Commodities and Commodity Derivatives

Modeling and Pricing for Agriculturals, Metals and Energy

Hélyette Geman

John Wiley & Sons, Ltd

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To Arnaud, Laure, Nathanaël and Don
To the memory of my parents

Contents

Foreword

NASSIM NICHOLAS TALEB

It is a great honor to find myself writing this foreword for my thesis director's book – although once someone's student always her student, it feels awkward to comment on one's supervisor's work. I was also pleased to find myself among its first readers, and benefit from its contents, as this work contains the first scientific compendium ever written on the intricacies linked to the physical nature of commodities.

Commodity options are not just interesting; they harbor all the pathologies encountered in the practice of derivatives, to the point of perhaps teaching us how to value derivatives with a different, deeper approach. Methodologists consider that, to understand a phenomenon, there are two routes. The first consists in examining the regular, the ordinary and the well-behaved, and excluding the unusual. The other consists in examining pathologies, the abnormal cases, then closing in on the ordinary as the exception. Economists, alas, have traditionally tended to use the first method, by pushing the exceptions under the rug – while physicists and other hard scientists tend to resort to the second as a way to satisfy their curiosity about the world.

Why are commodity options the most interesting, and the least misunderstood, of all derivatives? And why are they the exception that would teach us about derivatives? I will attempt here to make a short list and show how we can generalize into the wrinkles of *all* options, including the generally perceived theory-friendly financial ones.

First, the temporal dimension. The action of borrowing and lending is hardly predictable in commodities. They are heavily grounded in their physical nature. We have been taught that securities are derived by arbitrage arguments that allow us to seamlessly borrow and lend, in order to move the asset and liability across the temporal dimension. This makes the passage from the spot to the forward (or future) seamless, smooth and direct. In the arbitrage relationship, the forward equals the spot times some function of the differential between the yield r_1 and the cost of carry r_2. Accordingly, the forward contract is a mere extension of the spot, with stochasticity entering on occasion with one or both of the rates r_1 and r_2 being nondeterministic.

Now consider that you are trading in products that are not transferable into the future. Arbitrage becomes hardly possible – and, with it, the arbitrage argument. You may be dealing with a perishable commodity, like, say, an agricultural product. Storage can cause shrinkage in quantity, as with, say, electricity. The forward might not be born yet, as in the case of cattle. Forward oil may still be in the ground and might cost no

carry to the producer, whereas the arbitrageur would have to bear onerous storage costs. The relationship might hold, owing to the activities of the producer, but for arbitrage reasons.

How do you deal with it? Clearly, you need to treat every expiration like a separate underlying security. And you need to be careful about any arbitrage involving physical delivery. How does it apply to the other derivatives? Consider currency options. Currencies, I was told when I started trading two decades ago, were "clean". No worry, you just satisfy a forward obligation by buying spot and lending it, or vice versa. But every crisis, all except for one currency, the now defunct deutschmark, started behaving like commodities. They become impossible to borrow, sometimes, as was the case of the Irish punt, in 1992, commanding as high as several thousand percent interest rates. The Canadian dollar, the New Zealand and Australian currencies, all behaved unexpectedly outside the textbook. Emerging market currencies almost always behave like commodities.

Had I been trained in commodities I would not have been squeezed on the occasion; I would have considered such possibility unlikely but a present risk. And every underlying security bears that risk, with no exception: bonds become impossible to find to satisfy a delivery obligation; stocks with heavy short interest become unavailable for the borrower. The only products that seem to escape such problem are options on Futures.

The second point is the geographic limitation. While a security that you borrow is an abstract item, a mere balance sheet entry, commodities present location specificity that can make arbitrages arduous. You can own all the oil you need in Rotterdam; but, if your delivery is in New York tomorrow, you will have a problem. In electricity, shortage in one part of a continent can rarely be compensated with excess elsewhere.

How does it apply to other derivatives? Consider the "safe" currencies again. Say that you have the currencies available in a Brazilian bank but that you have an offshore delivery obligation. The bank calls you to explain that the government forced exchange controls and that delivery will not be possible. You will have an immediate need to find offshore Brazilian currencies. There have been similar pressures with pricing differentials problems with almost all currencies.

The third point is the intricacy of storage. Commodities are "heavy", unlike financial products. If you are expecting delivery and do not line up a warehouse you will be in trouble. Environmental agencies will not even let you dump your oil in the ocean. Cows are expensive to feed.

Do we have equivalent problems with cleaner derivatives? Of course: consider bonds that may be costly to own relative to your cost of carry, particularly when you have to borrow at a prohibitive short rate to fund them.

Fourth, the meat of the problem: dynamic hedging. Clearly, it is not possible to dynamically hedge a security that you cannot short, sometimes cannot easily own and that can be severely illiquid. Transaction costs can be monstrous. Fat tails and gaps thwart the argument that an option is a redundant security because it is safely replicated with a stream of dynamic hedges. We have enough evidence of large deviations to realize that dynamic hedging is not attainable in practice.

Then how do people value options? Clearly, options trade and we still manage to price them using risk-neutral probabilities. How do we do it? We practitioners consider an option as simply the expected value of its pay-off under some probability distribution, but not necessarily using dynamic hedging arguments.

As President of the Bachelier Finance Society, Professor Geman organized an international conference in Louis Bachelier's honor in the summer of 2000 which featured, among others, such prominent speakers as Paul Samuelson, Robert Merton, Henri McKean and Steve Ross. Reading this book, I come to realize that we have finally vindicated Louis Bachelier: Commodities are teaching us that we do not dynamically hedge.

Enjoy this impressive book and the exciting discovery of the world of commodities.

Preface

"The world is hungry for commodities", was the headline of the April 2004 issue of a widely read economic publication. And, indeed, there is no day when daily financial newspapers do not dedicate many columns to commodity-related issues, from cotton to nickel, coffee and freight while an unprecedented rise in oil prices has inflamed all markets.

How to define a commodity? An economist would say that it is a consumption asset whose *scarcity*, whether in the form of exhausting underground reserves or depleted stocks, has a major impact on the world and country-specific economic development. A banker would observe that it is not a financial security, giving rise to a stream of cash flows and priced by net present value arguments. An ecologist would suggest that it is a natural good whose original integrity should be preserved. An academic would argue that, given the current volatility of all currencies, including the most established ones, a commodity is an exemplary *numéraire* with respect to which portfolio values should be measured. Indeed, oil-producing countries concerned with the decline of the dollar and uncertain about the long-term health of the euro recently proposed that a barrel of oil be priced against an average of the dollar and the euro (*Financial Times*, February 2004). A valid suggestion would be to go the other way around and suggest a barrel of oil or a million Btus of natural gas as a "universal numéraire" relative to which all currencies would be expressed (the role gold used to play decades ago).

Commodities constitute the only spot markets which have existed nearly throughout the history of humankind. Over the centuries, even millennia, the scope of commodities available for human existence has grown from essential agricultural commodities to include metals and energy. The nature of trading has evolved from *barter* organized on town marketplaces in the absence of any monetary vehicle, to more elaborate *forward contracting* between producers and merchants, then to organized Futures markets with clearing houses guaranteeing the creditworthiness of transactions. The specification of contracts has evolved from "plain-vanilla" forwards to exotic options and structured products allowing originators and intermediaries to hedge away the risks residing outside the domain of their primary expertise.

Some key properties of commodity markets contrasting them with stock and bond markets include the following:

- Commodity spot prices are defined by the intersections of *supply and demand* curves in a given location, as opposed to the net present value of receivable cash flows.

- Demand for commodities is generally inelastic to prices, given the indispensable nature of the good. *Inventories* when they exist in sufficient volumes allow a smooth balance of supply and demand over time to be created. Hence, their importance in the discussion conducted in the economic literature for decades.
- Physical transactions – which were the only ones prior to the introduction of financial trades – still have a crucial importance today. Among other virtues, they provide a reference spot price or index against which derivative transactions are financially settled.
- Supply is defined by production and inventory. But, in the case of energy commodities, underground *reserves* also play a role since they have an impact on long-term prices.
- Financial transactions (forwards, Futures, options) represent today a huge volume. They involve prices closely related to spot prices in particular because physical delivery is a choice that is left to the buyer. Consequently, the understanding of spot markets and their characteristics is a necessary step in the analysis of commodities and commodity derivatives.
- For most energy commodities, the balancing of supply and demand now takes place both at the regional level and at the world level. This explains the explosion of shipping and freight markets and the emergence of new trading strategies such as the rerouting of an LNG (Liquid Natural Gas) tanker to countries where gas prices exhibit a momentary spike.
- Commodities represent today a new asset class in its own right. Many institutional investors and funds are increasingly turning to it for diversification benefits *and* for the returns generated.

The goal of the book is to present the three fundamental groups of commodities: agriculturals, metals and energy, with a particular emphasis on the third one in the context of deregulation of gas and electricity markets worldwide. However, the importance of the first two groups should not be dismissed: today, soybean exports from the US to China are bigger than the exports of airplanes while, in the case of Brazil, soybeans represent one-third of the shipments to China. The technical discussions will systematically emphasize the differences (or similarities), with the corresponding properties of stocks and bond markets. For instance, a crucial difference between securities and commodities is the *physical delivery* attached to spot, forward contracts and Futures positions not closed prior to maturity and translates into good transfer, with the corresponding constraints for both parties in terms of shipping arrangements, warehousing and so forth. The famous Sugar Quay in London gets its name from the fact that buyers of forward contracts used to be called on the phone to look out of the window and contemplate the product of their sugar transaction sitting in a barge on the Thames. At the other end of the commodity spectrum, sellers of forward contracts and options on electricity learned the hard way what physical delivery means during the crisis of June 1998: the combination of a lengthy heatwave and transmission disruption drove electricity prices up to thousands of dollars in the Midwestern part of the United States, and sent to bankruptcy firms which had sold power options without fully envisioning their unique features.

Another key difference between security and commodity markets is the existence, in the latter case, of *quantity risk*. Investors owning stocks or bonds are only concerned by

equity markets going down or interest rates going up (i.e., by the *price risk* attached to the instruments they are holding). Coffee producers and power generators know that their revenues are not only affected by random moves in the spot price of coffee or electricity, but also by the *variability of demand* due to changes in consumption, in growth development worldwide and in weather conditions.

This explains why the variety of exotic options which are now familiar in securities markets, such as Asian, exchange or spread, are the most appropriate options in commodity markets. In the latter case, moreover, contracts quasi-unknown in financial markets, such as "take or pay" or "swing", are playing a key role since they are designed to provide a hedge against *volumetric risk*. These options trade today as individual financial instruments, after having been included at no cost in contracts signed for decades between gas and electricity producers and end-users. Lastly, these options naturally appear in the so-called "real options approach" to the valuation of power plants, gas storage facilities or pipelines, as will be discussed at the end of the book.

We will try to cover a variety of theoretical and practical issues related to Futures and options markets, ranging from the mechanics of Futures trading to the discussion of equilibrium relationships between Futures and options prices, on the one hand, and spot prices, on the other hand, under some equilibrium (e.g., no-arbitrage) assumption.

The guiding thread, beyond the qualitative properties and knowledge of fundamental trading rules prevailing, respectively, in agricultural, metals and commodity markets, is to try to bring together the fundamental results from economic theory, the constraints of physical delivery and the lessons learned in modern finance and option pricing. The mathematics are kept to a minimal admissible level of formalism in order not to obscure the economic message. Readers should also get some insights about "weather to buy or sell" coffee or sugar Futures contracts, looking at the climate pattern – late frost versus warm and dry weather – prevailing across Brazil in August. Or they may wisely conclude to never plan a trip to Florida without first scrutinizing the frozen Orange Juice Futures prices posted on the New York Cotton Exchange.

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First and foremost, I want to express my gratitude to Sophie Lémann, my Assistant in the Finance Department at ESSEC. Her editing talent and relentless optimism have allowed this work to move forward across oceans, airports and power conferences.

Then, I would like to thank those who helped me in various ways, including Philippe van den Abeele, Omar Alami, Martine Azara, Nilgon Baykara, Carole Boussiba, Naïma Esskali, Raj Gupta, Benjamin Geman, Aymeric Khalife, Stelios Kuruvakalis, George Martin, Jean-Pierre Mateille, Alessandro Moro, Julien Mutin, Steve Ohana, Patrick Perfetti, Patrick Slama, Jean Winghart.

Last but not least, I wish to thank Etienne Amic and Chris Harris, each of whom I invited to write chapters in his domain of unique expertise, oil and metals respectively. It is my admiration for them which explains why, throughout the book, the brilliant trader is referred to as "he", instead of the more politically correct "she".

1

Fundamentals of Commodity Spot and Futures Markets: Instruments, Exchanges and Strategies

1.1 THE IMPORTANCE OF COMMODITY SPOT TRADING

Commodity price risk is an important element of the world physionomy at this date, as it has an impact on the economy of both developed and developing countries: in a rough approximation, one can state that the latter include most commodity producing countries, the former being originators, marketers and manufacturers. All parties are still involved in activities of spot trading with physical delivery while the formidable development of liquid derivative markets – forward, Futures contracts and options – has paved the way for risk management and optimal design of supply and demand contracts.

Every commodity is traded on a spot market. In the old days, buyers and sellers used to meet on the marketplace where transactions led to immediate delivery. In the 18th and 19th centuries, potato growers in the state of Maine started selling their crops at the time of planting in order to finance the production process. In a parallel manner, numerous forward transactions were taking place in Chicago for cereals and agricultural products and in London for metals. A need for standardization in terms of quantity, quality, delivery date emerged and led to the establishment of the New York Cotton Exchange (NYCE) in 1842 and the Chicago Board Of Trade (CBOT) in 1848. The clearing house, unique counterparty for buyers and sellers of Futures contracts was the effective signal that the Exchange was operating. As of today, some of these clearing houses are owned by independent shareholders, others are primarily owned by market participants as in the case of the London Metal Exchange (LME) and the International Petroleum Exchange (IPE). Different qualities of the same commodity may be traded on different exchanges. The most famous examples include: coffee which in its "arabica" variety is traded on the New York Coffee, Sugar and Cocoa Exchange (CSCE), while the "robusta" type is traded on the London International Financial Futures Exchange (LIFFE); and oil which is traded on the New York Mercantile Exchange (NYMEX) as Western Texas Intermediate (WTI) and on the IPE in its Brent variety.

Let us observe that the fact that any transaction on commodities may be physical (delivery of the commodity) or financial (a cash flow from one party to the other at maturity and no exchange of the underlying good) is in sharp contrast to bonds and stock markets where all trades are financial. However, physical and financial commodity markets are, as expected, strongly related. Price and volatility observed in "paper" transactions are correlated to the analogous quantities in the physical market, both

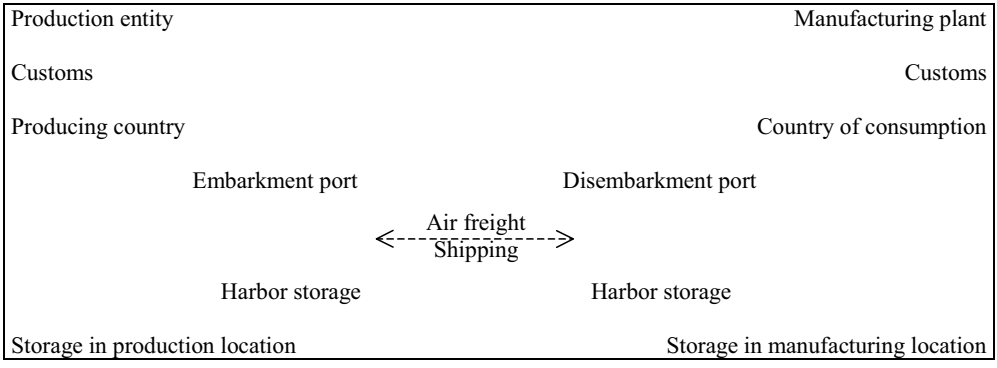
because of the physical delivery that may take place at maturity of a Futures contract and the existence of spot forward relationships that will be discussed throughout the book.

Lastly, let us note that the last two decades have experienced dramatic changes in world commodity markets. Political upheavals in some countries, economic mutation, new environmental regulation, a huge rise in the consumption of commodities in countries such as China and other structural changes have contributed to increase the volatility of supply and prices. This has made hedging activities (through forwards, Futures and options) indispensable for many sectors of the economy, the airline industry in particular being an important example.

As mantioned before, we call spot trading any transaction where delivery either takes place immediately (which is rarely the case in practice) or if there is a minimum lag, due to technical constraints, between the trade and delivery. Beyond that minimal lag, the trade becomes a forward agreement between the two parties and is properly documented by a written contract which specifies, among other things, who among the buyer and seller is responsible for shipping, unloading the goods and other transportation-related issues.

Consider a standard situation where the seller is a producer (e.g., of copper) and the buyer a manufacturer: in general, they never meet and, even if they did, would rarely agree on prices, timing and so forth. Hence the existence of intermediaries who play the role of go-between, are prepared to take delivery of goods that may not resell immediately and organize the storage and shipping.

We can represent the different phases of the physical execution of a trade as:



The document that represents the ownership of the good is called a *bill of lading*. It is issued either by the captain of the transportation ship or by the transporter in charge. That transportation contract may eventually be traded. It can bear the label "shipped" or "to be shipped"; the latter terminology indicates that the merchandise has been embarked, leading to the qualification *clean on board*.

Responsibility for commercial execution

It may lie in the hands of the seller, or the buyer, or the intermediary (since, in practice, many intermediaries will play a role, in particular because of the lags in the timing of different operations).

The responsibility will take different forms:

- For the exporter, sale Free On Board (FOB).
- For the commercial intermediary, purchase FOB out of the dock or in warehouse.
- For the importer, purchase on the dock or in warehouse.

Note that the commercial responsibility may be fragmented in the course of contract execution. For instance, a manufacturer who buys metals under a FOB specification is responsible for organizing the shipping but the armator is in charge of managing the shipping and holds the corresponding risk.

Major risks in commodity spot transaction

Four major types of risk may be identified in commodity spot markets:

- *Price risk*, which will be discussed throughout the book and for which the first examples of hedging strategies are presented in Section 1.2.
- *Transportation risk*, which is described below.
- *Delivery risk*, which concerns the quality of the commodity that is delivered and for which there is no financial hedge that may be put in place. The only coverage is provided by a very customized contract or by a solid long-term relationship with the originator.
- *Credit risk*, which is present all along until the final completion of the trade.

Risk attached to the transportation of commodities

1. The first category of risks concerns the deterioration, partial or total, of goods during transportation. Two types of risks are usually recognized in this category:
   - "ordinary" risks;
   - "extraordinary" risks such as wars, riots and strikes.

   The expeditor of the goods or the FOB buyer directly holds the transportation risk, unless they purchase an insurance contract to be covered. Different companies specialized in freight insurance (such as the famous Lloyds of London) propose various types of contracts. We need to keep in mind that transportation risk is an important one as it includes the entire community – the tanker that sank in Alaska being a sad example. If no specific insurance coverage has been purchased, the company that bears the liability must put in place some kind of *self-insurance* process as do some major oil companies today.
2. Cost of transportation risk: All Futures exchanges around the world quote FOB prices. Consequently, if a trade (e.g., on sugar) is settled for delivery 12 months later with the CIF price as a reference, the seller needs to hedge his position not only against a decline in sugar prices by, say, selling Futures on the New York Coffee, Cocoa and Sugar Exchange, but also against changes in the shipping cost. The latter risk will be hedged by entering into a Forward Freight Agreement (described in Section 1.5). Consequently, the two components of the CIF price will get hedged in totally different exchanges.

1.2 FORWARD AND FUTURES CONTRACTS

A forward contract may be generically described as an agreement struck at date 0 between two parties to exchange at some fixed future date a given quantity of a commodity for an amount of dollars defined at date 0. A Futures contract has the same general features as a forward contract but is transacted through a Futures exchange. The clearing house standing behind that exchange essentially takes away any credit risk from the positions of the two participants engaged in the transaction. This default risk is almost reduced to zero through margin deposits or initial margins that need to be made before entering into any contract, as well as the daily margin calls required to keep a contract alive if its market value has declined from the previous day.

Futures contracts serve many purposes. Their first role has been to facilitate the trading of various commodities as financial instruments. But they have from the start been providing a hedging vehicle against *price risk*: a farmer selling his crops in January through a Futures contract maturing at time T of the harvest (say, September) for a price $F^T(0)$ defined on 1 January has secured at the beginning of the year this amount of revenue. Hence, he may allocate the proceeds to be received to the acquisition of new machinery or storage facilities and, more generally, design his investment plans for the year independently of any news of corn oversupply possibly occurring over the 9-month period.

It is noticeable in many markets, ranging from agricultural commodities to electricity, that Futures contracts are used as a substitute for the spot market by hedge funds, Commodity Trading Advisors (CTAs) or any class of investors wishing to take a position in commodities, both because it takes away the physical constraints of spot trading and provides the flexibility of short and long positions, hence the choice of positive or negative exposure to a rise in prices. This will be discussed in detail in Chapter 14.

What follows describes in detail the mechanisms of forward and Futures contracts with their various characteristics as well as the way exchanges operate. The different classes of participants, the mechanism of *price discovery* and the crucial relationships, if they exist, between spot prices and Futures prices under some form of equilibrium assumptions are described in detail.

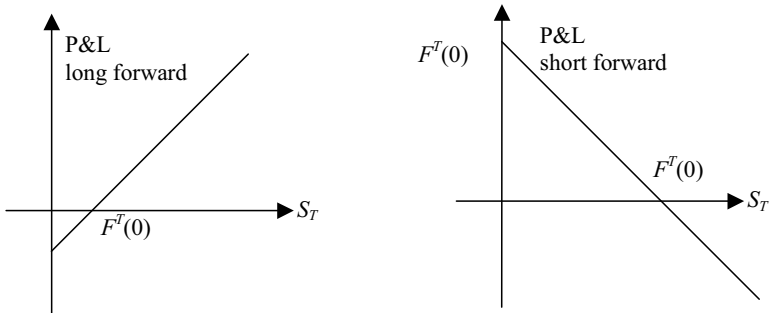
Forward contracts

A forward contract is an agreement signed between two parties A and B at time 0, according to which party A has the obligation of delivering at a fixed future date T an underlying asset and party B the obligation of paying at that date an amount fixed at date 0, denoted $F^T(0)$ and called the forward price for date T for the asset. Note that this price is not a price in the sense of the price of a stock, but rather a reference value in the contractual transaction. If the underlying asset is traded in a liquid market, the no-arbitrage condition between spot and forward markets at maturity implies that:

$$S(T) = F^T(T)$$

If the value at date T of the Futures contract maturing at that date was different from the spot price, an arbitrage opportunity would be realized by buying in one market and selling immediately in the other.

Keeping in mind that the buyer of the forward contract may immediately sell at maturity in the spot market at the price S_T the commodity which was delivered to him against the payment of $F^T(0)$ dollars, the respective Profit and Loss (P&L) of party A (called long forward) and party B (called short forward) are depicted by the following graphs:



Obviously, the contract is a zero-sum game between the buyer and the seller. Note also that, by definition, both P&Ls are expressed in dollars at date T.

For practical purposes, party A represents an economic agent who wants to hedge against a possible rise in the price of the underlying asset between dates 0 and T and locks in at date 0 a purchase price equal to $F^T(0)$. Party B, conversely, fears a collapse of this price or expects to profit from a rise. The price $F^T(0)$ represents their estimation at date 0 of how much the underlying asset S will be worth at date T together with the risk premium they are willing to pay (or receive). We will come back to this discussion in Chapter 2.

Should parties A and B enter into this T maturity contract at a future date t in the interval $(0, T)$, the price $F^T(t)$ on which they will agree is likely to be different from $F^T(0)$ and translates the changes between dates 0 and t in the expectations perceived by the market of the commodity future spot price $S(T)$.

Futures contracts

They are analogous to forward contracts in terms of their definition but present some key differences from them:

- They are "standardized" in terms of their characteristics (maturity, quantity of the underlying commodity, quality or variety).
- They are traded on an exchange, such as NYMEX or the IPE; hence, they carry no counterparty risk since both the buyer and the seller of the Futures deal with the clearing house of the exchange which is in principle fully trustworthy.
- They require the payment of margin deposits in order to be able to start placing orders on the exchange.
- They are marked-to-market daily and the participants have to adjust their positions: for instance, if a participant has a long position in a Futures contract acquired at the price $F^T(0)$ and if the price $F^T(\text{day } 1)$ is lower, then this participant has experienced a loss between days 0 and 1 equal to $F^T(\text{day } 1) - F^T(0)$. In order to keep his

position, he has to adjust it by adding at the end of day 1 a cash amount equal to his loss.

Forward and Futures prices on the same underlying asset with the same time to expiry are different because of taxes, transaction costs and other important elements, such as the impact of credit risk on the one hand and stochastic interest rates on the other hand. In practice, they remain very close to each other since the fluctuations of the underlying commodity represent the most important explanatory factor. Except when stated otherwise, we will view the two prices as the same in a first-order approximation.

We can recap the similarities and differences between the fundamental types of transactions prevailing in commodity markets as depicted in the following diagram:

Spot trading	Forward contracts	Futures contracts
◆ *commercial contract* ◆ flexible covenants	◆ *bilateral agreement* ◆ flexible covenants	◆ *standardized instrument* ◆ necessity of a physical delivery or termination of the position before maturity
◆ juridical commitments of the buyer and seller until execution of the contract	◆ replace spot transactions on many occasions (e.g., in the case of a non-storable commodity such as electricity)	◆ buyer and seller only refer to the clearing house
⇩	⇩	⇩
◆ long transaction ◆ illiquid and discontinuous market ◆ allows the transfer of goods in conditions suiting the demand	◆ form of contracting totally appropriate for commodities ◆ credit risk fully present ◆ flexibility regarding the optimal transfer of goods	◆ central clearing mechanism generating "market prices" ◆ price transparency ◆ liquidity ◆ low transaction costs

1.3 THE ACTORS IN FUTURES MARKETS

Hedgers

Futures markets were originally set up to meet the needs of hedgers, namely farmers who wanted to lock in advance a fixed price for their harvests. Commodity Futures are still widely used by producers and users of commodities for hedging purposes. Suppose that the date of analysis is January and company XYZ knows that it will have to buy on 25 September (date T) of the same year one million tons of fuel. In order to hedge against the possible increase in fuel price between January and the end of September, airline company XYZ will buy (equivalently, *enter a long position in*) Futures contracts written on fuel, maturity September and in an amount corresponding to the necessary quantity of fuel. By doing so, the airline company has locked in at the beginning of the

year the price $F^T(0)$ it will pay in September and has done so with no cash flow payment at the beginning of the year.

Another possible hedge (as we will see later) would be to buy options – again in the appropriate quantity – written on the fuel as the underlying, with maturity T and strike price $k = F^T(0)$ for example.[1] In this case, the resulting cost of fuel in September for company XYZ will be either $F^T(0)$ or, strictly, less if the market spot price is very low. This alternative is strictly superior for the hedger at maturity, but at inception of the option contract in January company XYZ will have to pay the premium of the options involved in the hedge.

Returning to Futures contracts, *basic risk* refers to the risk remaining after the hedge has been put in place and essentially represents the difference between the Futures price – should the Futures position be closed prematurely – and the spot price. It also includes other components such as:

- The price of cleaning the local grade of the commodity into a grade deliverable in a Futures contract (or the premium for a superior grade).
- The price of transportation to or from the delivery point in the Futures contract.
- The physical cost of storage, including insurance, between the time of the harvest and the delivery date of the Futures contract.

Speculators

While hedgers want to avoid exposure to adverse movements of the price of a commodity which is part of their manufacturing process in the economy, speculators wish to get exposure to commodity price moves (i.e., take risks in order to make profits). Using the same example as before, a bank ABC which has no "natural" exposure to the price of fuel may decide to take a position either in a Futures contract on fuel or in an option written on fuel and, by doing so, will create for itself exposure to the fuel price; obviously, the nature of the position – long or short in either instrument – will be determined by the "view" that bank ABC has on the subsequent moves of the fuel price. It is in fact "betting" that this price will go up or will go down and counting on the corresponding profits the bank will generate. Unsurprisingly, commodities are becoming increasingly attractive to investors and hedge fund managers who view them as an *alternative asset class* allowing one to reduce the overall risk of a financial portfolio and enhance the return as well. Futures are the obvious instrument – because of their liquidity, because of the low transaction costs on the exchange, because of the absence of credit risk – to take positions reflecting an anticipation of a price rise by purchasing Futures or a decline by selling Futures. Most of the liquidity is generated by the combined activity of speculators and hedgers.

Arbitragers

Arbitragers represent a third important – but smaller in size – group of participants in futures and options markets. An arbitrage is a *riskless* profit realized by simultaneously

[1] For practical purposes, this will be the strike most often chosen when hedging with an option since everyone has the forward position in mind as the alternative hedge.

entering into several transactions in two or more markets. Arbitrage opportunities are very desirable but not easy to uncover and they do not last for long. If a given instrument is underpriced, buying activity will cause the price to rise up to a value which is viewed by the market as the "fair" price and at which there will be no more excess demand.

In this book, as in the fundamental models of option pricing, the values of Futures and options contracts will be based on the assumption of no-arbitrage opportunities. Given the importance of this concept, we are going to propose several definitions of what is called in daily language "no free lunch".

Definition 1 A portfolio P is called an "arbitrage portfolio" if it satisfies simultaneously the following conditions:

$$\begin{cases} V_p(0) = 0 \\ V_p(H) \geq 0 \quad \text{in all states of the world at date } H \\ V_p(H) > 0 \quad \text{in some states of the world} \end{cases}$$

where H is a date later than date 0.

Definition 2 A market is said to be arbitrage-free if there exists no arbitrage opportunity; in other words, for any portfolio P:

$$\begin{cases} V_p(0) = 0 \\ \text{and} \qquad\qquad\qquad\qquad \Rightarrow V_p(H) = 0 \quad \text{in all states of the world} \\ V_p(H) \geq 0 \quad \text{in all states of the world} \\ \qquad\qquad \text{at date } H > 0 \end{cases}$$

In other words, if a portfolio *requires a null investment* and is *riskless* (there is no possible loss at the horizon H), then its terminal value at date H has to be zero. These are the various expressions of the no free lunch property: if you start with no money and *take no risk*, your final wealth will be zero. In the above statements, the assumption of "riskless" is crucial: if you have no money, you can always borrow from your banker and invest the proceeds in a hedge fund. Your final wealth may be quite large or very negative once you have repaid your loan, but rarely equal to zero.

Comments

For practical purposes, traders searching for arbitrage opportunities are realistically looking, in fact, for "quasi-riskless" strategies generating profits. Such a strategy should be as simple as possible and, hence, involve only spot and forward positions which have *linear payoffs and P&Ls* in the underlying price, making the nullification of risk possible by the resolution of a *single* equation in the price S of the underlying commodity. When we move to positions involving options, we will see that the convexity (or concavity) of the P&L (i.e., the gamma of the option), together with the sensitivity to volatility (i.e., the vega), make this "quasi-riskless" feature much more problematic. Hence, it is indeed in Futures markets that the fundamental concept of arbitrage finds its origin,