h \frac{b_1^3}{b_1^2 + b_2^2}$ $J = \frac{(b_1 + b_2)t_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f h^2}{12} \frac{b_1^3 b_2^3}{b_1^3 + b_2^3}$	<p>If $t_f = t_w = t$:</p> $J = \frac{t^3}{3}(b_1 + b_2 + h)$
	$e = \frac{3b^2 t_f}{6bt_f + ht_w}$ $J = \frac{2bt_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f b^3 h^2}{12} \frac{3bt_f + 2ht_w}{6bt_f + ht_w}$	<p>If $t_f = t_w = t$:</p> $e = \frac{3b^2}{6b + h}$ $J = \frac{t^3}{3}(2b + h)$ $C_w = \frac{tb^3 h^2}{12} \frac{3b + 2h}{6b + h}$

Fig. 1.9 Shear flow distribution over thin-walled open sections



For the thin-walled angle section, the torsion stress in the flange is given by

$$\tau_f = T \cdot t_f / J$$

The torsion stress in the web of the section is given by

$$\tau_w = T \cdot t_w / J$$

The angle of twist is given by

$$\varphi = TL/GJ$$

where G = shear modulus and is given by

$$G = E/2(1 + \nu)$$

1.5 Torsion of Uniform Thin-Walled Closed Sections

Let

q = shear flow in t/m over the periphery of the thin-walled closed section.

Then

$$\begin{aligned} T &= \oint q \cdot ds \cdot r \\ &= q \oint r ds \\ &= 2q \oint dA \\ &= 2qA \end{aligned}$$

where $r \cdot ds = 2$ sectional area, see Fig. 1.10; A = enclosed area of the section.

Hence

$$T = 2 \cdot q \cdot A$$

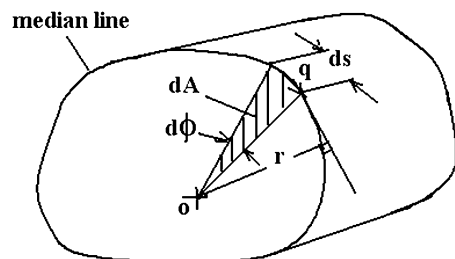
and

$$q = T/2A$$

Thus, when a torque T is applied to a closed thin-walled uniform member, the shear stress is given by

$$\tau = q/t = T/2At$$

Fig. 1.10 Shear flow over a closed thin-walled section



The rate of twist, θ , is calculated by equating the internal energy to the external work, i.e.,

$$1/2 \cdot T \cdot \theta = \oint \tau^2 / 2G \cdot dV$$

But $dV = t \cdot ds$ for a unit length of the member and

$$\tau = T/2At$$

Hence

$$\theta = d\phi/dL = 1/4A^2G \cdot \oint Tds/t = 1/2AG \cdot \oint qds/t$$

In general

$$\theta = T/GJ_t$$

where $J_t =$ torsion constant of the thin-walled closed section and is given by

$$J_t = 4A^2 / \oint ds/t$$

1.6 Basic Equations of Torsion of Thin-Walled Closed Sections

1.6.1 Shear Flow and Stress

Let

$q =$ shear flow in t/m around the periphery of the section, see Fig. 1.11. Hence the shear flow is given by

$$q = T/2At$$

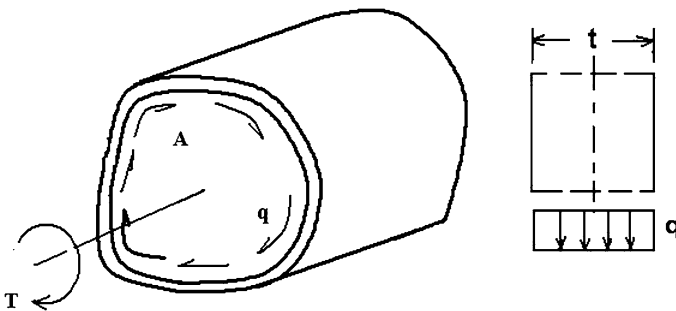


Fig. 1.11 Shear stress over the thickness of a closed thin-walled section