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Giorgio Israel
Ana Millán Gasca

The World as a Mathematical Game

John von Neumann and Twentieth Century Science

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Authors:
Giorgio Israel
Dipartimento di Matematica
Università di Roma "La Sapienza"
P.le A. Moro, 5
00185 Roma
Italy
e-mail: giorgio.israel@uniroma1.it

Ana Millán Gasca
Dipartimento di Matematica
Università Roma Tre
Largo S. Leonardo Murialdo, 1
00146 Roma
Italy
e-mail: ana.millan@uniroma3.it

Translated from the Italian by Ian McGilvray

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Abbreviations

AF	Air Force
AEC	Atomic Energy Commission
AMP	Applied Mathematics Panel
AMS	American Mathematical Society
AOD	Army Ordnance Department
BRL	AOD Ballistic Research Laboratory
CalTech	California Institute of Technology
CIA	Central Intelligence Agency
ECP	Electronic Computer Project
EDVAC	Electronic Discrete Variable Computer
ENIAC	Electronic Numerator, Integrator, Analyser and Computer
FAS	Federation of American Scientists
IAS	Institute of Advanced Study, Princeton
IBM	International Business Machines
ICBM	Intercontinental Ballistic Missile
JNCW	*John von Neumann: Collected works*, edited by A. Taub, New York, Macmillan, 6 volumes, 1961–63.
JNLC	John von Neumann Papers, Manuscript Division, Library of Congress, Washington D.C.
MAA	Mathematical Association of America
MIT	Massachusetts Institute of Technology
NBO	Navy Bureau of Ordnance
NDRC	National Defense Research Committee
NRC	National Research Council
ONR	Office of Naval Research
OSRD	Office of Scientific Research and Development
RAND	Research and Development Corporation
RCA	Radio Corporation of America
SAGE	Semiautomatic Ground Environment
SIAM	Society for Industrial and Applied Mathematics
UN	United Nations Organization

Introduction

> *Matters at court devour my time & energies, as always. His Majesty becomes daily more capricious. At times he will forget my name, and look at me with that frown, which all who know him so well, as if he does not recognize me at all; then suddenly will come an urgent summons, and I must scamper up to the palace with my star charts & astrological tables. For he puts much innocent faith in this starry scrying, which, as you know well, I consider a dingy business. He demands written reports upon various matters, such as for instance the nativity of the Emperor Augustus and of Mohammed, and the fate which is to be expected for the Turkish Empire, and, of course, that which so exercises everyone at court these days, the Hungarian question: his brother Matthias grows ever more brazen in his pursuit of power.*
>
> – John Banville, *Kepler*.

In the lands situated in the heart of Europe, swept by the conflicts of the early twentieth century – just as they were three centuries before, at the time of Kepler – began the life of John von Neumann, a figure who perhaps more than any other is representative of twentieth century science. Von Neumann was a brilliant scientist, the author of fundamental contributions to mathematics and theoretical physics, and he played a vital role in the extraordinary display of power of twentieth-century science, as expressed by the development of computers, automation, space travel, and the use of atomic energy.

In fact, the triumphs of science – and also its miseries – which so deeply mark our era, are actually the outcome of a long historical process that begins symbolically with the work of Galileo. After the Scientific Revolution, the role of natural philosophy in modern society thinking and life became increasingly important. Even when this role was expressed in essentially theoretical and intellectual forms – that is, without any visible practical fall-out on everyday life – scientific breakthroughs, and above all the best-known of them all, Newton's mechanics and his theory of universal gravitation, formed the most solid basis for faith in progress, typical of the Enlightenment period and later of the whole of modern thought.[1]

1) On the principle of omnipotence characterizing modern scientific thought, see Israel 2001.

The links between von Neumann's own epoch and the distant age of the birth of modern science are closer than they might seem. In the first place – despite the emergence of relativistic and even sceptical views –, because science has continued to draw inspiration from the ideal of achieving a unitary, coherent, objective and universal image of the world. And secondly because, except for a few important differences, Kepler's description of the servitude and difficulties of his life as a "scientist" also evokes the delicate role that befell the so-called "experts" or scientific advisors after World War II. Kepler was appointed imperial mathematician to the court of Kaiser Rudolph II in 1601: in view of his knowledge of astrology, his main task was to advise the emperor on the management of a wide range of personal and political matters, and to deliver his prophecies in the form of "written reports". Von Neumann, although born in Budapest, in Hungary – that European frontier land between East and West on which the concerns of Rudolph II were focused – rose to the highest rank possible for a scientist at the service of the United States government. The dramatic upheavals that began to shake Europe in the 1930s led him, like many other scientists, to move to the United States, where he became a member of the select and exclusive Atomic Energy Commission. And it may well be said that several of von Neumann's writings prompted by his government service could well be considered prophecies concerning the times in which we live.

But here the analogies cease and differences arise. Indeed, the growing role of science in society and the increasing interweaving of science and technical knowledge which, towards the end of the nineteenth century, culminated in the burgeoning of technology, allowed less scope for the figure of the scientific scholar serving as an advisor at the mercy of the "king's" whims. The Age of Enlightenment had put forward the idea that the government of society must be founded on scientific bases, and indeed it is the scientific *élite* itself who should lead society in accordance with these principles. The *Idéologues* school led by the mathematician Condorcet[2] dreamed of discovering and applying the mathematical laws that would rationally and justly regulate the making of decisions in courtrooms, in assemblies, and in elections, or else would be used to govern the economy.[3] This highly ambitious project came to nought: significatively enough, Napoleon opposed the rights of human subjectivity and history to the Enlightenment's claim to base the government of society on the principles of pure rationality.[4] Nevertheless, the seeds had been sown of a dialectics that is still

2) On these topics see Moravia 1974, 1986.

3) Condorcet was the author of a program aimed at the establishment of a "social mathematics", that is, a mathematics suitable for treating all the problems involved in managing society and the economy. In this connection, see Baker 1975, Israel 1993a, 1996b.

4) In a speech delivered to the Council of State on 20 December 1812, Napoleon Bonaparte, in the following terms, attacked the movement of the *Idéologues* and its claim to construct a science of society: «It is *ideology*, this shady metaphysics that, subtly seeking the prime causes, sets out to establish on these bases the legislation of peoples, instead of appropriating laws to the knowledge of the human heart and lessons of history, that must be blamed for all the misfortunes that have befallen our fair France. [. . .] When one is called upon to regenerate a state, it is necessary to follow constantly opposing principles. History paints the human heart: it is in history that one must seek the advantages and disadvantages of the different legislations.» (N. Bonaparte, *Correspondance*, 32

being discussed and is still unresolved. And it was on the basis of the highly developed prestige and role of science that a multi-faceted genius, with an extraordinary capacity for navigating not only the paths of science but also those of government, was given the chance to succeed, albeit in isolation, where the followers of Enlightenment had failed. Von Neumann was not only a respected government advisor but, acting in this capacity, he succeeded in communicating and even putting into practice his idea that the governance of worldly matters must be guided by a universal logic within which each individual must move in accordance with a rational strategy directed to achieve the best result, taking into account the fact that also the other individuals are pursuing the same aim.

The life and scientific activity of von Neumann may be divided into two main periods, the time before and the time after his move to the United States. These two periods correspond to widely differing scientific interests. The European period is characterized by fundamental contributions to the great scientific issues of the early twentieth century, and will be dealt with in the chapters 1 and 2 of the book. The American period instead reflects the more usual and consolidated image of this scientist: it is no coincidence that the Hungarian emigré scientist is generally referred to by the Anglicized version of his first name, John (or Johnny). In this period science attained a far-reaching influence, not only in philosophy and culture, but also in the social and economic fields, and even in politics. This was the period that saw the birth of "big science", that is, scientific practice based on large-scale research projects linked to technological development and carried out by very large groups of specialists working in different fields, in large research centres provided with highly complex equipment and infrastructures and receiving very substantial funding. Chapters 4 and 5 are devoted to the contributions made by von Neumann to this line of development. Chapter 3 aims at outlining his overall scientific outlook, which in our view was consolidated already in the early years after his move to the United States. We will then follow the unfolding and development of this scientific outlook in the widely differing fields in which he was active: digital computers, the theory of automata, meteorology, game theory, and many other aspects of mathematical modelling.

Von Neumann died suddenly in 1957, aged only 54, as a result of a bone cancer that rapidly laid him low. Several important projects, such as his theory of automata, conceived at the height of his extraordinary scientific career, thus did not get off the ground. And yet, to view his life from a vantage point that is now becoming increasingly remote from the end of the Cold War affords an overview of twentieth century science. This book sets out to illustrate this vast and complex historical panorama without overstepping the inevitable bounds imposed by the need for a measured and non-specialist treatment.[5]

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voll., Paris, 1858–69, vol. 24, n. 19390: 398–399). It is instructive to compare this passage with the claim made by François Quesnay, the founder of the physiocratic movement and one of the fathers of the scientific conception of economics: «Let us not seek lessons in the history of nations or human dismay, which portrays to us only an abyss of disorder» (Quesnay 1767).

5) An overview of sources and scholarship regarding von Neumann is provided before the bibliographical references at the end of the book.

No special knowledge is required, though some extremely difficult scientific concepts are discussed. To give a simple and at the same time exhaustive treatment of these issues would have required a book three times as large. In a few places, we opted to avoid trying to explain everything exhaustively. In these cases, we have tried to convey an "impressionistic" idea of these concepts, so that readers can get a general drift of the arguments, and then to guide readers who wish to go deeper into these matters to specialist publications that will help them. It is also possible to skip some of these passages without missing anything very important. Indeed the main aim of the book is not technical but to acquaint as large a reading public as possible with the scientific and cultural significance of one of the greatest figures of twentieth century science and technology.

Chapter 1

János Neumann's Early Years

1.1 A Jewish family in early twentieth century Budapest

Our hero was born in Budapest on 28 December 1903. János Lájos Neumann was his name in Hungarian, Jancsi for short. He was the eldest son of Miksa and Margit Neumann, members of the large Jewish community resident in the capital. Hungary at the time was still part of the Austro-Hungarian Empire, within which it enjoyed a substantial political autonomy governed by rules established in 1867, when Franz Joseph I was crowned King of Hungary.

During the second half of the nineteenth century, Hungary enjoyed a period of relative peace and industrial development, and had increasingly taken on the appearance of a modern European country, even though its economy continued to be based essentially on agriculture. The aristocracy had retained its position as the ruling class and governed the country's destiny from the capital, although its foreign and military policy depended on Vienna, the centre of the Empire. The Hungarian establishment displayed great dynamism that did not appear to be hindered by the new social tensions caused by the contrast between the backwardness of the peasant world, by the formation of an industrial proletariat or by the problems entrained by the coexistence of numerous national minorities, mostly of Slav origin (Croats, Romanians, Slovaks, Serbs).

The Jewish minority was relatively large and played quite an important role.[6] At the beginning of the century, about half the population of Budapest was made up of Jewish families. Many of these, initially living off the land in the poorer rural areas, later moved to the capital and improved their social position as the country's economy developed. Jancsi's mother, Margit, came from a well-to-do family: grandfather Jakab Kann, although of humble origins, had made his fortune trading in farm implements.

6) In 1919 almost a million Jews formed 4,5% of Hungary's population. See McCagg 1972, 1997, Rozenblit 2001, Gerö 2007.

Miksa Neumann was a lawyer who, at the birth of his eldest son, had been employed for some time as the director of one of the largest Hungarian banks, the Magyar Jelzáloghitel Bank (Hungarian Mortgage Bank). In 1907 his second son, Mihály, was born and in 1911 the third, Miklós.

As well as being the political centre of Hungary, Budapest, where most of the aristocracy resided, was also its cultural centre and reflected both the liveliness and the contradictions of an age of rapid economic and social development. The ruling class of the time was particularly open and ready to absorb the more enterprising and capable. Nevertheless, the educated social sectors were riven by divisions and contradictions. In particular, society was split into the "official" sector, which held traditional political and social views, and a minority of sympathizers with a constantly growing socialist movement, which planned to carry out a profound reform of the country. The radical party, composed mainly of physicians, lawyers and businessmen, many of whom were of Jewish origin, championed reform and universal suffrage and moved closer and closer to the socialists.[7]

The position of the Neumann family, like that of other well-to-do Jewish families in the capital, was both extraordinarily promising and unstable. The professional career of a Budapest Jew was apparently limited only by his personal capability. And yet the difficult situation regarding the country's equilibrium, the growing political weakness of the Empire, and the ever-present risk of new waves of anti-Semitism, threatened to perturb this favourable situation at any moment. Miksa Neumann was determined to enter the establishment, without, however, renouncing his religion, although the Neumann family did not observe Jewish religious rules.[8] Like other Jewish bankers and industrialists in the capital, in 1913 he procured a noble title granted by the emperor Franz Joseph. The family surname thus became Margittai Neumann, where "Margittai" stood for "of Margitta"). In the German version of János's surname, the particle "von" was added as the translation of the final "i", in reference to the acquired noble title.

1.2 A young talent in Hungarian mathematics *belle époque*

János Neumann received an excellent education and took full advantage of the exciting cultural atmosphere of Budapest at the beginning of the twentieth century. He was

7) Péter Hának has described Vienna as "the garden" and Budapest as "the workshop" (Hának 1998). See Jászi 1929, Horvath 1966, de Ferdinandy 1967, and Gerö 1995.

8) Grandfather Jakab Kann was a strict observer of Jewish religious practices and cultivated Talmudic studies. Miksa's family lived in a non-religious fashion, even exchanging gifts at Christmas time, although they did not fail to observe Jewish feast days. According to János' younger brother, Miklós, Jewish festivities were celebrated in the family as "non-denominational family reunions". In the United States, Miklós Neumann adopted the name of Nicholas A. Vonneuman and his recollections of his brother are contained in Vonneuman 1987. Information from interviews with him and other people regarding von Neumann's youth are gathered in Heims 1980, Chapter 2.

taught by private tutors until the age of ten and then pursued secondary school studies at the Lutheran Gymnasium, one of the best teaching establishments in the capital. The school had excellent teachers and permitted differential religious instruction, so that János was able to study the Hebrew language and literature with a rabbi. At the same time, he was given specific private tuition in mathematics. A scholar of excellent reputation, László Rátz, who had played an active role in the organization of modern mathematics syllabuses in Hungarian schools (as a member of the 1906 Mathematics Reform Commission), was professor of mathematics at the Lutheran Gymnasium. Rátz informed the family of the extraordinary mathematical ability displayed by the young János right from the first few months of attendance at the Gymnasium and advised his father to ensure that the young student received supplementary tuition. He saw to it personally that a private tutor was found: to do so he consulted a professor of mathematics at Budapest Technical University, Jószef Kürschák, who recommended several teachers capable of developing János' enormous talent.

During the closing decades of the nineteenth century, mathematical studies in Hungary underwent spectacular growth at all levels of the educational system. Growing scope was allocated to mathematics in technical high school syllabuses. The University of Budapest, the Technical University (established in 1871), and the University of Kolozsvár (established in 1872), catered to the growing demand for secondary school teachers and engineers. University circles played an important role in modernizing the country's mathematical culture: a typical figure was Gyula König, professor at the Budapest Technical University, whose textbooks and institutional activities made a strong contribution to the development of Hungarian mathematical research. A national group of mathematicians benefiting from autonomous institutions was rapidly forming in those years.

In this way, Hungary became part of the process of renewal of the scientific institutions that had changed the face of European science during the second half of the nineteenth century. Previously, the academies had been the main driving force behind scientific activities. From now on, although the academies retained an important role, specific scientific disciplines began to acquire an autonomous organization, in particular mathematics. One first step in this direction was the founding of periodicals specifically dedicated to mathematics, beginning with pioneering ones like *Annales des mathématiques pures et appliqués*, founded by the French mathematician Joseph Diaz Gergonne in 1810, and *Journal für die reine und angewandte Mathematik*, founded by August Leopold Crelle in 1832. Later the first mathematical scientific societies were set up: in 1864 the Moscow Mathematical Society and in 1865 the London Mathematical Society. Towards the end of the century there was a tendency to set up national mathematical societies, each with its own review, such as the Société Mathématique de France (founded in 1873) and the Deutsche-Mathematiker Vereinigung (founded in 1890). These societies gathered together all those in the country with an interest in mathematics: university professors, secondary and technical school teachers, engineers, and the military.

At the turn of the century, this model was extended to other countries of less prominent mathematical tradition and played a key role in fostering the exchange of scientific information and in the development of original research inside individual countries. In Hungary, König, with other colleagues of the Technical University, founded the journal *Müegyetemi Lapok*, which was short-lived owing to lack of funds and subscriptions. However, several other initiatives destined to endure were consolidated. In 1885 an association of mathematicians was set up in Budapest by König and other scholars, including Baron Loránd Eötvös, a distinguished physicist who was also Minister of Education in 1894 and 1895. In 1891 an offshoot of this society was the Hungarian Mathematical and Physics Society – later known as the János Bolyai Mathematical Society – which initially counted nearly three hundred members. A few months before the constitution of the society the review *Mathematikai és Physikai Lapok* (Journal of mathematics and physics) began publication. However, the decision to use the Hungarian language was a serious obstacle to communication with mathematicians in other countries. The Academy of Sciences helped overcome this obstacle in 1883 by founding the review *Mathematische und Naturwissenschaftliche Berichte aus Ungarn*, which contributed to disseminating the work of Hungarian mathematicians; on the other hand, a large number of the latter had begun publishing articles in foreign reviews.[9]

Two initiatives represented a valuable contribution to the process of training young Hungarian mathematicians: the foundation in 1894 of the review *Középiskolai Matematikai Lapok* (Mathematical journal for the secondary school) by Dániel Arany, who was succeeded three years later by Rátz, János Neumann's mathematics teacher; and the organization, the same year, by the Mathematics and Physics Society, of a "mathematics competition among pupils" for the Eötvös prize, in honour of the Minister of Education. The above-mentioned review proposed exercises and problems in elementary mathematics and had the same structure as other European reviews circulating widely among teachers and students in their final school years, such as *Nouvelles Annales des Mathématiques*, published in France for candidates sitting for the awesome entrance examinations to the Paris *Grandes écoles*, such as the École Polytechnique and the École Normale.

Symbolic of the development of mathematics in the country at the turn of the century was the institution of the Bolyai prize by the Academy of Sciences – in honour of the famous Hungarian mathematician, János Bolyai, one of the inventors of non-Euclidean geometry – first awarded to the great French mathematician Henri Poincaré. Those years marked the beginning of the career of two mathematicians, Lipót Fejér and Frigyes Riesz, whose ideas were to lay the cornerstone of Hungarian mathematics research in the first half of the century. Both were Jews (although Riesz later converted to Calvinism), which seriously affected their careers. In Hungary at that time, as in Germany and Italy, there were already a large number of Jewish mathematicians. Science had never been a main concern in Jewish cultural tradition, although as a result of the emancipation and gradual conquest of civil rights in the various European countries, a growing number of Jewish intellectuals had access to

9) An overview of the organization of the national mathematical communities can be found in Grattan-Guinness (ed.) 1994; on Hungary see Szénassy 1992 and Hersh, John-Steiner 1993.

academic careers and thus developed an interest in scientific disciplines. However, it was very difficult for a Central European Jewish intellectual to obtain a university chair: the choice of mathematics was partly due, as David Rowe pointed out, to the autonomous and ideologically "neutral" nature of mathematics in the realm of knowledge (Rowe 1986). The ideal profession for those who had not yet obtained full equality of rights and were potential migrants had to be universalistic, international, and apolitical: and nothing approached this model more closely than the scientific disciplines, in particular mathematics (Israel, Nastasi 1998, Israel 2004d).

An anecdote related by Reuben Hersh and Vera John-Steiner clearly illustrates the rapidity with which Hungarian mathematics developed in relation to the broader cultural and social context (Hersh, John-Steiner 1993). In 1905, Poincaré went to Budapest to receive the Bolyai Prize and was given red carpet treatment, perhaps above all because he was a cousin of Raymond Poincaré, a French politician who later became President of the Republic and was three times Prime Minister under the Third Republic. Many ministers and VIPs had come to greet him but Poincaré, as soon as he got off the train, asked: "Where is Fejér?". The welcoming committee were surprised and asked: "Who is Fejér?" and Poincaré promptly retorted: "Fejér is the most important Hungarian mathematician, one of the most important mathematicians in the world". One year later, Fejér – who had not yet been given any teaching post – was appointed Professor at the University of Kolozsvár, and in 1911 was awarded a chair at the University of Budapest. Also Riesz, who had studied at Zurich, Göttingen, and Paris, was given a chair at the University of Kolozsvár, which, after Transylvania was transferred to Romania in 1920, was moved to the city of Szeged.

In the early decades of the century, the main core of Hungarian mathematicians was divided into two groups: Fejér's group at the University of Budapest, to which Marcel Riesz, Frigyes' brother, György Pólya, Ottó Szász, Mihály Fekete and Gábor Szegö belonged, and that of the University of Szeged, composed of Riesz, Alfréd Haar, Béla Kerékjártó and László Kalmár. Fejér's main interest lay in classic mathematical analysis. His most significant contribution was contained in his Ph.D. thesis and had been summarized in the review "Comptes rendus de l'Académie des Sciences de Paris", and in an extended version in *Mathematikai és Physikai Lapok* (Fejér 1900, 1902). It consisted of a new generalized method for summing Fourier series. Fourier series (introduced in the early nineteenth century by the French mathematician Joseph Fourier) are infinite sums of combinations of trigonometric functions, and one fundamental theorem of the theory affords the possibility of expressing each function satisfying fairly elementary hypotheses as the sum of such a series. This makes it possible to conceive of many phenomena as the superposition of a number of elementary periodic phenomena. The Fourier series found their first important application in the theory of heat and soon became a central topic in nineteenth century analysis and mathematical physics. Nevertheless the development of Fourier analysis was hindered by the difficulty involved in establishing rigorous conditions of convergence (that is, the conditions in which the series has a "sum"). Fejér's work addressed this particular aspect and led to a revival of Fourier analysis. Even though Fejér continued later to publish valuable contributions,[10] his scientific career was strongly hindered by the vicissitudes of Hungarian politics in the period between the two world wars, and in particular by racial persecution of Jews.

Frigyes Riesz had a more straightforward career. For various reasons he managed to avoid the difficulties of those years, even though he only obtained the Budapest university chair after the war, because Fejér was already professor in the capital and there was a law against having another Jewish professor in the country's main university. Together with Haar, Riesz set up a research institute at Szeged, the Bolyai Institute, and also founded the review *Acta Scientiarum Mathematicarum*. He wrote numerous articles on the new approach to the problems of mathematical analysis, which had begun to emerge at the end of the century and made use of tools and analogies drawn from other branches of mathematics, in particular algebra and topology. The forerunner of this new research approach in the late nineteenth century was the Italian mathematician Vito Volterra, the author of fundamental contributions to the calculus of variations and the theory of differential and integral equations. However, Volterra did not give priority to the abstract approach in his research on what he had called "functions of lines" theory; his scientific outlook was still that of classical analysis, linked to geometry and physics. In the early twentieth century, the abstract approach received a huge impulse from the work of David Hilbert at Göttingen and Maurice Fréchet in Paris. The new methods, which covered a wide range of topics, opened up the way to a new branch of mathematics known as "functional analysis" of which more will be said later. The group of mathematicians who elaborated on Hilbert's research included Riesz, and the German Erhard Schmidt and Ernst Fischer. The former provided a demonstration for an important theorem, now known as the Riesz-Fischer theorem, and carried on work on the linear spaces of functions and the theory of operators defined on such spaces.[11]

Hungarian mathematics was thus quite open to the new trends emerging in early twentieth century mathematics. In addition to classic and functional analysis studies, another branch of research undergoing significant development in this country was what is usually called today "discrete" mathematics (in opposition to the mathematics of the continuum). This branch consists of a complex of mathematical techniques designed to address problems of a "finite" nature, such as classical combinatorial problems: for example, all possible configurations of a certain type that may be found in a finite (or discrete) set are studied and enumerated, together with the operations that may be defined on such a set, the correspondences between finite sets, and so on. Research in this field has a longstanding tradition: it was, however, overshadowed from the seventeenth century on by the supremacy of Newton's and Leibniz' infinitesimal

10) See, for instance, Fejér 1904.

11) This was a typical new chapter of functional analysis, which emerged as a development of Hilbert's theory of integral equations (Bernkopf, M. 1966–67). Moreover, also Volterra's contribution was closely linked to the theory of integral equations (Israel 1984, 1991). Schmidt's work includes extending the concept of eigen-function to integral equations with non-symmetric nuclei (Schmidt 1907). In the same year Riesz obtained (Riesz 1907) the well-known theorem today called Riesz-Fischer's theorem. It was actually re-demonstrated the same year by Fischer using the notion of median convergence (Fischer 1907a, Fischer 1907b). In a subsequent fundamental work Riesz introduced the concepts of weak convergence and strong convergence (Riesz 1910).

calculus, which placed the mathematics of the continuum at the focus of research, namely problems having an infinite nature and in particular those in which infinity has the property of being "continuous" (which meant, in the rough and ready insights characterizing the early phases of the research, without "interruptions" or "gaps"). At the beginning of the twentieth century, in relation to the growing influence of the algebraic approach and the development of set theory, discrete mathematics underwent a renewal in which an important role was played by Hungarian mathematics. Following the path opened by Gyula König's seminal work on set theory,[12] the pioneers in this field were König's son, Dénes – the author of a fundamental book on the theory of finite and infinite graphs, published in German in 1936 (König 1936) –, László Kalmár and György Pólya.

This brilliant prospect for Hungarian mathematics was clouded by the defeat of Austro-Hungary in the Great War, which had a negative effect on the flourishing world of Hungarian mathematics, as well as on every other aspect of the country's life. The Bolyai prize, received by Hilbert in 1910, was no longer awarded owing to the devaluation of the fund supporting it. Many young mathematicians were left jobless and were obliged to give private lessons to make ends meet. This was the case of two young mathematicians in Fejér's group, who were private tutors to János Neumann, first Gäbor Szegö then, after the latter left Budapest, Mihály Fekete.

This was how von Neumann came into contact with the brilliant mathematical tradition of his own country which had such a strong influence on the direction taken by his research. His studies together with Fekete were fruitful: even before he had completed high school, the two wrote an article in collaboration, in which they generalized one of Fejér's results on the localization of the roots of Chebyshev's polynomials, published in 1922 in the journal of the Union of German Mathematicians *Jahresbericht der Deutschen Mathematiker-Vereinigung* (Fekete, Neumann 1922). Jancsi continued his secondary studies, gaining full marks and, in the end, was awarded the Eötvös national prize like his illustrious predecessors: Fejér, Haar, Marcel Riesz, his tutor Szegö, Kalmár, and the physicist Tódor Kármán (who changed his name to Theodore von Kármán after migrating to the United States) and Ede Teller (Edward Teller).

1.3 Lights and shadows. The von Neumann generation

The formative years of the young Jancsi were extremely encouraging. His lively mind, which had already set him apart from his peers, was stimulated by brilliant teachers who introduced him to a wide range of philosophic and scientific topics. Numerous anecdotes are told about the quality of this true "enfant prodige", his astonishing memory, his insatiable thirst for knowledge, his critical mind and his penchant for discussion. One of the more vivid ones is told by the physicist Eugene P. Wigner (whose original name in Hungarian was Jenö Pál). In an interview he gave to Thomas

12) For an extensive bibliography on Gyula König's work in this and other fields see Szénássy 1992.

Kuhn, now a famous Nobel physics laureate, this is how he remembers his school companion Jancsi:

> *We often took walks and he told me about mathematics and about set theory and this and that. It was amazing. And he loved to talk about mathematics – he went on and on and I drank it in.*
>
> *He was inexhaustible on such occasions in telling me about set theory, number theory, and other mathematical subjects. It was really wonderful. He never thought of going home* [...] *He was phenomenal, also in his desire to talk.* [...]
>
> *Particularly from having known Jancsi von Neumann, I realized what the difference was between a first-rate mathematician and someone* [*like me*].[13]

However, even though the personal qualities of this young man were astonishing, there is no doubt that the world he was being formed in was also particularly stimulating and reflected the dynamic quality of a dizzily expanding, culturally rich and lively, society. In the preceding section we saw Hungarian mathematics as a vigorous reality. Therefore, although von Neumann's training was destined to be moulded mainly by input from the Göttingen school of mathematics and its leader David Hilbert, it would be an error not to take into account the influence the Hungarian environment had on him. The importance of functional analysis and set theory was perceived by von Neumann in Hungary even before it was in Germany, and his very marked attention to combinatorial analysis and finite and discrete mathematics were strongly rooted in the research of Kalmár and Dénes König.

We shall have occasion later to discuss in detail one of his greatest contributions to twentieth century science: the foundation of the modern mathematical theory of games. And yet in this case also it is clear that his early interest in this topic is linked to his Hungarian experience.[14] This link must be considered not only in a strictly technical sense but in the broader cultural and conceptual sense. As we shall see, for him game theory was to represent a very general form of mathematical analysis of the problems of conflict between individuals pursuing contrasting ends. In other words, it was no longer to refer only to ordinary parlour games, but to the much wider and more "serious" field of social and economic conflict. Von Neumann is unanimously considered one of the scientists of the last century who paid most attention to social and economic applications and topics, and though the birth of this interest is often traced back to his commitment to the field of military applications, which began during World War II, this is not enough to account for the special passion he displayed for this topic and which is not to be found in many other scientists, for whom military

13) Interview of Wigner by T.S. Kuhn, *Archives for the History of Quantum Physics*, American Philosophical Society Library, Philadelphia, Pa., tapes 92b and 93a. See Heims 1980, 42–43.

14) The cited book by Dénes König on graph theory (König 1936) contains two interesting references to von Neumann to which we shall return and which refer also to game theory.

and industrial commitments represented a parenthesis after which they returned to their pure research studies.

The interest in socioeconomic theory and technological and industrial problems openly displayed from the 1940s on was rooted in von Neumann's conviction that they could be subjected to exact scientific treatment – a conviction that can be traced back to the cultural climate of Budapest and in particular to the atmosphere of enlightened rationalism current in his own family under the influence of Miksa Neumann. His father's economic and social activities brought János into contact with more worldly matters, the successes and hazards of real life. Miksa Neumann's home was assiduously frequented by colleagues, businessmen, and well-known Budapest intellectuals. This is what his brother Nicholas A. Vonneumann – to use the American version of his name – remembers of those years:

> *Around the dinner table we listened frequently to my father's comments on his own profession and business activities, as a lawyer and banker engaged in financing, and his underlying theories, namely, financial statement analysis and criteria for credit qualifications; and the practical applications, namely, selecting investment opportunities or borrowers. This usually led to a general discussion of the underlying theories also of other professions in relation to their respective practical applications. Father also used to discuss the technological aspects which his industrial applications involved.*[15]

This portrays a vivid picture of the typical faith in the scientific and technical rationality of a social class undergoing and managing an impelling social and economical expansion process. However, it would be an error to define this attitude as a kind of positivistic optimism. As we will see, throughout his life, von Neumann's activities were based on a practically unbounded confidence in the power of reasoning, and of scientific and mathematical reasoning in particular. However, this confidence was accompanied by a lack of faith, or scepticism, and not just latent or implicit, regarding man's capacity to follow more reasonable and constructive paths. Faced with the irrational tendency that mankind often displays when it chooses the more disastrous and destructive alternatives, scientific discernment and rational determination guiding optimal behaviour seemed to be the only way of preventing the potentially detrimental effects of this human tendency, or at least of keeping it within realistic limits. Also this aspect of his mindset – of which obvious traces may be found in his scientific work on social science – has its roots in his Hungarian and European experience. The power of scientific and mathematical reasoning in the context of man's destructive behaviours were the light and shadows of his early life.

Indeed, Jancsi's childhood was not immune from serious upheavals which left their mark on his personality. The year he entered the Lutheran Gymnasium, World War I broke out and the subsequent defeat of the Austro-Hungarian empire brought a tremendous catastrophe for Hungary in its wake. This florid Central European state

15) Glimm, Impagliazzo, Singer (eds.) 1990, 20.

lost many of its territories and more than three million Hungarians were transferred under the sovereignty of three new states – Czechoslovakia, Romania and Yugoslavia. Charles IV, the grandson of Emperor Franz Joseph, who had succeeded to the throne on the former's death in 1916, abdicated in October 1918. Shortly before, Count Mihály Károlyi had been nominated prime minister of Hungary, but Károlyi failed in his attempt to found a democratic republic with the support of the liberal, radical and socialist forces and, in early 1919, power passed to the communists of Béla Kun. During the five months of the Hungarian Soviet republic the Neumann family sought refuge in Austria, between Vienna and Abbazia, on the Adriatic coast. This experience profoundly affected the political views of the young János and contributed to arousing in him a powerful hostility towards communism. It is significant in this regard that in 1955 he should acknowledge that

> *My opinions have been violently opposed to Marxism ever since I remember, and quite in particular since I had about a three months taste of it in Hungary in 1919.*[16]

Bankers, aristocrats, and conservative military in Vienna and Szeged, with the help of the countries of the Entente, prepared a counter-revolutionary movement that, after the invasion of Hungarian territory by the Romanian army, seized power under the leadership of Miklós Horthy. The situation in which the Neumann family found itself on their return to Budapest was anything but reassuring because of the powerful waves of repression and anti-semitism that followed the advent of the Horthy regime. The participation of many Jewish intellectuals – including Béla Kun himself – in revolutionary movements became a pretext for unleashing persecution. Horthy introduced a number of anti-semitic measures although, very pragmatically, he protected Jewish bankers and industrialists who were useful to him in the reconstruction of the country, in particular the ennobled élite. In this way, the Neumanns continued their everyday life without too many problems. In 1921 János passed the rigorous tests of loyalty to the regime and was admitted to a course of mathematical studies at the University of Budapest, despite severe restrictions placed on the number of Jews admitted to the University that had been set by parliament one year previously (5% of the total number of students).

Nevertheless, from that year on, János Neumann's life began to take a different course from that of his country. He first embarked upon an intense training phase in Germany, attending the University of Budapest only at exam time. In this and other aspects, the path followed by him was similar to that of many other Hungarian mathematicians and scientists of the time, most of whom were Jews. In an interview with Steve J. Heims, the economist William J. Fellner, born in Budapest and who later migrated to the United States, claimed that «many traits of von Neumann noted by Americans are just those of a good Budapester of his time and his social class; that he

16) This passage is contained in a hand-corrected draft referring to "Nomination of John von Neumann to be a member of the United States Atomic Energy Commission", 8 March 1955 (John von Neumann papers at the Library of Congress, JNLC, see Aspray 1990, 247).

was very much a Budapest type» (Heims 1980, 26–27). Von Neumann's personality was strongly affected by events in this early part of his life – both the political vicissitudes and the extraordinary cultural experience of that time. In particular, it is easy to understand the significant role played in this context by the struggle for success, the ineluctable need to stand out in the professional and social environment, and the inclination towards a committed public life – aspects characteristic of his personality that also distinguish it from the classical image of the scientist as a "wise man" shut up in his study or laboratory and remote from worldly matters.

Von Neumann himself described the salient features of his own experience and that of his generation in the following terms, as reported by his Polish mathematician friend Stanislaw M. Ulam: «An external pressure on the whole society of this part of Central Europe, a subconscious feeling of extreme insecurity in individuals, and the necessity of producing the unusual or facing extinction» (Ulam 1958, 1). This situation often led to a psychological state that could be described as the constant feeling that one was "living out of one's suitcase" and which encouraged an "internationalistic" attitude in perfect harmony with the most modern and advanced trends in early twentieth century scientific research. These were the characteristics and problems of an entire generation of high calibre Hungarian intellectuals who made an exceptional contribution to European culture. A generation that included not only the scientists we have already mentioned but also physicists like Eugene P. Wigner, Leo Szilard and Dénes Gábor, as well as the mathematician Abraham Wald, the economist Nicholas Kaldor (the Hungarian Finance Minister, who later became a naturalized Englishman), and the already cited Fellner, the sociologists Oszkar Jászi and Karl Mannheim, the philosopher György Lukács,and the writer Arthur Koestler. Some of these were politically active in Budapest, while others, like von Neumann, preferred to distance themselves from the events taking place in their country.

The 1920s witnessed the beginning of a gradual wave of migration from Hungary by many mature scientists, as well as many talented young ones desirous of completing their studies, all now aware that staying in Budapest would lead to a dead end. Their destinations were above all the centres of Germanic culture like Berlin, Göttingen, and Zurich. Indeed, for a country like Hungary – tormented by tension with the Slavic east represented by the less well educated classes and by the constant threat from Russia – the German area culture represented a chance for the country's inclusion in the modern and advanced currents of western Europe. Subsequently, in the 1930s, many of these scientists migrated to the United States, first because of the difficulty of finding teaching posts in Germany, due to the inflated numbers of young researchers, and immediately afterwards because of the rise of Hitler's national socialism. For example, von Kármán moved to Göttingen, then to Aachen, and ultimately to CalTech (California Institute of Technology). Marcel Riesz obtained a post as professor at Lund (Sweden), where he set up an important school of mathematics. Pólya went to Zurich, before settling in Stanford, in the United States. Fekete migrated to Israel. Gábor moved to the UK. Many others among von Neumann's peers, such as Teller, Wigner, Szilard and Paul Halmos, settled like him in the United States, forming a group of "illustrious immigrants" – to use an expression coined by