Financial Modeling of the Equity Market

From CAPM to Cointegration

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From CAPM to Cointegration

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John Wiley & Sons, Inc.
FJF
To my wife Donna and my children,
Francesco, Patricia, and Karly

SMF
To the memory of Bertrand Russell to whom I owe
the foundation of my intellectual development

PNK
To my best friend, my wife, and my love—Carmen
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his book is about financial modeling for equity asset management. We take a broad view of financial modeling, encompassing pure modeling as well as model engineering and financial optimization. Our perspective is that of an asset management firm. When reasoning and making decisions about modeling, a firm needs to grasp all the aspects related to modeling. This includes not only the mathematical models per se but also methods for model estimation, the optimization process that translates model forecasts into active strategies, and methods that help mitigate eventual inadequacies of the models being used.

Our perspective is similar to that of physical engineering, where the knowledge of a few abstract laws of physics is a far cry from building an automobile or an airplane. We broadly define financial modeling as theoretical financial and mathematical principles as well as statistical methods that allow for representing and forecasting financial data, procedures for estimating and testing these representations, and methods for engineering and optimizing financial strategies. Without a methodology for engineering, estimating, and testing financial strategies, a financial model is of little use.

In this book we offer an up-to-date treatment of financial modeling for asset management, presenting and discussing a number of developments at the forefront of equity modeling technology: robust estimation, robust optimization, the analysis of transaction costs, linear and non-linear dynamic models, and model risk mitigation techniques.

Since the downturn in the U.S. equity market in 2002, there has been an increased use of financial modeling and optimization in equity portfolio management. Under pressure to boost returns and reduce costs, asset management firms have begun to look with increasing attention at quantitative techniques. Not only has the diffusion of quantitative methods in equity portfolio management broadened since the turn of the century, but the variety of models and depth of use have also increased.

Three trends are worth pointing out. First, there is a greater use of predictive models. Predictive models assume that it is possible to make conditional forecasts of expected returns, an objective that was previously considered not achievable by classical financial theory. Second, in
order to exploit forecasts, optimization techniques are now being used. Previously, optimization technologies were considered too brittle for safe deployment in asset management. Third, as a consequence of a greater use of predictive models and optimization, there is a growing interest in “robust” methods—particularly methods for robust estimation and robust optimization—as well as a heightened attention to the analysis of transaction costs.

Two technology trends have also facilitated the deployment of modeling in equity asset management. First, the continuous decrease in the cost of computers coupled with a parallel increase in computational power makes the necessary computing power affordable even to small firms. Second, statistical software packages now offer a broad variety of general and specialized econometric building blocks. The availability of these software packages proved to be a powerful enabler for the deployment of modeling.

The book is divided into four parts. In Part One we cover modern portfolio theory, numerical optimization methods, the analysis of transaction costs, and the handling of nonnormal distributions in portfolio allocation applications through the consideration of higher moments. We present important recent theoretical advances as well as the basic modeling techniques. In Part One these methods are applied in the classical one-period mean-variance and utility-maximization frameworks. This allows us to give an up-to-date treatment of modern portfolio theory and to explain new methods of analysis of transaction costs, numerical optimization, and the handling of higher moments in a unified and concrete framework.

In Part Two we introduce robust methodologies. As mentioned above, robust techniques have become fundamental in the practical deployment of modern portfolio theory. We discuss both the classical and more recent methods for forecasting expected return and risk. In particular, we address topics including dimensionality reduction and the robust estimation of the covariance matrix of returns. Part Two provides a comprehensive presentation of robust methodologies for estimation and optimization.

In Part Three we discuss the motivation for adopting predictive models and present several families of models. We begin with an analysis of the empirical evidence of feedbacks in financial markets. We then describe the statistical properties of models that allow to capture these feedbacks, including regressive and autoregressive models, state-space models, and nonlinear hidden variable, regime-switching models. We discuss cointegration and its many different representations, including dynamic factor analysis. We also elaborate on the process and the pitfalls of the model selection process.
In Part Four we discuss current methods for estimating dynamic models. We close with a discussion on how to mitigate model risk in a dynamic framework.

Two appendices provide complementary mathematical details for the interested reader. Appendix A discusses solutions of difference equations. Appendix B presents a number of mathematical facts on regressions, correlations, and copulas. In several chapters throughout the book we make use of the MSCI World Index and its individual constituents (country indices) in various illustrations. Appendix C provides some basic statistics and properties of this data set.

The purpose of this book is to serve as a working tool for practitioners who use financial modeling in their work and for students who are pursuing careers in finance. Since most of the subjects are advanced in nature, we have tried to offer an intuitive and simplified treatment of most mathematical topics, although at no time have we compromised mathematical rigor. When we feel the subject is too technical, we offer references to the original work. In summary, we feel the book should be of interest to practitioners, students, and researchers who need an updated and integrated view of equity modeling.

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Acknowledgments

In writing a book that covers a wide range of technical topics in financial modeling drawing from a wide range of fields in applied mathematics and financial econometrics, we were fortunate to have received comments from the following individuals:

- Rustam Ibragimov at Harvard University, Department of Economics, reviewed Chapters 2, 4, 5, 8, 9, 10, 11, 12, 14, 16, and Appendix B.
- Bernd Hanke of Goldman Sachs Asset Management reviewed Chapters 2, 3, 4, 5, and 7.
- Russell Fogler of Fogler Research and Management reviewed Chapter 3, 9, 12, 13, 16, and 17.
- Harry Farrell of TRG Management reviewed Chapters 2, 3, 4, 7, and 9.
- Dessislava Pachamanova of Babson College reviewed Chapters 6 and 9.
- Stan Beckers of KULeuven reviewed Chapters 5 and 7.
- Volker Ziemann of Edhec Risk and Asset Management Research Center reviewed Chapters 11 and 12.
- Yu Zhu of China Europe International Business School reviewed Chapters 2 and 4.
- Thomas Philips of OTA Asset Management reviewed Chapters 2 and 7.
- Donald Goldfarb and Garud Iyengar of Columbia University reviewed Chapter 9 and allowed us to use their illustration in that chapter.
- Eric Sorensen, Eddie Qian, and Ronald Hua of Panagora reviewed Chapters 4, 6, and 9.
- Jarrod Wilcox of Wilcox Investments reviewed Chapters 2 and 7.
- Jeff Miller of Millennium Partners reviewed Chapters 4 and 8.
- Massimo Guidolin, Senior Economist at the Federal Reserve Bank of St. Louis, reviewed Chapter 16.
- Sadayuki Ono of the University of York reviewed Chapter 16.
- Milan Borkovec and Ian Domowitz of ITG Inc. reviewed Chapter 3.
- John M. Manoyan of CYMALEX Advisors reviewed Chapter 6.
- Sebastian Ceria and Robert Stubbs of Axioma reviewed Chapter 9.
- Da-shan Huang of Kyoto University reviewed Chapters 6 and 9.
Reviews and editorial comments on the entire manuscript were made by Caroline Jonas of The Intertek Group and Carmen Manoyan.

We thank Morgan Stanley Capital International, Inc., http://www.msci.com, for providing us with the MSCI World Index dataset used in some of the examples throughout the book. In particular, we are indebted to Nicholas G. Keyes for preparing and for answering all our questions in regards to the dataset.

Our thanks go to Deepti Bathula for her assistance in preparing various computational illustrations in Parts One and Two of the book.

Megan Orem typeset the book and provided editorial assistance. We appreciate her patience and understanding in working through several revisions of the chapters and several reorganizations of the table of contents.
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Since the sharp stock market downturn in the United States in 2000, we have witnessed a progressive increase of the depth and breadth of financial modeling at many asset management firms. The need to reduce costs and to rely on a more predictable and repeatable business model were behind this change. This book discusses some of the major trends and innovations that characterize the modeling and selection of equity portfolios. It addresses the two major issues of modeling today: (1) the need to adopt increasingly sophisticated models to capture profit opportunities and (2) the need for robust and reliable solutions and methodologies, at the same time.

HISTORICAL PERSPECTIVE ON THE FINANCIAL MODELING OF THE EQUITY MARKET

Investment management as we know it today is a relatively recent discipline. Until the 18th century, wealth was essentially physical wealth associated with land ownership or privileges, such as the right to impose tariffs or exploit natural resources. Throughout the Middle Ages in Western Europe, lending money to realize a return was considered usury and condemned by the Church. Nevertheless, the same period saw the development of important international banks, such as the Peruzzi and Bardi banks based in Florence. Interestingly enough, these banks were brought down when the English king Edward III defaulted completely on 1 million gold florins in loans in 1339.

The first exchange for trading financial contracts opened in Antwerp in the 16th century, but it was the opening of the stock exchange in Paris in 1720, followed by that in London in 1792, and New York in 1801 that ushered in the era of financial trading and investment as we know it.
today. Social, economic, and political developments were behind the change. The Industrial Revolution greatly accelerated the pace of the creation and destruction of capital and brought with it the need for continuous investment. While land was quite a permanent form of wealth, factories had to be built from scratch, required the continuous replacement of machinery, and lasted only a comparatively short period of time. The creation of a relatively stable and independent legal and social order, a development that took place in the 18th and 19th centuries, was also a powerful enabler of the creation of financial wealth.

Financial markets and their ability to create and destroy wealth fascinated people and created two opposing views of financial trading. On one hand, investing in financial assets was associated with gambling and speculation. Even a profoundly rational economic thinker like John Maynard Keynes had an essentially speculative view of financial markets, dominated, he believed, by the “animal spirit.” Keynes himself was a successful investor. This view of investment as a form of gambling was reflected in the language. As recently as the 1970s, the French and Italian expressions for investing in stocks were respectively “jouer à la Bourse” and “giocare in Borsa,” that is, “gambling in the Exchanges.”

On the other hand, there was the view that markets are perfectly rational, transparent vehicles that serve to channel savings to the most productive destinations. People were truly fascinated by the fact that the independent action of myriads of individual investors led to the discovery of the “true value” of a financial contract. This view led to concentrating analytical efforts on analyzing the financial status of companies. The monumental treatise of Graham and Dodd on financial analysis is perhaps the most complete expression of this view; published in 1934, it has remained mandatory reading for financial analysts to this day.

In a sense, the development of modern investment management is the progressive blending of these two initially irreconcilable views. There are explanations for why it took so long to arrive at a reasonably comprehensive understanding of financial markets. It is perhaps useful to briefly follow this development as it will give us the opportunity to discuss the key components of financial modeling and quantitative techniques that were to progressively become a part of the investment management process.

We will briefly outline the technical and scientific aspects of this development, but it should be noted that broad cultural and social

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1 Benjamin Graham (1894–1976) is often called “the father of value investing.” His book Security Analysis, written together with David Dodd and published in 1934 by McGraw-Hill, has been considered a bible for serious investors ever since its appearance.
issues were also at work. The latter profoundly influenced economic thinking. The 18th and 19th centuries witnessed the development of the concept of free markets. Markets are as old as civilization itself. Trade routes, such as the long-distance trade route connecting ancient Egypt to Afghanistan, were established as earlier as 2250 BCE. However, such exchanges did not give rise to a merchant class; they were fixed price affairs with the price regulated by temple or palace. Following the collapse of the Roman Empire in the West, it was only toward the end of the Middle Ages that economic activity and trading resumed in full earnest in Europe. And it was only at the end of the 18th century in, for example, England and post-Revolutionary France, that the concept of a modern state with an independent and stable legal system began to develop.

This development brought rules that encouraged economic and entrepreneurial activity and with it, the creation of a new wealth, less dependent on privileges. In the 19th century these developments were associated with the idea of individual freedom. As a consequence, the virtues of free markets became an article of faith. This is reflected in the language of economics that opposes the idea of perfect markets to markets with defects and imperfections. To draw a parallel in physics, the notion of an idealized perfect gas was developed about at the same time but it would have been considered ludicrous to consider real gases as gases with defects and imperfections!

From the scientific point of view, the major obstacles to a better understanding of financial markets were:

- A need for the concepts and mathematics of probability and statistics and, more in general, of uncertainty (these developed only much later)
- A need to perform onerous computations, made possible only by the relatively recent development of high-performance computers

Any phenomenon related to human behavior is essentially uncertain. Because finance and economics are deeply influenced by human behavior and human decision-making processes, the development of a quantitative theory of finance depended critically on the development of a quantitative theory of uncertainty. This task was achieved in full earnest only with the recent development of probability theory. A logically rigorous formulation was first developed in the first three decades of the 20th century. Before this time, probability theory was plagued by internal contradictions that made its application problematic.

When Louis Bachelier discussed his now famous thesis on the theory of speculation in Paris in 1900, he was in advance of his times. Bachelier introduced a number of concepts that were not understood in his time, such as Brownian motion to describe stock price behavior or arbitrage arguments to price options. Unfortunately for Bachelier, his reasoning was too economic to satisfy mathematicians and too mathematical to satisfy economists. When Albert Einstein introduced Brownian motion in physics in 1905, five years after Bachelier had introduced the same concept in economics, Einstein’s theory was hailed as a major scientific advance.

Economics had to wait until the second half of the 20th century to see probability theory accepted as a mainstream tool in financial analysis. Acceptance went through a slow process that progressively introduced probabilistic notions in the logical structure of economic theory. Only when probability theory was blended with the key economic concepts of supply and demand and with the theory of financial decision-making through the work of Arrow and Debreu did probabilistic reasoning become a mainstream tool for economists. Despite this major step forward, the path to modern financial econometrics was still long and arduous.

Between 1950 and 1960, three major developments took place. First, in 1952 Harry Markowitz outlined the theory of investment as the maximization of a risk-return trade-off. Second, assuming that investors behave as theorized by Markowitz, between 1962 and 1964, William Sharpe, John Lintner, and Jan Mossin introduced the first asset pricing theory, the capital asset pricing model (CAPM). Third, in 1965 Fama and Samuelson introduced the concept of efficient financial markets together with the notion that “properly anticipated prices fluctuate randomly.” This idea had been introduced by Bachelier 65 years earlier,

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3 Despite his genial intuitions, Bachelier did not enjoy a successful academic career.
5 Harry M. Markowitz, “Portfolio Selection,” Journal of Finance (March 1952), pp. 77–91. The principles in Markowitz’s article were then expanded in his book Portfolio Selection, Cowles Foundation Monograph 16 (New York: John Wiley & Sons, 1959).
but Fama and Samuelson put the concept into a more general framework of how financial markets process information.

It was believed that the above major milestones in the development of modern asset management and financial econometrics entailed the following three key conclusions:

- Logarithms of prices can be represented as unpredictable multivariate random walks.
- Markets exhibit a risk-return trade-off, where risk has to be computed taking into account correlations between stocks.
- There is no possibility of earning an excess returns in the sense that any return in excess of the risk-free rate offered by a risky security is determined by the risk-return relationship of the market for that risk.

These conclusions were enormously important for the asset management community. The ensuing debate focused on two issues:

- The predictability versus the nonpredictability of asset prices
- The paradox introduced by the concepts that (1) markets are efficient because investors can anticipate prices, but (2) investing resources in acquiring the ability to anticipate prices is futile as it does not bring any reward.

It was argued that if prices are not predictable, it was difficult to justify the asset management industry: it would simply not make sense to pay manager fees to obtain returns that could be obtained through a simple buy-and-hold strategy. For 14 years, between 1988 and 2002, the Wall Street Journal was to run a competition between experienced asset managers and pure random stock picking, personified by the random throwing of a dart. On average, professional managers realized an average 10.2% investment gain, while the darts managed just a 3.5% gain.⁸

The asset management community was split between those who claimed that regardless of the theory of efficient markets, a good manager could bring excess returns using intuition, judgment or information not available to other market participants, and those who maintained that because markets are efficient the best investment policy was buy-and-hold (i.e., passive). In hindsight we can say that the debate was ill-conceived. It was to slow down the development of a more scientific approach to asset management. Let us see why.

Consider predictability. Technically, we call a process *predictable* if there is some dependence of future distributions (and therefore expected values) on past data. For example, a multivariate Gaussian random walk (see Chapter 7) is not predictable because conditional expected values of drifts and correlations are identical to the unconditional constant drifts and correlations. A lot of research was devoted to proving that, without overturning the notion of market efficiency, there might be subtle patterns that allow predictability. The theory of martingales was thus introduced in asset pricing theory.

All the reasoning about martingales and market efficiency is logically correct but misses one fundamental point: Any random walk model is an approximate model that is to this day very difficult to estimate. If we look at a random walk from the point of view of information, we see that a multivariate random walk conveys *a lot* of information in drifts and correlations. The random walk model of stock prices is, therefore, far from being uninformative.

The idea that no analysis was required to arrive at this model was a misconception, to say the least. Anyone who takes seriously the notion that markets reward risk cannot be indifferent to finding the optimal risk-return combination. This was the essential pragmatic teaching of Markowitz. But in the 1960s, approximate but robust estimates of drifts and correlation matrices were extremely difficult (not to say impossible) to obtain. The dispute over subtle patterns of predictability delayed the widespread acceptance of a much more fundamental paradigm of stable structures of risk and returns.

A 2000/2001 report on quantitative methods in investment management found that major asset management firms still believed that the key benefit of modeling was the discipline it brought to the investment process. That is to say, the major benefit of quantitative methods was that it persuaded asset managers that the idea of risk-return optimization was real. This is more than half a century after Markowitz!

A preoccupation for logical details—even in the absence of insufficient empirical data—is a major difference between economics and the physical sciences. Physics and engineering never use more mathematics than strictly needed and make extensive use of data. The opposition of these views is illustrated by an anecdote reported at the beginning of Chapter 13 on model selection. When physicists of the Santa Fe Institute asked the economist Kenneth Arrow why economists use such sophisticated mathematics, Arrow reportedly answered that economists needed to use sophisticated mathematics precisely because of the scarcity of

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data. The assumption was that sophisticated mathematics would allow the absolute certainty of logical coherence.

Another, and perhaps even more important, point is that the theoretical assumption that logarithms of prices behave as multivariate random walks subject to risk-return constraints is a very strong assumption. Not only is the random walk hypothesis very far from being uninformative, it is actually a strong hypothesis on the structure of financial markets. In fact, the random walk hypothesis entails that drifts and volatility are time-invariant—a strong hypothesis. Should drifts and volatility vary with time, the random walk hypothesis would be at best an approximation. As we will see in Chapter 10, a simple econometric analysis shows that, over long time horizons, prices do not behave as time-invariant random walks.

Yet the debate on asset pricing continued to focus on the complicated details of martingale asset pricing, efficient versus inefficient markets, and so on, when it should have been clear that any time-invariant model of prices was untenable. At most, the random walk model could be only a temporarily valid approximation. Though the assumption of random walk behavior is difficult to reject for individual stock price processes, the assumption of multivariate random walk behavior is easy to reject.

The real problem is how to glean information from very noisy time series data. It was not fully realized that the assumption of absence of predictability cannot lead per se to a tenable theory of asset pricing. When combined with the assumption that risk is remunerated, these theoretical assumptions would imply the ability to capture a stable structure of drifts and volatilities that do not change with time. Such permanent structures do not exist in reality.

The last decade has witnessed a significant shift in financial econometrics. Academics have abandoned the preoccupation of staying within the basic paradigms of the nonpredictability of asset prices. It is clear by now that random walks are at best an approximation. If we estimate the parameters of a multivariate random walk from realistic price data, we obtain randomly varying quantities. Financial econometrics has abandoned the efforts to prove that they are meaningless and is now trying to extract information from these distributions. The aim of financial modeling is to provide the tools to extract this information and use it in a sound decision-making process. Our objective in this book is to explain and illustrate how this is done for the equity market.
CENTRAL THEMES OF THE BOOK

Three major lines of development have shaped modern financial econometrics and asset management theory. First, robust optimization and estimation. This line of development includes many advanced methods to optimize in a single- and multiperiod framework, estimate the correlation matrix, and mitigate model risk.

A second line of development is embodied in the quest for predictors. Predictors are variables of various natures such as economic quantities, financial ratios, or the lagged values of the same prices. These developments lead to the use of Vector Autoregressive (VAR) models and to strategies based on dynamic factorization and cointegration.

The third line of development attempts to represent states of the market using hidden variables. This approach leads to models such as Markov-switching models and GARCH models, whose interest resides essentially on their explanatory power. However, these techniques are data hungry and therefore difficult to deploy in practice.

The adoption of modeling techniques by asset management firms has greatly increased over the last five years. Models to predict expected returns are routinely used at major firms. In most cases, it is a question of relatively simple models based on factors or predictor variables. However, autoregressive models, cointegration and state-space models are also being used and experimented with. Nonlinear models such as neural networks and genetic algorithms are also being deployed, but a lack of transparency continues to hold back their wider diffusion in the industry.

In trying to address the question as to what techniques are actually being used in financial modeling, we will restrict our analysis to models of stock prices and returns, which is the subject of the book. We can reasonably state that financial modeling is presently characterized by three major challenges:

- The need to extract meaningful information from very noisy time series
- The need to make more robust both estimation and optimization
- The need to arrive at a sound decision-making process, possibly through formal optimization

As mentioned, today’s financial econometrics is no longer deeply influenced by the debate on market efficiency and forecastability: it is now widely accepted that there is some forecastability in the market but that extracting this forecasting information is difficult. Forecastability is no longer considered a market imperfection but the natural result of the interaction of multiple interacting agents with different capabilities and motiva-