The IXth International Workshop on "Intelligent Statistical Quality Control" took place in September 2007 in Beijing, China, and was hosted by Professor Quan-lin Li (chairman) and Professor Wu Su (co-chairman), Department of Industrial Engineering, Tsinghua University, Beijing, P. R. China. The workshop itself was jointly organized by Professors H.-J. Lenz, P.-T. Wilrich and Quan-Lin Li.

The twenty-three papers in this volume are carefully selected, reviewed and revised for this volume, and are divided into three parts: Part I: “On-line Control – Control Charts”, Part II: “On-line Control – Surveillance Sampling and Sampling Plans” and Part III: “Off-line Control”.


Tsung and Wang make a plea for adaptive charts in their paper entitled “Adaptive Charting Techniques: Literature Review and Extensions”. They compare the performance of a double-sided directionally variant chart with conventional multivariate charts. The detection power of unpredictable shifts and robustness are the two great advantages of their charts.

Reynolds and Stoumbos† jointly author the paper “Multivariate Monitoring of the Process Mean and Variability Using Combinations of Shewhart and MEWMA Control Charts”. They analyze the problem of simultaneously control charting the mean and the variability of multivariate normal variables. They recommend a combination of MEWMA charts that includes one chart based on the squared deviations from target.

Colosimo, Mammarella and Petrò in their paper on “Quality Control of Manufactured Surfaces” study multivariate surface monitoring where the quality characteristic is the response to one or more location variables (in time or space). They present a new method which is based on a combined spatial autoregressive regression model.

In “Statistical Process Control for Semiconductor Manufacturing Processes” Higashide, Nishina, Kawamura and Ishii consider SPC for the semiconductor manufacturing chemical industry where automatic process adjustment and process maintenance are widely used. Two case studies are presented related to adjustment and maintenance of auto-correlated processes.
Cheng and Thaga in their paper “The MAX-CUSUM Chart” propose a single CUSUM control chart capable of detecting changes in both mean and standard deviation. They make a comparative study with other single charts like the Max-EWMA chart and Max chart based on the ARL criterion.

Hryniewicz and Szediw propose a new control chart based on Kendall’s $\tau$ in their study entitled “Sequential Signals on a Control Chart Based on Nonparametric Statistical Tests”. In the case of a Gaussian auto-regressive production process this chart behaves in a similar way to the well known autocorrelation chart, but it is more robust in non-Gaussian cases.

A slightly different perspective on control charts is taken by Golosnoy, Okhrin, Ragulin and Schmid in “On the Application of SPC in Finance”. The field of interest is a fast on-line detection of changes of the optimal portfolio of a financial investor. Different types of EWMA and CUSUM control charts are analyzed by an extensive Monte Carlo simulation study using the ARL.

In Part II “On-line Control – Surveillance Sampling and Sampling Plans” Frisén presents a paper on “Principles for Multivariate Surveillance”. She reviews general approaches and makes suggestions on the special challenges of evaluating multivariate surveillance methods.

Woodall, Grigg and Burkom present an overview paper entitled “Research Issues and Ideas on Health-Related Surveillance” and compare surveillance methods used in health-care with industrial quality control.

In his paper “Surveillance sampling schemes” Baillie proposes a new type of sampling scheme for the simultaneous acceptance inspection of a number of large lots of similar size. For a beta prior distribution of the process fractions nonconforming, the expected proportion of lots accepted and the expected number of items inspected per lot are derived analytically.

Matsuura and Shinozaki in “Selective Assembly for Maximizing Profit in the Presence and Absence of Measurement Error” study the selective assembly of two mating components. They assume that the two component dimensions are normally distributed with equal variance, and that measurement error, if any, is Gaussian, too. It is shown numerically that the expected profit based on a density optimal partition decreases with increasing variance of the measurement error.

In his paper “A New Approach to Bayesian Sampling Plans” Wilrich designs a new (adaptive) Bayesian sampling plan for inspection by attributes based on a beta binomial model. The lot acceptance decision is directly based on the posterior distribution of the fraction nonconforming in the lot. The author illustrates that the adaptive single Bayesian plan dominates the equivalent ISO plans.
In Part III “Off-line Control” von Collani and Baur in “Stochastic Modelling as a Tool for Quality Assessment and Quality Improvement Illustrated by Means of Nuclear Fuel Assemblies” are concerned with modelling the quality assessment of fuel rods. Their Bernoulli-space model enables an accurate prediction of the performance of fuel rods, and supports a safe increase of the burn-up of nuclear fuel.

Mastrangelo, Kumar and Forrest in “Hierarchical Modeling for Monitoring Defects” propose a hierarchical linear model for linking the impact of process variables to defect rates. Process data drawn from the various gates are used to estimate the defect rates. Additionally, the output from the sub-models may be monitored with a control chart that is ‘oriented’ towards yield.

Göb and Müller on “Conformance Analysis of Population Means under Restricted Stratified Sampling” analyze risk-based auditing. The authors propose a restricted stratified sampling plan as an alternative for auditing.

In “Data Quality Control based on metric data models” Köppen and Lenz consider metric variables linked by the four arithmetic operators due to balance equations. Assuming a multivariate Gaussian distribution and an error in the variables model estimation of the unknown (latent) variables, the authors use MCMC-simulation to determine the “exact” distributions in non-normal cases and under cross-correlation.

Process capability studies are an important part of modern off-line control. Spiring picks up this topic in his contribution “The Sensitivity of Common Capability Indices to Departures from Normality”. The author devises a procedure to analyze the robustness of $C_{pqw}$ to departures from normality, and discusses its impact.

In his paper “A note on the estimation of restricted scale parameters of Gamma distributions”, Chang derives an admissible estimator for the scale parameter of a Gamma distribution. Simulation results illustrate the improvement of the new estimator compared with the traditional ones.

Kametani, Nishina and Suzuki investigate whether or not there exists an empirical relationship between the maturity of quality concerning the environment and the environmental lifestyle. The survey supports such a hypothesis for Japan.

Yasui, Ojima and Suzuki reconsider the Box and Meyer statistic in their study “On Identifying Dispersion Effects in Unreplicated Fractional Factorial Experiments”. The distribution of the statistic under the null hypothesis is derived for unreplicated fractional factorial experiments. The power of the test for the detection of a single active dispersion effect is evaluated.
Suzuki, Yasui and Ojima look at tournament systems in sports. In their paper “Evaluating Adaptive Paired Comparison Experiments” they remind the reader that in the incomplete case forming pairs is crucial. They propose an evaluation method, a new criterion, and give examples.

The study ”Approximated Interval Estimation in the Staggered Nested Designs for Precision Experiments” is authored by Yamasaki, Okuda, Ojima, Yasui and Suzuki. For the interval estimation of reproducibility a staggered nested precision experiment is proposed and evaluated by a Monte-Carlo simulation experiment.

The quality of any workshop is primarily shaped by the quality of papers that are presented at the meeting and their subsequent revision and submission for publication. The editors would like to express their deep gratitude to the members of the scientific programme committee, who did a superb job concerning the recruiting of invited speakers and the refereeing of the papers:

Mr David Baillie, United Kingdom
Prof. Elart von Collani, Germany
Prof. Olgierd Hryniewicz, Poland
Prof. Hans-J. Lenz, Germany
Prof. Quan-lin Li, P. R. China
Prof. Yoshikazu Ojima, Japan
Prof. Peter-Th. Wilrich, Germany
Prof. William H. Woodall, U.S.A.

We would like to thank very much our colleague Quan-lin Li and his students of the Department of Industrial Engineering, Tsinghua University, Beijing, who very efficiently supported the organization of the workshop: Mrs. Qinqin Zhang, Mrs. Rui Liu, Mr. Junjie Wu and Mr. Shi Chen.

Moreover, we again thank Physica-Verlag, Heidelberg, for their continuing efficient collaboration.

Finally, we are very sad to announce that three former participants have passed away: Professors Poul Thyregod, Zachary G. Stoumbos and Edward G. Schilling. Poul Thyregod was a permanent member of the programme committee for many years until he retired. Furthermore, he acted as the host of the third workshop at the Technical University of Denmark, Lyngby, 1986.

Berlin, February 2009

Hans - J. Lenz
Peter - Th. Wilrich
Wolfgang Schmid
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Part I

On-line Control

Control Charts
Control Charting Normal Variance – Reflections, Curiosities, and Recommendations

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Summary. Following an idea of Box, Hunter & Hunter (1978), the consideration of the log of the sample variance $S^2$ became quite popular in SPC literature concerned with variance monitoring. The sample standard deviation $S$ and the range $R$ are the most common statistics in daily SPC practice. SPC software packages that are used in semiconductor industry offer exclusively $R$ and $S$ control charts. With Castagliola (2005) one new log based transformation started in 2005. Again, the search for symmetry and quasi-normality served as reason to look for a new chart statistic. Symmetry of the chart statistic could help in setting up two-sided control charts. Here, a comparison study is done that looks especially to the two-sided setup, straightens out the view of the available set of competing statistics used for variance monitoring and, eventually, leads to recommendations that could be given in order to choose the right statistic.

1 Introduction

Monitoring the mean of normally distributed random variable is the most popular task within Statistical Process Control (SPC). For setting up the related control charts, one has to identify the underlying variance. Its value influences the chart design and performance heavily. Thus, it seems to be reasonable to check continuously the assumption about a certain variance level. Of course, there are further objectives of applying variance control charts, e.g., in the field of monitoring uniformity.

Before starting with estimating or monitoring variance, one may (or should) take into account that there could be several variance components. This was already considered in, e.g., Yashchin (1994), Woodall & Thomas (1995), and Srivastava (1997). This paper will focus on one variance component.

A further important topic is the amount of data that are collected at a given time point. The usual assumption is that the size of the sample could

†AMTC is a joint venture of Qimonda, AMD and Toppan Photomasks.
be chosen freely. Three major types could be distinguished here: (i) only one data point per time, (ii) a small (possibly arbitrary) number larger than one, and (iii) a large number. To illustrate all three cases consider two examples from daily practice in a mask shop like AMTC (Advanced Mask Technology Center), a company that is producing photomasks for wafer fabs. On certain monitor masks a larger number (100 and more) of lines are measured in order to monitor both mean level and variation. These lines (the so-called critical dimensions or abbreviated CD) have nominally the same size. Then average and standard deviation are calculated and used in a control chart. For an appropriate SPC setup for the mean one has to regard the mask-to-mask variation, consequently situation (i) is present. For monitoring variation on the mask one has to deal with (iii). These CD values are usually determined with a CD SEM (scanning electron microscope). Now, the CD SEM is monitored by repeating measurements at some fixed locations on a further specific mask. The number of repetitions has to be small, because the measured structures become more and more contaminated so that the CD size will change. Thus, situation (ii) is present. In case (i), typical approaches are relying on the Moving Range or certain measures of deviation from a given mean value $\mu_0$ (see Acosta-Mejía & Pignatiello Jr. (2000) or also Domangue & Patch (1991)). This is not studied here. Nevertheless, it is an interesting and challenging topic. In case (iii) certain approximations will work quite well. In this paper, case (ii) will be studied in deeper detail.

In the first decades of executing SPC, $R$ (range) and $MR$ (moving range) control charts dominated (they survived in a lot of SPC software packages, quality circles etc.). Besides, control charts based on the sample variance $S^2$ and its square root $S$ were considered. Later on, EWMA (exponentially weighted moving average) and CUSUM (cumulative sum) control charts were constructed for nearly all considered statistics.

In the late 20th century a number of papers were written that compared several of these charts. It is possible that one of the items on the list "General Trends and Research Ideas" (in SPC) in Woodall & Montgomery (1999) led to plenty of papers about monitoring variance. Woodall & Montgomery (1999) stated: "There has been a trend toward more research on monitoring process variability, but more work is needed on this topic." Thereby, a couple of transformations such as the popular $\log S^2$ and the recently introduced $a + b \log(S^2 + c)$ ($a, b, c$ are chosen to get nearly normality; Castagliola (2005) was the beginning of a whole family of publications) should improve shape and, hopefully, the performance of the considered schemes.

Eventually, by monitoring variance one could be interested in detecting increases and/or decreases. While an increased variance level indicates mostly some trouble that should be detected and removed, a decreased variance could weaken the performance of a simultaneously operated mean chart.

Now, this paper will focus on

- EWMA (exponentially weighted moving average) charts
- in two-sided fashion.
The latter is chosen because all these normalizing transformations are motivated by the search for symmetry (and normality). Essentially, this seems to be reasonable only for two-sided schemes. Given the two-sided case, EWMA control charts are more practicable by design. For applying CUSUM charts, one has to combine two single charts. And this is, of course, a disadvantage for application despite the (slightly) better performance. However, some remarks about CUSUM charting for monitoring variance are given in the remaining part of this section.

There is a large number of papers about CUSUM charts for monitoring normal variance. Refer to Amin & Wilde (2000) for a Crosier-type CUSUM chart based on log $S^2$. In Acosta-Mejía, Pignatiello Jr. & Rao (1999) and already Tuprah & Ncube (1987) several CUSUM charts based on $S$, $S^2$, and $R$ are compared. See also Box & Ramírez (1991), Srivastava & Chow (1992), and recently Poetrodjojo, Abdollahian & Debnath (2002) for more comparison studies. Already in Page (1963) CUSUM charts employing $R$ were analyzed. Hawkins (1981) considered a CUSUM chart for a normalized version of $|X - \mu_0|/\sigma_0^{1/2}$. Acosta-Mejía et al. (1999) reviewed a couple of more normal approximations. Lowry, Champ & Woodall (1995) compared CUSUM-type variance charts (using $S^2$, $S$, and $R$) with classical $S$ and $R$ Shewhart charts extended with runs rules (different from the Western Electric recommended ones). Finally, not only in Chang & Gan (1995) and in Poetrodjojo et al. (2002) it was pointed out that $S^2$ CUSUM is the optimal scheme in the one-sided setup if looking at the famous worst case criterion due to Lorden (1971).

See Table 1 for a small example.

Table 1. Slightly modified and shortened update of Table 5 in Chang & Gan (1995) – the EWMA schemes are also one-sided and equipped with a lower reflecting barrier, see Knoth (2005) for more details.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>CUSUM-$S^2$</th>
<th>CUSUM-ln $S^2$</th>
<th>EWMA-$S^2$</th>
<th>EWMA-ln $S^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>1.1</td>
<td>27.9</td>
<td>30.2</td>
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</tr>
<tr>
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<td>13.8</td>
<td>12.9</td>
<td>13.8</td>
</tr>
<tr>
<td>1.3</td>
<td>7.75</td>
<td>8.15</td>
<td>7.86</td>
<td>8.26</td>
</tr>
<tr>
<td>1.4</td>
<td>5.47</td>
<td>5.63</td>
<td>5.57</td>
<td>5.76</td>
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<td>1.5</td>
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<td>4.30</td>
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</tr>
<tr>
<td>2</td>
<td>2.08</td>
<td>2.11</td>
<td>2.11</td>
<td>2.22</td>
</tr>
</tbody>
</table>

The boldly written rows in both tables (1 and 2) are indicating the case the chart was optimized for. For further examples see, e.g., Lowry et al. (1995) who (numerically) demonstrated that CUSUM charts based on $S^2$ dominate those based on $S$ or $R$. This is not really surprising because of the support
by theoretical results for the $S^2$ version. To illustrate the same for detecting decrease see Table 2.

Table 2. Update of Table 2 of Acosta-Mejía et al. (1999).

<table>
<thead>
<tr>
<th>$\sigma$</th>
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<th>CUSUM-ln $S^2$</th>
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<td></td>
<td>$k_l = 0.793$</td>
<td>$k_l^\log = 0.375$</td>
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<td>200.02</td>
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<tr>
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</tbody>
</table>

To make it clear, it is evident that for the one-sided charts the original sample variance, $S^2$, should be used.

For fixed mean $\mu_0$ one should use instead of $S^2 = 1/(n-1)\sum(X_i-\bar{X})^2$ the statistic $\tilde{S}^2 = 1/n\sum(X_i-\mu_0)^2$. The latter was called CP CUSUM (change point CUSUM) in Acosta-Mejía et al. (1999).

Finally, Srivastava & Chow (1992) gave also some results for a Shiryaev-Roberts procedure for monitoring normal variance.

2 Two-sided EWMA charts for monitoring variance

The first papers dealing with EWMA control charts for monitoring the variance came from Wortham & Ringer (1971) and, a decade later, Sweet (1986), where only rough recommendations for the control chart design parameters were given. Thereafter, Domangue & Patch (1991), Crowder & Hamilton (1992), MacGregor & Harris (1993), and Mittag, Stemann & Tewes (1998) investigated EWMA control charts based on $S^2$, $S$ and the natural log of the $S^2$. Ng & Case (1989) evaluated EWMA charts based on $R$ and $MR$. Additionally, in papers by Srivastava (1994), Gan (1995), Reynolds Jr. & Stoumbos (2001), and Knoth (2007) the joint monitoring of mean and variance with EWMA schemes was considered.

It is interesting that EWMA smoothing of log $S^2$ (the natural log) has reached great popularity. There are different reasons for this phenomenon. Box et al. (1978) and others had recommended this transformation and so started Crowder & Hamilton (1992) with an EWMA chart smoothing log $S^2$ instead of $S^2$ itself or $S$ and $R$, respectively. Mainly, it is the transition from a scale-change model to a level-change model that motivates the log-transformation. Now, changes in the scale do not affect the variance of the new chart statistic. Furthermore, log $S^2$ is nearly normally distributed and so one can transfer the known results from the EWMA mean control chart. Finally, the distribution of log $S^2$ is more symmetric than that of $S^2$ so that two-sided control charts
are simpler to design. This attained a certain climax with papers following Castagliola (2005) who tuned \( \log S^2 \) by choosing suitable constants \( a, b, \) and \( c \) to get with \( a + b \log(S^2 + c) \) a nearly normally distributed random variable. By the way, Castagliola (2005) did not simply choose \( a, b, \) and \( c \) to create a statistic that has in the in-control case mean 0, variance 1, and – as one might expect – skewness 0. He considered a three-parameter log-normal distribution, that is, a random variable \( Y \) where \( a + b \log(Y + c) \) is exactly standard normally distributed. Then, \( a, b, \) and \( c \) are chosen to match the first three moments of \( Y \) and \( S^2 \). The resulting variable \( a + b \log(S^2 + c) \), however, is skewed and has a different curtosis than a normal distribution. See Table 3 for some numerical results.

Here, all these different statistics will be compared and studied in more detail. To begin with, some basic notation will be introduced now.

Let \( \{X_{ij}\} \) be a sequence of subgroups of independent and normally distributed data. Each subgroup \( i \) consists of \( n \) observations \( X_{i1}, \ldots, X_{in} \). The subgroup size \( n \) is larger than 1 (called case (ii) in the Introduction). The following change point model is considered for the variance \( \sigma^2 \):

\[
\sigma^2 = \begin{cases} 
\sigma_0^2 & , \ i < m \\
\sigma_1^2 \neq \sigma_0^2 & , \ i \geq m
\end{cases}
\]

The parameters \( \sigma_0^2 \) and \( \sigma_1^2 \) denote the in- and out-of-control values of the monitored variance. Then, based on the sequence of sequentially observed data one wants to detect the unknown integer value \( m \), which is called the change point. In the sequel, the subscript \( m \) will refer to the actual change point position. Thus, e.g., \( E_m(\cdot) \) denotes the expectation under a change point \( m \) (\( m = \infty \) stands for no change at all). Without loss of generality the variance \( \sigma_0^2 \) is set to 1.

The basic idea of EWMA control charts is to smooth an appropriate statistic such as the average for mean monitoring. For variance monitoring, the following statistics will be studied.

\[
R_i = \max_j X_{ij} - \min_j X_{ij},
\]

\[
S_i^2 = \frac{1}{n-1} \sum_{j=1}^{n} (X_{ij} - \bar{X}_i)^2 = \left( \bar{X}_i = \frac{1}{n} \sum_{j=1}^{n} X_{ij} \right),
\]

\[
S_i = \sqrt{S_i^2} , \ IS_i = \log S_i^2 , \ abcS_i = a + b \log(S_i^2 + c) .
\]

To get a first impression about the differences, the density functions for all 5 competitors are plotted in Figure 1 for \( n = 5 \). In order to allow better judgment, the densities are standardized under the in-control model. While it is quite simple to get numerical results for all statistics derived from \( S^2 \) (here the function \texttt{dchisq()} in the statistics software R was used), it is quite demanding for the (normal) range \( R \). In standard text books one can find
tables (and formulas) that give the mean and the standard deviation of $R$ for given sample size $n$ – the famous constants $d_2$ and $d_3$. It is not easy to find results for the cumulative distribution (cdf) or density function (pdf) of $R$. Here, a result of Bland, Gilbert, Kapadia & Owen (1966) was taken:

$$P(R/\sigma \leq r) = \int_{-\infty}^{\infty} n \phi(x)(\Phi(x + r) - \Phi(x))^{n-1} \, dx.$$ 

To get the density, the term above was differentiated by $r$ and – as already done for the cdf – the integral was evaluated with numerical quadrature.

In Figure 1 the pdf for all five statistics is drawn for 3 process states, in-control ($\sigma = 1$) and two out-of-control states ($\sigma = 0.8$ and $\sigma = 1.25$). In the in-control case, the third moment $\gamma_1$ and the fourth moment $\gamma_2$ (here the excess curtosis is calculated) of all considered chart statistics, except of $S^2$, are collected in Table 3. Recall that the first and second moment do not affect the control chart performance. For $S^2$, $\gamma_1 = \sqrt{8/(n-1)}$ and $\gamma_2 = 12/(n-1)$. The

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\log S^2$</th>
<th>$abcS$</th>
<th>$S$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma_1$</td>
<td>$\gamma_2$</td>
<td>$\gamma_1$</td>
<td>$\gamma_2$</td>
</tr>
<tr>
<td>3</td>
<td>-1.1395</td>
<td>2.4000</td>
<td>0.5572</td>
<td>-0.3206</td>
</tr>
<tr>
<td>4</td>
<td>-0.9170</td>
<td>1.6125</td>
<td>0.3752</td>
<td>-0.3947</td>
</tr>
<tr>
<td>5</td>
<td><strong>-0.7802</strong></td>
<td><strong>1.1875</strong></td>
<td><strong>0.2746</strong></td>
<td><strong>-0.3803</strong></td>
</tr>
<tr>
<td>6</td>
<td>-0.6879</td>
<td>0.9312</td>
<td>0.2119</td>
<td>-0.3478</td>
</tr>
<tr>
<td>7</td>
<td>-0.6209</td>
<td>0.7626</td>
<td>0.1697</td>
<td>-0.3142</td>
</tr>
<tr>
<td>8</td>
<td>-0.5699</td>
<td>0.6442</td>
<td>0.1398</td>
<td>-0.2837</td>
</tr>
<tr>
<td>9</td>
<td>-0.5293</td>
<td>0.5569</td>
<td>0.1176</td>
<td>-0.2572</td>
</tr>
<tr>
<td>10</td>
<td>-0.4962</td>
<td>0.4901</td>
<td>0.1007</td>
<td>-0.2344</td>
</tr>
<tr>
<td>11</td>
<td>-0.4686</td>
<td>0.4374</td>
<td>0.0874</td>
<td>-0.2147</td>
</tr>
<tr>
<td>12</td>
<td>-0.4450</td>
<td>0.3949</td>
<td>0.0768</td>
<td>-0.1978</td>
</tr>
<tr>
<td>13</td>
<td>-0.4247</td>
<td>0.3597</td>
<td>0.0681</td>
<td>-0.1834</td>
</tr>
<tr>
<td>14</td>
<td>-0.4069</td>
<td>0.3303</td>
<td>0.0610</td>
<td>-0.1703</td>
</tr>
<tr>
<td>15</td>
<td>-0.3911</td>
<td>0.3053</td>
<td>0.0550</td>
<td>-0.1591</td>
</tr>
</tbody>
</table>

Sample variance $S^2$ (see also Figure 1) is obviously the “least” normal one. The case $n = 5$ (the $n$ chosen in Figure 1 and written in bold in Table 3) leads to $\gamma_1 = \sqrt{2}$ and $\gamma_2 = 3$ which are considerably larger than the corresponding values in Table 3.

The densities of $S$, $R$, and $abcS$ look quite similar. Only the numbers in Table 3 demonstrate that $abcS$ is less skewed and $S$’s curtosis is closer to a normal one. The original log transformation, the log $S^2$ is “less” normal than the previous candidates. There is no reason to prefer log $S^2$ in terms of symmetry or normality. Its charm comes from the scale to a level change.
Fig. 1. Standardized density plots for n=5
solid line - in-control model, dotted line - out-of-control models:
left $\sigma=0.8$, right $\sigma=1.25$
model transformation. Thus, the competing process states are only shifts of the chart statistic density. Eventually, $S^2$ distinguishes itself by the widest variety among the three given states. Moreover, it is the “least” normal statistic. Apparently, it is too early to predict the most powerful basis for an EWMA control chart.

By writing $V_i$ as a dummy for the related variance estimator, the EWMA sequence with smoothing constant $\lambda \in (0, 1]$ is given by:

\begin{align*}
Z_0 &= z_0 = E_\infty(V_i), \\
Z_i &= (1 - \lambda) Z_{i-1} + \lambda V_i, \quad i \geq 1.
\end{align*}

Note that both Crowder & Hamilton (1992) and Castagliola (2005) started their EWMA schemes not at $E_\infty(V_i)$, but at $(\sigma_0^2 = 1)$

\[
\begin{array}{ll}
z_0 = 
\begin{cases}
\log \sigma_0^2 = 0 & (\log S^2) \\
(a + b \log(\sigma_0^2 + c) = 0.211 & (abcS)
\end{cases}.
\end{array}
\]

which are larger than

\[
E_\infty(V_i) = \begin{cases}
E_\infty(\log S^2) = -0.2704 & (\log S^2) \\
E_\infty(a + b \log(S^2 + c)) = 0.0075 & (abcS)
\end{cases}.
\]

Thus, they gave their schemes a head-start for detecting increases. Here, all schemes are started from their in-control mean level $E_\infty(Z_i) = E_\infty(V_i)$.

As already announced, two-sided EWMA charts will be considered. Thus, an alarm is given as soon as

\[Z_i < c_l \lor Z_i > c_u.\]

Or, written as stopping time,

\[L = \min \{ i \in \mathbb{N} : Z_i \notin [c_l, c_u] \} .\]

Recall that (using the independence)

\[Z_i = (1 - \lambda) \sigma_0^2 + \lambda \sum_{j=1}^{i} (1 - \lambda)^{i-j} V_j ,
\]

\[Var(Z_i) = \frac{\lambda}{2-\lambda} (1 - (1-\lambda)^{2i}) Var(V_i) .
\]

The thresholds given in the above alarm rule are not normalized.

All EWMA control charts will be evaluated in terms of their Average Run Length (ARL). This is nothing else than $E_1(L)$ and $E_\infty(L)$ for the (special) change point $m = 1$ and no-change point situation, respectively.

Many papers were written about calculating the ARL. Most of the numerical approaches could be embedded into the class of procedures for solving
Fredholm integral equations of the second kind. Thereby, the following integral equation that characterizes the ARL $L$ as function of the starting value $z_0 = z$ (see (1)) should be solved.

$$L(z) = 1 + \int_{c_l}^{c_u} L(x) \frac{1}{\lambda} f \left( \frac{x - (1 - \lambda)z}{\lambda} \right) \, dx \quad , \quad z \in [c_l, c_u]. \quad (3)$$

For more details about this integral equation see Crowder (1987), Champ & Rigdon (1991), or Knoth (2005). Popular approaches for solving (3) are

- applying mid point rule to the integral (equivalent to the famous Markov chain approach due to Brook & Evans (1972) and already proposed by Page (1963)) or
- evaluating the integral with Gauss-Legendre quadrature (see Crowder (1987) for the first application to EWMA ARL) or Simpson rule.

As pointed out in Knoth (2005), these (and other) approaches are not accurate for EWMA charts built on characteristics with bounded support such as for $S^2$, $S$, $R$, and also $abcS = a + b \log(S^2 + c)$. Collocation is the approach that is used here. For more computational details see Knoth (2005). Castagliola (2005) did not mention how he actually had solved the corresponding integral equation for $abcS$.

In the following section, an ARL based comparison is done for all five two-sided EWMA charts in order to find out an appropriate choice of the statistic to be deployed in the EWMA smoothing.

## 3 Comparison study

The variance EWMA control charts under consideration are tuned for two situations. In the first case, the chart should detect small changes as fast as possible. The related out-of-control $\sigma$ values are $4/5 = 0.8$ and $5/4 = 1.25$ (recall that $\sigma_0 = 1$). In the second case these two values are $2/3 \approx 0.667$ and $3/2 = 1.5$. Moreover, the so-called in-control ARL, $E_{\infty}(L)$, should be 500 for all schemes. To ensure a certain symmetry of the ARL function $L_\sigma = L_\sigma(z = z_0)$ in $\sigma$, the idea of unbiased ARL functions proposed by Acosta-Mejía et al. (1999) is applied. The idea is simple: The maximum of the ARL function should be attained for $\sigma = \sigma_0$. This was realized like in Knoth (2005). It has to be mentioned that none of the five schemes provides symmetry by choosing symmetrical control limits $(c_l, c_u)$ – see Table 5 for the final control limits. The smoothing parameter $\lambda$ was chosen to give the smallest value for

$$L_{0.8} + L_{1.25} \quad \text{and} \quad L_{0.667} + L_{1.5} \quad , \quad \text{respectively.}$$

This led to the following values for $\lambda$ searched on $\{0.02, 0.03, \ldots, 0.99\}$.

It is comfortable that these values do not differ considerably. In Figure 2 an illustration is given for the optimal $\lambda$ search. Here, the ARL functions for
Table 4. Optimal values for smoothing $\lambda$.

<table>
<thead>
<tr>
<th>case</th>
<th>$R$</th>
<th>$S^2$</th>
<th>$S$</th>
<th>$lS$</th>
<th>$abcS$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{0.8} + L_{1.25}$</td>
<td>0.06</td>
<td>0.07</td>
<td>0.06</td>
<td>0.05</td>
<td>0.07</td>
</tr>
<tr>
<td>$L_{0.667} + L_{1.5}$</td>
<td>0.15</td>
<td>0.17</td>
<td>0.15</td>
<td>0.12</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Fig. 2. Comparison of ARL profiles for $S^2$ EWMA control charts with various $\lambda$ values. The arrows in the bottom line indicate the corresponding area of optimality. The gray vertical lines mark the $\sigma$ values used for the tuning.

various $\lambda$ of only the $S^2$ based EWMA chart are drawn. As usual, the smaller the change that should be detected, the smaller $\lambda$ has to be chosen.

Now, the ARL $L_\sigma$ function is calculated for $\sigma \in [0.25, 1.75]$. In Figure 3 all 5 schemes were compared on a log scale for the ARL values (the log scale was already used in Figure 2). Their control limits are listed in Table 5.

Table 5. Control limits and “center” lines for $L_{0.8} + L_{1.25}$ optimal charts.

<table>
<thead>
<tr>
<th>chart parameter</th>
<th>statistic</th>
<th>$R$</th>
<th>$S^2$</th>
<th>$S$</th>
<th>$lS$</th>
<th>$abcS$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_n$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_\infty(V_i)$</td>
<td></td>
<td></td>
<td>1</td>
<td>0.940</td>
<td>-0.270</td>
<td>0.008</td>
</tr>
<tr>
<td>$c_l$</td>
<td></td>
<td></td>
<td>0.688</td>
<td>0.788</td>
<td>-0.618</td>
<td>-0.468</td>
</tr>
</tbody>
</table>
Despite or because of the log scale, no big differences between the five competitors could be seen (except the tails). Therefore, in Figure 4 the difference between the $S^2$ scheme and each of the competitors is plotted (on original scale). Additionally, in Table 6 some ARL numbers are collected.

From Figure 4 one can conclude that the $S^2$ EWMA procedure does a good job for a large range of possible values for $\sigma$. The two log based variants provide the best behavior for small $\sigma$ values. However, the log $S^2$ EWMA has the worst performance for detecting increases and small up to moderate decreases. Even the $R$ EWMA is better. The numbers in Table 6 support these conclusions. Based on the results given, one would take $S^2$ or $S$. To put it in other words, there is no reason to apply the more artificially looking log based charts in practice. Additionally, for small sample sizes like $n = 5$, the range $R$ provides similar power like the other statistics.

How does the picture change by optimizing for larger changes? See now Figure 5. Again, on the log scale the profiles look quite similar. Therefore, Figure 6 displays the difference between $S^2$ EWMA ARL and the remaining schemes.

Now, the $S^2$ variant beats all competitors for a long range within $\sigma < \sigma_0$. $S$ gives the best performance for small increases. For $\sigma > 1.3$ all schemes, except log $S^2$, are very close to each other. Again, one may conclude that there is no need to choose one of the log schemes.
Fig. 4. Difference between the ARL function of all 5 EWMA charts and $S^2$ EWMA – the $L_{0.8} + L_{1.25}$ case. The arrows in the bottom line indicate the corresponding area of optimality. The gray vertical lines mark the $\sigma$ values used for the tuning.

Table 6. ARL table for the $L_{0.8} + L_{1.25}$ competition.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$LS^2$</th>
<th>$abcS^2$</th>
<th>$S^2$</th>
<th>$S$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td><strong>6.348</strong></td>
<td>6.617</td>
<td>7.943</td>
<td>6.881</td>
<td>7.033</td>
</tr>
<tr>
<td>0.6</td>
<td>8.977</td>
<td><strong>8.651</strong></td>
<td>9.779</td>
<td>9.004</td>
<td>9.215</td>
</tr>
<tr>
<td>0.7</td>
<td>13.97</td>
<td><strong>12.72</strong></td>
<td>13.48</td>
<td>13.09</td>
<td>13.43</td>
</tr>
<tr>
<td>0.75</td>
<td>18.59</td>
<td><strong>16.63</strong></td>
<td>17.03</td>
<td>16.90</td>
<td>17.37</td>
</tr>
<tr>
<td>0.8</td>
<td>26.65</td>
<td>23.66</td>
<td><strong>23.40</strong></td>
<td>23.62</td>
<td>24.32</td>
</tr>
<tr>
<td>0.9</td>
<td>85.00</td>
<td>78.31</td>
<td><strong>73.43</strong></td>
<td>74.34</td>
<td>76.78</td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td>500.000</td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>84.25</td>
<td>80.39</td>
<td>77.66</td>
<td><strong>76.12</strong></td>
<td>79.54</td>
</tr>
<tr>
<td>1.2</td>
<td>30.76</td>
<td>27.02</td>
<td><strong>25.19</strong></td>
<td>26.19</td>
<td>27.30</td>
</tr>
<tr>
<td>1.25</td>
<td>23.02</td>
<td>19.65</td>
<td><strong>17.95</strong></td>
<td>19.16</td>
<td>19.92</td>
</tr>
<tr>
<td>1.3</td>
<td>18.46</td>
<td>15.40</td>
<td><strong>13.79</strong></td>
<td>15.06</td>
<td>15.63</td>
</tr>
<tr>
<td>1.4</td>
<td>13.39</td>
<td>10.80</td>
<td><strong>9.308</strong></td>
<td>10.55</td>
<td>10.92</td>
</tr>
<tr>
<td>1.5</td>
<td>10.67</td>
<td>8.396</td>
<td><strong>6.976</strong></td>
<td>8.155</td>
<td>8.424</td>
</tr>
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<td>8.978</td>
<td>6.928</td>
<td><strong>5.564</strong></td>
<td>6.677</td>
<td>6.887</td>
</tr>
</tbody>
</table>
Fig. 5. ARL profiles of all 5 EWMA charts – the $L_{0.667} + L_{1.5}$ case. The arrows in the bottom line indicate the corresponding area of optimality. The gray vertical lines mark the $\sigma$ values used for the tuning.

Fig. 6. Difference between the ARL function of all 5 EWMA charts and $S^2$ EWMA – the $L_{0.667} + L_{1.5}$ case. The arrows in the bottom line indicate the corresponding area of optimality. The gray vertical lines mark the $\sigma$ values used for the tuning.
The ARL profiles in Figure 3 and 5 are nearly symmetric for all five competitors with ARL values that are slightly smaller for $\sigma < \sigma_0$ than for $\sigma > \sigma_0$. Thus, by utilizing the concept of unbiased ARL function the potential control chart user would get a nearly symmetric scheme for all considered charts. The question remains open whether it is reasonable to ask for a control chart design that provides ARL performance symmetric in $\sigma = \sigma_0$. The results obtained here and their practical meaning will be summarized in the next section.

4 Conclusions

It was and is quite popular to deploy the log transformation in order to get an appropriate symmetric control chart for monitoring normal variance. It turned out that the log schemes are not more symmetric than the older schemes based on $S^2$, $S$, and $R$. Moreover, the best performance in terms of the ARL profile is given by the $S^2$ and $S$ EWMA charts. The first log example, log $S^2$ is even beaten by $R$. Generally speaking, each of the charts, except the log $S^2$, is usable. In practice, variation is mostly evaluated in terms of $S$ or $R$. Then nothing could be said against their usage in an EWMA chart. If the sample size $n$ increases, then $S$ is the favorite in practice. Eventually, take the $S^2$ scheme and get the best.

References


