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Perhaps no other single architect has had an impact on the face of Western architecture as has Andrea Palladio (1508-1580). The characteristic forms used in his villas, basilicas and palazzos were adopted and adapted for widespread use first in England and then in the United States and are now as representative of those lands as they are of the Veneto which saw their birth. But perhaps an even greater legacy of Palladio is his *Four Books of Architecture*, one of the first treatises on architecture that was richly illustrated and intended for a readership of architects and builders rather than intellectuals. Thanks to the clarity and scope of the *Four Books*, they can still be studied with profit today. What makes the *Four Books* of enduring interest is that Palladio set forth his canons of architecture, that is, the rules he used to create his architectural forms, from the details and proportions of the orders to the layout of floor plans for various building types. These rules, rather than remaining specific to a single building type at a unique moment in time, have been studied and abstracted and reapplied to find new, fresh applications. This is the aspect we most wish to honor with this special issue of the *Nexus Network Journal*, entitled “Canons of Form-Making,” dedicated to the quincentenary of Palladio’s birth.

The issue opens with Stephen R. Wassell’s “Andrea Palladio (1508-1580)”. This brief biography was originally written for “The Year of Palladio” website of the Institute for Classical Architecture and Classical America (http://www.classicist.org/resources/year-of-palladio/). The editors therefore wish to thank the ICA&CA for permitting us to publish the biography in this special issue of the NNJ. For inclusion in this issue, the endnotes and bibliography have been expanded from the original version to highlight numerous publications concerning relationships between architecture and mathematics in Palladio’s oeuvre. Of course, we can now add to this bibliography the following three articles devoted to Palladio!

Lionel March’s “Palladio, Pythagoreanism and Renaissance Mathematics” is an analysis of the very first building presented in the crucial second book of Palladio’s treatise, the Palazzo Antonini. In this article March applies his expert knowledge of the types of mathematics (including what we would now call numerology) that were important to Renaissance scholars and practitioners, in order to gain a better understanding of Palladio’s design methodologies and canons.

Coincidentally, Buthayna Eilouti’s “A Formal Language for Palladian Palazzo Façades Represented by a String Recognition Device” also analyzes Palazzo Antonini, as well as the rest of the nine designs Palladio includes in book II, chapter 3. But while March focuses on mathematics known to Palladio, Eilouti analyzes the nine façades using modern mathematics related to computer science, namely regular languages and finite state automata.

In “A Perspective Analysis of the Proportions of Palladio’s Villa Rotonda: Making the Invisible Visible”, Tomás Salgado-García examines the very familiar Villa Rotonda from the viewpoint of perspective to show that Palladio’s proportions are not buried as abstract concepts visible only on the drawing board but reveal themselves visually in the building.

Canons of architecture existed of course in antiquity, long before Palladio set his own down in writing, but in the face of the lack of written documentation, they are tantalizingly hidden. New mathematical tools can help researchers uncover those canons. In “The Doric Order as Fractal”, Carl Bovill uses the technique of iterated function system (IFS) to analyze the Doric temple, and shows that the characteristics of self-similarity and self-
affinity that result from it indicate that the Greeks based their canons of architecture on what they observed in nature.

Michael Duddy also examines the Doric order in “Roaming Point Perspective: A Dynamic Interpretation of the Visual Refinements of the Greek Doric Temple”. He examines the very subtle canons for making corrections to structures or elements so that they look like they should ideally, and presents a new theory that holds that these corrections were efficacious as the observer changed position, not just from a static, “ideal” viewing point.

Roberto Castiglia and Marco Giorgio Bevilacqua take us to Albania to discover canons of Islamic architecture. In “The Turkish Baths at Elbasan: Architecture, Geometry and Well-Being”, Castiglia and Bevilacqua report on the results of a survey campaign coordinated by the University of Pisa, which has provided precise information of the geometric rules that underlie the complex plans and the domes and vaults of the hamman.

All of these studies show that mathematics is a powerful tool for the architect, both during the original design process and in the after-the-fact study of existing monuments. Anat David-Artman believes that mathematics is not just powerful but vital. In “Mathematics as a Vital Force in Architecture”, the work of biologist Hans Dreisch is used to forge an analogy between the principle of individuation and equipotential in life forms and in architecture.

Regarding didactics, in “The Use of Linear Fractional Transformations to Produce Building Plans” Christopher Stone gives a step-by-step explanation of how mappings on a complex plane can be used to generate floor plans.

In this issue’s Geometer’s Angle column, geometer Rachel Fletcher turns her attention once more to Palladio, as she has done twice before for Nexus readers and conference participants. In part three of her examination of Dynamic Root Rectangles, she explains the properties of root-three rectangles and applies them to the plans of Palazzo della Torre in Verona and Villa Mocenigo, Marocco in Treviso.

This issue concludes with two book reviews. Kim Williams reviews Andrea Palladio: The Villa Cornaro in Piombino Dese edited by Branko Mitrović and Stephen R. Wassell. Sylvie Duvernoy reviews Architettura e Musica nella Venezia del Rinascimento edited by Deborah Howard and Laura Moretti.

This is not only an anniversary year for Palladio, but one for the Nexus community as well. This is the tenth year of publication of the Nexus Network Journal, and it was ten years ago that the post-Nexus 1998 conference workshop was a tour of Palladio’s villas. We co-editors, who led the 1998 workshop, learned much on that tour and as you can see, are still exploring the depths of Palladio’s oeuvre ten years on!

Happy birthday, Andrea Palladio!

[Signature]

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Stephen R. Wassell
Department of Mathematics
and Computer Science
Sweet Briar College
Sweet Briar, VA 24595 USA
wassell@sbc.edu

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proportions, geometry, design
theory, classical architecture

Research

Andrea Palladio (1508-1580)

Abstract. A brief description of Palladio’s life and works. The
focus is on the evolution of his design methodology, including
the growing importance of proportion to his approach. Selected
mathematical details are cited in the endnotes, and the list of
references includes many publications focused on the
relationships between architecture and mathematics in Palladio’s
designs.

All over the western world, hundreds of thousands of houses, churches and
public buildings with symmetrical fronts and applied half-columns topped
by a pediment descend from the designs of Andrea Palladio. He is the most
imitated architect in history, and his influence on the development of
English and American architecture probably has been greater that that of all
other Renaissance architects combined.

[Ackerman 1966: 19]

Half a millennium ago in Padua, a prominent city in the Veneto region of Italy,
decades before the future architect would adopt the moniker Palladio, Andrea di Pietro
dalla Gondola (1508-1580) was born into a family of modest means, his father Pietro being
a mill worker. In 1521 Palladio was apprenticed to a Paduan stonemason, Bartolomeo
Cavezza, but he broke away from Cavezza in 1524 and moved to Vicenza. To this day his
adopted city celebrates Palladio as its most famous citizen.

Fig. 1. Villa Godi (photograph by the author)

Here Palladio joined the Pedemuro bottega, a workshop of stonemasons that enjoyed a
steady stream of sculptural and architectural commissions, due in no small part to Vicenza’s
wealth. The workshop’s consistent use of “protoclassical elements” with a “rare and unusual
virtuosity in the range of artisan skills” undoubtedly influenced Palladio, although the workshop’s classical “references have the effect of quotations embedded in vernacular architecture.” Palladio inherited his first architectural commission, Villa Godi (begun c. 1537, fig. 1), from Pedemuro at about the same time that he left the bottega.

During these formative years, Palladio developed the origins of his classical architectural vocabulary, deriving elements from many skilled architects, scholars, and practitioners in the surrounding region. These included the Pedemuro master Giovanni di Giacomo da Porlezza; the prominent Paduan patron Alvise Cornaro and his architectural circle, most notably Giovanni Maria Falconetto, Michele Sanmicheli, Jacopo Sansovino, Giulio Romano, and Sebastiano Serlio, whose nascent treatise on architecture was available to Palladio, books III and IV having been published by 1540. Palladio’s drawings of classical elements such as capitals and entablatures from this time period show his desire to exercise his growing vocabulary, and it is telling that he later modified a number of these drawings after seeing the original buildings with his own eyes.

What was clearly missing in his early years was the first-hand knowledge of the Roman architectural sources upon which any respectable classical language must be based. Instrumental in bridging this gap was Giangiorgio Trissino, an aristocrat, writer, and humanist, who recognized Palladio’s tremendous potential and facilitated his first trip to Rome in 1541. Palladio returned to Rome a number of times, with and without Trissino, where he meticulously researched, drew, and recorded copious amounts of architectural information, from ancient Roman sources to Renaissance masters such as Bramante, Raphael, and Michelangelo, from intricate details to overall plans and elevations. Trissino also had formed an academy of sorts at his estate Villa Trissino in Cricoli (near Vicenza), on which Palladio had worked during his Pedemuro years. Here Trissino provided a humanistic education to promising scholars. He became a mentor to Palladio in the late 1530s and probably created the young architect’s pseudonym. Through his study of the classics with Trissino, especially the architectural treatise of Vitruvius, and with the first-hand knowledge of Rome he acquired over several years, Palladio transformed his design approach substantially.

Let us first consider his earlier designs, during the 1540s, before the full extent of his education had taken force. Palladio makes scant use of the orders and other classical elements in many of his early villas. Instead he exhibits an innate interest in geometry as design medium, using simple forms such as the circle and semicircle to adorn his early façades, e.g., Villa Valmarana at Vigardolo (begun 1541, fig. 2) and Villa Poiana (begun c. 1548, fig. 3).

Symmetry is a constant stabilizing force early on and remains so throughout his career. Palladio’s façade motif comprised of a three bay arcade surmounted by a pediment can inherently be viewed as a formal abstraction, a template from his toolkit – one that he realized at least four times, twice with rustication, once with orders, and once with minimal treatment; cf. Villa Pisani at Bagnolo (begun c. 1542-45), Villa Caldogno (begun c. 1545), Villa Gazzotti (begun 1541-42), and Villa Saraceno (begun c. 1545-8).
Palladio makes use of the orders, on palazzos and a small number of villas, to convey the importance of the owners, yet a major formal role of the classical elements is to regulate the steady rhythm of the principal façade (see, e.g., Palazzo Civena [begun c. 1540], Palazzo Thiene [begun c. 1542-6], and Palazzo Iseppo Porto [begun c. 1549]). In his interiors Palladio demonstrates a natural genius for shaping space while addressing programmatic concerns; of particular note is his use of vaults, individually and in combination, which, whether frescoed or left monochrome, read beautifully in their form, geometry, and structural grace.

Towards the end of the 1540s and into the 1550s, as Palladio’s classical vocabulary developed, there emerged a more advanced approach to his praxis. While still utilizing his mastery of geometry and creative ingenuity, Palladio fully embraced the use of the classical orders, which he integrated in more sophisticated ways. The previous uniform rhythm of pilasters or columns along the entire front façade becomes richer through the use of column groupings and varying intercolumniations, or it gives way completely to a central
pedimented zone containing the orders. Entablatures become more articulated and prominent. Orders of different scales are combined. Intercolumniations are reduced in order to better conform to Vitruvian specifications. Buildings from this time period include the famous Basilica loggias (begun 1549) that adorn the main piazza of Vicenza and helped establish Palladio as the preeminent architect of the Veneto, Palazzo Chiericati (begun c. 1551), Villa Pisani at Montagnana (begun c. 1552), and Villa Cornaro (begun c. 1552, fig. 4).

In combination with his more sophisticated use of the orders, Palladio also takes strides in unifying the various components of plan, elevation, and section, an approach he would later describe in a sort of general maxim:

> Beauty will derive from a graceful shape and the relationship of the whole to the parts, and of the parts among themselves and to the whole, because buildings must appear to be like complete and well-defined bodies, of which one member matches another and all the members are necessary for what is required.^3

This unified approach, which Palladio further refined during the 1550s and 1560s, was due in large part to the influence of his highly accomplished patron, Daniele Barbaro, a Venetian patrician, scholar, and humanist, who started working with Palladio soon after Trissino died in 1550. Palladio provided illustrations for Barbaro’s translation and commentary of Vitruvius, published in 1556. Barbaro’s commentaries are quite involved on topics concerning proportions, from the theory of the orders in books III and IV, to music in book V, to room ratios in book VI.
Palladio undoubtedly heightened his command of classical Roman architectural theory, as well as ancient Greek arithmetic and geometry, through his relationship with Barbaro.4

Villas from Palladio’s later period include Villa Barbaro (begun c. 1556, fig. 5), Villa Malcontenta (begun c. 1558, fig. 6), Villa Emo (begun c. 1560, fig. 7), and the design widely considered to be his masterpiece, Villa Rotonda (begun c. 1566, fig. 8). His approach to design in this period has been the inspiration for much analysis. In “The Mathematics of the Ideal Villa” Colin Rowe compares Villa Malcontenta to Le Corbusier’s Villa Stein, focusing mainly on proportional considerations [Rowe 1982: 1–27]. Villa Emo is one that most easily fits into Rudolf Wittkower’s “harmonic proportions” formulation of Palladio’s design theory, which has generated much interest and scrutiny since its publication about fifty years ago.5 It is true that such proportional analyses can be used to argue that the beauty of Palladio’s architecture is not necessarily tied to his use of the orders, and that architecture devoid of ornamentation can still delight the eye if aesthetically pleasing proportions are incorporated.6 It is impossible to deny, however, that classical architecture has stood the test of time, and to this day classical designers look to the timeless beauty of Palladio’s oeuvre for guidance and inspiration.7
Fig. 6. Villa Malcontenta (photograph by the author)

Fig. 7. Villa Emo (photograph by the author)
The death of Sansovino in 1570 left open the position of proto of the procurators of San Marco, i.e., the primary architect of Venice. Although Palladio never officially filled this position, during the 1560s and 1570s he succeeded in landing major commissions in the capital city, aided by his relationship with the Barbaro brothers, Daniele, who also died in 1570, and Marc’Antonio. Rather than adapt to Venetian styles, Palladio applied the architectural lexicon that he had derived from Roman sources and had mastered through years of study and practice. Palladio designed the front façade (commissioned 1562) of San Francesco della Vigna in a style that solves the problem of unifying the height of the central nave with the lower sides, by overlapping a classical temple front spanning the whole elevation with a colossal order fronting the nave. Variations on this same theme are seen on two churches in Venice fully designed by Palladio (albeit not completed until after his death), San Giorgio Maggiore (church begun 1566, fig. 9) and Il Redentore (begun 1577). The interiors of these two may seem a bit austere to some observers, since the only ornamentation is that which is implicit in the orders and concomitant classical elements, but this underscores the importance he placed upon them. Their use in ecclesiastical architecture had already been established, having made the transition from pagan and secular sources in ancient Rome, where they had been used for the glorification of the gods or of the state. Palladio’s now fully Roman classical vocabulary was simply the only means of ornamentation acceptable to him. His genius lay in his inventiveness and creativity in assembling the elements of this vocabulary in order to shape space, articulate solid, and modulate light with elegance, grace, and beauty.
The work that secured Palladio such a prominent place in the history of architecture is not made of brick or stone, however. It is his treatise, *I quattro libri dell’architettura*, first published in 1570 but translated and republished myriad times since, which ensured that his influence would be felt centuries after his death in 1580. It became the *de facto* primary source book for classical architecture, since Palladio included a plethora of painstakingly detailed and amply dimensioned architectural drawings, from designs to details, which would be the inspiration for many later architects. He also presents his design philosophies in substantial detail, and the importance he placed on ratio and proportion (recall “the relationship of the whole to the parts, and of the parts among themselves and to the whole”) cannot be overstated. The specifications of the five orders in book I are exhaustively dimensioned in terms of a module based on the column diameter, in some cases to such precision as to be beyond what could actually be achieved or perceived, which underscores his theoretical approach. Later in book I Palladio lists his seven preferred room types, namely the circle, the square, and rectangles with the following length-to-width ratios: $\sqrt{2}:1$, 4:3, 3:2, 5:3, and 2:1. Of course, Palladio’s own designs in book II, as well as those of Roman buildings in book IV, are ready made for creative borrowing and extensive analysis.

Palladio’s later works in Vicenza include Palazzo Valmarana (begun c. 1565), his first palace with a colossal order; Palazzo Barbarano (begun c. 1570); Palazzo Porto Breganze (begun c. 1571, fig. 10); Loggia del Capitaniato (begun c. 1571, fig. 11), which is located across the piazza from the Basilica; and Teatro Olimpico (begun c. 1580). The last of these Palladio designed for the Accademia Olimpica, of which he was one of the earliest members, and he completed his design for the theater just prior to his death in August of 1580.
Palladio and his wife, Allegradonna, had five children, three of whom survived their father. Palladio’s body was buried in the Dominican church of Santa Corona in Vicenza, but his remains were exhumed in the nineteenth century and moved to prominent tomb in a new, neoclassical civic cemetery.

Palladio certainly benefited from the renaissance of intellectual thought that surrounded him in cinquecento Italy. The studies occurring in the various circles to which Palladio belonged over the years, such as those associated with Alvise Cornaro, Giangiorno Trissino, and Daniele Barbaro, were widely ranging. Palladio himself was at times stone mason, architect, engineer, archaeologist and architectural historian. Through meticulous research in Rome and elsewhere, he was able to develop an authentic classical vocabulary from ancient and contemporary sources, which he incorporated with seemingly boundless care and ingenuity in order to design an impressively large number of exceptionally beautiful and sturdy buildings. In the final analysis, however, the exceptional beauty of his architecture depends on an inborn artistic ability that cannot be quantified or otherwise explained by the influences of those around him. There is a huge difference between classically true and truly beautiful, and it is Palladio’s innate mastery of aesthetics that is his greatest legacy.

Acknowledgment

This biography of Palladio was originally written for “The Year of Palladio” website of the Institute for Classical Architecture and Classical America (http://www.classicist.org/resources/year-of-palladio/), and the bibliography accompanying the original includes several sources for further reading. The endnotes and bibliography have now been expanded from the original version to highlight numerous publications concerning relationships between architecture and mathematics in Palladio’s oeuvre.
Notes

1. The first two quotations are from [Puppi 1975: 27]; the third is from [Boucher 1994: 20].
2. This first-hand architectural information eventually became source material for Palladio’s first two publications, Le antichità di Roma and Descrizione della chiese di Roma, both published in 1554, as well as for book IV of his treatise, I quattro libri dell’architettura, published in 1570.
3. Palladio’s treatise, book I, chapter 1, p. 6–7 [Palladio 1997: 7]; similar statements are found in book II, chapters 1–2, p. 3–4 [Palladio 1997: 77–78]. For an example from Palladio’s work, at Villa Cornaro not only do the exterior columns reflect the placement of the interior walls and columns; the main sala’s length-to-width ratio equals not only its width-to-height ratio but also the ratio of the lower-storey column diameter to the upper-storey column diameter. See [Mitrović and Wassell 2006: 28–29, 46]; the ratios cited are based on dimensions from their 2003 survey of Villa Cornaro, on which their book is based.
4. Note Puppi’s quotes of Barbaro in the following: “For Barbaro the scientific basis of knowledge was to be found in mathematics, … [and] he concludes that ‘some arts have more of science and others less’, and the ‘more worthy’ are ‘those wherein the art of numeracy, geometry, and mathematics is required’”; see [Puppi 1975: 18]. Puppi cites his quotes as D. Barbaro, 1556, p. 7, i.e., from Barbaro’s translation and commentary of Vitruvius [Barbaro 1556].
5. [Wittkower 1988: 104 ff]. Wittkower bases his analysis on the dimensions given in Palladio’s treatise. Two articles providing more in-depth analyses related to Wittkower’s appear in the Journal of the Society of Architectural Historians: [Howard and Longair 1982] and [Mitrović 1990]. A more exhaustive study of potential proportional sources for Palladio is available in [March 1998]; indeed, Palladio’s corpus certainly supports March’s claim: “In truth the Renaissance might be called the era of conspicuous erudition in which patrons, scholars and artists displayed their breadth of classical learning in various works and commissions” [March 1998: xii]. For example, although Palladio’s use of ad triangulum references (inherently involving \(\sqrt{3}\)) on the Villa Rotonda cannot be explained through Wittkowerian means, the length-to-width ratio of the four large corner rooms (26:15) is a canonical ancient Greek approximation for \(\sqrt{3}\), as Mitrović stated in his first JSAH article [Mitrović 1990: 285]. Moreover, the diameter of the central circular room, 30, completes a virtual 30º-60º-90º triangle. (The dimensions 15, 26, and 30 are those given by Palladio in his treatise, book II, chapter 3, p. 19 [Palladio 1997: 95].) To state this using modern notation, the angles of a triangle with side lengths 15, 26, and 30 are, to the hundredth of a degree, 30.00º, 60.07º, and 89.93º; cf. [Wassell 1999: 124–125].
6. See, for example, two articles in the Journal of the Society of Architectural Historians: [Millon 1972] and [Payne 1994].
7. See the chapter “Palladianism Today” in [Mitrović 2004: 171–187], where he carefully analyzes and counters arguments against the use of classicism today, especially regarding the issue of appropriateness to time, and offers several examples of contemporary Palladian architecture.
8. The principal competitor, over the years, to Palladio’s treatise has been [Vignola 1563]. While some prefer Vignola’s canon of the five orders, his treatise lacked the plethora of drawings available in books II through IV of I quattro libri dell’architettura.
10. See book I, chapter 21, p. 52 of [Palladio 1997: 57]; the full text is:

   There are seven types of room that are the most beautiful and well proportioned and turn out better: they can be made circular, though these are rare; or square; or their length will equal the diagonal of the square of the breadth; or a square and a third; or a square and a half; or a square and two-thirds; or two squares.
Palladio also gives constructions for the arithmetic, geometric, and harmonic means in book I, chapter 23, p. 53–54 [Palladio 1997: 58–59], and he recommends choosing from them to find a vaulted room’s height given its length and width.

11. Palladio included his Basilica amongst the public architecture in book III, which is otherwise dominated by bridge design.

**Bibliography**

**Sources published before 1800**


Barbaro, Daniele. 1556. *I dieci libri dell’architettura di M. Vitruvio, tradotti et commentati*. Venice.


**Sources published after 1800**


*About the author*

Stephen R. Wassell received a B.S. in architecture in 1984, a Ph.D. in mathematics (mathematical physics) in 1990, and an M.C.S. in computer science in 1999, all from the University of Virginia. He is a Professor of Mathematical Sciences at Sweet Briar College, where he joined the faculty in 1990. Steve’s primary research focus is on the relationships between architecture and mathematics. He has co-authored two books, one with Kim Williams entitled *On Ratio and Proportion* (a translation and commentary of Silvio Belli, *Della proportione et proportionalità*), and one with Branko Mitrović entitled *Andrea Palladio: Villa Cornaro in Piombino Dese* (see review in this issue). Steve’s overall aim is to explore and extol the mathematics of beauty and the beauty of mathematics.