
Asymmetric Passive Components in Microwave Integrated Circuits

HEE-RAN AHN

 **WILEY-
INTERSCIENCE**

A JOHN WILEY & SONS, INC., PUBLICATION

Asymmetric Passive Components in Microwave Integrated Circuits

Asymmetric Passive Components in Microwave Integrated Circuits

HEE-RAN AHN

 **WILEY-
INTERSCIENCE**

A JOHN WILEY & SONS, INC., PUBLICATION

Copyright © 2006 by John Wiley & Sons, Inc. All rights reserved.

Published by John Wiley & Sons, Inc., Hoboken, New Jersey.
Published simultaneously in Canada.

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 750-4470, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, or online at <http://www.wiley.com/go/permission>.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services or for technical support, please contact our Customer Care Department within the United States at (800) 762-2974, outside the United States at (317) 572-3993 or fax (317) 572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic formats. For more information about Wiley products, visit our web site at www.wiley.com.

Library of Congress Cataloging-in-Publication Data:

Ahn, Hee-Ran, 1956–

Asymmetric passive components in microwave integrated circuits / by Hee-Ran Ahn.

p. cm.

“A Wiley-Interscience publication.”

Includes bibliographical references and index.

ISBN-13: 978-0-471-73748-3

ISBN-10: 0-471-73748-8

1. Microwave integrated circuits. I. Title.

TK7876.A38 2006

621.381'32–dc22

2005056772

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Contents

Preface	xi
1 Introduction	1
1.1 Asymmetric Passive Components	1
1.2 Circuit Parameters	2
1.3 Asymmetric Four-Port Hybrids	3
1.3.1 Asymmetric Ring Hybrids	3
1.3.2 Asymmetric Branch-Line Hybrids	4
1.4 Asymmetric Three-Port Power Dividers	5
1.5 Asymmetric Two-Port Components	6
References	6
2 Circuit Parameters	10
2.1 Scattering Matrix	10
2.1.1 Transmission-Line Theory	11
2.1.2 Basis-Dependent Scattering Parameters of a One-Port Network	12
2.1.3 Voltage- and Current-Basis Scattering Matrices of n -Port Networks	14
2.1.4 Complex Normalized Scattering Matrix	17
2.2 Scattering Parameters of Reduced Multiports	18
2.2.1 Examples of Reduced Multiports	21
2.3 Two-Port Network Analysis Using Scattering Parameters	23

2.4	Other Circuit Parameters	29
2.4.1	ABCD Parameters	29
2.4.2	Open-Circuit Impedance and Short-Circuit Admittance Parameters	36
2.4.3	Conversion Matrices of Two-Port Networks Terminated in Arbitrary Impedances	40
2.5	Analyses of Symmetric Networks	43
2.5.1	Analyses with Even- and Odd-Mode Excitations	43
2.5.2	Useful Symmetric Two-Port Networks	45
2.5.3	Properties of Symmetric Two-Port Networks	47
2.6	Analyses with Image Parameters	47
2.6.1	Image Impedances	47
2.6.2	Image Propagation Constants	49
2.6.3	Symmetrical and Common Structures	50
	Exercises	52
	References	54
3	Conventional Ring Hybrids	56
3.1	Introduction	56
3.2	Original Concept of the 3-dB Ring Hybrid	57
3.3	Conventional Ring Hybrids	62
3.3.1	Coupled Transmission Lines	62
3.3.2	Ring Hybrids with Coupled Transmission Lines	68
3.3.3	Wideband Ring Hybrids	71
3.3.4	Symmetric Ring Hybrids with Arbitrary Power Divisions	74
3.3.5	Conventional Lumped-Element Ring Hybrids	77
3.3.6	Mixed Small Ring Hybrids	80
3.4	Conventional 3-dB Uniplanar Ring Hybrids	84
3.4.1	Uniplanar T-Junctions	85
3.4.2	Transitions	86
3.4.3	Wideband Uniplanar Baluns	86
3.4.4	Uniplanar Ring Hybrids	88
	Exercises	90
	References	91

4	Asymmetric Ring Hybrids	93
4.1	Introduction	93
4.2	Derivation of Design Equations of Asymmetric Ring Hybrids	93
4.3	Small Asymmetric Ring Hybrids	99
4.4	Wideband or Small Asymmetric Ring Hybrids	100
4.4.1	Microstrip Asymmetric Ring Hybrids	100
4.4.2	Uniplanar Asymmetric Ring Hybrids	102
4.5	Miniaturized Ring Hybrids Terminated in Arbitrary Impedances	106
4.5.1	Asymmetric Lumped-Element Ring Hybrids	106
	Exercises	122
	References	122
5	Asymmetric Branch-Line Hybrids	125
5.1	Introduction	125
5.2	Origin of Branch-Line Hybrids	125
5.3	Multisection Branch-Line Couplers	127
5.4	Branch-Line Hybrids for Impedance Transforming	132
5.5	Asymmetric Four-Port Hybrids	139
5.5.1	Analyses of Asymmetric Four-Port Hybrids	139
5.5.2	Conventional–Direction Asymmetric Branch-Line Hybrids	140
5.5.3	Anti-Conventional-Direction Asymmetric Branch-Line Hybrids	147
	Exercises	150
	References	151
6	Conventional Three-Port Power Dividers	154
6.1	Introduction	154
6.2	Three-Port 3-dB Power Dividers	155
6.3	Three-Port Power Dividers with Arbitrary Power Divisions	156
6.4	Symmetric Analyses of Asymmetric Three-Port Power Dividers	160

6.5	Three-Port 3-dB Power Dividers Terminated in Complex Frequency-Dependent Impedances	163
6.6	Three-Port 45° Power Divider/Combiner	167
	Exercises	168
	References	168
7	Three-Port 3-dB Power Dividers Terminated in Different Impedances	170
7.1	Introduction	170
7.2	Perfect Isolation Condition	171
7.3	Analyses	173
7.4	Scattering Parameters of Three-Port Power Dividers	177
7.5	Lumped-Element Three-Port 3-dB Power Dividers	186
7.6	Coplanar Three-Port 3-dB Power Dividers	188
	Exercises	189
	References	190
8	General Design Equations for N-Way Arbitrary Power Dividers	192
8.1	Introduction	192
8.2	General Design Equations for Three-Port Power Dividers	193
8.2.1	Coplanar Three-Port Power Divider Terminated in 50 Ω , 60 Ω , and 70 Ω	196
8.2.2	Determining Z_{Ad}	197
8.3	General Design Equations for N -Way Power Dividers	199
8.3.1	Analyses of N -Way Power Dividers	200
	Exercises	204
	References	204
9	Asymmetric Ring-Hybrid Phase Shifters and Attenuators	206
9.1	Introduction	206
9.2	Scattering Parameters of Asymmetric Ring Hybrids	207
9.3	Asymmetric Ring-Hybrid Phase Shifters	209
9.3.1	Uniplanar Asymmetric Ring-Hybrid -135° Phase Shifter	216

9.4	Asymmetric Ring-Hybrid Attenuator with Phase Shifts	216
9.4.1	Microstrip Asymmetric Ring-Hybrid 4-dB Attenuator with 45° Phase Shift	220
	Exercises	222
	References	223
10	Ring Filters and Their Use in a New Measurement Technique for Inherent Ring-Resonance Frequency	225
10.1	Introduction	225
10.2	Ring Filters	226
10.2.1	Analyses of Ring Filters	226
10.2.2	Measurements	230
10.3	New Measurement Technique for Inherent Ring-Resonance Frequency	230
10.3.1	Lossless Case	230
10.3.2	Loss Case	234
10.4	Conclusions	237
	Exercises	238
	References	238
11	Small Impedance Transformers, CVTs and CCTs, and Their Applications to Small Power Dividers and Ring Filters	240
11.1	Small Transmission-Line Impedance Transformers	240
11.2	Mathematical Approach for CVTs and CCTs	241
11.2.1	CVTs and CCTs	242
11.2.2	Microstrip CVTs and CCTs	247
11.2.3	Bounded Length of CVTs and CCTs	248
11.2.4	Phase Responses of CVTs and CCTs	251
11.3	CVT3PDs and CCT3PDs	253
11.3.1	Isolation Circuits of CVT3PDs and CVT3PDs	254
11.3.2	Design of CVT3PDs and CCT3PDs	256
11.4	Asymmetric Three-Port 45° Power Divider Terminated in Arbitrary Impedances	258
11.4.1	Asymmetric 45° Power Divider Terminated in 30 Ω, 60 Ω, and 50 Ω	259

11.5	CVT and CCT Ring Filters	261
11.5.1	Analyses of Ring Filters	262
	Exercises	266
	References	267
Appendix A: Symbols and Abbreviations		269
Appendix B: Conversion Matrices		272
Appendix C: Derivation of the Elements of a Small Asymmetric Ring Hybrid		276
Appendix D: Trigonometric Relations		279
Appendix E: Hyperbolic Relations		281
Index		283

Preface

This book was written primarily as an advanced text in microwave engineering for graduate students. Since it concentrates on the principle of circuit element designs, it is of value to engineers in industry who want to design advanced microwave circuits. To understand the text, transmission line theory, circuit matrices, and microwave circuit theory are the necessary background.

There are three classes of components in microwave integrated circuits: one is passive and the others are active and nonreciprocal ferrite components. This book treats the passive components intensively, especially asymmetric components. An epoch-making development was the even- and odd-mode excitation analyses suggested in 1956 by J. Reed and G. J. Wheeler, and their analyses were based on the symmetrical structures. Therefore, only symmetric passive designs have been developed until now. However, the passive components with arbitrary termination impedances are needed to reduce circuit size since they allow the elimination of the matching networks needed to obtain the desired output performances when the symmetric components are integrated with other elements. This results in components that are no longer symmetrical, so conventional design methods may not be used. Therefore, new design methods are needed. In this book, asymmetric design methods are illustrated for components such as asymmetric ring hybrids, asymmetric branch-line hybrids, asymmetric three-port power dividers, asymmetric ring-hybrid phase shifters and attenuators, asymmetric ring filters, and asymmetric impedance transformers, which are the basic and indispensable elements for integration with other active and/or passive devices.

The book is made up of eleven chapters: Chapter 1 provides a brief introduction to asymmetric passive components in microwave integrated circuits and their short history, starting with the asymmetric ring hybrids first described in 1994 by the author. Microwave circuit parameters such as scattering, $ABCD$, impedance, admittance, and image parameters are explained in Chapter 2. Basis-independent or basis-dependent scattering parameters are discussed, and an easy method of analyzing any network with multiports is described in more detail. A particular

feature of this chapter is a conversion table between various circuits matrices characterizing two-port networks terminated in arbitrary impedances.

Chapters 3 and 4 cover ring hybrids. The design method of conventional ring hybrids, is given in Chapter 3, and asymmetric ring hybrids are treated in Chapter 4, where their design equations and a method to reduce their size are explained. Asymmetric branch-line hybrids are discussed in Chapter 5. Branch-line and ring hybrids are both four-port components, but they have been treated differently in conventional analyses. However, it is suggested in this book that they are not different and that the name *ring hybrid* or *branch-line hybrid* may be determined by the power division directions. Therefore, asymmetric branch-line hybrids may be designed using the same method as that used for asymmetric ring hybrids.

Chapters 6 to 8 treat three-port power dividers. Since they are very useful components in various applications, they are studied in more detail. Conventional three-port power dividers are described in Chapter 6. Asymmetric three-port power dividers are covered in Chapters 7 and 8: the case of equal power division in Chapter 7 and that of arbitrary power division in Chapter 8.

Chapters 9 to 11 cover asymmetric two-port components. They are asymmetric ring-hybrid phase shifters and attenuators, ring filters, and asymmetric impedance transformers. The asymmetric impedance transformers are not only small but can also produce arbitrary phase shifters of less than 90° . Due to these distinct properties, they can be used for various applications, such as small three-port power dividers, small ring filters, and asymmetric three-port 45° power dividers. Several applications are described in the latter part of Chapter 11; other applications are left for the readers.

Several people deserve acknowledgment for their help in completing the book. Professor Ingo Wolff at Duisburg–Essen University in Germany encouraged me to write the habilitation thesis that has served as a source for the book; Professor Tatsuo Itoh at UCLA in the United States, Professor Wolfgang Menzel at Ulm University in Germany, and Professor Kwyro Lee at KAIST (Korea Advanced Institute of Science and Technology) in Korea reviewed the book despite their busy schedules; and Professor Bumman Kim at POSTECH (Pohang University of Science and Technology) in Korea helped me concentrate on completing the book. I am especially appreciative of the interest and help of Professor Kai Chang at Texas A&M University. Without their interest and assistance, this task could not have been accomplished.

HEE-RAN AHN

Pohang University Science and Technology (POSTECH)

Introduction

1.1 ASYMMETRIC PASSIVE COMPONENTS

In microwave integrated circuits, there are three classes of components: one passive, the others active and nonreciprocal ferrite components. Power dividers, phase shifters, impedance transformers, and filters are typical passive components, and their conventional structures have been symmetric. However, asymmetric structures, with arbitrary termination impedances, are strongly preferable since they allow elimination of the matching networks needed to obtain the output performances desired when symmetric components are integrated with other elements.

The study of asymmetric passive components started with asymmetric ring hybrids in 1991 [1–5], and the design equations of asymmetric 3-dB ring hybrids were derived in 1994 by Ahn et al. [6]. Shortly after, those of asymmetric ring hybrids with arbitrary power divisions were synthesized assuming conditions of perfect isolation [7,8]. In 2000, a new concept [9], which stated that ring hybrids are not different from branch-line hybrids [10–13], allowed two types of asymmetric branch-line hybrids to be investigated and their design equations derived. In this book, asymmetric ring hybrids and branch-line hybrids are discussed in Chapters 3 to 5.

For asymmetric three-port power dividers, Ahn and Wolff [14,15] derived a perfect isolation condition that had never been analyzed for these types of conventional three-port power dividers. Using perfect isolation conditions 3-dB three-port power dividers terminating in different impedances could be constructed and their design equations derived. However, the design equations were available only for three-port power dividers with equal power division. In 2000, general design equations were derived for three-port power dividers with both arbitrary termination impedances and arbitrary power divisions [16,17]. In this

book, the design of asymmetric three-port power dividers with arbitrary power divisions is treated in Chapters 7 and 8.

Development of the monolithic microwave integrated circuit (MMIC) technique has created a strong incentive to reduce circuit size. Three-port power dividers are key components in microwave integrated circuits, and there have been many trials to reduce their size. One of them was to adapt lumped-element circuits to be equivalent to transmission-line sections [1–6,14]. However, the bandwidths of three-port power dividers with lumped elements were not sufficient, so wideband impedance transformers were required. For this, two types of small wideband impedance transformers were introduced: the constant-voltage standing-wave-ratio transmission-line impedance transformer (CVT) and the constant-conductance transmission-line impedance transformer (CCT) [17–19]. CVTs and CCTs have arbitrary phase shifts of less than 90° , which differs considerably from conventional impedance transformers, which have only odd multiples of 90° phase shifts. Thus, the distinct characteristics of CVTs and CCTs allow construction of small CVT and CCT 3-dB power dividers (CVT3PD and CCT3PDs), three-port 45° power dividers, and CVT and CCT ring filters. One CCT3PD has the smallest recorded, and three-port 45° power dividers are very important in getting rid of third harmonics of active devices. The key components of CVTs and CCTs are treated in Chapter 11 as asymmetric two-port components.

In addition to CVTs and CCTs, asymmetric ring-hybrid phase shifters and attenuators [20–22] and ring filters are asymmetric two-port components and are considered in Chapters 9 and 10. Since asymmetric phase shifters and attenuators consist of asymmetric ring hybrids, they may be used as impedance transformers together with their original functions. Conventional 180° phase shifters were used for building linearizers in high-power systems, but they had narrowband properties. For this, ring filters [23] were introduced as wideband 180° transmission lines. Each ring filter consists of a ring and two short stubs, and it is possible to measure an inherent ring resonance frequency by having the two short stubs differ in length. This measurement technique is treated in Chapter 10.

1.2 CIRCUIT PARAMETERS

The measured quantities of passive components at microwave frequencies are almost always the scattering parameters. The scattering matrix discussed in Chapter 2 is admirably suitable for the description of a large class of passive microwave components and is used as much as possible throughout the book. In many cases it leads to a complete understanding of a microwave device while avoiding the need to construct a formal electromagnetic boundary-value problem for the structure. The entries of the scattering matrix of an n -port junction are a set of quantities related to incident and reflected waves at the ports of the junction that describe the performance of a network under any termination conditions specified. The coefficients along the main diagonal of the scattering matrix are reflection coefficients, whereas those along the off-diagonal are transmission coefficients.

A scattering matrix exists for every linear, passive, and time-invariant network, and it is possible to deduce important general properties of junctions containing a number of ports by invoking such junction properties as reciprocity and power conservation. Since the entries of the scattering matrix \mathbf{S} , impedance matrix \mathbf{Y} , and admittance matrix \mathbf{Z} of a symmetric network are linear combinations of the circuit eigenvalues, their direct evaluation or measurement provides an alternative formulation of network parameters. Therefore, the relation between the scattering matrix and other circuit matrices is important and is described in Chapter 2.

1.3 ASYMMETRIC FOUR-PORT HYBRIDS

Many different types of power dividers, with and without isolation between output ports, are used for various applications. They perform a variety of functions, such as splitting and combining power in mixers (hybrids), sampling power from sources for level control, separating incident and reflected signals in network analyzers, and dividing power among a number of loads. Certain power dividers, which provide isolation between their output ports, are branch-line hybrids, ring hybrids, and parallel-coupled directional couplers, and their two outputs are in phase or out of phase by 90° or 180° . These power dividers are shown in Fig. 1.1(a), where the direction of power flow is indicated when power is fed into port ①. As shown, the direction of the ring hybrid is the same as that of the parallel-coupled directional coupler, but the two output signals of the ring hybrid are in phase or 180° out of phase, whereas those of the parallel-coupled directional coupler are 90° out of phase. The branch-line hybrid in Fig. 1.1(b) is same as that in Fig. 1.1(c), in that the two output signals are out of phase by 90° , but the power division directions are different from each other. Thus, the branch-line coupler (hybrid) is called a *forward coupler*, whereas the parallel-coupled directional coupler is called a *backward coupler*.

1.3.1 Asymmetric Ring Hybrids

The first conventional ring hybrid to be treated in Chapter 3 was investigated by Tyrrel in 1947 [24]. Tyrrel tried to explain ring hybrids using the concept of waveguide T-junctions, and described two types of hybrid circuits, one involving a ring or loop transmission line and the other relying on the symmetry properties of certain four-arm junctions. After he described the fundamental characteristics of distributed circuit hybrids, a number of workers discussed the performance of practical wideband realizations constructed in coaxial line [25–27] and stripline [28]. One of them was that two coupled-line filters were used for a wideband ring hybrid in the 1950s. In 1961, Pon [29] derived design equations for ring hybrids with arbitrary power divisions. In 1968, March [28] developed a wideband ring hybrid, adapting one coupled-line filter instead of a three-quarter-wavelength transmission line, which causes narrowband responses.

In the 1980s, as uniplanar techniques emerged for MMIC applications, there were several researchers developed small broadband ring hybrids which employed

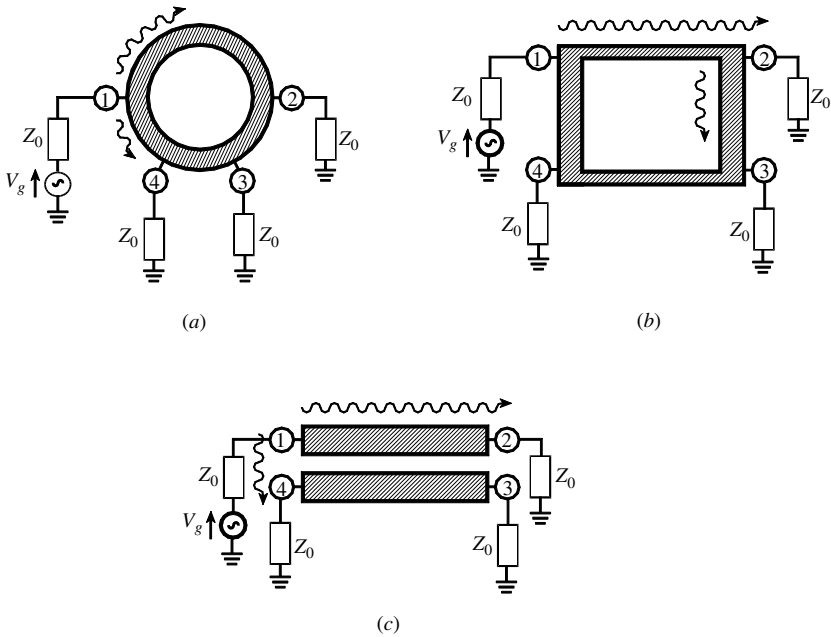


FIGURE 1.1 Four-port power dividers and their power flows: (a) ring hybrid; (b) branch-line hybrid; (c) parallel-coupled transmission-line directional coupler.

a combination of coplanar waveguides and slotlines using only one-sided substrates [30–32]. Since the first ring hybrid was introduced, ring hybrids have been studied and used for various applications in microwave equipment. Thus, they are indispensable components in various MICs (microwave integrated circuits) or MMICs (monolithic microwave integrated circuits), such as balanced mixers, balanced amplifiers, frequency discriminators, phase shifters, feeding networks in antenna arrays, and so on. The important conventional ring hybrids are introduced and discussed in Chapter 3.

In practice, ring hybrids are used together with other active and passive devices. Thus, to obtain a desired performance, additional matching networks are necessary for conventional ring hybrids. Therefore, ring hybrids terminating in arbitrary impedances can reduce the size of MICs significantly. Since no symmetry plane is available for asymmetric ring hybrids terminating in arbitrary impedance, the conventional method of even- and odd-mode excitation analyses cannot be used, so new design methods are required. In Chapter 4 we discuss how to derive design equations for asymmetric ring hybrids [8] and how to reduce the loss produced by inductors of asymmetric lumped-element ring hybrids [6].

1.3.2 Asymmetric Branch-Line Hybrids

Branch-line directional couplers, which originated with Mumford [33], consist of two adjacent transmission lines with one or more coupling elements between

them. One of these two lines is the main or primary line and the other one is the secondary line. A small fraction of the energy in the main line is transferred, through the coupling elements, to the secondary line. The mechanisms of the power transmission and isolation from the primary line to the secondary line are discussed in Chapter 5. If the coupling elements are branch lines and hybrid T-junctions are used for the directional couplers, they are called *branch-line hybrids*, and one-stage branch-line hybrids are narrowband. So to increase the bandwidth, multiple branch-line hybrids [34] are needed and the design of such a directional coupler and the calculation of its frequency response are also covered in Chapter 5. The branch-line directional coupler is particularly desirable since the design constants are readily found and its frequency response can be calculated. Branch-line hybrids may be used for impedance transformers with one or two sections [35,36]; their design methods are treated in Chapter 5.

Branch-line hybrids have been studied for a long time. However, these studies have focused on symmetric branch-line hybrids [33–40]. If branch-line hybrids are terminated in arbitrary impedances, they are no longer symmetric and a new design method is needed. In the latter part of Chapter 5, the isolation mechanism of asymmetric four-port hybrids and design equations for asymmetric branch-line hybrids are treated.

1.4 ASYMMETRIC THREE-PORT POWER DIVIDERS

The history of three-port power dividers began in 1960 with Wilkinson [41], who described a device that separated one signal into n equiphase–equiamplitude signals. Theoretically perfect isolation between all output ports is achieved at one frequency. With $n = 2$, his circuit may be reduced to a three-port power divider. In 1965, Parad and Moynihan [42] presented a hybrid with the output signals in phase and an arbitrary amplitude difference. The perfect three-port hybrid property was again achieved at one frequency. In 1968, Cohn [43] presented a class of equal-power dividers with isolation and impedance matching at any number of frequencies. In 1971, Ekinge [44] described a three-port hybrid consisting of n sections in cascade, each section composed of two coupled lossless transmission lines with a certain electrical length and an intermediate resistor. His design seems to be similar to that of Cohn [43]. However, Cohn treated equal-power-split three-port hybrids, whereas Ekinge dealt with three-port hybrids with arbitrary power split.

For the design of three-port power dividers, it is very important to determine the isolation conditions or the values of isolation resistors. A number of papers have dealt with how to get isolation between output signals. Parad and Moynihan [42] suggested one of many isolation conditions, and Cohn, Ekinge, et al. [43–45] had to use intensive optimizing methods to derive the isolation resistances. In this book, a perfect isolation condition without optimization is discussed in Chapter 7.

In addition to the above, an optimization method for 3-dB three-port power dividers terminated in complex frequency-dependent impedances was

suggested [45] and is discussed in Chapter 6. The output signals of conventional three-port power dividers are mostly in phase. However, if they are out of phase by 45° power dividers are particularly important to reduce unwanted intermodulation harmonic frequencies [46]. Such three-port power dividers can be built in two ways: The first is simply to use a delay line, and the second uses the small impedance transformers, CVTs and CCTs. These two ways are introduced in Chapters 6 and 11, respectively.

Since Wilkinson, studies on three-port hybrids have continued [42–48] to focus on symmetrical structures for which conventional even- and odd-mode excitation methods [49] can be used. If three-port power dividers are terminated in arbitrary impedances, they are no longer symmetric, and new ideas to derive design equations are needed. In earlier work [44], Ekinge mentioned that all the termination impedances had to be equal to each other, or at least the two output termination impedances be the same for the equal power division. However, Ahn and Wolff [14] showed that all the termination impedances could differ from each other despite equal power division. This is described in Chapter 7. General design equations that are available to any type of N -way power dividers with arbitrary power divisions and arbitrary termination impedances are discussed in Chapter 8.

1.5 ASYMMETRIC TWO-PORT COMPONENTS

Phase shifters, filters, and impedance transformers are two-port components. The phase shifter, as a general-purpose device in microwave components, finds use in a variety of communication and radar systems, microwave instrumentation, measurement systems, and industrial applications. Several different forms of phase shifters have been suggested, but most applications use a 3-dB hybrid power divider with symmetric reflection terminations. However, conventional ring-hybrid phase shifters require an additional transmission-line section to utilize symmetrical reflection terminations [50]. To reduce the size, a new design method for ring-hybrid phase shifters was presented without this transmission-line section [20]. In Chapter 9, the method is introduced and asymmetric ring-hybrid phase shifters and attenuators are presented and discussed.

Conventional impedance transformers have only odd multiples of 90° phase shift [51]. However, impedance transformers with arbitrary phase shifts are needed to reduce the size of microwave integrated circuits. The ring filters discussed in Chapter 10 have 180° phase shifts, and the CVTs and CCTs in Chapter 11 have arbitrary phase shifts of less than 90° . Asymmetric phase shifters, attenuators, impedance transformers, and ring filters are treated as asymmetric two-port components in Chapters 10 and 11.

REFERENCES

1. H.-R. Ahn, I.-S. Chang, and S.-W. Yun, A Study on the Miniaturization of 3-dB Ring Hybrid and Power Divider Using Lumped-Element Circuit, *J. KITE*, Vol. 28-A, No. 1, January 1991, pp. 15–22.

2. H.-R. Ahn, I.-S. Chang, and S.-W. Yun, A Study on the Miniaturization of 3-dB Ring Hybrid Having Arbitrary Termination Impedances Using Lumped-Element Circuit, *MTT Korean Chapter KITE-S Dig.*, Vol. 14, No. 2, October 1991, pp. 104–109.
3. H.-R. Ahn, I.-S. Chang, and S.-W. Yun, A Study on the Miniaturization of 3-dB Ring Hybrid Having Arbitrary Termination Impedances Using Lumped Equivalent Circuit, *J. KITE*, Vol. 29-A, No. 3, March 1992, pp. 25–32.
4. H.-R. Ahn, I.-S. Chang, and S.-W. Yun, A Study on the Miniaturization of 3-dB Ring Hybrid Having Arbitrary Termination Impedances Using Lumped Equivalent Circuit, *J. Telecommun. Rev.*, No. 5, May 1992, pp. 112–125.
5. H.-R. Ahn, I.-S. Chang, and S.-W. Yun, Lumped Element 3-dB 180° Hybrid with Asymmetrically Terminated Impedances, *J. KITE*, Vol. 31-A, No. 6, June 1994, pp. 18–25.
6. H.-R. Ahn, I.-S. Chang, and S.-W. Yun, Miniaturized 3-dB Ring Hybrid Terminated by Arbitrary Impedances, *IEEE Trans. Microwave Theory Tech.*, Vol. 42, December 1994, pp. 2216–2221.
7. H.-R. Ahn, I. Wolff, and I.-S. Chang, Arbitrary Termination Impedances, Arbitrary Power Division and Small-Sized Ring Hybrids, *IEEE MTT-S Dig.*, June 1997, pp. 285–288.
8. H.-R. Ahn, I. Wolff, and I.-S. Chang, Arbitrary Termination Impedances, Arbitrary Power Division and Small-Sized Ring Hybrids, *IEEE Trans. Microwave Theory Tech.*, Vol. 44, December 1997, pp. 2241–2247.
9. H.-R. Ahn and I. Wolff, Asymmetric Four-Port and Branch-Line Hybrids, *IEEE Trans. Microwave Theory Tech.*, Vol. 48, September 2000, pp. 1585–1588.
10. H.-R. Ahn and I. Wolff, 3-dB Branch-Line Hybrids with Arbitrary Termination Impedance Values, *IEICE Trans. Electron.*, Vol. E82-C, No. 7, July 1999, pp. 1324–1326.
11. H.-R. Ahn and I. Wolff, Arbitrary Power Division Branch-Line Hybrid Terminated by Arbitrary Impedances, *IEE Electron. Lett.*, Vol. 35, No. 7, April 1999, pp. 572–273.
12. H.-R. Ahn and I. Wolff, 3-dB Branch-Line Hybrid Terminated by Arbitrary Impedances, *IEE Electron. Lett.*, Vol. 34, No. 11, May 1998, pp. 1109–1110.
13. H.-R. Ahn and I. Wolff, Asymmetric Four-Port Hybrids, Asymmetric 3-dB Branch-Line Hybrids, *Asia-Pacific Microwave Conf. Proc.*, Yokohama, Japan, December 1998, pp. 677–680.
14. H.-R. Ahn and I. Wolff, Three-Port 3-dB Power Divider Terminated by Different Impedances and Its Application to MMIC's, *IEEE Trans. Microwave Theory Tech.*, Vol. 47, June 1999, pp. 786–794.
15. H.-R. Ahn and I. Wolff, Three-Port 3-dB Power Divider Terminated by Arbitrary Impedances, *IEEE MTT-S Dig.*, Baltimore, June 1998, pp. 781–784.
16. H.-R. Ahn and I. Wolff, General Design Equations of Three-Port Power Dividers, *IEEE MTT-S Dig.*, Boston, June 2000, pp. 1137–1140.
17. H.-R. Ahn and I. Wolff, General Design Equations of Three-Port Power Dividers, Small-Sized Impedance Transformers, and Their Applications to Small-Sized Three-Port 3-dB Power Divider, *IEEE Trans. Microwave Theory Tech.*, Vol. 49, July 2001, pp. 1277–1288.
18. H.-R. Ahn and I. Wolff, Small-Sized Impedance Transformers, *IEEE MTT/AP/EMC Korea Chapter KEES Dig.*, Vol. 23, No. 2, September 2000, pp. 157–160.

19. H.-R. Ahn and I. Wolff, Miniaturized Impedance Transformers, *MIOP Dig.*, May 2001, pp. 274–278.
20. H.-R. Ahn and I. Wolff, Asymmetric Ring Hybrid Phase-Shifters and Attenuators, *IEEE Trans. Microwave Theory Tech.*, Vol. 50, April 2002, pp. 1146–1155.
21. H.-R. Ahn and I. Wolff, Asymmetric Ring-Hybrid Phase Shifters, *IEEE MTT/AP/EMC Korea Chapter KEES Dig.*, Vol. 23, No. 2, September 2000, pp. 165–168.
22. H.-R. Ahn and I. Wolff, Small-Sized Ring-Hybrid Phase Shifters, *MIOP Dig.*, May 2001, pp. 274–278.
23. H.-R. Ahn and I. Wolff, Novel Ring Filter as a Wide-Band 180° Transmission-Line, *EUMC Proc.*, Vol. III, October 1999, pp. 95–98.
24. W. A. Tyrrel, Hybrid Circuits for Microwaves, *Proc. IRE*, Vol. 35, November 1947, pp. 1294–1306.
25. T. Morita and L. S. Sheingold, A Coaxial Magic-T, *IRE Trans. Microwave Theory Tech.*, Vol. 1, November 1953, pp. 17–23.
26. V. I. Albanese and W. P. Peyser, An Analysis of a Broad-Band Coaxial Hybrid Ring, *IRE Trans. Microwave Theory Tech.*, Vol. 6, October 1958, pp. 369–373.
27. W. V. Tyminski and A. E. Hylas, A Wide-Band Hybrid Ring for UHF, *Proc. IRE*, Vol. 41, January 1953, pp. 81–87.
28. S. March, Wideband Stripline Hybrid Ring, *IEEE Trans. Microwave Theory Tech.*, Vol. 16, June 1968, pp. 361–362.
29. C. Y. Pon, Hybrid-Ring Directional Coupler for Arbitrary Power Divisions, *IRE Trans. Microwave Theory Tech.*, Vol. 9, November 1961, pp. 529–535.
30. T. Hirota, Y. Tarusawa, and H. Ogawa, Uniplanar MMIC Hybrids: A Proposed New MMIC Structure, *IEEE Trans. Microwave Theory Tech.*, Vol. 35, June 1987, pp. 576–581.
31. C.-H. Ho, L. Fan, and K. Chang, Broad-Band Uniplanar Hybrid-Ring and Branch-Line Couplers, *IEEE Trans. Microwave Theory Tech.*, Vol. 41, December 1993, pp. 2116–2124.
32. C.-H. Ho, L. Fan, and K. Chang, New Uniplanar Coplanar Waveguide Hybrid-Ring Couplers and Magic-T's, *IEEE Trans. Microwave Theory Tech.*, Vol. 42, December 1994, pp. 2440–2448.
33. W. W. Mumford, Directional Couplers, *Proc. IRE*, Vol. 35, February 1947, pp. 159–165.
34. J. Reed, The Multiple Branch Waveguide Coupler, *IRE Trans. Microwave Theory Tech.*, Vol. 6, October 1958, pp. 398–403.
35. R. K. Gupta, S. E. Anderson, and W. Getsinger, Impedance-Transforming 3-dB Hybrids, *IEEE Trans. Microwave Theory Tech.*, Vol. 35, December 1987, pp. 1303–1307.
36. S. Kumar, C. Tannous, and T. Danshin, A Multisection Broadband Impedance Transforming Branch-Line Hybrid, *IEEE Trans. Microwave Theory Tech.*, Vol. 43, November 1995, pp. 2517–2523.
37. R. Levy, Analysis of Practical Branch-Guide Directional Couplers, *IEEE Trans. Microwave Theory Tech.*, Vol. 17, May 1969, pp. 289–290.
38. H. J. Riblet, Mathematical Theory of Directional Couplers, *Proc. IRE*, Vol. 35, December 1947, pp. 1307–1313.

39. R. Levy and L. F. Lind, Synthesis of Symmetrical Branch-Guide Directional Couplers, *IEEE Trans. Microwave Theory Tech.*, Vol. 4, February 1968, pp. 80–89.
40. H. J. Riblet, The Application of a New Class of Equal-Ripple Functions to Some Familiar Transmission-Line Problems, *IEEE Trans. Microwave Theory Tech.*, Vol. 12, July 1964, pp. 415–421.
41. E. J. Wilkinson, An n -Way Hybrid Power Divider, *IRE Trans. Microwave Theory Tech.*, Vol. 8, January 1960, pp. 116–118.
42. L. I. Parad and R. L. Moynihan, Split-Tee Power Divider, *IRE Trans. Microwave Theory Tech.*, Vol. 8, January 1965, pp. 91–95.
43. S. B. Cohn, A Class of Broadband Three-Port TEM-Mode Hybrids, *IRE Trans. Microwave Theory Tech.*, Vol. 16, February 1968, pp. 110–116.
44. R. B. Ekinge, A New Method of Synthesizing Matched Broad-Band TEM-Mode Three-Ports, *IEEE Trans. Microwave Theory Tech.*, Vol. 19, January 1971, pp. 81–88.
45. S. Rosloniec, Three-Port Hybrid Power Dividers Terminated in Complex Frequency-Dependent Impedances, *IEEE Trans. Microwave Theory Tech.*, Vol. 44, August 1996, pp. 1490–1493.
46. H. Hayashi, H. Okazaki, A. Kanda, T. Hirota, and M. Muraguch, Millimeter-Wave-Band Amplifier and Mixer MMIC's Using a Broad-Band 45° Power Divider/Combiner, *IEEE Trans. Microwave Theory Tech.*, Vol. 46, June 1998, pp. 811–818.
47. B. Kopp, Asymmetric Lumped Element Power Splitters, in *IEEE MTT-S Dig.*, 1989, pp. 333–336.
48. D. Köther, B. Hopf, Th. Sporkmann, and I. Wolff, MMIC Wilkinson Couplers for Frequencies Up to 110 GHz, in *IEEE MTT-S Dig.*, 1995, pp. 663–665.
49. J. Reed and G. J. Wheeler, A Method of Analysis of Symmetrical Four-Port Networks, *IRE Trans. Microwave Theory Tech.*, Vol. 4, October 1956, pp. 346–352.
50. J. F. White, Diode Phase Shifters for Array Antennas, *IEEE Trans. Microwave Theory Tech.*, Vol. 22, June 1974, pp. 658–674.
51. L. Young, The Quarter-Wave Transformer Prototype Circuit, *IRE Trans. Microwave Theory Tech.*, Vol. 8, September 1960, pp. 483–489.

Circuit Parameters

2.1 SCATTERING MATRIX

A large class of passive microwave components may be characterized by any of various sets of parameters, such as short-circuit admittance parameters, open-circuit impedance parameters, hybrid parameters, transmission parameters, scattering parameters, and so on. However, not all of these parameters will always exist because most parameters are partially defined with respect to zero or infinite loading at the ports. The scattering parameters, on the other hand, are defined in terms of some finite stable loadings at the ports. Because of this, they always exist for all linear passive networks.

The scattering parameters originated from the theory of transmission lines and form a matrix of transformation between variables which are linear combinations of the voltages and currents in a network. Since they are particularly suitable for problems of power transfer in networks designed to be terminated by complex loads, the scattering formalism finds convenient application in problems that involve insertion loss (e.g., filters, attenuators, power dividers, hybrids) and matching networks. Because the scattering parameters are closely associated with the power transfer properties of a network, they are indispensable in the design of microwave networks and permit the formulation of concise and useful expressions for energy constraints in passive structures. They are, therefore, particularly suitable for realization in the frequency domain. In addition, the scattering parameters possess another important property useful in general passive synthesis studies.

For the study of the scattering parameters, in this chapter we first treat how to get reflection coefficients from transmission-line theory and then apply the concepts to those of an n -port network. Since the scattering parameters are normalized to n complex loads on the complex plane, it will be shown that this

normalization can be extended to entire complex loads and to general n -port loads on the imaginary axis.

2.1.1 Transmission-Line Theory

The most important property of a transmission line is that electromagnetic fields can be uniquely related to voltages and currents. For this reason, circuit theory concepts are used for analyses of the structures, and the lumped-element equivalent circuit of a small section of transmission line with length dz is derived as shown in Fig. 2.1. It consists of series inductance per unit length L , series resistance per unit length R , shunt conductance per unit length G , and shunt capacitance per unit length C . Wave equations for $V(z)$ and $I(z)$ in Fig. 2.1 are obtained as

$$\frac{d^2V(z)}{dz^2} - \gamma^2V(z) = 0, \quad (2.1a)$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2I(z) = 0, \quad (2.1b)$$

where $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$ is the complex propagation constant, and α and β are phase and attenuation constants, respectively.

Assuming that $\alpha = 0$, the solutions for $V(z)$ and $I(z)$ in (2.1) are

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} = V_i(z) + V_r(z), \quad (2.2a)$$

$$I(z) = I_0^+ e^{-j\beta z} - I_0^- e^{j\beta z} = I_i(z) - I_r(z), \quad (2.2b)$$

where V_0^+ , V_0^- , I_0^+ , and I_0^- are voltage and current amplitudes traveling in positive and negative z -directions at $z = 0$. The total voltage $V(z)$ or current $I(z)$ along the line may be considered as the sum of an incident wave traveling in the positive z -direction and a reflected wave traveling in the negative z -direction, and the negative sign associated with the reflected current $I_r(z)$ indicates that the positive direction for $I_r(z)$ is opposite that for $I_i(z)$.

Figure 2.2 shows a uniform lossless transmission line that is connected between a load impedance Z_L and a voltage source V_g with an internal impedance

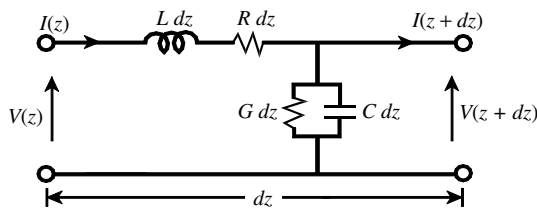


FIGURE 2.1 Lumped-element equivalent circuit of a transmission line of length dz .

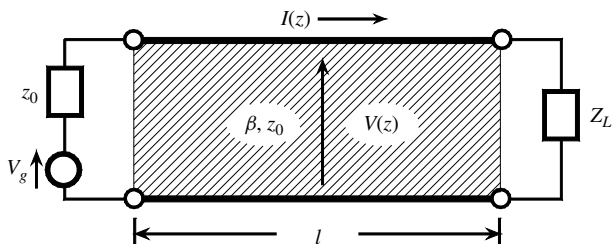


FIGURE 2.2 Uniform lossless transmission line.

z_0 . When $Z_L \neq z_0$, reflections occur at $z = 0$, and the relations between voltages and currents and for z_0 and Z_L are given as

$$V(0) = V_i(0) + V_r(0), \quad (2.3a)$$

$$I(0) = I_i(0) - I_r(0), \quad (2.3b)$$

$$z_0 = \frac{V_r(0)}{I_r(0)} = \frac{V_i(0)}{I_i(0)}, \quad (2.3c)$$

$$Z_L = \frac{V(0)}{I(0)}. \quad (2.3d)$$

The ratio of reflected voltage to incident voltage at $z = 0$ is defined as the *voltage-basis reflection coefficient* $\Gamma^V = V_r(0)/V_i(0)$, and the ratio of reflected current to incident current at $z = 0$ is defined as the *current-basis reflection coefficient* $\Gamma^I = I_r(0)/I_i(0)$. By (2.2) and (2.3), these coefficients can easily be found and they are given by

$$\Gamma^I = (Z_L + z_0)^{-1}(Z_L - z_0), \quad (2.4a)$$

$$\Gamma^V = -(Y_L + y_0)^{-1}(Y_L - y_0), \quad (2.4b)$$

where $y_0 = 1/z_0$ and $Y_L = 1/Z_L$. For real z_0 , $\Gamma^I = \Gamma^V$.

The most interesting conclusion from the relations in (2.4) is that reflections occur when $Z_L \neq z_0$ and that there is no reflected voltage or current wave when $Z_L = z_0$. Thus, when $Z_L \neq z_0$, Γ^I and Γ^V are finite, and standing waves of voltage and current exist along the transmission line. When $Z_L = z_0$, all the energy of the incident wave is transferred to the load, which cannot be distinguished from a transmission line with infinite length and characteristic impedance z_0 . This concept is very important for the design of matching circuits.

2.1.2 Basis-Dependent Scattering Parameters of a One-Port Network

Since an equivalent impedance is found at $z = -l$ in Fig. 2.2, a one-port equivalent network is obtained from the uniform transmission line in Fig. 2.2. Thus,

the concepts related to the reflection coefficients of the transmission line can be applied to those of the one-port network N . The one-port network N of Fig. 2.3(a) is characterized by its driving point impedance $Z(p)$, where p denotes $j\omega$. It is driven by a voltage source $V_g(p)$ in series with its *reference impedance* $z(p)$. The amount of power transferred from the source into the one-port network N depends on the impedance $Z(p)$, and the maximum power absorbed by the one-port network N is obtained with $Z(j\omega) = z_*(j\omega)$, where $z_*(j\omega)$ is the complex conjugate of $z(j\omega)$. When $Z(j\omega) = z_*(j\omega)$, the one-port network N is said to be *conjugately matched* to the load.

Like the case of transmission-line theory, the actual terminal voltage $V(p)$ in Fig. 2.3(a) is the sum of an *incident voltage* $V_i(p)$ and a *reflected voltage* $V_r(p)$, and the actual terminal current $I(p)$ is that of an *incident current* $I_i(p)$ and a *reflected current* $I_r(p)$. The incident current and voltage are those that appear under the optimal power-matching conditions. Thus, they are completely dependent on the loads, and Fig. 2.3(b) shows that the one-port network N is conjugately matched. From the Fig. 2.3(b), the relations between voltages and currents are obtained as

$$V_i(p) = [z(p) + z_*(p)]^{-1} V_g(p) z_*(p), \quad (2.5a)$$

$$I_i(p) = [z(p) + z_*(p)]^{-1} V_g(p). \quad (2.5b)$$

The matched load $z_*(p)$ is found from (2.5) as

$$V_i(p) = z_*(p) I_i(p). \quad (2.6)$$

As in (2.2), the reflected voltage $V_r(p)$ and current $I_r(p)$ are given as

$$V_r(p) = V(p) - V_i(p), \quad (2.7a)$$

$$-I_r(p) = I(p) - I_i(p). \quad (2.7b)$$

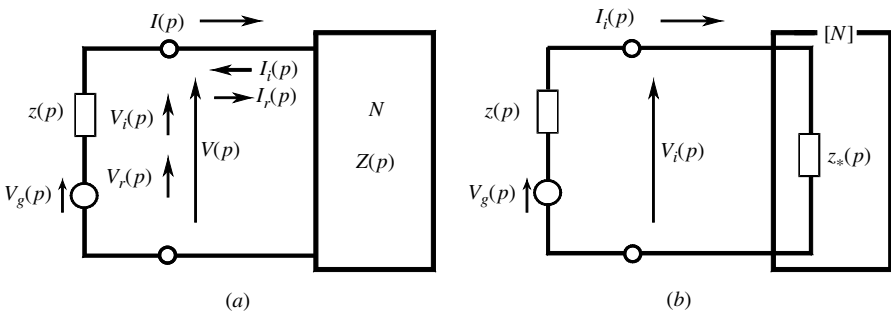


FIGURE 2.3 One-port networks: (a) characterized by its impedance $Z(p)$; (b) with the optimal power-matching condition.

The *voltage- and current-basis reflection coefficients* $\Gamma^V(p)$ and $\Gamma^I(p)$ are defined according to the relations

$$V_r(p) = \Gamma^V(p)V_i(p), \quad (2.8a)$$

$$I_r(p) = \Gamma^I(p)I_i(p). \quad (2.8b)$$

The reflection coefficients $\Gamma^V(p)$ and $\Gamma^I(p)$ are also referred to as *voltage- and current-basis scattering parameters*, respectively, of the one-port network N . In a similar way, $\Gamma^I(p)$ and $\Gamma^V(p)$ are

$$\Gamma^I(p) = [Z(p) + z(p)]^{-1}[Z(p) - z_*(p)], \quad (2.9)$$

$$\Gamma^V(p) = -[Y(p) + y(p)]^{-1}[Y(p) - y_*(p)]. \quad (2.10)$$

Comparing (2.9) with (2.10) yields

$$\Gamma^V(p)z_*(p) = z(p)\Gamma^I(p). \quad (2.11)$$

$\Gamma^I(p)$ is the same as $\Gamma^V(p)$ only when the reference impedance $z(p)$ is real, but they are in general different. On the real-frequency axis, they differ only by the phase, which is equal to twice the angle of the reference impedance $z(p)$. Both reflection coefficients are zero under optimal power-matching conditions, $Z(p) = z_*(p)$, and are related to the reference impedance $z(p)$ itself. It means that the incident waves see the impedance $z_*(p)$ and that the reflected waves see the reference impedance $z(p)$ itself.

2.1.3 Voltage- and Current-Basis Scattering Matrices of n -Port Networks

The reflection coefficients or scattering parameters of a one-port network have been discussed in detail. These concepts can be extended easily to an n -port network, and the scattering matrix of an n -port network is merely the matrix version of reflection coefficients of the one-port network. Figure 2.4 shows an n -port network N and its open-circuit impedance matrix $Z(p)$. Each of the n ports of N is loaded by a passive impedance $z_k(p)$ in series with a voltage source $V_{gk}(p)$. Since all elements of $z_k(p)$ are strictly passive, $z_k(p) + z_{k*}(p)$ cannot be identically zero.

The voltages, currents, and sources in Fig. 2.4 are represented by the vectors and related by the equation

$$\mathbf{V}_g(\mathbf{p}) = \mathbf{V}(\mathbf{p}) + \mathbf{z}(\mathbf{p})\mathbf{I}(\mathbf{p}) = [\mathbf{Z}(\mathbf{p}) + \mathbf{z}(\mathbf{p})]\mathbf{I}(\mathbf{p}), \quad (2.12)$$

where

$$\mathbf{V}(\mathbf{p}) = \begin{bmatrix} V_1(p) \\ V_2(p) \\ \vdots \\ V_n(p) \end{bmatrix}, \quad \mathbf{I}(\mathbf{p}) = \begin{bmatrix} I_1(p) \\ I_2(p) \\ \vdots \\ I_n(p) \end{bmatrix}, \quad \mathbf{V}_g(\mathbf{p}) = \begin{bmatrix} V_{g1}(p) \\ V_{g2}(p) \\ \vdots \\ V_{gn}(p) \end{bmatrix}, \quad (2.13)$$

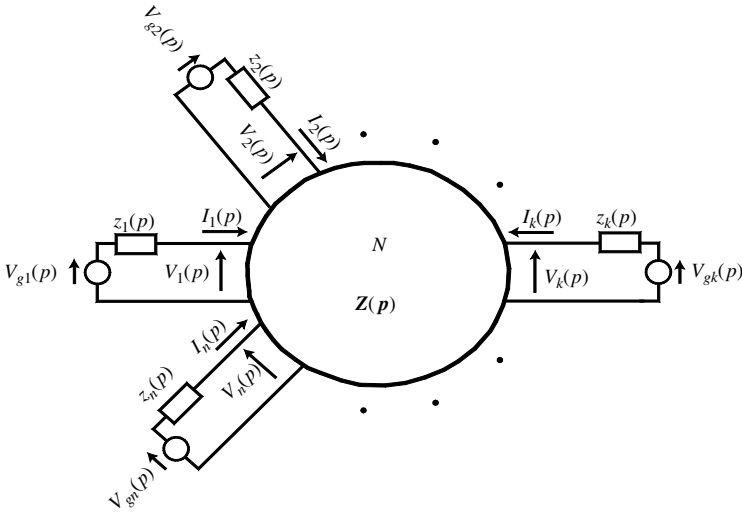


FIGURE 2.4 An n -port network N for current quantities.

and the *reference impedance matrix* $\mathbf{z}(\mathbf{p})$ of N is given as

$$\mathbf{z}(\mathbf{p}) = \begin{bmatrix} z_1(\mathbf{p}) & 0 & \dots & 0 \\ 0 & z_2(\mathbf{p}) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z_n(\mathbf{p}) \end{bmatrix}, \quad (2.14)$$

whose kk th element is the reference impedance $z_k(\mathbf{p})$ of the k th port.

The concepts used for a one-port network can be applied to the scattering matrix of n -port network N . The *incident voltage vector* and *incident current vector* represent voltages $V_{i1}(\mathbf{p}), V_{i2}(\mathbf{p}), \dots, V_{in}(\mathbf{p})$ and currents $I_{i1}(\mathbf{p}), I_{i2}(\mathbf{p}), \dots, I_{in}(\mathbf{p})$, respectively, and they would appear at the terminals of the n -ports under the optimal power-matching conditions. Figure 2.5 shows the n -port network N terminated in optimal loads. Like the one-port case, the incident-voltage and incident-current vectors are represented by the equations

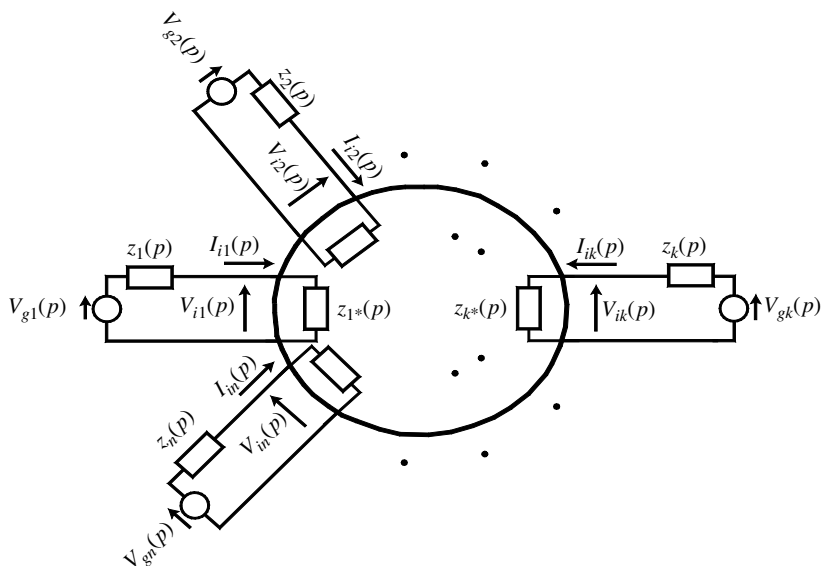
$$\mathbf{V}_i(\mathbf{p}) = \mathbf{z}_*(\mathbf{p})\mathbf{I}_i(\mathbf{p}), \quad (2.15a)$$

$$\mathbf{V}_g(\mathbf{p}) = [\mathbf{z}(\mathbf{p}) + \mathbf{z}_*(\mathbf{p})]\mathbf{I}_i(\mathbf{p}), \quad (2.15b)$$

where $\mathbf{z}(\mathbf{p}) + \mathbf{z}_*(\mathbf{p})$ is not identically singular with the passive loads. The *reflected voltage vector* $\mathbf{V}_r(\mathbf{p})$ and the *reflected current vector* $\mathbf{I}_r(\mathbf{p})$, defined by the difference between the actual quantities and incident quantities, are given as

$$\mathbf{V}_r(\mathbf{p}) = \mathbf{V}(\mathbf{p}) - \mathbf{V}_i(\mathbf{p}), \quad (2.16a)$$

$$-\mathbf{I}_r(\mathbf{p}) = \mathbf{I}(\mathbf{p}) - \mathbf{I}_i(\mathbf{p}). \quad (2.16b)$$


 FIGURE 2.5 Matched n -port network for current quantities.

The matrix S^V relating the ratio of the reflected-voltage vector $V_r(\mathbf{p})$ to the incident-voltage vector $V_i(\mathbf{p})$,

$$V_r(\mathbf{p}) = S^V(\mathbf{p})V_i(\mathbf{p}), \quad (2.17)$$

is called the *voltage-basis scattering matrix*. Similarly, the matrix S^I relating the reflected-current vector $I_r(\mathbf{p})$ to the incident-current vector $I_i(\mathbf{p})$,

$$I_r(\mathbf{p}) = S^I(\mathbf{p})I_i(\mathbf{p}), \quad (2.18)$$

is called the *current-basis scattering matrix*. The elements of $S^V(\mathbf{p})$ and $S^I(\mathbf{p})$ are referred to as the current- and voltage-basis scattering parameters of the n -port network. Using (2.15), (2.16), and (2.18), the $S^I(\mathbf{p})$ is, in terms of $Z(\mathbf{p})$ and $z(\mathbf{p})$, derived as

$$\begin{aligned} S^I(\mathbf{p}) &= U_n - [Z(\mathbf{p}) + z(\mathbf{p})]^{-1}[z(\mathbf{p}) + z_*(\mathbf{p})] \\ &= [Z(\mathbf{p}) + z(\mathbf{p})]^{-1}[Z(\mathbf{p}) - z_*(\mathbf{p})], \end{aligned} \quad (2.19)$$

based on

$$I_r(\mathbf{p}) = -I(\mathbf{p}) + I_i(\mathbf{p}) = -[Z(\mathbf{p}) + z(\mathbf{p})]^{-1}V_g(\mathbf{p}) + I_i(\mathbf{p}), \quad (2.20)$$

where U_n denotes an identity matrix of order n .