Risk and Financial Management
Risk and Financial Management
Mathematical and Computational Methods

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John Wiley & Sons, Ltd
This book is dedicated to:

Daniel
Dafna
Oren
Oscar and
Bettina
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Another finance book to teach what market gladiators/traders either know, have no time for or can’t be bothered with. Yet another book to be seemingly drowned in the endless collections of books and papers that have swamped the economic literate and illiterate markets ever since options and futures markets grasped our popular consciousness. Economists, mathematically inclined and otherwise, have been largely compensated with Nobel prizes and seven-figures earnings, competing with market gladiators – trading globalization, real and not so real financial assets. Theory and practice have intermingled accumulating a wealth of ideas and procedures, tested and remaining yet to be tested. Martingale, chaos, rational versus adaptive expectations, complete and incomplete markets and whatnot have transformed the language of finance, maintaining their true meaning to the mathematically initiated and eluding the many others who use them nonetheless.

This book seeks to provide therefore, in a readable and perhaps useful manner, the basic elements or economic language of financial risk management, mathematical and computational finance, laying them bare to both students and traders. All great theories are based on simple philosophical concepts, that in some circumstances may not withstand the test of reality. Yet, we adopt them and behave accordingly for they provide a framework, a reference model, inspiring the required confidence that we can rely on even if there is not always something to stand on. An outstanding example might be complete markets and options valuation – which might not be always complete and with an adventuresome valuation of options. Market traders make seemingly risk-free arbitrage profits that are in fact model-dependent. They take positions whose risk and rewards we can only make educated guesses at, and make venturesome and adventuresome decisions in these markets based on facts, fancy and fanciful interpretations of historical patterns and theoretical–technical analyses that seek to decipher things to come.

The motivation to write this book arose from long discussions with a hedge fund manager, my son, on a large number of issues regarding markets behaviour, global patterns and their effects both at the national and individual levels, issues regarding psychological behaviour that are rendering markets less perfect than what we might actually believe. This book is the fruit of our theoretical and practical contrasts and language – the sharp end of theory battling the long and wily practice of the market gladiator, each with our own vocabulary and misunderstandings. Further, too many students in computational finance learn techniques, technical analysis and financial decision making without assessing the dependence of such
analyses on the definition of uncertainty and the meaning of probability. Further, defining ‘uncertainty’ in specific ways, dictates the type of technical analysis and generally the theoretical finance practised. This book was written, both to clarify some of the issues confronting theory and practice and to explain some of the ‘fundamentals, mathematical’ issues that underpin fundamental theory in finance.

Fundamental notions are explained intuitively, calling upon many trading experiences and examples and simple equations-analysis to highlight some of the basic trends of financial decision making and computational finance. In some cases, when mathematics are used extensively, sections are starred or introduced in an appendix, although an intuitive interpretation is maintained within the main body of the text.

To make a trade and thereby reach a decision under uncertainty requires an understanding of the opportunities at hand and especially an appreciation of the underlying sources and causes of change in stocks, interest rates or assets values. The decision to speculate against or for the dollar, to invest in an Australian bond promising a return of five % over 20 years, are risky decisions which, inordinately amplified, may be equivalent to a gladiator’s fight for survival. Each day, tens of thousands of traders, investors and fund managers embark on a gargantuan feast, buying and selling, with the world behind anxiously betting and waiting to see how prices will rise and fall. Each gladiator seeks a weakness, a breach, through which to penetrate and make as much money as possible, before the hordes of followers come and disturb the market’s equilibrium, which an instant earlier seemed unmovable. Size, risk and money combine to make one richer than Croesus one minute and poorer than Job an instant later. Gladiators, too, their swords held high one minute, and history a minute later, have played to the arena. Only, it is today a much bigger arena, the prices much greater and the losses catastrophic for some, unfortunately often at the expense of their spectators.

Unlike in previous times, spectators are thrown into the arena, their money fated with these gladiators who often risk, not their own, but everyone else’s money – the size and scale assuming a dimension that no economy has yet reached.

For some, the traditional theory of decision-making and risk taking has fared badly in practice, providing a substitute for reality rather than dealing with it. Further, the difficulty of problems has augmented with the involvement of many sources of information, of time and unfolding events, of information asymmetries and markets that do not always behave competitively, etc. These situations tend to distort the approaches and the techniques that have been applied successfully but to conventional problems. For this reason, there is today a great deal of interest in understanding how traders and financial decision makers reach decisions and not only what decisions they ought to reach. In other words, to make better decisions, it is essential to deal with problems in a manner that reflects reality and not only theory that in its essence, always deals with structured problems based on specific assumptions – often violated. These assumptions are sometimes realistic; but sometimes they are not. Using specific problems I shall try to explain approaches applied in complex financial decision processes – mixing practice and theory. The approach we follow is at times mildly quantitative, even though much of the new approach to finance is mathematical and computational and requires an
extensive mathematical proficiency. For this reason, I shall assume familiarity with basic notions in calculus as well as in probability and statistics, making the book accessible to typical economics and business and maths students as well as to practitioners, traders and financial managers who are familiar with the basic financial terminology.

The substance of the book in various forms has been delivered in several institutions, including the MASTER of Finance at ESSEC in France, in Risk Management courses at ESSEC and at Bar Ilan University, as well as in Mathematical Finance courses at Bar Ilan University Department of Mathematics and Computer Science. In addition, the Montreal Institute of Financial Mathematics and the Department of Finance at Concordia University have provided a testing ground as have a large number of lectures delivered in a workshop for MSc students in Finance and in a PhD course for Finance students in the Montreal consortium for PhD studies in Mathematical Finance in the Montreal area. Throughout these courses, it became evident that there is a great deal of excitement in using the language of mathematical finance but there is often a misunderstanding of the concepts and the techniques they require for their proper application. This is particularly the case for MBA students who also thrive on the application of these tools. The book seeks to answer some of these questions and problems by providing as much as possible an interface between theory and practice and between mathematics and finance. Finally, the book was written with the support of a number of institutions with which I have been involved these last few years, including essentially ESSEC of France, the Montreal Institute of Financial Mathematics, the Department of Finance of Concordia University, the Department of Mathematics of Bar Ilan University and the Israel Port Authority (Economic Research Division). In addition, a number of faculty and students have greatly helped through their comments and suggestions. These have included, Elias Shiu at the University of Iowa, Lorne Switzer, Meir Amikam, Alain Bensoussan, Avi Lioui and Sebastien Galy, as well as my students Bernardo Dominguez, Pierre Bour, Cedric Lespiau, Hong Zhang, Philippe Pages and Yoav Adler. Their help is gratefully acknowledged.
PART I

Finance and Risk Management
CHAPTER 1

Potpourri

1.1 INTRODUCTION

Will a stock price increase or decrease? Would the Fed increase interest rates, leave them unchanged or decrease them? Can the budget to be presented in Transylvania’s parliament affect the country’s current inflation rate? These and so many other questions are reflections of our lack of knowledge and its effects on financial markets performance. In this environment, uncertainty regarding future events and their consequences must be assessed, predictions made and decisions taken. Our ability to improve forecasts and reach consistently good decisions can therefore be very profitable. To a large extent, this is one of the essential preoccupations of finance, financial data analysis and theory-building. Pricing financial assets, predicting the stock market, speculating to make money and hedging financial risks to avoid losses summarizes some of these activities. Predictions, for example, are reached in several ways such as:

- ‘Theorizing’, providing a structured approach to modelling, as is the case in financial theory and generally called fundamental theory. In this case, economic and financial theories are combined to generate a body of knowledge regarding trades and financial behaviour that make it possible to price financial assets.
- Financial data analysis using statistical methodologies has grown into a field called financial statistical data analysis for the purposes of modelling, testing theories and technical analysis.
- Modelling using metaphors (such as those borrowed from physics and other areas of related interest) or simply constructing model equations that are fitted one way or another to available data.
- Data analysis, for the purpose of looking into data to determine patterns or relationships that were hitherto unseen. Computer techniques, such as neural networks, data mining and the like, are used for such purposes and thereby make more money. In these, as well as in the other cases, the ‘proof of the pudding is in the eating’. In other words, it is by making money, or at least making...
it possible for others to make money, that theories, models and techniques are validated.

- Prophecies we cannot explain but sometimes are true.

Throughout these ‘forecasting approaches and issues’ financial managers deal practically with uncertainty, defining it, structuring it and modelling its causes, explainable and unexplainable, for the purpose of assessing their effects on financial performance. This is far from trivial. First, many theories, both financial and statistical, depend largely on how we represent and model uncertainty. Dealing with uncertainty is also of the utmost importance, reflecting individual preferences and behaviours and attitudes towards risk. Decision Making Under Uncertainty (DMUU) is in fact an extensive body of approaches and knowledge that attempts to provide systematically and rationally an approach to reaching decisions in such an environment. Issues such as ‘rationality’, ‘bounded rationality’ etc., as we will present subsequently, have an effect on both the approach we use and the techniques we apply to resolve the fundamental and practical problems that finance is assumed to address. In a simplistic manner, uncertainty is characterized by probabilities. Adverse consequences denote the risk for which decisions must be taken to properly balance the potential payoffs and the risks implied by decisions – trades, investments, the exercise of options etc. Of course, the more ambiguous, the less structured and the more uncertain the situations, the harder it is to take such decisions. Further, the information needed to make decisions is often not readily available and consequences cannot be predicted. Risks are then hard to determine. For example, for a corporate finance manager, the decision may be to issue or not to issue a new bond. An insurance firm may or may not confer a certain insurance contract. A Central Bank economist may recommend reducing the borrowing interest rate, leaving it unchanged or increasing it, depending on multiple economic indicators he may have at his disposal. These, and many other issues, involve uncertainty. Whatever the action taken, its consequences may be uncertain. Further, not all traders who are equally equipped with the same tools, education and background will reach the same decision (of course, when they differ, the scope of decisions reached may be that much broader). Some are well informed, some are not, some believe they are well informed, but mostly, all traders may have various degrees of intuition, introspection and understanding, which is specific yet not quantifiable. A historical perspective of events may be useful to some and useless to others in predicting the future. Quantitative training may have the same effect, enriching some and confusing others. While in theory we seek to eliminate some of the uncertainty by better theorizing, in practice uncertainty wipes out those traders who reach the wrong conclusions and the wrong decisions. In this sense, no one method dominates another: all are important. A political and historical appreciation of events, an ability to compute, an understanding of economic laws and fundamental finance theory, use of statistics and computers to augment one’s ability in predicting and making decisions under uncertainty are only part of the tool-kit needed to venture into trading speculation and into financial risk management.
1.2 THEORETICAL FINANCE AND DECISION-MAKING

Financial decision making seeks to make money by using a broad set of economic and theoretical concepts and techniques based on rational procedures, in a consistent manner and based on something more than intuition and personal subjective judgement (which are nonetheless important in any practical situation). Generally, it also seeks to devise approaches that may account for departures from such rationality. Behavioural and psychological reasons, the violation of traditional assumptions regarding competition and market forces and exchange combine to alter the basic assumptions of theoretical economics and finance.

Finance and financial instruments currently available through brokers, mutual funds, financial institutions, commodity and stock markets etc. are motivated by three essential problems:

- Pricing the multiplicity of claims, accounting for risks and dealing with the negative effects of uncertainty or risk (that can be completely unpredictable, or partly or wholly predictable)
- Explaining, and accounting for investors’ behaviour. To counteract the effects of regulation and taxes by firms and individual investors (who use a wide variety of financial instruments to bypass regulations and increase the amount of money investors can make).
- Providing a rational framework for individuals’ and firms’ decision making and to suit investors’ needs in terms of the risks they are willing to assume and pay for. For this purpose, extensive use is made of DMUU and the construction of computational tools that can provide ‘answers’ to well formulated, but difficult, problems.

These instruments deal with the uncertainty and the risks they imply in many different ways. Some instruments merely transfer risk from one period to another and in this sense they reckon with the time phasing of events to reckon with. One of the more important aspects of such instruments is to supply ‘immediacy’, i.e. the ability not to wait for a payment for example (whereby, some seller will assume the risk and the cost of time in waiting for that payment). Other instruments provide a ‘spatial’ diversification, in other words, the distribution of risks across a number of independent (or almost independent) risks. For example, buying several types of investment that are less than perfectly correlated, maintaining liquidity etc. By liquidity, we mean the cost to instantly convert an asset into cash at its fair price. This liquidity is affected both by the existence of a market (in other words, buyers and sellers) and by the cost of transactions associated with the conversion of the asset into cash. As a result, risks pervading finance and financial risk management are varied; some of them are outlined in greater detail below.

Risk in finance results from the consequences of undesirable outcomes and their implications for individual investors or firms. A definition of risk involves their probability, individual and collective and consequences effects. These are relevant to a broad number of fields as well, each providing an approach to the
measurement and the valuation of risk which is motivated by their needs and by the set of questions they must respond to and deal with. For these reasons, the problems of finance often transcend finance and are applicable to the broad areas of economics and decision-making. Financial economics seeks to provide approaches and answers to deal with these problems. The growth of theoretical finance in recent decades is a true testament to the important contribution that financial theory has made to our daily life. Concepts such as financial markets, arbitrage, risk-neutral probabilities, Black–Scholes option valuation, volatility, smile and many other terms and names are associated with a maturing profession that has transcended the basic traditional approaches of making decisions under uncertainty. By the same token, hedging which is an important part of the practice finance is the process of eliminating risks in a particular portfolio through a trade or a series of trades, or contractual agreements. Hedging relates also to the valuation-pricing of derivatives products. Here, a portfolio is constructed (the hedging portfolio) that eliminates all the risks introduced by the derivative security being analyzed in order to replicate a return pattern identical to that of the derivative security. At this point, from the investor’s point of view, the two alternatives – the hedging portfolio and the derivative security – are indistinguishable and therefore have the same value. In practice too, speculating to make money can hardly be conceived without hedging to avoid losses.

The traditional theory of decision making under uncertainty, integrating statistics and the risk behaviour of decision makers has evolved in several phases starting in the early nineteenth century. At its beginning, it was concerned with collecting data to provide a foundation for experimentation and sampling theory. These were the times when surveys and counting populations of all sorts began. Subsequently, statisticians such as Karl Pearson and R. A. Fisher studied and set up the foundations of statistical data analysis, consisting of the assessment of the reliability and the accuracy of data which, to this day, seeks to represent large quantities of information (as given explicitly in data) in an aggregated and summarized fashion, such as probability distributions and moments (mean, variance etc.) and states how accurate they are. Insurance managers and firms, for example, spend much effort in collecting such data to estimate mean claims by insured clients and the propensity of certain insured categories to claim, and to predict future weather conditions in order to determine an appropriate insurance premium to charge. Today, financial data analysis is equally concerned with these problems, bringing sophisticated modelling and estimation techniques (such as linear regression, ARCH and GARCH techniques which we shall discuss subsequently) to bear on the application of financial analysis.

The next step, expounded and developed primarily by R. A. Fisher in the 1920s, went one step further with planning experiments that can provide effective information. The issue at hand was then to plan the experiments generating the information that can be analysed statistically and on the basis of which a decision could, justifiably, be reached. This important phase was used first in testing the agricultural yield under controlled conditions (to select the best way to grow plants, for example). It yielded a number of important lessons, namely that the
procedure (statistical or not) used to collect data is intimately related to the kind of relationships we seek to evaluate. A third phase, expanded dramatically in the 1930s and the 1940s consisted in the construction of mathematical models that sought to bridge the gap between the process of data collection and the need of such data for specific purposes such as predicting and decision making. Linear regression techniques, used extensively in econometrics, are an important example. Classical models encountered in finance, such as models of stock market prices, currency fluctuations, interest rate forecasts and investment analysis models, cash management, reliability and other models, are outstanding examples.

In the 1950s and the 1960s the (Bayes) theory of decision making under uncertainty took hold. In important publications, Raiffa, Luce, Schlaiffer and many others provided a unified framework for integrating problems relating to data collection, experimentation, model building and decision making. The theory was intimately related to typical economic, finance and industrial, business and other problems. Issues such as the value of information, how to collect it, how much to pay for it, the weight of intuition and subjective judgement (as often used by behavioural economists, psychologists etc.) became relevant and integrated into the theory. Their practical importance cannot be understated for they provide a framework for reaching decisions under complex situations and uncertainty. Today, theories of decision making are an ever-expanding field with many articles, books, experiments and theories competing to provide another view and in some cases another vision of uncertainty, how to model it, how to represent certain facets of the economic and financial process and how to reach decisions under uncertainty. The DMUU approach, however, presumes that uncertainty is specified in terms of probabilities, albeit learned adaptively, as evidence accrues for one or the other event. It is only recently, in the last two decades, that theoretical and economic analyses have provided in some cases theories and techniques that provide an estimate of these probabilities. In other words, while in the traditional approach to DMUU uncertainty is exogenous, facets of modern and theoretical finance have helped ‘endogenize’ uncertainty, i.e. explain uncertain behaviours and events by the predictive market forces and preferences of traders. To a large extent, the contrasting finance fundamental theory and traditional techniques applied to reach decisions under uncertainty diverge in their attempts to represent and explain the ‘making of uncertainty’. This is an important issue to appreciate and one to which we shall return subsequently when basic notions of fundamental theory including rational expectations and option pricing are addressed.

Today, DMUU is economics, finance, insurance and risk motivated. There are a number of areas of special interest we shall briefly discuss to better appreciate the transformations of finance, insurance and risk in general.

### 1.3 INSURANCE AND ACTUARIAL SCIENCE

Actuarial science is in effect one of the first applications of probability theory and statistics to risk analysis. Tetens and Barrois, already in 1786 and 1834
respectively, were attempting to characterize the ‘risk’ of life annuities and fire insurance and on that basis establish a foundation for present-day insurance. Earlier, the Gambling Act of 1774 in England (King George III) laid the foundation for life insurance. It is, however, to Lundberg in 1909, and to a group of Scandinavian actuaries (Borch, 1968; Cramer, 1955) that we owe much of the current mathematical theory of insurance. In particular, Lundberg provided the foundation for collective risk theory. Terms such as ‘premium payments’ required from the insured, ‘wealth’ or the ‘firm liquidity’ and ‘claims’ were then defined. In its simplest form, actuarial science establishes exchange terms between the insured, who pays the premium that allows him to claim a certain amount from the firm (in case of an accident), and the insurer, the provider of insurance who receives the premiums and invests and manages the moneys of many insured. The insurance terms are reflected in the ‘insurance contract’ which provides legally the ‘conditional right to claim’. Much of the insurance literature has concentrated on the definition of the rules to be used in order to establish the terms of such a contract in a just and efficient manner. In this sense, ‘premium principles’ and a wide range of operational rules worked out by the actuarial and insurance profession have been devised. Currently, insurance is gradually being transformed to be much more in tune with market valuation of insurable contracts and financial instruments are being devised for this purpose. The problems of insurance are, of course, extremely complex, with philosophical and social undertones, seeking to reconcile individual with collective risk and individual and collective choices and interests through the use of the market mechanism and concepts of fairness and equity. In its proper time setting (recognizing that insurance contracts express the insured attitudes towards time and uncertainty, in which insurance is used to substitute certain for uncertain payments at different times), this problem is of course, conceptually and quantitatively much more complicated. For this reason, the quantitative approach to insurance, as is the case with most financial problems, is necessarily a simplification of the fundamental issues that insurance deals with.

Risk is managed in several ways including: ‘pricing insurance, controls, risk sharing and bonus-malus’. Bonus-malus provides an incentive not to claim when a risk materializes or at least seeks to influence insured behaviour to take greater care and thereby prevent risks from materializing. In some cases, it is used to discourage nuisance claims. There are numerous approaches to applying each of these tools in insurance. Of course, in practice, these tools are applied jointly, providing a capacity to customize insurance contracts and at the same time assuming a profit for the insurance firm.

In insurance and finance (among others) we will have to deal as well with special problems, often encountered in practical situations but difficult to analyse using statistical and analytical techniques. These essentially include dependencies, rare events and man-made risks. In insurance, correlated risks are costlier to assume while insuring rare and extremely costly events is difficult to assess. Earthquake and tornado insurance are such cases. Although, they occur, they do so with small probabilities. Their occurrence is extremely costly for the insurer,
however. For this reason, insurers seek the participation of governments for such insurance, study the environment and the patterns in weather changes and turn to extensive risk sharing schemes (such as reinsurance with other insurance firms and on a global scale). Dependencies can also be induced internally (endogenously generated risks). For example, when trading agents follow each other’s action they may lead to the rise and fall of an action on the stock market. In this sense, ‘behavioural correlations’ can induce cyclical economic trends and therefore greater market variability and market risk. Man-made induced risks, such as terrorists’ acts of small and unthinkable dimensions, also provide a formidable challenge to insurance companies. John Kay (in an article in the Financial Times, 2001) for example states:

The insurance industry is well equipped to deal with natural disasters in the developed world: the hurricanes that regularly hit the south-east United States; the earthquakes that are bound to rock Japan and California from time to time. Everyone understands the nature of these risks and their potential consequences. But we are ignorant of exactly when and where they will materialize. For risks such as these, you can write an insurance policy and assess a premium.

But the three largest disasters for insurers in the past 20 years have been man-made, not natural. The human cost of asbestos was greater even than that of the destruction of the World Trade Center. The deluge of asbestos-related claims was the largest factor in bringing the Lloyd’s insurance market to its knees.

By the same token, the debacle following the deregulation of Savings and Loans in the USA in the 1960s led to massive opportunistic behaviours resulting in huge losses for individuals and insurance firms. These disasters have almost uniformly involved government interventions and in some cases bail-outs (as was the case with airlines in the aftermath of the September 11th attack on the World Trade Center). Thus, risk in insurance and finance involves a broad range of situations, sources of uncertainty and a broad variety of tools that may be applied when disasters strike. There are special situations in insurance that may be difficult to assess from a strictly financial point of view, however, as in the case of man-made risks. For example, environmental risks have special characteristics that are affecting our approach to risk analysis:

- Rare events: Relating to very large disasters with very small probabilities that may be difficult to assess, predict and price.
- Spillover effects: Having behavioural effects on risk sharing and fairness since persons causing risks may not be the sole victims. Further, effects may be felt over long periods of time.
- International dimensions: having power and political overtones.

For these reasons, some of the questions raised in conjunction with environmental risk that are of acute interest today are numerous, including among others:
Who pays for it?
What prevention if at all?
Who is responsible if at all?

By the same token, the future of genetic testing promises to reveal information about individuals that, hitherto has been unknown, and thereby to change the whole traditional approach to insurance. In particular, randomness, an essential facet of the insurance business, will be removed and insurance contracts could/would be tailored to individuals’ profiles. The problems that may arise subsequent to genetic testing are tremendous. They involve problems arising over the power and information asymmetries between the parties to contracts. Explicitly, this may involve, on the one hand, moral hazard (we shall elaborate subsequently) and, on the other, adverse selection (which will see later as well) affecting the potential future/non-future of the insurance business and the cost of insurance to be borne by individuals.

1.4 UNCERTAINTY AND RISK IN FINANCE

Uncertainty and risk are everywhere in finance. As stated above, they result from consequences that may have adverse economic effects. Here are a few financial risks.

1.4.1 Foreign exchange risk

*Foreign exchange risk* measures the risk associated with unexpected variations in exchange rates. It consists of two elements: an internal element which depends on the flow of funds associated with foreign exchange, investments and so on, and an external element which is independent of a firm’s operations (for example, a variation in the exchange rates of a country).

Foreign exchange risk management has focused essentially on short-term decisions involving accounting exposure components of a firm’s working capital. For instance, consider the case of captive insurance companies that diversify their portfolio of underwriting activities by reinsuring a ‘layer’ of foreign risk. In this case, the magnitude of the transaction exposure is clearly uncertain, compounding the exchange and exposure risks. Bidding on foreign projects or acquisitions of foreign companies will similarly entail exposures whose magnitudes can be characterized at best subjectively. Explicitly, in big-ticket export transactions or large-scale construction projects, the exporter or contractor will first submit a bid $B(T)$ of say 100 million which is denominated in SUS (a foreign currency from the point of view of the decision maker) and which, if accepted, would give rise to a transaction exposure (asset or liability) maturing at a point in time $T$, say 2 years ahead. The bid will in turn be accepted or rejected at time $t$, say 6 months ahead ($0 < t < T$), resulting in the transaction exposure which is uncertain until the resolution (time) standing at the full amount $B(T)$ if the bid is accepted, or
being cancelled if the bid is rejected. Effective management of such uncertain exposures will require the existence of a futures market for foreign exchange allowing contracts to be entered into or cancelled at any time \( t \) over the bidding uncertainty resolution horizon \( 0 < t < T \). The case of foreign acquisition is a special case of the above more general problem with uncertainty resolution being arbitrarily set at \( t = T \). Problems in long-term foreign exchange risk management – that is, long-term debt financing and debt refunding – in a multi-currency world, although very important, is not always understood and hedged. As global corporations expand operations abroad, foreign currency-denominated debt instruments become an integral part of the opportunities of financing options. One may argue that in a multi-currency world of efficient markets, the selection of the optimal borrowing source should be a matter of indifference, since nominal interest rates reflect inflation rate expectations, which, in turn, determine the pattern of the future spot exchange rate adjustment path. However, heterogeneous corporate tax rates among different national jurisdictions, asymmetrical capital tax treatment, exchange gains and losses, non-random central bank intervention in exchange markets and an ever-spreading web of exchange controls render the hypothesis of market efficiency of dubious operational value in the selection process of the least-cost financing option. How then, should foreign debt financing and refinancing decisions be made, since nominal interest rates can be misleading for decision-making purposes? Thus, a managerial framework is required, allowing the evaluation of the uncertain cost of foreign capital debt financing as a function of the 'volatility' (risk) of the currency denomination, the maturity of the debt instrument, the exposed exchange rate appreciation/depreciation and the level of risk aversion of the firm.

To do so, it will be useful to distinguish two sources of risk: internal and external. Internal risk depends on a firm’s operations and thus that depends on the exchange rate while external risk is independent of a firm’s operations (such as a devaluation or the usual variations in exchange rates). These risks are then expressed in terms of:

- Transaction risk, associated with the flow of funds in the firm
- Translation risk, associated with in-process, present and future transactions.
- Competition risk, associated with the firm’s competitive posture following a change in exchange rates.

The actors in a foreign exchange (risk) market are numerous and must be considered as well. These include the firms that import and export, and the intermediaries (such as banks), or traders. Traders behave just as market makers do. At any instant, they propose to buy and sell for a price. Brokers are intermediaries that centralize buy and sell orders and act on behalf of their clients, taking the best offers they can get. Over all, foreign exchange markets are competitive and can reach equilibrium. If this were not the case, then some traders could engage in arbitrage, as we shall discuss later on. This means that some traders will be able to make money without risk and without investing any money.
1.4.2 Currency risk

Currency risk is associated with variations in currency markets and exchange rates. A currency is not risky because its depreciation is likely. If it were to depreciate for sure and there were to be no uncertainty as to its magnitude and timing—there would not be any risk at all. As a result, a weak currency can be less risky than a strong currency. Thus, the risk associated with a currency is related to its randomness. The problems thus faced by financial analysts consist of defining a reasonable measure of exposure to currency risk and managing it. There may be several criteria in defining such an exposure. First, it ought to be denominated in terms of the relevant amount of currency being considered. Second, it should be a characteristic of any asset or liability, physical or financial, that a given investor might own or owe, defined from the investor’s viewpoint. And finally, it ought to be practical. Currency risks are usually associated with macroeconomic variables (such as the trade gap, political stability, fiscal and monetary policy, interest rate differentials, inflation, leadership, etc.) and are therefore topics of considerable political and economic analysis as well as speculation. Further, because of the size of currency markets, speculative positions may be taken by traders leading to substantial profits associated with very small movements in currency values. On a more mundane level, corporate finance managers operating in one country may hedge the value of their contracts and profits in another foreign denominated currency by assuming financial contracts that help to relieve some of the risks associated with currency (relative or absolute) movements and shifts.

1.4.3 Credit risk

Credit risk covers risks due to upgrading or downgrading a borrower’s creditworthiness. There are many definitions of credit risk, however, which depend on the potential sources of the risk, who the client may be and who uses it. Banks in particular are devoting a considerable amount of time and thoughts to defining and managing credit risk. There are basically two sources of uncertainty in credit risk: default by a party to a financial contract and a change in the present value (PV) of future cash flows (which results from changes in financial market conditions, changes in the economic environment, interest rates etc.). For example, this can take the form of money lent that is not returned. Credit risk considerations underlie capital adequacy requirements (CAR) regulations that are required by financial institutions. Similarly, credit terms defining financial borrowing and lending transactions are sensitive to credit risk. To protect themselves, firms and individuals turn to rating agencies such as Standard & Poors, Moody’s or others (such as Fitch Investor Service, Nippon Investor Service, Duff & Phelps, Thomson Bank Watch etc.) to obtain an assessment of the risks of bonds, stocks and financial papers they may acquire. Furthermore, even after a careful reading of these ratings, investors, banks and financial institutions proceed to reduce these risks by risk management tools. The number of such tools is of course very large. For