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Quantum Decoherence

Poincaré Seminar 2005

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Foreword

This book is the sixth in a series of lectures of the *Séminaire Poincaré*, which is directed towards a large audience of physicists and of mathematicians.

The goal of this seminar is to provide up-to-date information about general topics of great interest in physics. Both the theoretical and experimental aspects are covered, with some historical background. Inspired by the Bourbaki seminar in mathematics in its organization, hence nicknamed "Bourbaphi", the Poincaré Seminar is held twice a year at the Institut Henri Poincaré in Paris, with contributions prepared in advance. Particular care is devoted to the pedagogical nature of the presentations so as to fulfill the goal of being readable by a large audience of scientists.

This volume contains the eighth such Seminar, held in 2005. It is devoted to Quantum Decoherence. A broad perspective on the subject is provided by the contributions of W.H. Zurek (introductory), H.D. Zeh and E. Joos (historical), together with clear (precise) up-to-date presentations of the recent experiments on decoherence both in the mesoscopic systems of atomic physics, by J.M. Raimond and S. Haroche, and in the "quantronic" or condensed matter context, by D. Esteve et al. Finally the question of quantum codes and error corrections is discussed in the contribution of J. Kempe.

We hope that the publication of this series will serve the community of physicists and mathematicians at graduate student or professional level.

We thank the Commissariat à l'Énergie Atomique (Division des Sciences de la Matière), the Centre National de la Recherche Scientifique (Sciences Physique et Mathématiques), and the Daniel Iagolnitzer Foundation for sponsoring the Seminar. Special thanks are due to Chantal Delongeas for the preparation of the manuscript.

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Poincaré Seminar 2005

Decoherence and the Transition from Quantum to Classical – Revisited

Wojciech Hubert Zurek

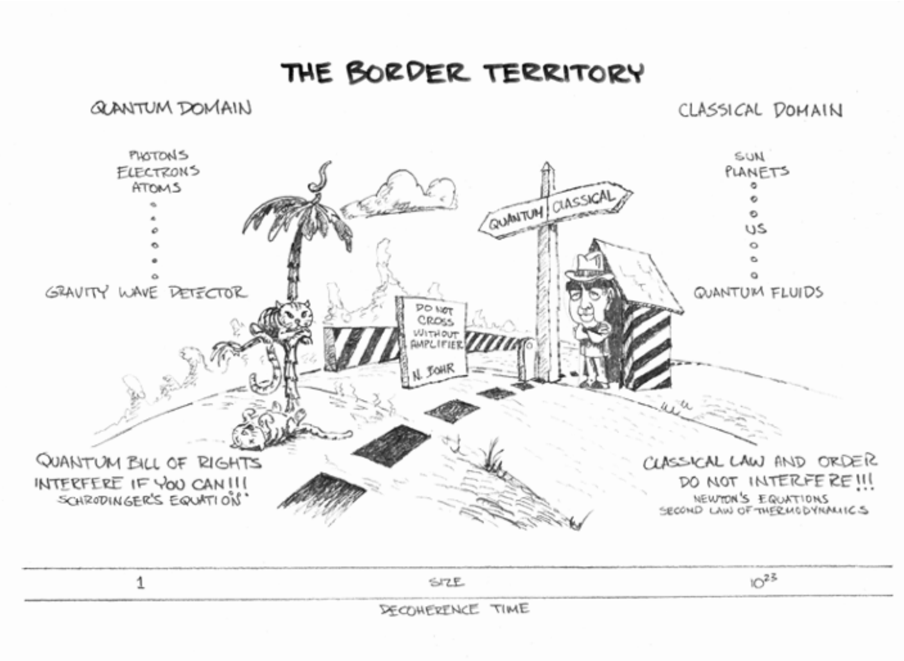
Abstract. The environment surrounding a quantum system can, in effect, monitor some of the systems observables. As a result, the eigenstates of these observables continuously decohere and can behave like classical states.

This paper has a somewhat unusual origin and, as a consequence, an unusual structure. It is built on the principle embraced by families who outgrow their dwellings and decide to add a few rooms to their existing structures instead of starting from scratch. These additions usually "show," but the whole can still be quite pleasing to the eye, combining the old and the new in a functional way. What follows is such a "remodeling" of the paper I wrote a dozen years ago for *Physics Today* (1991). The old text (with some modifications) is interwoven with the new text, but the additions are set off in boxes throughout this article and serve as a commentary on new developments as they relate to the original. The references appear together at the end.

In 1991, the study of decoherence was still a rather new subject, but already at that time, I had developed a feeling that most implications about the system's "immersion" in the environment had been discovered in the preceding 10 years, so a review was in order. While writing it, I had, however, come to suspect that the small gaps in the landscape of the border territory between the quantum and the classical were actually not that small after all and that they presented excellent opportunities for further advances.

Indeed, I am surprised and gratified by how much the field has evolved over the last decade. The role of decoherence was recognized by a wide spectrum of practicing physicists as well as, beyond physics proper, by material scientists and philosophers. The study of the predictability sieve, investigations of the interface between chaotic dynamics and decoherence, and most recently, the tantalizing glimpses of the information-theoretic nature of the quantum have elucidated our understanding of theubert Universe.

Not all of the new developments are reported in this review: Some of the most recent (and, conceivably, most far-reaching) are still too "fresh", and, hence,



too difficult to describe succinctly. The role of redundancy of the imprint left by the preferred observables of the system on the states of the environment in the emergence of the objective classical properties from the quantum substrate, or the concept of the environment – assisted invariance (or envariance) that allows one to give a fully quantum justification of Born's rule connecting amplitudes with probabilities are beyond the scope of this minireview.

Finally, I have some advice to the reader. I believe this paper should be read twice: first, just the old text alone; then – and only then – on the second reading, the whole thing. I would also recommend to the curious reader two other overviews: the draft of my *Reviews of Modern Physics* paper (Zurek 2001a) and Les Houches Lectures coauthored with Juan Pablo Paz (Paz and Zurek 2001).

Introduction

Quantum mechanics works exceedingly well in all practical applications. No example of conflict between its predictions and experiment is known. Without quantum physics, we could not explain the behavior of the solids, the structure and function of DNA, the color of the stars, the action of lasers, or the properties of superfluids. Yet nearly a century after its inception, the debate about the relation of quantum physics to the familiar physical world continues. Why is a theory that seems to account with precision for everything we can measure still deemed lacking?

The only "failure" of quantum theory is its inability to provide a natural framework for our prejudices about the workings of the Universe. States of quantum systems evolve according to the deterministic, linear Schrödinger equation

$$i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle \ . \tag{1}$$

That is, just as in classical mechanics, given the initial state of the system and its Hamiltonian H, one can, at least in principle, compute the state at an arbitrary time. This deterministic evolution of $|\psi\rangle$ has been verified in carefully controlled experiments. Moreover, there is no indication of a border between quantum and classical at which Equation (1) would fail (see cartoon on the opener to this article).

There is, however, a very poorly controlled experiment with results so tangible and immediate that it has enormous power to convince: Our perceptions are often difficult to reconcile with the predictions of Equation (1). Why? Given almost any initial condition, the Universe described by $|\psi\rangle$ evolves into a state containing many alternatives that are never seen to coexist in our world. Moreover, while the ultimate evidence for the choice of one alternative resides in our elusive "consciousness," there is every indication that the choice occurs much before consciousness ever gets involved and that, once made, it is irrevocable. Thus, at the root of our unease with quantum theory is the clash between the principle of superposition – the basic tenet of the theory reflected in the linearity of Equation (1) – and everyday classical reality in which this principle appears to be violated.

The problem of measurement has a long and fascinating history. The first widely accepted explanation of how a single outcome emerges from the multitude of potentialities was the Copenhagen Interpretation proposed by Niels Bohr (1928), who insisted that a classical apparatus is necessary to carry out measurements. Thus, quantum theory was not to be universal. The key feature of the Copenhagen Interpretation is the dividing line between quantum and classical. Bohr emphasized that the border must be mobile so that even the "ultimate apparatus" – the human nervous system – could in principle be measured and analyzed as a quantum object, provided that a suitable classical device could be found to carry out the task.

In the absence of a crisp criterion to distinguish between quantum and classical, an identification of the classical with the macroscopic has often been tentatively accepted. The inadequacy of this approach has become apparent as a result of relatively recent developments: A cryogenic version of the Weber bar – a gravity-wave detector – must be treated as a quantum harmonic oscillator even though it may weigh a ton (Braginsky et al. 1980, Caves et al. 1980). Nonclassical squeezed states can describe oscillations of suitably prepared electromagnetic fields with macroscopic numbers of photons (Teich and Saleh 1990). Finally, quantum states associated with the currents of superconducting Josephson junctions involve macroscopic numbers of electrons, but still they can tunnel between the minima of the effective potential corresponding to the opposite sense of rotation (Leggett et al. 1987, Caldeira and Leggett 1983a, Tesche 1986).

If macroscopic systems cannot be always safely placed on the classical side of the boundary, then might there be no boundary at all? The Many Worlds Interpretation (or more accurately, the Many Universes Interpretation), developed by Hugh Everett III with encouragement from John Archibald Wheeler in the 1950s, claims to do away with the boundary (Everett 1957, Wheeler 1957). In this interpretation, the entire universe is described by quantum theory. Superpositions evolve forever according to the Schrödinger equation. Each time a suitable interaction takes place between any two quantum systems, the wave function of the universe splits, developing ever more "branches."

Initially, Everett's work went almost unnoticed. It was taken out of mothballs over a decade later by Bryce DeWitt (1970) and DeWitt and Neill Graham (1973), who managed to upgrade its status from "virtually unknown" to "very controversial." The Many Worlds Interpretation is a natural choice for quantum cosmology, which describes the whole Universe by means of a state vector. There is nothing more macroscopic than the Universe. It can have no a priori classical subsystems. There can be no observer "on the outside." In this universal setting, classicality must be an emergent property of the selected observables or systems.

At first glance, the Many Worlds and Copenhagen Interpretations have little in common. The Copenhagen Interpretation demands an a priori "classical domain" with a border that enforces a classical "embargo" by letting through just one potential outcome. The Many Worlds Interpretation aims to abolish the need for the border altogether. Every potential outcome is accommodated by the ever-proliferating branches of the wave function of the Universe. The similarity between the difficulties faced by these two viewpoints becomes apparent, nevertheless, when we ask the obvious question, "Why do I, the observer, perceive only one of the outcomes?" Quantum theory, with its freedom to rotate bases in Hilbert space, does not even clearly define which states of the Universe correspond to the "branches." Yet, our perception of a reality with alternatives – not a coherent superposition of alternatives – demands an explanation of when, where, and how it is decided what the observer actually records. Considered in this context, the Many Worlds Interpretation in its original version does not really abolish the border but pushes it all the way to the boundary between the physical Universe and consciousness. Needless to say, this is a very uncomfortable place to do physics.

In spite of the profound nature of the difficulties, recent years have seen a growing consensus that progress is being made in dealing with the measurement problem, which is the usual euphemism for the collection of interpretational conundrums described above. The key (and uncontroversial) fact has been known almost since the inception of quantum theory, but its significance for the transition from quantum to classical is being recognized only now: Macroscopic systems are never isolated from their environments. Therefore – as H. Dieter Zeh emphasized (1970) – they should not be expected to follow Schrödinger's equation, which is applicable only to a closed system. As a result, systems usually regarded as classical suffer (or benefit) from the natural loss of quantum coherence, which "leaks out" into the environment (Zurek 1981, 1982). The resulting decoherence cannot be ignored when one addresses the problem of the reduction of the quantum mechanical wavepacket: Decoherence imposes, in effect, the required "embargo" on the potential outcomes by allowing the observer to maintain only records of the alternatives sanctioned by decoherence and to be aware of only one of the branches – one of the "decoherent histories" in the nomenclature of Murray Gell-Mann and James Hartle (1990) and Hartle (1991).

The aim of this paper is to explain the physics and thinking behind decoherence and environment-induced superselection. The reader should be warned that this writer is not a disinterested witness to this development (Wigner 1983, Joos and Zeh 1985, Haake and Walls 1986, Milburn and Holmes 1986, Albrecht 1991, Hu et al. 1992), but rather, one of the proponents. I shall, nevertheless, attempt to paint a fairly honest picture and point out the difficulties as well as the accomplishments.

Decoherence in Quantum Information Processing

Much of what was written in the introduction remains valid today. One important development is the increase in experimental evidence for the validity of the quantum principle of superposition in various contexts including spectacular double-slit experiments that demonstrate interference of fullerenes (Arndt et al. 1999), the study of superpositions in Josephson junctions (Mooij et al.1999, Friedman et al. 2000), and the implementation of Schrödinger "kittens" in atom interferometry (Chapman et al. 1995, Pfau et al. 1994), ion traps (Monroe et al. 1996) and microwave cavities (Brune et al. 1996). In addition to confirming the superposition principle and other exotic aspects of quantum theory (such as entanglement) in novel settings, some of these experiments allow – as we shall see later – for a controlled investigation of decoherence.

The other important change that influenced the perception of the quantum-to-classical "border territory" is the explosion of interest in quantum information and computation. Although quantum computers were already being discussed in the 1980s, the nature of the interest has changed since Peter Shor invented his factoring algorithm. Impressive theoretical advances, including the discovery of quantum error correction and resilient quantum computation, quickly followed, accompanied by increasingly bold experimental forays. The superposition principle, once the cause of trouble for the interpretation of quantum theory, has become the central article of faith in the emerging science of quantum information processing. This last development is discussed elsewhere in this volume, so I shall not dwell on it here.

The application of quantum physics to information processing has also transformed the nature of interest in the process of decoherence: At the time of my original review (1991), decoherence was a solution to the interpretation problem – a mechanism to impose an effective classicality on de facto quantum systems. In quantum information processing, decoherence plays two roles. Above all, it is a threat to the quantumness of quantum information. It invalidates the quantum superposition principle and thus turns quantum computers into (at best) classical computers, negating the potential power offered by the quantumness of the algorithms. But decoherence is also a necessary (although, until recently, tacitly taken for granted) ingredient in quantum information processing, which must, after all, end in a "measurement."

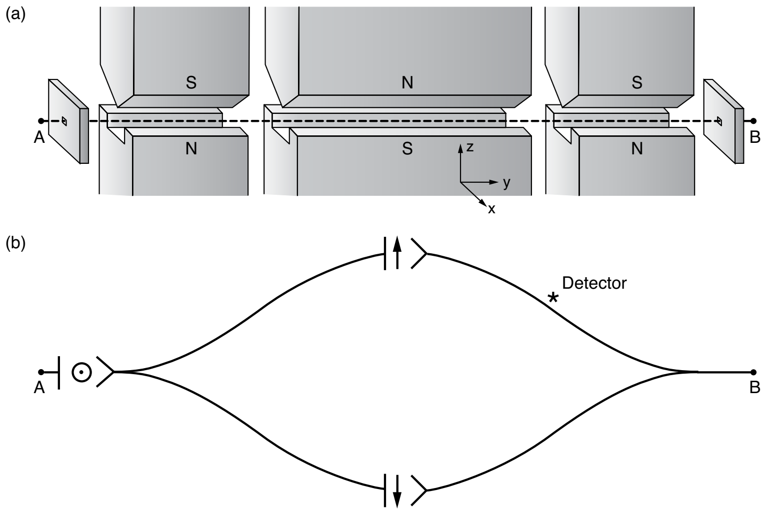


FIGURE 1. A Reversible Stern-Gerlach Apparatus.
The "gedanken" reversible Stern-Gerlach apparatus in (a) splits a beam of atoms into two branches that are correlated with the component of the spin of the atoms (b) and then recombines the branches before the atoms leave the device. Eugene Wigner (1963) used this gedanken experiment to show that a correlation between the spin and the location of an atom can be reversibly undone. The introduction of a one-bit (two-state) quantum detector that changes its state when the atom passes nearby prevents the reversal: The detector inherits the correlation between the spin and the trajectory, so the Stern-Gerlach apparatus can no longer undo the correlation. (This illustration was adapted with permission from Zurek 1981.)

The role of a measurement is to convert quantum states and quantum correlations (with their characteristic indefiniteness and malleability) into classical, definite outcomes. Decoherence leads to the environment-induced superselection (einselection) that justifies the existence of the preferred pointer states. It enables one to draw an effective border between the quantum and the classical in straightforward terms, which do not appeal to the "collapse of the wavepacket" or any other such deus ex machina.

Correlations and Measurements

A convenient starting point for the discussion of the measurement problem and, more generally, of the emergence of classical behavior from quantum dynamics is the analysis of quantum measurements due to John von Neumann (1932). In contrast to Bohr, who assumed at the outset that the apparatus must be classical (thereby forfeiting claim of quantum theory to universal validity), von Neumann analyzed the case of a quantum apparatus. I shall reproduce his analysis for the simplest case: a measurement on a two-state system $\mathcal{S}$ (which can be thought of as an atom with spin 1/2) in which a quantum two-state (one bit) detector records the result.

The Hilbert space $\mathcal{H}_S$ of the system is spanned by the orthonormal states $|\uparrow\rangle$ and $|\downarrow\rangle$, while the states $|d_\uparrow\rangle$ and $|d_\downarrow\rangle$ span the $\mathcal{H}_D$ of the detector. A two-dimensional $\mathcal{H}_D$ is the absolute minimum needed to record the possible outcomes. One can devise a quantum detector (see Figure 1) that "clicks" only when the spin is in the state $|\uparrow\rangle$, that is,

$$|\uparrow\rangle|d_\downarrow\rangle \rightarrow |\uparrow\rangle|d_\uparrow\rangle \ , \tag{2}$$

and remains unperturbed otherwise.

I shall assume that, before the interaction, the system was in a pure state $\psi_\mathcal{S}$ given by

$$|\psi_\mathcal{S}\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle \ , \tag{3}$$

with the complex coefficients satisfying $|\alpha|^2 + |\beta|^2 = 1$. The composite system starts as

$$|\Phi^i\rangle = |\psi_\mathcal{S}\rangle|d_\downarrow\rangle \ , \tag{4}$$

Interaction results in the evolution of $|\Phi^i\rangle$ into a correlated state $|\Phi^c\rangle$:

$$|\Phi^i\rangle = (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \Rightarrow \alpha|\uparrow\rangle|d_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle = |\Phi^c\rangle \ . \tag{5}$$

This essential and uncontroversial first stage of the measurement process can be accomplished by means of a Schrödinger equation with an appropriate interaction. It might be tempting to halt the discussion of measurements with Equation (5). After all, the correlated state vector $|\Phi^c\rangle$ implies that, if the detector is seen in the state $|d_\uparrow\rangle$, the system is guaranteed to be found in the state $|\uparrow\rangle$. Why ask for anything more?

The reason for dissatisfaction with $|\Phi^c\rangle$ as a description of a completed measurement is simple and fundamental: In the real world, even when we do not know the outcome of a measurement, we do know the possible alternatives, and we can safely act as if only one of those alternatives has occurred. As we shall see in the next section, such an assumption is not only unsafe but also simply wrong for a system described by $|\Phi^c\rangle$.

How then can an observer (who has not yet consulted the detector) express his ignorance about the outcome without giving up his certainty about the "menu" of the possibilities? Quantum theory provides the right formal tool for the occasion: A density matrix can be used to describe the probability distribution over the alternative outcomes.

Von Neumann was well aware of these difficulties. Indeed, he postulated (1932) that, in addition to the unitary evolution given by Equation (1), there should be an ad hoc "process 1"—a nonunitary reduction of the state vector—that would take the pure, correlated state $|\Phi^c\rangle$ into an appropriate mixture: This process makes the outcomes independent of one another by taking the pure-state density matrix:

$$\begin{aligned}\rho^c \quad = \quad |\Phi^c\rangle\langle\Phi^c| \quad &= \quad |\alpha|^2|\uparrow\rangle\langle\uparrow||d_\uparrow\rangle\langle d_\uparrow| + \alpha\beta^*|\uparrow\rangle\langle\downarrow||d_\uparrow\rangle\langle d_\downarrow| \\ &\quad + \alpha^*\beta|\downarrow\rangle\langle\uparrow||d_\downarrow\rangle\langle d_\uparrow| + |\beta|^2|\downarrow\rangle\langle\downarrow||d_\downarrow\rangle\langle d_\downarrow| \,, \qquad (6)\end{aligned}$$

and canceling the off-diagonal terms that express purely quantum correlations (entanglement) so that the reduced density matrix with only classical correlations emerges:

$$\rho^r = |\alpha|^2|\uparrow\rangle\langle\uparrow||d_\uparrow\rangle\langle d_\uparrow| + |\beta|^2|\downarrow\rangle\langle\downarrow||d_\downarrow\rangle\langle d_\downarrow| \,. \qquad (7)$$

Why is the reduced ρ^r easier to interpret as a description of a completed measurement than ρ^c? After all, both ρ^r and ρ^c contain identical diagonal elements. Therefore, both outcomes are still potentially present. So what – if anything – was gained at the substantial price of introducing a nonunitary process 1?

The Question of Preferred Basis: What Was Measured?

The key advantage of ρ^r over ρ^c is that its coefficients may be interpreted as classical probabilities. The density matrix ρ^r can be used to describe the alternative states of a composite spin-detector system that has classical correlations. Von Neumann's process 1 serves a similar purpose to Bohr's "border" even though process 1 leaves all the alternatives in place. When the off-diagonal terms are absent, one can nevertheless safely maintain that the apparatus, as well as the system, is each separately in a definite but unknown state, and that the correlation between them still exists in the preferred basis defined by the states appearing on the diagonal. By the same token, the identities of two halves of a split coin placed in two sealed envelopes may be unknown but are classically correlated. Holding one unopened envelope, we can be sure that the half it contains is either "heads" or "tails" (and not some superposition of the two) and that the second envelope contains the matching alternative.

By contrast, it is impossible to interpret ρ^c as representing such "classical ignorance." In particular, even the set of the alternative outcomes is not decided by ρ^c! This circumstance can be illustrated in a dramatic fashion by choosing $\alpha = -\beta = 1/\sqrt{2}$ so that the density matrix ρ^c is a projection operator constructed from the correlated state

$$|\Phi^c\rangle = (|\uparrow\rangle|d_\uparrow - |\downarrow\rangle|d_\downarrow\rangle)\sqrt{2}\ . \tag{8}$$

This state is invariant under the rotations of the basis. For instance, instead of the eigenstates of $|\uparrow\rangle$ and $|\downarrow\rangle$ of $\hat{\sigma}_z$ one can rewrite $|\Phi^c\rangle$ in terms of the eigenstates of $\hat{\sigma}_x$:

$$|\odot\rangle = (|\uparrow\rangle + |\downarrow\rangle)\sqrt{2}\ , \tag{9a}$$

$$|\otimes\rangle = (|\uparrow\rangle - |\downarrow\rangle)\sqrt{2}\ . \tag{9b}$$

This representation immediately yields

$$|\Phi^c\rangle = (|\odot\rangle|d_\odot\rangle - |\otimes\rangle|d_\otimes\rangle)/\sqrt{2}\ , \tag{10}$$

where

$$|d_\odot\rangle = |d_\downarrow\rangle - d_\uparrow\rangle/\sqrt{2} \quad \text{and} \quad |d_\otimes\rangle = |d_\uparrow\rangle + d_\downarrow\rangle\ \sqrt{2}\ , \tag{11}$$

are, as a consequence of the superposition principle, perfectly “legal” states in the Hilbert space of the quantum detector. Therefore, the density matrix

$$\rho^c = |\Phi^c\rangle\langle\Phi^c|$$

could have many (in fact, infinitely many) different states of the subsystems on the diagonal.

This freedom to choose a basis should not come as a surprise. Except for the notation, the state vector $|\Phi^c\rangle$ is the same as the wave function of a pair of maximally correlated (or entangled) spin-1/2 systems in David Bohm's version (1951) of the Einstein-Podolsky-Rosen (EPR) paradox (Einstein et al. 1935). And the experiments that show that such nonseparable quantum correlations violate Bell's inequalities (Bell 1964) are demonstrating the following key point: The states of the two spins in a system described by $|\Phi^c\rangle$ are not just unknown, but rather they cannot exist before the “real” measurement (Aspect et al. 1981, 1982). We conclude that when a detector is quantum, a superposition of records exists and is a record of a superposition of outcomes – a very nonclassical state of affairs.

Missing Information and Decoherence

Unitary evolution condemns every closed quantum system to “purity.” Yet, if the outcomes of a measurement are to become independent events, with consequences that can be explored separately, a way must be found to dispose of the excess information. In the previous sections, quantum correlation was analyzed from the point of view of its role in acquiring information. Here, I shall discuss the flip side of the story: Quantum correlations can also disperse information throughout the degrees of freedom that are, in effect, inaccessible to the observer. Interaction with the degrees of freedom external to the system – which we shall summarily refer to as the environment – offers such a possibility.

Reduction of the state vector, $\rho^c \Rightarrow \rho^r$, decreases the information available to the observer about the composite system $\mathcal{SD}$. The information loss is needed if the outcomes are to become classical and thereby available as initial conditions to predict the future. The effect of this loss is to increase the entropy $\mathcal{H} = -Tr\rho \ln \rho$ by an amount

$$\Delta\mathcal{H} = \mathcal{H}(\rho^r) - \mathcal{H}(\rho^c) = (|\alpha|^2 \ln |\alpha|^2 + |\beta|^2 \ln |\beta|^2) \ . \tag{12}$$

Entropy must increase because the initial state described by ρ^c was pure, $\mathcal{H}(\rho^c) = 0$, and the reduced state is mixed. Information gain – the objective of the measurement – is accomplished only when the observer interacts and becomes correlated with the detector in the already precollapsed state ρ^r.

To illustrate the process of the environment-induced decoherence, consider a system $\mathcal{S}$, a detector $\mathcal{D}$, and an environment $\mathcal{E}$. The environment is also a quantum system. Following the first step of the measurement process – establishment of a correlation as shown in Equation (5) – the environment similarly interacts and becomes correlated with the apparatus:

$$|\Phi^c\rangle|\mathcal{E}\rangle = (\alpha|\uparrow\rangle|d_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle)\mathcal{E}_0\rangle \Rightarrow \alpha|\uparrow\rangle|d_\uparrow\rangle|\mathcal{E}_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle|\mathcal{E}_\downarrow\rangle = |\Psi\rangle \ . \tag{13}$$

The final state of the combined $\mathcal{S}DE$ "von Neumann chain" of correlated systems extends the correlation beyond the $\mathcal{S}D$ pair. When the states of the environment $\mathcal{E}_i\rangle$ corresponding to the states $|d_\uparrow\rangle$ and $|d_\downarrow\rangle$ of the detector are orthogonal, $\langle\mathcal{E}_i|\mathcal{E}_{i'}\rangle = \delta_{ii'}$, the density matrix for the detector-system combination is obtained by ignoring (tracing over) the information in the uncontrolled (and unknown) degrees of freedom

$$\begin{aligned}\rho_{\mathcal{D}S} &= Tr_{\mathcal{E}}|\Psi\rangle\langle\Psi| = \Sigma_i\langle\mathcal{E}_i|\Psi\rangle\langle\Psi|\mathcal{E}_{i'}\rangle \\ &= |\alpha|^2|\uparrow\rangle\langle\uparrow||d_\uparrow\rangle\langle d_\uparrow| + |\beta|^2|\downarrow\rangle\langle\downarrow||d_\downarrow\rangle\langle d_\downarrow| = \rho^r \ .\end{aligned} \tag{14}$$

The resulting ρ^r is precisely the reduced density matrix that von Neumann called for. Now, in contrast to the situation described by Equations (9)–(11), a superposition of the records of the detector states is no longer a record of a superposition of the state of the system. A preferred basis of the detector, sometimes called the "pointer basis" for obvious reasons, has emerged. Moreover, we have obtained it – or so it appears – without having to appeal to von Neumann's nonunitary process 1 or anything else beyond the ordinary, unitary Schrödinger evolution. The preferred basis of the detector – or for that matter, of any open quantum system – is selected by the dynamics.

Not all aspects of this process are completely clear. It is, however, certain that the detector-environment interaction Hamiltonian plays a decisive role. In particular, when the interaction with the environment dominates, eigenspaces of any observable Λ that commutes with the interaction Hamiltonian,

$$[\Lambda, H_{int}] = 0 \ . \tag{15}$$

invariably end up on the diagonal of the reduced density matrix (Zurek 1981, 1982). This commutation relation has a simple physical implication: It guarantees that the pointer observable Λ will be a constant of motion, a conserved quantity under the evolution generated by the interaction Hamiltonian. Thus, when a system is in an eigenstate of Λ, interaction with the environment will leave it unperturbed.

In the real world, the spreading of quantum correlations is practically inevitable. For example, when in the course of measuring the state of a spin-1/2 atom (see Figure 1b), a photon had scattered from the atom while it was traveling along one of its two alternative routes, this interaction would have resulted in a correlation with the environment and would have necessarily led to a loss of quantum coherence. The density matrix of the $\mathcal{SD}$ pair would have lost its off-diagonal terms. Moreover, given that it is impossible to catch up with the photon, such loss of coherence would have been irreversible. As we shall see later, irreversibility could also arise from more familiar, statistical causes: Environments are notorious for having large numbers of interacting degrees of freedom, making extraction of lost information as difficult as reversing trajectories in the Boltzmann gas.

Quantum Discord – A Measure of Quantumness

The contrast between the density matrices in Equations (6) and (7) is stark and obvious. In particular, the entanglement between the system and the detector in ρ^c is obviously quantum – classical systems cannot be entangled. The argument against the "ignorance" interpretation of ρ^c still stands. Yet we would like to have a quantitative measure of how much is classical (or how much is quantum) about the correlations of a state represented by a general density matrix. Such a measure of the quantumness of correlation was devised recently (Zurek 2000, Ollivier and Zurek 2002). It is known as quantum discord. Of the several closely related definitions of discord, we shall select one that is easiest to explain. It is based on mutual information – an information-theoretic measure of how much easier it is to describe the state of a pair of objects $(\mathcal{S}, D)$ jointly rather than separately. One formula for mutual information $\mathcal{I}(S : D)$ is simply

$$\mathcal{I}(S : D) = H(S) + H(D) - H(S, D) \ ,$$

where $\mathcal{H}(S)$ and $\mathcal{H}(D)$ are the entropies of $\mathcal{S}$ and $\mathcal{D}$, respectively, and $\mathcal{H}(S, D)$ is the joint entropy of the two. When $\mathcal{S}$ and $\mathcal{D}$ are not correlated (statistically independent),

$$\mathcal{H}(S, D) = H(S) + H(D) \ ,$$

and $\mathcal{I}(S : D) = 0$. By contrast, when there is a perfect classical correlation between them (for example, two copies of the same book), $\mathcal{H}(S, D) = H(S) = H(D)$ $=I(S : D)$. Perfect classical correlation implies that, when we find out all about one of them, we also know everything about the other, and the conditional entropy $\mathcal{H}(S|D)$ (a measure of the uncertainty about $\mathcal{S}$ after the state of $\mathcal{D}$ is found out) disappears. Indeed, classically, the joint entropy $\mathcal{H}(S, D)$ can always be decomposed into, say, $\mathcal{H}(D)$, which measures the information missing about $\mathcal{D}$, and the conditional entropy $\mathcal{H}(S|D)$. Information is still missing about $\mathcal{S}$ even after the state of $\mathcal{D}$ has been determined: $\mathcal{H}(S, D) = H(D) + H(S|D)$. This expression for