QUANTITATIVE INVESTMENT ANALYSIS WORKBOOK

Second Edition

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John Wiley & Sons, Inc.

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PART I

LEARNING OUTCOMES, SUMMARY OVERVIEW, AND PROBLEMS

CHAPTER 1

THE TIME VALUE OF MONEY

LEARNING OUTCOMES

After reading chapter 1, you should be able to do the following:

- Explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for distinct types of risk.
- Calculate the future value (FV) or present value (PV) of a single sum of money.
- Distinguish between the stated annual interest rate and the effective annual rate.
- Calculate the effective annual rate, given the stated annual interest rate and the frequency of compounding.
- Solve time value of money problems when compounding periods are other than annual.
- Calculate the FV or PV of an ordinary annuity and an annuity due.
- Calculate the PV of a perpetuity.
- Calculate an unknown variable, given the other relevant variables, in time value of money problems.
- Calculate the FV or the PV of a series of uneven cash flows.
- Draw a time line, specify a time index, and solve problems involving the time value of money as applied, for example, to mortgages and savings for college tuition or retirement.
- Explain the cash flow additivity principle in time value of money applications.

SUMMARY OVERVIEW

In chapter 1, we have explored a foundation topic in investment mathematics, the time value of money. We have developed and reviewed the following concepts for use in financial applications:

- The interest rate, r, is the required rate of return; r is also called the discount rate or opportunity cost.
- An interest rate can be viewed as the sum of the real risk-free interest rate and a set of premiums that compensate lenders for risk: an inflation premium, a default risk premium, a liquidity premium, and a maturity premium.
- The future value, FV, is the present value, PV, times the future value factor, $(1 + r)^N$.
- The interest rate, r, makes current and future currency amounts equivalent based on their time value.

- The stated annual interest rate is a quoted interest rate that does not account for compounding within the year.
- The periodic rate is the quoted interest rate per period; it equals the stated annual interest rate divided by the number of compounding periods per year.
- The effective annual rate is the amount by which a unit of currency will grow in a year with interest on interest included.
- An annuity is a finite set of level sequential cash flows.
- There are two types of annuities, the annuity due and the ordinary annuity. The annuity due has a first cash flow that occurs immediately; the ordinary annuity has a first cash flow that occurs one period from the present (indexed at $t = 1$).
- On a time line, we can index the present as 0 and then display equally spaced hash marks to represent a number of periods into the future. This representation allows us to index how many periods away each cash flow will be paid.
- Annuities may be handled in a similar fashion as single payments if we use annuity factors instead of single-payment factors.
- The present value, PV, is the future value, FV, times the present value factor, $(1 + r)^{-N}$.
- The present value of a perpetuity is A/r, where A is the periodic payment to be received forever.
- It is possible to calculate an unknown variable, given the other relevant variables in time value of money problems.
- The cash flow additivity principle can be used to solve problems with uneven cash flows by combining single payments and annuities.

PROBLEMS

1. The table below gives current information on the interest rates for two two-year and two eight-year maturity investments. The table also gives the maturity, liquidity, and default risk characteristics of a new investment possibility (Investment 3). All investments promise only a single payment (a payment at maturity). Assume that premiums relating to inflation, liquidity, and default risk are constant across all time horizons.

Investment	Maturity (in years)	Liquidity	Default Risk	Interest Rate (%)
1	2	High	Low	2.0
2	2	Low	Low	2.5
3	7	Low	Low	r_3
4	8	High	Low	4.0
5	8	Low	High	6.5

Based on the information in the above table, address the following:

A. Explain the difference between the interest rates on Investment 1 and Investment 2.
B. Estimate the default risk premium.
C. Calculate upper and lower limits for the interest rate on Investment 3, r_3.

2. A client has a $5 million portfolio and invests 5 percent of it in a money market fund projected to earn 3 percent annually. Estimate the value of this portion of his portfolio after seven years.
3. A client invests $500,000 in a bond fund projected to earn 7 percent annually. Estimate the value of her investment after 10 years.
4. For liquidity purposes, a client keeps $100,000 in a bank account. The bank quotes a stated annual interest rate of 7 percent. The bank's service representative explains that the stated rate is the rate one would earn if one were to cash out rather than invest the interest payments. How much will your client have in his account at the end of one year, assuming no additions or withdrawals, using the following types of compounding?

   A. Quarterly
   B. Monthly
   C. Continuous

5. A bank quotes a rate of 5.89 percent with an effective annual rate of 6.05 percent. Does the bank use annual, quarterly, or monthly compounding?
6. A bank pays a stated annual interest rate of 8 percent. What is the effective annual rate using the following types of compounding?

   A. Quarterly
   B. Monthly
   C. Continuous

7. A couple plans to set aside $20,000 per year in a conservative portfolio projected to earn 7 percent a year. If they make their first savings contribution one year from now, how much will they have at the end of 20 years?
8. Two years from now, a client will receive the first of three annual payments of $20,000 from a small business project. If she can earn 9 percent annually on her investments and plans to retire in six years, how much will the three business project payments be worth at the time of her retirement?
9. To cover the first year's total college tuition payments for his two children, a father will make a $75,000 payment five years from now. How much will he need to invest today to meet his first tuition goal if the investment earns 6 percent annually?
10. A client has agreed to invest €100,000 one year from now in a business planning to expand, and she has decided to set aside the funds today in a bank account that pays 7 percent compounded quarterly. How much does she need to set aside?
11. A client can choose between receiving 10 annual $100,000 retirement payments, starting one year from today, or receiving a lump sum today. Knowing that he can invest at a rate of 5 percent annually, he has decided to take the lump sum. What lump sum today will be equivalent to the future annual payments?
12. A perpetual preferred stock position pays quarterly dividends of $1,000 indefinitely (forever). If an investor has a required rate of return of 12 percent per year on this type of investment, how much should he be willing to pay for this dividend stream?
13. At retirement, a client has two payment options: a 20-year annuity at €50,000 per year starting after one year or a lump sum of €500,000 today. If the client's required rate of return on retirement fund investments is 6 percent per year, which plan has the higher present value and by how much?

14. You are considering investing in two different instruments. The first instrument will pay nothing for three years, but then it will pay $20,000 per year for four years. The second instrument will pay $20,000 for three years and $30,000 in the fourth year. All payments are made at year-end. If your required rate of return on these investments is 8 percent annually, what should you be willing to pay for:

    A. The first instrument
    B. The second instrument (use the formula for a four-year annuity)

15. Suppose you plan to send your daughter to college in three years. You expect her to earn two-thirds of her tuition payment in scholarship money, so you estimate that your payments will be $10,000 a year for four years. To estimate whether you have set aside enough money, you ignore possible inflation in tuition payments and assume that you can earn 8 percent annually on your investments. How much should you set aside now to cover these payments?
16. A client is confused about two terms on some certificate-of-deposit rates quoted at his bank in the United States. You explain that the stated annual interest rate is an annual rate that does not take into account compounding within a year. The rate his bank calls APY (annual percentage yield) is the effective annual rate taking into account compounding. The bank's customer service representative mentioned monthly compounding, with $1,000 becoming $1,061.68 at the end of a year. To prepare to explain the terms to your client, calculate the stated annual interest rate that the bank must be quoting.
17. A client seeking liquidity sets aside €35,000 in a bank account today. The account pays 5 percent compounded monthly. Because the client is concerned about the fact that deposit insurance covers the account for only up to €100,000, calculate how many months it will take to reach that amount.
18. A client plans to send a child to college for 4 years starting 18 years from now. Having set aside money for tuition, she decides to plan for room and board also. She estimates these costs at $20,000 per year, payable at the beginning of each year, by the time her child goes to college. If she starts next year and makes 17 payments into a savings account paying 5 percent annually, what annual payments must she make?
19. A couple plans to pay their child's college tuition for 4 years starting 18 years from now. The current annual cost of college is C$7,000, and they expect this cost to rise at an annual rate of 5 percent. In their planning, they assume that they can earn 6 percent annually. How much must they put aside each year, starting next year, if they plan to make 17 equal payments?
20. You are analyzing the last five years of earnings per share data for a company. The figures are $4.00, $4.50, $5.00, $6.00, and $7.00. At what compound annual rate did EPS grow during these years?

CHAPTER 2

DISCOUNTED CASH FLOW APPLICATIONS

LEARNING OUTCOMES

After reading chapter 2, you should be able to do the following:

- Calculate and interpret the net present value (NPV) and the internal rate of return (IRR) of an investment.
- Contrast the NPV rule to the IRR rule.
- Distinguish between money-weighted and time-weighted rates of return.
- Calculate the money-weighted and time-weighted rates of return of a portfolio.
- Calculate bank discount yield, holding period yield, effective annual yield, and money market yield for a U.S. Treasury bill.
- Convert among holding period yields, money market yields, and effective annual yields.
- Calculate bond-equivalent yield.

SUMMARY OVERVIEW

In chapter 2, we applied the concepts of present value, net present value, and internal rate of return to the fundamental problem of valuing investments. We applied these concepts first to corporate investment, the well-known capital budgeting problem. We then examined the fundamental problem of calculating the return on a portfolio subject to cash inflows and outflows. Finally we discussed money market yields and basic bond market terminology. The following summarizes the chapter's key concepts:

- The net present value (NPV) of a project is the present value of its cash inflows minus the present value of its cash outflows. The internal rate of return (IRR) is the discount rate that makes NPV equal to 0. We can interpret IRR as an expected compound return only when all interim cash flows can be reinvested at the internal rate of return and the investment is maintained to maturity.
- The NPV rule for decision making is to accept all projects with positive NPV or, if projects are mutually exclusive, to accept the project with the higher positive NPV. With mutually exclusive projects, we rely on the NPV rule. The IRR rule is to accept all projects with an internal rate of return exceeding the required rate of return. The IRR rule can be affected by problems of scale and timing of cash flows.
- Money-weighted rate of return and time-weighted rate of return are two alternative methods for calculating portfolio returns in a multiperiod setting when the portfolio is subject to

additions and withdrawals. Time-weighted rate of return is the standard in the investment management industry. Money-weighted rate of return can be appropriate if the investor exercises control over additions and withdrawals to the portfolio.

- The money-weighted rate of return is the internal rate of return on a portfolio, taking account of all cash flows.
- The time-weighted rate of return removes the effects of timing and amount of withdrawals and additions to the portfolio and reflects the compound rate of growth of one unit of currency invested over a stated measurement period.
- The bank discount yield for U.S. Treasury bills (and other money-market instruments sold on a discount basis) is given by $r_{BD} = (F - P_0)/F \times 360/t = D/F \times 360/t$, where F is the face amount to be received at maturity, P_0 is the price of the Treasury bill, t is the number of days to maturity, and D is the dollar discount.
- For a stated holding period or horizon, holding period yield (HPY) = (Ending price − Beginning price + Cash distributions)/(Beginning price). For a U.S. Treasury bill, HPY = D/P_0.
- The effective annual yield (EAY) is $(1 + HPY)^{365/t} - 1$.
- The money market yield is given by $r_{MM} = \text{HPY} \times 360/t$, where t is the number of days to maturity.
- For a Treasury bill, money market yield can be obtained from the bank discount yield using $r_{MM} = (360 \times r_{BD})/(360 - t \times r_{BD})$.
- We can convert back and forth between holding period yields, money market yields, and equivalent annual yields by using the holding period yield, which is common to all the calculations.
- The bond-equivalent yield of a yield stated on a semiannual basis is that yield multiplied by 2.

PROBLEMS

1. Waldrup Industries is considering a proposal for a joint venture that will require an investment of C$13 million. At the end of the fifth year, Waldrup's joint venture partner will buy out Waldrup's interest for C$10 million. Waldrup's chief financial officer has estimated that the appropriate discount rate for this proposal is 12 percent. The expected cash flows are given below.

Year	Cash Flow
0	− C$13,000,000
1	C$3,000,000
2	C$3,000,000
3	C$3,000,000
4	C$3,000,000
5	C$10,000,000

   A. Calculate this proposal's NPV.
   B. Make a recommendation to the CFO (chief financial officer) concerning whether Waldrup should enter into this joint venture.

2. Waldrup Industries has committed to investing C$5,500,000 in a project with expected cash flows of C$1,000,000 at the end of Year 1, C$1,500,000 at the end of Year 4, and C$7,000,000 at the end of Year 5.

   A. Demonstrate that the internal rate of return of the investment is 13.51 percent.
   B. State how the internal rate of return of the investment would change if Waldrup's opportunity cost of capital were to increase by 5 percentage points.

3. Bestfoods, Inc. is planning to spend $10 million on advertising. The company expects this expenditure to result in annual incremental cash flows of $1.6 million in perpetuity. The corporate opportunity cost of capital for this type of project is 12.5 percent.

   A. Calculate the NPV for the planned advertising.
   B. Calculate the internal rate of return.
   C. Should the company go forward with the planned advertising? Explain.

4. Trilever is planning to establish a new factory overseas. The project requires an initial investment of $15 million. Management intends to run this factory for six years and then sell it to a local entity. Trilever's finance department has estimated the following yearly cash flows:

Year	Cash Flow
0	−$15,000,000
1	$4,000,000
2	$4,000,000
3	$4,000,000
4	$4,000,000
5	$4,000,000
6	$7,000,000

   Trilever's CFO decides that the company's cost of capital of 19 percent is an appropriate hurdle rate for this project.

   A. Calculate the internal rate of return of this project.
   B. Make a recommendation to the CFO concerning whether to undertake this project.

5. Westcott–Smith is a privately held investment management company. Two other investment counseling companies, which want to be acquired, have contacted Westcott–Smith about purchasing their business. Company A's price is £2 million. Company B's price is £3 million. After analysis, Westcott–Smith estimates that Company A's profitability is consistent with a perpetuity of £300,000 a year. Company B's prospects are consistent with a perpetuity of £435,000 a year. Westcott–Smith has a budget that limits acquisitions to a maximum purchase cost of £4 million. Its opportunity cost of capital relative to undertaking either project is 12 percent.

   A. Determine which company or companies (if any) Westcott–Smith should purchase according to the NPV rule.

B. Determine which company or companies (if any) Westcott–Smith should purchase according to the IRR rule.
C. State which company or companies (if any) Westcott–Smith should purchase. Justify your answer.

6. John Wilson buys 150 shares of ABM on 1 January 2002 at a price of $156.30 per share. A dividend of $10 per share is paid on 1 January 2003. Assume that this dividend is not reinvested. Also on 1 January 2003, Wilson sells 100 shares at a price of $165 per share. On 1 January 2004, he collects a dividend of $15 per share (on 50 shares) and sells his remaining 50 shares at $170 per share.

A. Write the formula to calculate the money-weighted rate of return on Wilson's portfolio.
B. Using any method, compute the money-weighted rate of return.
C. Calculate the time-weighted rate of return on Wilson's portfolio.
D. Describe a set of circumstances for which the money-weighted rate of return is an appropriate return measure for Wilson's portfolio.
E. Describe a set of circumstances for which the time-weighted rate of return is an appropriate return measure for Wilson's portfolio.

7. Mario Luongo and Bob Weaver both purchase the same stock for €100. One year later, the stock price is €110 and it pays a dividend of €5 per share. Weaver decides to buy another share at €110 (he does not reinvest the €5 dividend, however). Luongo also spends the €5 per share dividend but does not transact in the stock. At the end of the second year, the stock pays a dividend of €5 per share but its price has fallen back to €100. Luongo and Weaver then decide to sell their entire holdings of this stock. The performance for Luongo and Weaver's investments are as follows:

Luongo: Time-weighted return = 4.77 percent

Money-weighted return = 5.00 percent

Weaver: Money-weighted return = 1.63 percent

Briefly explain any similarities and differences between the performance of Luongo's and Weaver's investments.

8. A Treasury bill with a face value of $100,000 and 120 days until maturity is selling for $98,500.

A. What is the T-bill's bank discount yield?
B. What is the T-bill's money market yield?
C. What is the T-bill's effective annual yield?

9. Jane Cavell has just purchased a 90-day U.S. Treasury bill. She is familiar with yield quotes on German Treasury discount paper but confused about the bank discount quoting convention for the U.S. T-bill she just purchased.

A. Discuss three reasons why bank discount yield is not a meaningful measure of return.
B. Discuss the advantage of money market yield compared with bank discount yield as a measure of return.
C. Explain how the bank discount yield can be converted to an estimate of the holding period return Cavell can expect if she holds the T-bill to maturity.

CHAPTER 3

STATISTICAL CONCEPTS AND MARKET RETURNS

LEARNING OUTCOMES

After reading chapter 3, you should be able to do the following:

- Differentiate between a population and a sample.
- Explain the concepts of a parameter and a sample statistic.
- Explain the differences among the types of measurement scales.
- Define and interpret a frequency distribution.
- Define, calculate, and interpret a holding period return (total return).
- Calculate relative frequencies and cumulative relative frequencies, given a frequency distribution.
- Describe the properties of data presented as a histogram or a frequency polygon.
- Define, calculate, and interpret measures of central tendency, including the arithmetic mean, population mean, sample mean, weighted mean, geometric mean, harmonic mean, median, and mode.
- Describe and interpret quartiles, quintiles, deciles, and percentiles.
- Define, calculate, and interpret (1) a weighted average or mean (including portfolio return viewed as weighted mean), (2) a range and mean absolute deviation, (3) a sample and a population variance and standard deviation.
- Contrast variance with semivariance and target semivariance.
- Calculate the proportion of observations falling within a certain number of standard deviations of the mean, using Chebyshev's inequality.
- Define, calculate, and interpret the coefficient of variation.
- Define, calculate, and interpret the Sharpe ratio.
- Describe the relative locations of the mean, median, and mode for a nonsymmetrical distribution.
- Define and interpret skew, and explain the meaning of a positively or negatively skewed return distribution.
- Define and interpret kurtosis and explain the meaning of kurtosis in excess of 3.
- Describe and interpret sample measures of skew and kurtosis.

SUMMARY OVERVIEW

In chapter 3, we have presented descriptive statistics, the set of methods that permit us to convert raw data into useful information for investment analysis.

- A population is defined as all members of a specified group. A sample is a subset of a population.
- A parameter is any descriptive measure of a population. A sample statistic (statistic, for short) is a quantity computed from or used to describe a sample.
- Data measurements are taken using one of four major scales: nominal, ordinal, interval, or ratio. Nominal scales categorize data but do not rank them. Ordinal scales sort data into categories that are ordered with respect to some characteristic. Interval scales provide not only ranking but also assurance that the differences between scale values are equal. Ratio scales have all the characteristics of interval scales as well as a true zero point as the origin. The scale on which data are measured determines the type of analysis that can be performed on the data.
- A frequency distribution is a tabular display of data summarized into a relatively small number of intervals. Frequency distributions permit us to evaluate how data are distributed.
- The relative frequency of observations in an interval is the number of observations in the interval divided by the total number of observations. The cumulative relative frequency cumulates (adds up) the relative frequencies as we move from the first interval to the last, thus giving the fraction of the observations that are less than the upper limit of each interval.
- A histogram is a bar chart of data that have been grouped into a frequency distribution. A frequency polygon is a graph of frequency distributions obtained by drawing straight lines joining successive points representing the class frequencies.
- Sample statistics such as measures of central tendency, measures of dispersion, skewness, and kurtosis help with investment analysis, particularly in making probabilistic statements about returns.
- Measures of central tendency specify where data are centered and include the (arithmetic) mean, median, and mode (most frequently occurring value). The mean is the sum of the observations divided by the number of observations. The median is the value of the middle item (or the mean of the values of the two middle items) when the items in a set are sorted into ascending or descending order. The mean is the most frequently used measure of central tendency. The median is not influenced by extreme values and is most useful in the case of skewed distributions. The mode is the only measure of central tendency that can be used with nominal data.
- A portfolio's return is a weighted mean return computed from the returns on the individual assets, where the weight applied to each asset's return is the fraction of the portfolio invested in that asset.
- The geometric mean, G, of a set of observations $X_1, X_2, \ldots X_n$ is $G = \sqrt[n]{X_1 X_2 X_3 \ldots X_n}$ with $X_i \geq 0$ for $i = 1, 2, \ldots, n$. The geometric mean is especially important in reporting compound growth rates for time series data.
- Quantiles such as the median, quartiles, quintiles, deciles, and percentiles are location parameters that divide a distribution into halves, quarters, fifths, tenths, and hundredths, respectively.
- Dispersion measures such as the variance, standard deviation, and mean absolute deviation (MAD) describe the variability of outcomes around the arithmetic mean.
- Range is defined as the maximum value minus the minimum value. Range has only a limited scope because it uses information from only two observations.
- MAD for a sample is $\dfrac{\sum_{i=1}^{n} \left|X_i - \overline{X}\right|}{n}$ where $\overline{X}$ is the sample mean and n is the number of observations in the sample.