
FOUNDATIONS FOR GUIDED-WAVE OPTICS

Chin-Lin Chen
Purdue University
West Lafayette, Indiana



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PREFACE

Over the last 50 years, we have witnessed an extraordinary evolution and progress in optical science and engineering. When lasers were invented as light sources in 1960, cladded glass rods were proposed as transmission media in 1966; few, if any, would foresee their impact on the daily life of the modern society. Today, lasers and fibers are the key building blocks of optical communication systems that touch all walks of life. Photonic components are also used in consumer products, entertainment and medical equipment, not to mention the scientific and engineering instrumentation. Optical devices may be in the “bulk” or guided-wave optic forms. Guided-wave optic components are relatively new and much remains to be accomplished or realized. Thus engineering students and graduates should acquire a basic knowledge of principles, capabilities and limitations of guided-wave optic devices and systems in their education even if their specialization is not optics or photonics. The purpose of this book is to present an intermediate and in-depth treatment of integrated and fiber optics. In addition to the basic transmission properties of dielectric waveguides and optical fibers, the book also covers the basic principles of directional couplers, guided-wave gratings, arrayed-waveguide gratings and fiber optic polarization components. In short, the book examines most topics of interest to engineers and scientists.

The main objective of Chapter 1 is to introduce the nomenclature and notations. The rest of the book treats three major topics. They are the integrated optics (Chapters 2 to 8), fiber optics (Chapters 9 to 12) and the pulse evolution and broadening in optical waveguides (Chapters 13 and 14). Attempts are made to keep each chapter sufficiently independent and self-contained.

The book is written primarily as a textbook for advanced seniors, first-year graduate students, and recent graduates of engineering or physics. It is also useful for self-study. Like all textbooks, materials contained herein may be found in journal articles, research monographs and/or other textbooks. My aim is to assemble relevant materials in a single volume and to present them in a cohesive and unified fashion. It is not meant to be a comprehensive treatise that contains all topics of integrated optics and fiber optics. Nevertheless, most important elements of guided-wave optics are covered in this book.

Each subject selected is treated from the first principles. A rigorous analysis is given to establish its validity and limitation. Whenever possible, elementary

mathematics is used to analyze the subject matter. Detailed steps and manipulations are provided so that readers can follow the development on their own. Extensions or generalizations are noted following the initial discussion. If possible, final results are cast in terms of normalized parameters. Results or conclusions based on numerical calculations or experimental observations are explicitly identified. Convolutional theories that can't be established in simple mathematics are clearly stated without proof. Pertinent references are given so that readers can pursue the subject on their own. In spite of the analysis and mathematic manipulations, the emphasis of this book is physical concepts.

The book is based on the lecture notes written for a graduate course on integrated and fiber optics taught several times over many years at Purdue University. I apologize to students, past and present, who endured typos, corrections, and inconsistencies in various versions of the class notes. Their questions and comments helped immensely in shaping the book to the final form. I also like to thank my colleagues at Purdue for their encouragement, free advice, and consultation. I wish to acknowledge 3 friends in particular. They are Professors Daniel S. Elliott, Eric C. Furgason and George C. S. Lee.

Finally and more importantly, I like to express my sincere appreciation to my wife, Ching-Fong, for her enormous patience, constant encouragement, and steady support. She is the true force bonding three generations of Chen's together and the main pillar sustaining our family. Because of her, I am healthier and happier. I am forever indebted to her.

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September 2006*

BRIEF REVIEW OF ELECTROMAGNETICS AND GUIDED WAVES

1.1 INTRODUCTION

The telecommunication systems are a key infrastructure of all modern societies, and the optical fiber communications is the backbone of the telecommunication systems. An optical communication system is comprised of many optical, electrical, and electronic devices and components. The optical devices may be in the “bulk,” integrated, or fiber-optic form. Therefore, an understanding of the operation principles of these optical devices is of crucial importance to electrical engineers and electrical engineering students. This book is on integrated and fiber optics for optical communication applications. The subject matter beyond the introductory chapter is grouped into three parts. They are the integrated optics (Chapters 2–8), fiber optics (Chapters 9–12), and the propagation and evolution of optical pulses in linear and nonlinear fibers (Chapters 13 and 14).

The main purpose of Chapter 1 is to introduce the nomenclature and notations. In the process, we also review the basics of electromagnetism and essential theories of guided waves.

The theory of thin-film waveguides with constant index regions is relatively simple and complete and it is presented in Chapter 2. Most quantities of interest are

expressed in elementary functions. Examples include field components, the dispersion relation, the confinement factor, and power transported in each region. We also express the quantities in terms of the generalized parameters to facilitate comparison. In short, we use the step-index thin-film waveguides to illustrate the notion of guided waves and the basic properties of optical waveguides.

Many dielectric waveguides have a graded-index profile. Examples include optical waveguides built on semiconductors and lithium niobates. While the basic properties of graded-index waveguides are similar to that of step-index waveguides, subtle differences exist. In Chapter 3, we first analyze modes guided by linearly and exponentially tapered dielectric waveguides. We obtain closed-form expressions for fields and the dispersion relations for these waveguides. Then we apply the WKB (Wentzel, Kramers, and Brillouin) method and a numerical method to study optical waveguides with an arbitrary index profile.

So far, we have considered ideal waveguides made of loss-free materials and having a perfect geometry and index profile. While loss in dielectric materials may be very small, it is not zero. Obviously, no real waveguide structure or index profile is perfect either. As a result, waves decay as they propagate in real waveguides. In Chapter 4, we examine the effects of dielectric loss on the propagation and attenuation of guided modes and the perturbation on the waveguide properties by the presence of metallic films near or over the waveguide regions. The use of metal-clad waveguides as waveguide polarizers or mode filters is also discussed.

In practical applications, we may wish to pack the components densely so as to make the most efficient use of the available “real estate.” It is then necessary to reduce interaction between waveguides and to minimize cross talks. For this purpose, it is necessary to confine fields in the waveguide regions. To confine fields in the two transverse directions, geometric boundaries and/or index discontinuities are introduced in the transverse directions. This leads to three-dimensional waveguides such as channel waveguides and ridge waveguides. In Chapter 5, we examine the modes guided by three-dimensional waveguides with rectangular geometries. Two approximate methods are used to establish the dispersion of modes guided by rectangular dielectric waveguides. They are the Marcatili method and the effective index method. A detailed comparison of the two methods is presented in the last section.

Having discussed the propagation, attenuation, and fields of modes guided by isolated waveguides, we turn our attention to three classes of passive guided-wave components: the directional coupler devices, the waveguide grating devices and arrayed waveguide gratings. In Chapter 6, we discuss Marcatili’s improved coupled-mode equations for co-propagating modes and use these equations to establish the essential characteristics of directional coupling. Then we consider the switched $\Delta\beta$ directional couplers and their applications as switches, optical filters, and modulators.

Waveguide gratings are periodic topological structures or index variations built permanently on the waveguides. Periodic index variations induced by electrooptic or acoustooptic effects onto the waveguides are also waveguide gratings. These gratings are the building blocks of guided-wave components. Coupled-mode equations are

developed in Chapter 7 to describe the interaction of contrapropagating modes in the grating structures. Then we use these equations to study the operation of grating reflectors, grating filters, and distributed feedback lasers. Arrayed-waveguide gratings are briefly discussed in Chapter 8.

The transmission and input/output properties of single-mode fibers are discussed in Chapters 9 and 10. In Chapter 9, we study the transmission properties of linearly polarized (LP) modes in weakly guiding step-index fibers with a circular core. For these fibers, a rigorous analysis of fields is possible, and we obtain closed-form expressions for several quantities of interest. We discuss the phase velocity, group velocity, and the group velocity dispersion of LP modes. For obvious reasons, we are particularly interested in the intramodal dispersion of single-mode fibers. The generalized parameters of step-index fibers are also used in the discussion.

In most applications, it would be necessary to couple light into and out of fibers. Therefore, the input and output characteristics of fibers are of practical interest. In Chapter 10, we suppose that fibers are truncated, and we examine the fields radiated by LP modes from the truncated fibers. We also examine the excitation of LP modes in step-index fibers by uniform plane waves and Gaussian beams.

Ideal fibers would have a circular cross section and a rotationally symmetric index profile and are free from mechanical, electric, and magnetic disturbances. But no ideal fiber exists because of the fabrication imperfection and postfabrication disturbances. Real fibers are birefringent. In Chapter 11, we begin by tracing the physical origins of the fiber birefringence. Then we estimate the fiber birefringence due to noncircular cross section and that induced by mechanical, electrical, and magnetic disturbances. Lastly, we use Jones matrices to describe the birefringent effects in fibers under various conditions.

Most manufactured fibers have graded-index profiles. Naturally, we are interested in the propagation and dispersion of the modes guided by graded-index fibers. In Chapter 12, we concentrate on fibers having a radially inhomogeneous and angularly independent index profile. Of particular interest to telecommunications is the fundamental mode guided by the graded-index fibers. The notion of the mode field radius or spot size is discussed.

All fibers, ideal or real, are dispersive. As a result, optical pulses evolve as they propagate in linear fibers. In Chapter 13, we study the propagation and evolution of pulses in linear, dispersive waveguides and fibers. Three approaches are used to analyze the pulse broadening and distortion problems. The first approach is a straightforward application of the Fourier and inverse Fourier transforms. The concept of the impulse response of a transmission medium is then introduced. Finally, we recognize that fields are the product of the carrier sinusoids and the pulse envelope. The propagation of carrier sinusoids is simple and well understood. Our attention is mainly on the slow evolution of the pulse envelope. A linear envelope equation is developed to describe the evolution of the pulse envelope. A general discussion of the envelope distortion and frequency chirping is then presented.

In nonlinear dispersive fibers, both the pulse shape and spectrum evolve as the pulse propagates. However, if the nonlinear fibers have anomalous group velocity dispersion and if the input pulse shape, temporal width, and amplitude satisfy a well-defined relationship, the pulses either propagate indefinitely without distortion or they reproduce the original pulse shape, width, and peak amplitude periodically. These pulses are known as solitary waves or optical solitons. Naturally, the formation and propagation of the optical solitons are of interest to telecommunications and we study these subjects in Chapter 14. A nonlinear envelope equation, often referred to as the nonlinear Schrödinger equation, is developed to describe the evolution of pulses on nonlinear dispersive fibers. We rely on a simple and straightforward method to derive an expression for the fundamental solitons. From the expression for fundamental solitons, we extract the key properties and the basic parameters of the fundamental solitons. Higher-order solitons and interaction of fundamental solitons are briefly discussed.

1.2 MAXWELL'S EQUATIONS

To study the waves guided by optical waveguides and fibers, we begin with the time-dependent, source-free Maxwell equations:

$$\nabla \times \mathcal{E}(\mathbf{r}; t) = -\frac{\partial \mathcal{B}(\mathbf{r}; t)}{\partial t} \quad (1.1)$$

$$\nabla \times \mathcal{H}(\mathbf{r}; t) = \frac{\partial \mathcal{D}(\mathbf{r}; t)}{\partial t} \quad (1.2)$$

$$\nabla \cdot \mathcal{B}(\mathbf{r}; t) = 0 \quad (1.3)$$

$$\nabla \cdot \mathcal{D}(\mathbf{r}; t) = 0 \quad (1.4)$$

where $\mathcal{E}(\mathbf{r}; t)$, $\mathcal{D}(\mathbf{r}; t)$, $\mathcal{H}(\mathbf{r}; t)$, and $\mathcal{B}(\mathbf{r}; t)$ are the *electric field intensity* (V/m), *electric flux density* (C/m²), *magnetic field intensity* (A/m), and *magnetic flux density* (T or W/m²), respectively. They are real functions of position \mathbf{r} and time t . Although some dielectric waveguides and fibers may contain anisotropic materials, most optical waveguides and fibers of interest are made of isotropic, nonmagnetic dielectric materials. We confine our discussion in this book to isotropic, nonmagnetic materials only. For nonmagnetic and isotropic materials, the *constitutive relations* are

$$\mathcal{B}(\mathbf{r}; t) = \mu_0 \mathcal{H}(\mathbf{r}; t) \quad (1.5)$$

and

$$\mathcal{D}(\mathbf{r}; t) = \varepsilon_0 \mathcal{E}(\mathbf{r}; t) + \mathcal{P}(\mathbf{r}; t) \quad (1.6)$$

where ε_0 ($\approx 1/36\pi \times 10^{-9}$ F/m) and μ_0 ($= 4\pi \times 10^{-7}$ H/m) are the vacuum *permittivity* and *permeability*. $\mathcal{P}(\mathbf{r}; t)$ is the *electric polarization* of the medium [1–3].

It is convenient to use phasors to describe fields that vary sinusoidally in time. In the frequency domain, the Maxwell equations are

$$\nabla \times \mathbf{E}(\mathbf{r}; \omega) = -j\omega \mathbf{B}(\mathbf{r}; \omega) \quad (1.7)$$

$$\nabla \times \mathbf{H}(\mathbf{r}; \omega) = j\omega \mathbf{D}(\mathbf{r}; \omega) \quad (1.8)$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}; \omega) = 0 \quad (1.9)$$

$$\nabla \cdot \mathbf{D}(\mathbf{r}; \omega) = 0 \quad (1.10)$$

where $\mathbf{E}(\mathbf{r}; \omega)$, $\mathbf{H}(\mathbf{r}; \omega)$, and so forth are the phasor representation of $\mathcal{E}(\mathbf{r}; t)$, $\mathcal{H}(\mathbf{r}; t)$, and so forth and ω is the angular frequency. In general, $\mathbf{E}(\mathbf{r}; \omega)$, $\mathbf{D}(\mathbf{r}; \omega)$, $\mathbf{H}(\mathbf{r}; \omega)$, and $\mathbf{B}(\mathbf{r}; \omega)$ are complex functions of \mathbf{r} and ω . The time-domain field vectors and the corresponding frequency-domain quantities are related. For example,

$$\mathcal{E}(\mathbf{r}; t) = \text{Re}[\mathbf{E}(\mathbf{r}; \omega)e^{j\omega t}] \quad (1.11)$$

The constitutive relations in the frequency domain are, in lieu of (1.5) and (1.6),

$$\mathbf{B}(\mathbf{r}; \omega) = \mu_0 \mathbf{H}(\mathbf{r}; \omega) \quad (1.12)$$

$$\mathbf{D}(\mathbf{r}; \omega) = \varepsilon_0 \mathbf{E}(\mathbf{r}; \omega) + \mathbf{P}(\mathbf{r}; \omega) \quad (1.13)$$

We assume that the fields are weak enough that the nonlinear response of the medium is negligibly small. We will not be concerned with the second- and third-order polarizations until Chapter 14. In the first 13 chapters, we take the media as linear media. In simple, isotropic and linear media, $\mathbf{P}(\mathbf{r}; \omega)$ is proportional to and in parallel with $\mathbf{E}(\mathbf{r}; \omega)$. Then the electric flux density can be written as

$$\mathbf{D}(\mathbf{r}; \omega) = \varepsilon_0 [1 + \chi^{(1)}(\mathbf{r}; \omega)] \mathbf{E}(\mathbf{r}; \omega) = \varepsilon_0 \varepsilon_r(\mathbf{r}; \omega) \mathbf{E}(\mathbf{r}; \omega) \quad (1.14)$$

where $\chi^{(1)}(\mathbf{r}; \omega)$ is the *electric susceptibility*, and $\varepsilon_r(\mathbf{r}; \omega)$ is the *relative dielectric constant*. In optics literature, we often rewrite the above equation in terms of a *refractive index* $n(\mathbf{r}; \omega)$:

$$\mathbf{D}(\mathbf{r}; \omega) = \varepsilon_0 n^2(\mathbf{r}; \omega) \mathbf{E}(\mathbf{r}; \omega) \quad (1.15)$$

The relative dielectric constant and the refractive index may be functions of position and frequency. For example, different waveguide regions may have different ε_r and n .

To find the waves guided by a waveguide amounts to solving the Maxwell equations subject to the usual boundary conditions. Consider the boundary between media 1 and 2 as shown in Figure 1.1. Let $\mathbf{E}_i(\mathbf{r}; \omega)$, $\mathbf{H}_i(\mathbf{r}; \omega)$, and so forth be the field vectors in region i with an index n_i where $i = 1$ or 2 . $\hat{\mathbf{n}}_i$ is the *unit vector* normal to the boundary separating the two media and pointing in the outward direction

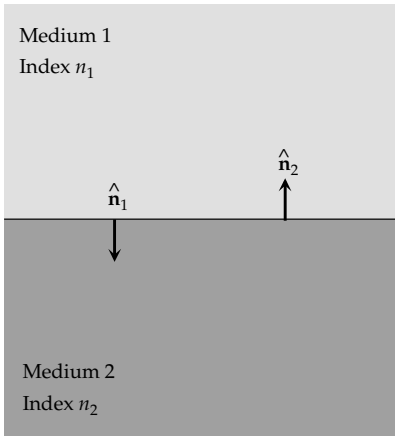


Figure 1.1 Unit vectors $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ normal to the boundary.

relative to region i . In the absence of the surface charge density and surface current density, the boundary conditions are as follows:

1. The tangential components of $\mathbf{E}(\mathbf{r}; \omega)$ and $\mathbf{H}(\mathbf{r}; \omega)$ are continuous at the boundary:

$$\hat{\mathbf{n}}_1 \times \mathbf{E}_1(\mathbf{r}; \omega) + \hat{\mathbf{n}}_2 \times \mathbf{E}_2(\mathbf{r}; \omega) = 0 \quad (1.16)$$

$$\hat{\mathbf{n}}_1 \times \mathbf{H}_1(\mathbf{r}; \omega) + \hat{\mathbf{n}}_2 \times \mathbf{H}_2(\mathbf{r}; \omega) = 0 \quad (1.17)$$

2. The normal components of $\mathbf{D}(\mathbf{r}; \omega)$ and $\mathbf{B}(\mathbf{r}; \omega)$ are also continuous at the boundary:

$$\hat{\mathbf{n}}_1 \cdot \mathbf{D}_1(\mathbf{r}; \omega) + \hat{\mathbf{n}}_2 \cdot \mathbf{D}_2(\mathbf{r}; \omega) = 0 \quad (1.18)$$

$$\hat{\mathbf{n}}_1 \cdot \mathbf{B}_1(\mathbf{r}; \omega) + \hat{\mathbf{n}}_2 \cdot \mathbf{B}_2(\mathbf{r}; \omega) = 0 \quad (1.19)$$

For brevity, we will drop the arguments $(\mathbf{r}; t)$ and $(\mathbf{r}; \omega)$ in the remaining discussion. In other words, we simply write \mathcal{E} , \mathcal{H} , \mathbf{E} , \mathbf{H} , and so forth in lieu of $\mathcal{E}(\mathbf{r}; \omega)$, $\mathcal{H}(\mathbf{r}; \omega)$, $\mathbf{E}(\mathbf{r}; \omega)$, and $\mathbf{H}(\mathbf{r}; \omega)$.

1.3 UNIFORM PLANE WAVES IN ISOTROPIC MEDIA

Waves are labeled as *plane waves* if the constant phase surfaces of the waves are planes. If the wave amplitude is the same everywhere on the constant phase plane, waves are identified as *uniform plane waves*. Consider uniform plane waves propagating in an arbitrary direction $\hat{\mathbf{k}}$ in free space. The electric and magnetic field intensities can be expressed as

$$\mathbf{E} = \mathbf{E}_0 e^{-j\mathbf{k} \cdot \mathbf{r}} \quad (1.20)$$

$$\mathbf{H} = \mathbf{H}_0 e^{-j\mathbf{k} \cdot \mathbf{r}} \quad (1.21)$$

where \mathbf{k} is the *wave vector* in free space. \mathbf{E}_0 and \mathbf{H}_0 are the amplitudes of the electric and magnetic field intensities, respectively. To determine the relation between various plane wave parameters, we substitute (1.20) and (1.21) into the time-harmonic Maxwell equations (1.7)–(1.10) and obtain

$$|\mathbf{k}| = k = \omega\sqrt{\mu_0\epsilon_0} \quad (1.22)$$

and

$$\mathbf{H}_0 = \frac{1}{\eta_0} \hat{\mathbf{k}} \times \mathbf{E}_0 \quad (1.23)$$

In (1.23), $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is the *intrinsic impedance* of free space.

The *Poynting vector* is

$$\mathbf{S} = \frac{1}{2} \text{Re}[\mathbf{E} \times \mathbf{H}^*] = \frac{|\mathbf{E}_0|^2}{2\eta_0} \hat{\mathbf{k}} \quad (1.24)$$

where * stands for the complex conjugation of a complex quantity.

For an isotropic medium with a refractive index n , the wave vector and the intrinsic impedance are $n\mathbf{k}$ and η_0/n , respectively. In lieu of (1.23) and (1.24), the field vectors and Poynting vector in the medium with index n are related through the following relations:

$$\mathbf{H}_0 = \frac{n}{\eta_0} \hat{\mathbf{k}} \times \mathbf{E}_0 \quad (1.25)$$

$$\mathbf{S} = \frac{n|E_0|^2}{2\eta_0} \hat{\mathbf{k}} \quad (1.26)$$

In linear isotropic media, \mathbf{E} and \mathbf{D} are in parallel. So are \mathbf{B} and \mathbf{H} . It is also clear from (1.23) to (1.26) that \mathbf{E} and \mathbf{D} are perpendicular to \mathbf{B} and \mathbf{H} . These field vectors are also perpendicular to \mathbf{k} and \mathbf{S} as depicted in Figure 1.2(a). Since electric and magnetic field vectors are transverse to the direction of propagation, uniform plane waves in isotropic media are *transverse electromagnetic (TEM) waves*. These remarks hold for isotropic media. But for anisotropic media, many statements have to be modified [see Fig. 1.2(b)] [2, 4].

1.4 STATE OF POLARIZATION

In the last section, we consider fields in the frequency domain. It is often instructive to examine the fields in the time domain as well. In the time-domain description, we can “visualize” the motion of a field vector as a function of time. We refer the evolution of the field vector in time as the *state of polarization*. Consider the electric field at a certain point. We suppose that the electric field is confined in a plane,

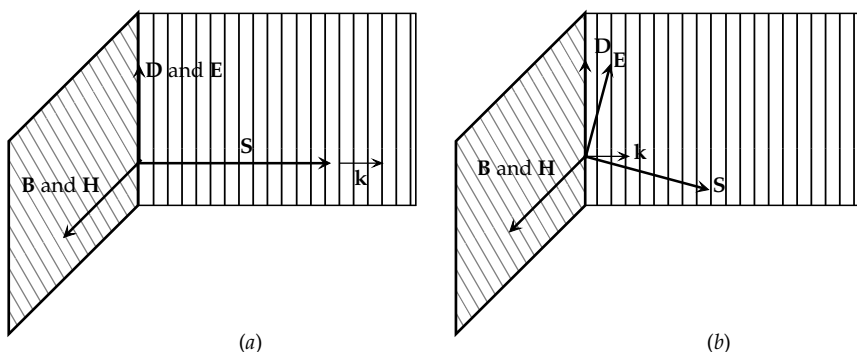


Figure 1.2 \mathbf{E} , \mathbf{D} , \mathbf{B} , \mathbf{H} , \mathbf{S} , and \mathbf{k} vectors of uniform plane waves in (a) isotropic and (b) anisotropic dielectric media.

which is taken as the xy plane for convenience. In the time-domain representation, the electric field at this point is

$$\mathcal{E} = \hat{\mathbf{x}}\mathcal{E}_x + \hat{\mathbf{y}}\mathcal{E}_y = \hat{\mathbf{x}}E_{x0} \cos(\omega t + \phi_x) + \hat{\mathbf{y}}E_{y0} \cos(\omega t + \phi_y) \quad (1.27)$$

where E_{x0} and E_{y0} are amplitudes of the two components, and they are real and positive quantities. ϕ_x and ϕ_y are the phases of the two components relative to an arbitrary time reference. $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors in the x and y directions. The corresponding frequency-domain representation is

$$\mathbf{E} = \hat{\mathbf{x}}E_x + \hat{\mathbf{y}}E_y = \hat{\mathbf{x}}E_{x0}e^{j\phi_x} + \hat{\mathbf{y}}E_{y0}e^{j\phi_y} \quad (1.28)$$

Depending on the amplitude ratio E_{y0}/E_{x0} and the phase difference $\Delta\phi = \phi_y - \phi_x$, the “tip” of the field vector may trace a linear, circular, or an elliptical trajectory in a left-hand or right-hand sense. If the two components are in time phase, that is, $\phi_x = \phi_y = \phi$, (1.27) can be simplified to

$$\mathcal{E} = (\hat{\mathbf{x}}E_{x0} + \hat{\mathbf{y}}E_{y0}) \cos(\omega t + \phi)$$

While the length of the field vector changes as a cosine function, the field vector points to a fixed direction $\hat{\mathbf{x}}E_{x0} + \hat{\mathbf{y}}E_{y0}$. In other words, the tip of \mathcal{E} moves along a straight line as time advances. We refer fields with $\phi_x = \phi_y$ as *linearly polarized* or *plane polarized fields* or *waves*.

If the two field components have the same amplitudes and are in time quadrature, that is, $E_{x0} = E_{y0}$ and $\Delta\phi = \phi_y - \phi_x = -\pi/2$, (1.27) becomes

$$\mathcal{E} = E_{x0}[\hat{\mathbf{x}} \cos(\omega t + \phi_x) + \hat{\mathbf{y}} \sin(\omega t + \phi_x)]$$

As time advances, the tip of \mathcal{E} traces a circular path. If the wave under consideration moves in the $+z$ direction, then the \mathcal{E} vector rotates in the counterclockwise sense for observers looking toward the source. If our right thumb points to the

direction of propagation, that is, the $+z$ direction, our right-hand fingers would curl in the same sense as the motion of the tip of the electric field vector. Thus waves having field components specified by $E_{x0} = E_{y0}$ and $\phi_y - \phi_x = -\pi/2$ and moving in the $+z$ direction are *right-hand circularly polarized waves* [3, 5–7].

Similarly, if $E_{x0} = E_{y0}$ and $\Delta\phi = \phi_y - \phi_x = +\pi/2$, the field in the time-domain representation is

$$\mathcal{E} = E_{x0}[\hat{\mathbf{x}} \cos(\omega t + \phi_x) - \hat{\mathbf{y}} \sin(\omega t + \phi_x)]$$

For waves propagating in the $+z$ direction, the tip of the field vector traces a circle in the clockwise direction to observers facing the approaching waves. In other words, the field vector rotates in the left-hand sense. The fields with $E_{x0} = E_{y0}$ and $\Delta\phi = \phi_y - \phi_x = +\pi/2$ are *left-hand circularly polarized waves*. This is the Institute of Electrical and Electronics Engineers' (IEEE) definition for the right-handedness or left-handedness of the waves. The terminology used in the physics and optics literature is exactly the opposite. In many books on physics and optics, waves having $\Delta\phi = -\pi/2$ and $\Delta\phi = +\pi/2$ are identified, respectively, as the left- and right-hand circularly polarized waves [3, 5–7].

In general, (1.27), or (1.28), describes *elliptically polarized fields*. To elaborate this point further, we combine (1.27) and (1.28) and obtain (Problem 1)

$$\frac{\mathcal{E}_x^2}{E_{x0}^2} + \frac{\mathcal{E}_y^2}{E_{y0}^2} - 2\frac{\mathcal{E}_x}{E_{x0}}\frac{\mathcal{E}_y}{E_{y0}}\cos\Delta\phi = \sin^2\Delta\phi \quad (1.29)$$

The equation describes a *polarization ellipse* inscribed into a $2E_{x0} \times 2E_{y0}$ rectangle as shown in Figure 1.3. The shape and the orientation of the ellipse depend on the amplitude ratio E_{y0}/E_{x0} and the phase difference $\Delta\phi$. The sense of rotation depends only on the phase difference. The major and minor axes of a polarization ellipse do not necessarily coincide with the x and y axes. Therefore, there is no simple way to relate the major and minor axes of the ellipse specified in (1.29) to E_{x0} and E_{y0} . By rotating the coordinates, (1.29) can be transformed to a canonical form for ellipses [3, 5]. In the canonical form, the lengths of major and minor axes, $2E_{mj}$ and $2E_{mn}$, are readily identified. We take E_{mj} and E_{mn} as positive and $E_{mj} \geq E_{mn}$. The shape of the ellipse may also be quantified in terms of the *ellipticity*:

$$\text{Ellipticity} = \frac{E_{mn}}{E_{mj}} \quad (1.30)$$

or the *visibility* (VS)

$$\text{VS} = \frac{E_{mj}^2 - E_{mn}^2}{E_{mj}^2 + E_{mn}^2} = \frac{1 - (E_{mn}^2/E_{mj}^2)}{1 + (E_{mn}^2/E_{mj}^2)} \quad (1.31)$$

The ellipticity and visibility of a polarization ellipse are functions of E_{mn}/E_{mj} .

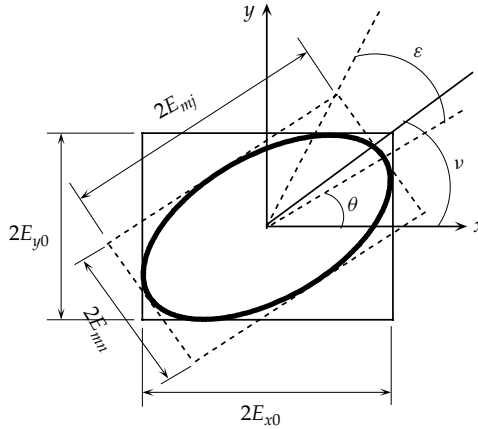


Figure 1.3 Parameters of elliptically polarized waves.

As shown in Figure 1.3, θ is the angle of the major axis relative to the x axis. We refer θ as the *azimuth* of the ellipse.

The sense of rotation of elliptically polarized field is the sense of motion of the tip of \mathcal{E} as a function of time. For $0 < \Delta\phi < \pi$, the tip of \mathcal{E} rotates in the same way as our left-hand fingers curl with the left thumb pointing in the direction of propagation. Thus fields with a phase difference $0 < \Delta\phi < \pi$ rotate in the left-hand sense. Fields with $\pi < \Delta\phi < 2\pi$ rotate in the right-hand direction. The limiting cases of $\Delta\phi = 0$ or π correspond to linearly polarized waves.

In summary, the state of polarization can be specified by E_{y0}/E_{x0} and $\Delta\phi$. It can also be cast in terms of E_{mj}/E_{mn} , θ , and the sense of rotation. The transformation from E_{y0}/E_{x0} and $\Delta\phi$ to E_{mn}/E_{mj} , θ , and the sense of rotation is facilitated by the following relations [3]:

$$\varepsilon_{\text{lr}} \frac{E_{mn}}{E_{mj}} = \tan \varepsilon \quad -\frac{\pi}{4} \leq \varepsilon \leq \frac{\pi}{2} \quad (1.32)$$

$$\sin 2\varepsilon = (\sin 2\nu) \sin \Delta\phi \quad (1.33)$$

$$\tan 2\theta = (\tan 2\nu) \cos \Delta\phi \quad 0 \leq \theta < \pi \quad (1.34)$$

$$\frac{E_{y0}}{E_{x0}} = \tan \nu, \quad 0 \leq \nu \leq \frac{\pi}{2} \quad (1.35)$$

In (1.32), ε_{lr} is $+1$ for the left-handed rotation and -1 for the right-handed rotation. Detail derivation for these relations is left as an exercise for the reader (Problem 2). The physical meanings of E_{y0}/E_{x0} , E_{mn}/E_{mj} , $\Delta\phi$, and θ are easily understood from Figure 1.3. Although ε and ν also have geometrical meaning of their own, as shown in Figure 1.3, we merely view ε and ν as the two auxiliary variables introduced to specify the ratios E_{y0}/E_{x0} and E_{mn}/E_{mj} .

As implied in (1.28), an elliptically polarized field can be considered as the superposition of two orthogonal linearly polarized fields. An elliptically polarized