# GROUNDING AND SHIELDING

CIRCUITS AND INTERFERENCE

# **FIFTH EDITION**

**Ralph Morrison** 





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# GROUNDING AND SHIELDING

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# CONTENTS

Preface to the Fifth Edition		xi	
1.	Volta	ge and Capacitance	1
	1.1.	Introduction	1
	1.2.	Charges and Electrons	2
	1.3.	The Electric Force Field	4
	1.4.	Field Representations	4
	1.5.	The Definition of Voltage	6
	1.6.	Equipotential Surfaces	8
	1.7.	The Electric Force Field between Two Conducting Plates	9
	1.8.	Electric Field Patterns	10
	1.9.	The Energy Stored in an Electric Field	13
	1.10.	Dielectrics	15
	1.11.	The D Field	15
	1.12.	Capacitance	17
	1.13.	Mutual Capacitance	18
	1.14.	Displacement Current	19
	1.15.	Energy Stored in a Capacitor	21
	1.16.	Forces in the Electric Field	21
	1.17.	Capacitors	22
2.	Magn	etics	23
	2.1.	Magnetic Fields	23
	2.2.	Biot and Savart's Law	24
	2.3.	The Solenoid	25
	2.4.	Faraday's Law and the Induction Field	25
	2.5.	Inductance	27
	2.6.	The Energy Stored in an Inductance	27
	2.7.	Magnetic Field Energy in Space	29
	2.8.	The Magnetic Circuit	30
	2.9.	A Magnetic Circuit with a Gap	32
	2.10.	Small Inductors	33
	2.11.	Self- and Mutual Inductance	34
	2.12.	Transformer Action	34
	2.13.	Hysteresis and Permeability	38

	2.14.	Eddy Currents	39
	2.15.	The Transport of Electrical Energy	40
	2.16.	Poynting's Vector	41
	2.17.	Transmission Lines Introduction	42
	2.18.	Transmission Line Operations	43
	2.19.	Transmission Line Field Patterns	44
	2.20.	Interfering Fields	47
3.	Utility	y Power and Facility Grounding	49
	3.1.	Introduction	49
	3.2.	History	49
	3.3.	Semantics	50
	3.4.	The Earth as a Conductor	51
	3.5.	The Neutral Connection to Earth	52
	3.6.	Ground Potential Differences	53
	3.7.	Field Coupling to Power Conductors	54
	3.8.	Neutral Conductors	55
	3.9.	<i>K</i> -Factor in Transformers	56
	3.10.	Ungrounded Power	57
	3.11.	A Request for Power	57
	3.12.	Earth Power Currents	58
	3.13.	Line Filters	58
	3.14.	Isolated Grounds	59
	3.15.	Facility Grounds—Some More History	61
	3.10. 2.17	Lightning	62 62
	3.17.	Lightning and Facilities	03
4.	Analo	og Circuits	65
	4.1.	Introduction	65
	4.2.	Instrumentation	65
	4.3.	History	67
	4.4.	The Basic Shield Enclosure	68
	4.5.	The Enclosure and Utility Power	69
	4.6.	The Two-Ground Problem	72
	4.7.	Instrumentation and the Two-Ground Problem	73
	4.8.	Strain-Gauge Instrumentation	75
	4.9.	The Floating Strain Gauge	76
	4.10.	The Thermocouple	78
	4.11.	The Basic Low-Gain Differential Amplifier	78
	4.12.	Shielding in Power Transformers	80
	4.13.	Calibration and Interference	82
	4.14.	The Guard Shield above 100kHz	82
	4.15.	Signal Flow Paths in Analog Circuits	83
	4.16.	Parallel Active Components	84

	4.17.	Feedback Stability—Introduction	84
	4.18.	Feedback Theory	85
	4.19.	Output Loads and Circuit Stability	86
	4.20.	Feedback Around a Power Stage	87
	4.21.	Constant-Current Loops	88
	4.22.	Filters and Aliasing Errors	88
	4.23.	Isolation and DC-to-DC Converters	89
	4.24.	Charge Converter Basics	91
	4.25.	Guard Rings	94
	4.26.	Thermocouple Effects	94
	4.27.	Guard Switching	94
	4.28.	Digital Control	95
5.	Radia	tion	97
	5.1.	Handling Radiation and Susceptibility	97
	5.2.	Radiation: What Is It?	98
	5.3.	The Dipole Antenna	101
	5.4.	Wave Impedance	102
	5.5.	Field Strength and Antenna Gain	103
	5.6.	Radiation from Loops	104
	5.7.	E-Field Coupling to a Loop	106
	5.8.	A Note on Sine Wave Analysis	106
	5.9.	Approximations for Pulses and Square Waves	107
	5.10.	Radiation from a Printed Circuit Board	111
	5.11.	The Sniffer and the Antenna	112
	5.12.	Solar Magnetic Storms	113
	5.13.	Radiation from the Earth	113
6.	Hard	ware	115
	6.1.	Cables with Foil Shields	115
	6.2.	Coaxial Cables	116
	6.3.	Low-Noise Cables	118
	6.4.	Transfer Impedance	118
	6.5.	Wave Guides	120
	6.6.	Electromagnetic Fields Over a Ground Plane	121
	6.7.	Skin Effect	122
	6.8.	Ohms per Square	123
	6.9.	Fields and Conductors	125
	6.10.	Conductive Enclosures—Introduction	126
	6.11.	Coupling through Enclosure Walls by an Induction Field	127
	6.12.	Reflection and Absorption of Field Energy at a	
		Conducting Surface	127
	6.13.	Independent Apertures	129
	6.14.	Dependent Apertures	130

6.15.	Honeycombs	131
6.16.	Summing Field Penetrations	131
6.17.	Power Line Filters	132
6.18.	Back Shell Connectors	134
6.19.	H-Field Coupling	135
6.20.	Gaskets	136
6.21.	Finger Stock	137
6.22.	Glass Apertures	137
6.23.	Guarding Large Transistors	138
6.24.	Mounting Components on Surfaces	139
6.25.	Zappers	139
7. Digita	al Electronics	141
7.1.	Introduction	141
7.2.	Circuit Board Material	143
7.3.	The Two-Sided Circuit Board	143
7.4.	Multilayer Circuit Boards	144
7.5.	Ground Planes and Digital Circuit Boards	146
7.6.	Clocked Logic	149
7.7.	The Transmission of a Single Logic Signal	151
7.8.	Decoupling Capacitors	154
7.9.	The Power Plane	156
7.10.	The Ground and Power Plane Capacitance	157
7.11.	Using Vias	158
7.12.	Decoupling Capacitors as Transmission Lines	159
7.13.	Characteristic Impedance Control	161
7.14.	Radiation from Digital Boards	163
7.15.	Measurement Problems—Ground Bounce	163
7.16.	High Clock Rates	164
7.17.	Balanced Transmission	166
7.18.	Ribbon Cable and Connectors	167
7.19.	Daughter Boards	168
7.20.	Mixing Analog and Digital Circuits	168
7.21.	Optical Isolation	169
7.22.	Gold Plating	169
7.23.	GHz Notes	170
8. Facili	ty Hardware	171
8.1.	Ground Planes	171
8.2.	A Facility Ground Plane Using Stringers	172
8.3.	Other Ground Planes	174
8.4.	Ground Planes and Remote Sites	175
8.5.	Extending Ground Planes	175
8.6.	Separately Derived Power	176

References Index		187
		185
8.11.	A True Story	183
8.10.	Motor Controllers	181
8.9.	Screen Rooms	179
8.8.	The Isolation Transformer	178
8.7.	Surge Protection	178

The first edition of *Grounding and Shielding* was published in 1967. The fourth edition was published in 1997. It is hard to imagine that another decade has passed and that I would be writing on the same subject for a fifth time. This fifth edition is a total rewrite because I wanted to present ideas in a different order and also to present new insight that was gained through teaching seminars and consulting. For example, I wanted to add material on printed circuit board design, a subject that was only touched upon in the fourth edition. I also left out material that was no longer relevant.

This book is intended for those interested in the real world of electronics. The new engineer can easily read this material although he or she may not recognize the importance of the approach or why some material is stressed. Engineers and circuit designers in the analog, digital, or power world who have seen a few problems will appreciate the need to understand interference and all it represents. There is some mathematics involved but it is not necessary that every equation be understood to get a great deal from the content of the book.

I have spent a career in electronics that started with vacuum tubes. I remember the first silicon diodes and the first transistors. I also remember thinking that 10 MHz would be impossible using solid sate devices. So much for impossibilities. Today, logic designers are considering clock speeds of 24 GHz. This time I believe it is possible.

In my early years I worked on handling low-level analog signals and I found out the importance of my physics background. That is what led me to write the first edition. Today, when I see the problems of digital design or facility layout I realize again how important physics is in understanding all electrical design. This is the reason I am writing a fifth edition. There are many new problem areas that need to be discussed. The book starts out with some very elementary physics. I hope the reader does not skip this material because it is at the heart of what is to follow.

There are several ideas that I want to emphasize in this book. The first idea is that a schematic or circuit diagram is only a rough plan or outline. It provides a basis for circuit analysis but it cannot present any geometric information. For example, component size and orientation are missing as are lead lengths and lead dress. As circuits get more complex this geometry is critical. The second idea I want to present is that all components are field operated. A capacitor stores electric field energy and an inductor stores magnetic field energy. A transistor requires a field in the semiconductor material. A transformer works by using both electric and magnetic fields. I want to show that the conductors in our circuits are there to place these fields into the various components. Third, I want to teach that a field in a component means the presence of field energy. Moving this energy around is the purpose of the conductors we use in our circuits. Trying to do this at a GHz is not simple. Understanding transmission lines is key to understanding how energy is moved in all circuits from dc to 100 GHz. Fourth, I want to teach that good engineering is a compromise. There are no perfect solutions. The issue is to solve the problem with available material in an economical manner. The problem is often that we expect too much from circuit symbols. These symbols are really just a starting point in design.

I want to thank the people that were helpful in making this book possible. My wife Elizabeth is a writer in her own right and I have gained from her many comments. I thank my editor George Telecki, who recognized the need for this book and offered me encouragement at every step along the way. The good words put in by Henry Ott and Rick Hartley gave important support. I would like to thank Dan Beeker of Freescale for his help and assistance. He recognizes the need for engineers to continue their education so that they can best use what technology has to offer.

My hope is that this book will provide the reader with a useful viewpoint, one that will solve problems. I hope that the approach taken in this book can find its way into the educational system where I feel it is strongly needed. I further wish that readers who agree with me will work with academia to make the needed changes.

My many thanks to all of you who have bought the early editions of this book. I trust you can gain by reading this fifth edition.

Pacifica, CA

**RALPH MORRISON** 

# Voltage and Capacitance

# **1.1. INTRODUCTION**

How does a circuit work? One answer is to do a sinusoidal analysis using Kirchhoff's laws. Another answer is to write a set of logic statements. These responses provide a small part of the answer. The full answer is buried in a mountain of details. In this book we are going to look at some of this detail but in a non-circuit way. We will take this new approach because circuit diagrams and circuit theory by their very nature must leave out a lot of pertinent detail. This detail is important for performance at high frequencies or for performance involving very small signals. It is also important when radiation, interference, or susceptibility are involved. Wire size, connection sequences, component orientation, and lead dress are often critical details. I like to call these details "circuit geometry." These details in geometry are closely related to how well a circuit works. Geometry is an issue in analog circuits, power circuits, and especially in digital circuits where clock rates are always rising.

When a circuit is put to practice there are many details that we take for granted. The components will most likely be connected together by strips or cylinders of copper. They will be soldered into eyelets or onto copper pads. Traces will go between layers on a printed circuit board using *vias*. These are the details in a design that are not questioned. There are details of a more subtle nature such as the thickness of a trace or ground plane or the dielectric constant of an epoxy board. In most cases we do not question how things are done because we tend to rely on accepted practice. Circuits built this way in the past have worked, so why make changes?

Taking things for granted is not always good engineering. Note that digital clock rates have changed from 1 MHz to 1 GHz in 20 years. That is three orders of magnitude! Imagine what would happen if automobile speeds went up one order of magnitude. That is 600 mph or jet aircraft speed. Even a modest increase in automobile speed would require extensive changes to the design of our roads and cities, not to mention extensive driver training.

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## 2 VOLTAGE AND CAPACITANCE

In electronics, an increase in speed does not pose a safety hazard. There are, however, differences in performance that should be understood. Often an effect is not sensed until the next generation in design is introduced. Understanding these effects requires an understanding of basic principles. The details we will look into do not appear on a circuit or schematic diagram. We will address many of these details that affect performance.

Electronics often makes use of power from the local utility. For reasons of safety the utility must earth one of the power conductors. Electronic hardware must interface with this power and also share this same earth connection. The result is often interference. I will discuss the relation between power distribution and circuit performance in later chapters.

A circuit diagram is only a plan or an organization of ideas. Circuit theory provides a basic overview of circuit performance. Circuit symbols are a part of the problem. They are necessarily very simple representations of complex objects. Every capacitor has a series resistance and an inductance and every inductor has series resistance and shunt capacitance. These considerations only begin to tell the entire story. For example, at high frequencies, dielectrics are nonlinear. For magnetic materials, permeability falls off with frequency. Thus circuit symbols can only convey limited information. Further, we do not have symbols for skin effect, transit time, radiation, or current flow patterns. A straight line on a diagram may actually be a very complex path in the actual circuit. In short, a schematic diagram provides little information on physical structure and this can limit our appreciation of what is actually happening.

The performance of many circuits or systems is closely related to how they are built. This is true for analog circuits as well as computers. I have often commented that it is not a question of whether there is an electrical connection but where the connection is to be made. In an analog circuit it is often important to know which end of a shield is grounded, not whether it is grounded. Here is good question! How should a circuit board ground plane be connected to the surrounding chassis? The answer to this question is not available from a schematic diagram. We will discuss this problem in the digital circuits chapter (Chapter 7).

A repeated theme discussed in this book relates to how signals and power are transported in circuits. This approach will lead to an understanding of many issues that are often poorly understood. In order to discuss the transport of electrical power and signals, the electric and magnetic fields must be discussed. To begin this discussion we introduce the electron. Don't despair. The time spent reviewing this area of physics will make it much easier to understand the ideas presented later in this book.

# **1.2. CHARGES AND ELECTRONS**

Circuit theory allows us to relate circuit voltages and the flow of current in a group of interconnected components. For RLC networks (ResistorInductor-Capacitor) this analysis is straightforward using Kirkkhoff's laws. The processes I want to discuss do not involve this approach. To understand the more subtle aspects of circuit performance we will use basic physics to explain many details that are often left to chance. Our starting point may seem a bit remote, but please read on. We first discuss the atom.

Atoms are composed of a nucleus of protons and neutrons surrounded by shells of electrons. The electrons have a negative charge and the matching protons in the nucleus have a positive charge. In a neutral atom the positive and negative charges are exactly equal. Each electronic shell is limited to a fixed number of electrons. The number of electrons in the outer shell says a lot about the character of the atom. As an example, copper has just one electron in its outer shell. This outer electron has a great deal of mobility and is involved in electrical activity. Because protons are comparatively heavy and because the shells of electrons shield them, they are not directly involved in the electronics we are going to consider.

Molecules are formed from atoms that bond together and share outer shell electrons. For an insulator, this bonding limits the outer electron mobility. Typical insulators might be nylon, air, epoxy, or glass. If two insulators are rubbed together, such as a silk cloth and a rubber wand, the difference in the mobility of the outer electrons will allow the transfer of a few electrons from the rod to the cloth. In this case, the silk cloth with extra electrons is called a negatively charged body. The rod is said to be positively charged. We will call the absence of negative charge a positive charge. In reality the positive charge stems from the immobile protons in the nucleus of atoms that do not have matching outer shell electrons. The absence of negative charges is the same as if there were fictitious positive charges on the surface of the insulator.

Experiments with charged bodies can demonstrate the nature of the forces that exist between charges (electrons). If one charged body repels another it is actually the fields of electrons that are involved. If you remember your physics class, these forces can be demonstrated using pith balls that hang by a string. Here the charges are attached to small masses and we can see the pith balls attract or repel each other.

The number of electrons involved in any of these experiments is extremely small. To illustrate this point I want to paraphrase the writing of Dr. Richard Feynman.<sup>1</sup> If two people are standing a few feet apart, what would be the force of repulsion if 1% of the electrons in each body were to repel each other? Would it be a few pounds? More! Would it be greater than their weight? More! Would it lift a building? More! Would it lift a mountain? The answer is astounding. The force would be great enough to lift the earth out of orbit. This is why gravity is called a weak force and the force between electrons is called a strong force. This also tells us something about nature. The percentage

<sup>&</sup>lt;sup>1</sup> *The Feynman Lectures on Physics*, Volume 2, p. 1-1. Addison-Wesley Publishing Company, Inc. Copyright 1964, California Institute of Technology.

of electrons involved in electrical activity is extremely small. We know that the forces in a circuit do not move the components or the traces. Obviously, since electrical forces are so large, electrical activity involves a very small percentage of the available electrons.

# **1.3. THE ELECTRIC FORCE FIELD**

When we encounter forces at a distance we us the expression *force field*. We experience a force field at all times as we live in the gravitational force field of the earth. Every mass has a force field, including the earth. The earth has the dominant field because the earth is so massive. The result is that each mass on earth is attracted toward the center of the earth. The attraction forces between individual objects on the earth are so small that they are very difficult to measure. On the earth's surface the force field is nearly constant. We would have to go out into space to see a reduction in the force of gravity.

The electrical force field is similar in many ways. Every electron carries with it an associated force field. This force field repels every other electron in the area. If a group of extra electrons are located on an isolated mass we call this mass a *charged body*. We refer to the extra electrons as a *charge*. If this mass is a conductor, the extra electrons will move apart until there is a balance of forces. On a conducting isolated sphere the electrons will move until they are evenly spaced over the entire outer surface. None of these excess electrons will remain on the inside the conductor. For a perfect insulator any extra electrons are not free to move about. Extra electrons on the inside of this material are called *trapped electrons*. It is also possible to have trapped absences of electrons. We can call these *holes*.

# **1.4. FIELD REPRESENTATIONS**

The electric force field in a volume of space can be measured by placing a small test charge in that space. A test charge can be formed using a small mass with a small excess of electrons on its surface. The force on this test charge has a magnitude and direction at each point in space. Having direction the force field is called a vector field. To be effective this test charge must be small enough so that it does not influence the charge distribution on the objects being measured. Performing this experiment is difficult, but fortunately we can deduce the field pattern without performing an actual test.

It is convenient to represent a force field by lines that follow the direction of the force. For an isolated conducting charged sphere, the lines of force are shown in Figure 1.1.



Figure 1.1. The force field lines around a positively charged conducting sphere

Note that the field exists everywhere between the lines. The lines are simply a way of showing the flow or shape of the field. In any practical example, the number of extra electrons that form the charge Q on the surface of a conductor is small compared to the number of electrons in the conductor. For a practical surface charge, the number of electrons is still so large that we can consider the charge as being continuously distributed over the surface of interest. This is the reason we will not consider the force field as resulting from individual electrons. From here on we will consider all charge distributions as being continuous. The total charge on the surface of the sphere in Figure 1.1 is Q. The charge density on the surface of the sphere is

$$\frac{Q}{A} = \frac{Q}{4\pi r^2}.$$
(1.1)

We will use the convention that a line starts on a unit of positive charge and terminates on a unit of negative charge. This unit can be selected so that the graphical representation of the field is useful. If the total charge is doubled, then the number of lines is doubled. For representations in this book, no attempt will be made to relate the number of lines to any specific amount of charge. In general we are interested in the shape of the field, areas of field concentration, and where the field lines terminate.

In Figure 1.1 the force on a small test charge q in the field of Q is proportional to the product of the two charges and inverse to the square of radius r or

$$f = \frac{qQ}{4\pi\varepsilon_{\rm o}r^2} = \frac{qQ}{\varepsilon_{\rm o}A} \tag{1.2}$$

where A is the area of the sphere. The constant  $\varepsilon_0$  is the permittivity of free space. Equation (1.2) is known as Coulomb's law. The force per unit charge or f/q is a measure of the electric field intensity. The letter *E* is used for this measure. The force field around a group of charges is referred to as an E field. Mathematically the E field around a charge is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$
 (1.3)

The E field falls off as the square of the distance r. Equation (1.3) could also be written as

$$E = \frac{Q}{\varepsilon_{o} A}$$
(1.4)

where A is the surface area of the sphere at the distance r. In Figure 1.1, the force field intensity E decreases as the field lines diverge. The forces are greatest at the surface of the sphere. Note that the field lines do not enter the sphere. This is because there are no excess charges inside the conductor. The field lines must terminate on the sphere perpendicular to its surface. If there were a tangential component of force on the surface the charges on the surface would be accelerated. If there were an absence of electrons on the surface, this absence of charge would also be accelerated. Remember the absence of negative charge can be considered the presence of a positive charge. For conductors, the mobility of a group of electrons is no different from the mobility of an absence of electrons. Except for the direction assigned to the force field, we will assume that positive and negative charges behave the same way. Figure 1.1 shows a sphere with a positive charge Q. If the charge were negative (the presence of electrons), the field lines would be shown with the arrows pointing inward.

The field lines in Figure 1.1 start at the surface of the sphere. If the charge Q were located at the center of the sphere and the sphere were removed, the field pattern at every value of r would be unchanged. A point charge Q implies an infinite charge density, which is impossible. Often it is mathematically convenient to consider the fields from point charges even though this can't exist.

# **1.5. THE DEFINITION OF VOLTAGE**

A test charge q in the field of a charge Q experiences a force given by Eq. (1.2). The work required to move the test charge a small distance  $\Delta d$  is  $f(\Delta d)$ .

The work to move it from infinity to a point  $r_1$  is the integral of force times distance from infinity to  $r_1$ . If we follow one of the field lines, the force is always tangent to the field lines. The work is

$$W = \int_{\infty}^{n} f \cdot dr = -\frac{qQ}{4\pi\varepsilon_{0}r_{1}}.$$
(1.5)

If we divide both sides of this equation by q we obtain the work per unit charge. This term has the familiar name *volts*. In equation form the voltage is given by

$$V = -\frac{Q}{4\pi\varepsilon_o r}.$$
 (1.6)

## DEFINITION

**Voltage difference.** The work required to move a unit charge between two points in space in an electric field.

In Eq. (1.5) we can make the assumption that the voltage at infinity is zero. This allows us to assign a voltage to points in space. In a circuit, the work required to move a unit charge between two conducting surfaces is called a potential difference or a voltage difference. It is important to realize that potential differences do exist between points in space. Of course it is difficult to place a voltmeter in space to get a measure of this voltage.

The voltage difference between two points in space is

$$V_2 - V_1 = \frac{1}{4\pi\varepsilon_o} \left( \frac{1}{r_2} - \frac{1}{r_1} \right).$$
(1.7)

# N.B.

A voltage difference cannot exist without the presence of an electric field.

In the presence of conductors, an electric field cannot exist without charges on the surface of these conductors. These charges are not apparent from a schematic diagram. When a circuit is in operation, there are surface charges everywhere there are voltage differences. These charges are independent of the moving charges in the circuit that we call *current*.