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DOMINIQUE PLACKO AND TRIBIKRAM KUNDU
DPSM FOR MODELING
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DPSM FOR MODELING ENGINEERING PROBLEMS

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We would like to dedicate this work to our wives:
Monique and Nupur

our daughters:
Amélie, Anne-Laure, Ina, and Auni

and our parents
CONTENTS

Preface xv

Contributors xix

Chapter 1 – Basic Theory of Distributed Point Source Method (DPSM) and Its Application to Some Simple Problems 1

D. Placko and T. Kundu

1.1 Introduction and Historical Development of DPSM, 1
1.2 Basic Principles of DPSM Modeling, 3
  1.2.1 The fundamental idea, 3
    1.2.1.1 Basic equations, 6
    1.2.1.2 Boundary conditions, 8
  1.2.2 Example in the case of a magnetic open core sensor, 9
    1.2.2.1 Governing equations and solution, 9
    1.2.2.2 Solution of coupling equations, 11
    1.2.2.3 Results and discussion, 13
1.3 Examples From Ultrasonic Transducer Modeling, 16
  1.3.1 Justification of modeling a finite plane source by a distribution of point sources, 17
  1.3.2 Planar piston transducer in a fluid, 18
    1.3.2.1 Conventional surface integral technique, 18
    1.3.2.2 Alternative DPSM for computing the ultrasonic field, 20
    1.3.2.3 Restrictions on $r_s$ for point source distribution, 29
  1.3.3 Focused transducer in a homogeneous fluid, 31
1.3.4 Ultrasonic field in a nonhomogeneous fluid in the presence of an interface, 32
  1.3.4.1 Pressure field computation in fluid 1 at point P, 33
  1.3.4.2 Pressure field computation in fluid 2 at point Q, 35
1.3.5 DPSM technique for ultrasonic field modeling in nonhomogeneous fluid, 38
  1.3.5.1 Field computation in fluid 1, 38
  1.3.5.2 Field in fluid 2, 42
1.3.6 Ultrasonic field in the presence of a scatterer, 43
1.3.7 Numerical results, 45
  1.3.7.1 Ultrasonic field in a homogeneous fluid, 45
  1.3.7.2 Ultrasonic field in a nonhomogeneous fluid – DPSM technique, 50
  1.3.7.3 Ultrasonic field in a nonhomogeneous fluid – surface integral method, 52
  1.3.7.4 Ultrasonic field in the presence of a finite-size scatterer, 53
References, 57

Chapter 2–Advanced Theory of DPSM—Modeling Multilayered Medium and Inclusions of Arbitrary Shape 59
T. Kundu and D. Placko

2.1 Introduction, 59
2.2 Theory of Multilayered Medium Modeling, 60
  2.2.1 Transducer faces not coinciding with any interface, 60
    2.2.1.1 Source strength determination from boundary and interface conditions, 62
  2.2.2 Transducer faces coinciding with the interface – case 1: transducer faces modeled separately, 64
    2.2.2.1 Source strength determination from interface and boundary conditions, 65
    2.2.2.2 Counting number of equations and number of unknowns, 68
  2.2.3 Transducer faces coinciding with the interface – case 2: transducer faces are part of the interface, 68
    2.2.3.1 Source strength determination from interface and boundary conditions, 69
  2.2.4 Special case involving one interface and one transducer only, 71
2.3 Theory for Multilayered Medium Considering the Interaction Effect on the Transducer Surface, 76
  2.3.1 Source strength determination from interface conditions, 78
  2.3.2 Counting number of equations and number of unknowns, 80
2.4 Interference between Two Transducers: Step-by-Step Analysis of Multiple Reflection, 80
2.5 Scattering by an Inclusion of Arbitrary Shape, 83
2.6 Scattering by an Inclusion of Arbitrary Shape – An Alternative Approach, 85
2.7 Electric Field in a Multilayered Medium, 87
2.8 Ultrasonic Field in a Multilayered Fluid Medium, 91
   2.8.1 Ultrasonic field developed in a three-layered medium, 93
   2.8.2 Ultrasonic field developed in a four-layered fluid medium, 94
Reference, 96

Chapter 3 – Ultrasonic Modeling in Fluid Media

T. Kundu, R. Ahmad, N. Alnauaimi, and D. Placko

3.1 Introduction, 97
3.2 Primary (Active) and Secondary (Passive) Sources, 100
3.3 Modeling Ultrasonic Transducers of Finite Dimension Immersed in a Homogeneous Fluid, 100
   3.3.1 Numerical results—ultrasonic transducers of finite dimension immersed in fluid, 107
3.4 Modeling Ultrasonic Transducers of Finite Dimension Immersed in a Nonhomogeneous Fluid, 111
   3.4.1 Obtaining the strengths of active and passive source layers, 112
      3.4.1.1 Computation of the source strength vectors when multiple reflections between the transducer and the interface are ignored, 113
      3.4.1.2 Computation of the source strength vectors considering the interaction effects between the transducer and the interface, 114
   3.4.2 Numerical results—ultrasonic transducer immersed in nonhomogeneous fluid, 116
3.5 Reflection at a Fluid–Solid Interface—Ignoring Multiple Reflections Between the Transducer Surface and the Interface, 117
   3.5.1 Numerical results for fluid–solid interface, 118
3.6 Modeling Ultrasonic Field in Presence of a Thin Scatterer of Finite Dimension, 118
3.7 Modeling Ultrasonic Field inside a Multilayered Fluid Medium, 120
3.8 Modeling Phased Array Transducers Immersed in a Fluid, 121
   3.8.1 Description and use of phased array transducers, 121
   3.8.2 Theory of phased array transducer modeling, 122
   3.8.3 Dynamic focusing and time lag determination, 124
   3.8.4 Interaction between two transducers in a homogeneous fluid, 125
   3.8.5 Numerical results for phased array transducer modeling, 126
      3.8.5.1 Dynamic steering and focusing, 127
      3.8.5.2 Interaction between two phased array transducers placed face to face, 129
3.9 Summary, 140
Reference, 141
Chapter 4 – Advanced Applications of Distributed Point Source Method – Ultrasonic Field Modeling in Solid Media

Sourav Banerjee and Tribikram Kundu

4.1 Introduction, 143

4.2 Calculation of Displacement and Stress Green’s Functions in Solids, 144
   4.2.1 Point source excitation in a solid, 145
   4.2.2 Calculation of displacement Green’s function, 147
   4.2.3 Calculation of stress Green’s function, 148

4.3 Elemental Point Source in a Solid, 149
   4.3.1 Displacement and stress Green’s functions, 150
   4.3.2 Differentiation of displacement Green’s function
       with respect to $x_1, x_2, x_3$, 151
   4.3.3 Computation of displacements and stresses in the solid
       for multiple point sources, 153
   4.3.4 Matrix representation, 155

4.4 Calculation of Pressure and Displacement Green’s Functions
   in the Fluid Adjacent to the Solid Half Space, 157
   4.4.1 Displacement and potential Green’s functions in the fluid, 158
   4.4.2 Computation of displacement and pressure in the fluid, 159
   4.4.3 Matrix representation, 161

4.5 Application 1: Ultrasonic Field Modeling Near Fluid–Solid
   Interface (Banerjee et al., 2007), 163
   4.5.1 Matrix formulation to calculate source strengths, 164
   4.5.2 Boundary conditions, 165
   4.5.3 Solution, 165
   4.5.4 Numerical results on ultrasonic field modeling near
       fluid–solid interface, 166

4.6 Application 2: Ultrasonic Field Modeling in a Solid Plate (Banerjee
   and Kundu, 2007), 180
   4.6.1 Ultrasonic field modeling in a homogeneous solid plate, 180
   4.6.2 Matrix formulation to calculate source strengths, 181
   4.6.3 Boundary and continuity conditions, 183
   4.6.4 Solution, 185
   4.6.5 Numerical results on ultrasonic field modeling in solid plates, 185

4.7 Application 3: Ultrasonic Fields in Solid Plates with Inclusion
   or Horizontal Cracks (Banerjee and Kundu, 2007a, b), 198
   4.7.1 Problem geometry, 198
   4.7.2 Matrix formulation, 200
   4.7.3 Boundary and continuity conditions, 201
   4.7.4 Solution, 202
   4.7.5 Numerical results on ultrasonic fields
       in solid plate with horizontal crack, 202

4.8 Application 4: Ultrasonic Field Modeling in Sinusoidally
   Corrugated Wave Guides (Banerjee and Kundu, 2006, 2006a), 204
   4.8.1 Theory, 204
4.8.2 Numerical results on ultrasonic fields in sinusoidal corrugated wave guides, 210

4.9 Calculation of Green’s Functions in Transversely Isotropic and Anisotropic Solid, 218
4.9.1 Governing differential equation for Green’s function calculation, 218
4.9.2 Radon transform, 222
4.9.3 Basic properties of Radon transform, 223
4.9.4 Displacement and stress Green’s functions, 224
References, 225

Chapter 5 – DPSM Formulation for Basic Magnetic Problems 231
N. Liebeaux and D. Placko

5.1 Introduction, 231
5.2 DPSM Formulation for Magnetic Problems, 233
  5.2.1 The Biot–Savart law as a DPSM current source definition, 233
    5.2.1.1 Wire of infinite length, 233
    5.2.1.2 Current loop, 234
  5.2.2 Current loops above a semi-infinite conductive target, 235
  5.2.3 Current loops above a semi-infinite magnetic target, 236
  5.2.4 Current loop circling a magnetic core, 237
    5.2.4.1 Geometry, 237
    5.2.4.2 DPSM formulation, 238
    5.2.4.3 Results, 240
  5.2.5 Finite elements simulation—comparisons, 241
5.3 Conclusion, 243
References, 244

Chapter 6 – Advanced Magnetodynamic and Electromagnetic Problems 247
D. Placko and N. Liebeaux

6.1 Introduction, 247
6.2 DPSM Formulation Using Green’s Sources, 248
  6.2.1 Green’s theory, 248
  6.2.2 Green’s function in free homogeneous space, 249
6.3 Green’s Functions and DPSM Formulation, 249
  6.3.1 Expressions of the magnetic and electric fields, 249
  6.3.2 Boundary conditions, 253
6.4 Example of Application, 256
  6.4.1 Target in aluminum \((\sigma = 50 \text{ Ms/m})\), frequency \(= 1000 \text{ Hz}\), 256
  6.4.2 Target in aluminum \((\sigma = 50 \text{ Ms/m})\), frequency \(= 100 \text{ Hz}\),
    inclined excitation loop, 260
  6.4.3 Dielectric target \((\varepsilon_r = 5)\), frequency \(= 3 \text{ GHz}\), 10° tilted excitation loop, 263
6.5 Conclusion, 270
References, 271
9.2.5 Eddy currents, 316
9.2.6 Polarization of dielectrics, 317
9.3 Principle of Electromagnetic Probe for NDE, 319
  9.3.1 Application of dielectric materials, 319
  9.3.2 Application to conductive materials, 320
     9.3.2.1 Magnetic method, 320
     9.3.2.2 Hybrid method, 323
9.4 Electromagnetic Method for Structural Health Monitoring
     (SHM) Applications, 327
  9.4.1 Generalities, 327
  9.4.2 Hybrid method, 327
  9.4.3 Electric method, 330
References, 331

Chapter 10 – Advanced Electromagnetic Problems With Industrial
Applications 333
M. Lemistre and D. Placko
10.1 Introduction, 333
10.2 Modeling the Sources, 334
   10.2.1 Generalities, 334
   10.2.2 Primary source, 335
   10.2.3 Boundary conditions, 335
10.3 Modeling a Defect Inside the Structure, 339
10.4 Solving the Inverse Problem, 345
10.5 Conclusion, 347
References, 347

Chapter 11 – DPSM Beta Program User’s Manual 349
A. Cruau and D. Placko
11.1 Introduction, 349
11.2 Glossary, 350
   11.2.1 Medium, 350
   11.2.2 Object, 350
   11.2.3 Interface, 351
   11.2.4 Boundary conditions (BC), 351
   11.2.5 Frontier, 351
   11.2.6 Workspace, 352
   11.2.7 Scalar and vector physical values, 352
11.3 Modeling Preparation, 352
11.4 Program Steps, 352
11.5 Conclusion, 368

Index 369
Distributed point source method (DPSM) is a newly developed mesh-free numerical/semianalytical technique that has been developed by the co-editors for solving a variety of engineering problems—ultrasonic, magnetic, electrostatic, electromagnetic, among others. This book gives the basic theory of this method and then solves the problems from different fields of engineering applications.

In the last few decades, numerical methods such as the finite element method (FEM) and boundary element method (BEM) have been used for solving a variety of science and engineering problems. In the finite element technique, the entire problem geometry is discretized into a number of finite elements. In the boundary element technique, only the boundary of the problem geometry is discretized into boundary elements. In general, for complex geometries and inhomogeneous materials, FEM is comparatively a more efficient technique. However, for homogeneous medium BEM is more efficient; this is because it requires a fewer number of elements, distributed over the boundary only. Unlike in FEM or BEM in DPSM it is not necessary to discretize or mesh the problem geometry or its boundary. Instead, point sources are placed near the boundaries (not necessarily on the boundaries) and interfaces to build a model that can correctly take into account the radiation condition (reflection and/or transmission of ultrasonic waves or other types of energy) at these interfaces. Thus, DPSM is a mesh-free technique.

DPSM can solve problems with both homogeneous and inhomogeneous media. For inhomogeneous medium two layers of sources on two sides of the interface are placed to model the field on both sides of the interface. Because DPSM does not require the discretization of the entire problem geometry and only uses the point source solutions, it is generally much faster than the conventional numerical techniques. It needs the basic analytical solution for point sources; therefore, this method is
semianalytical. This method has been jointly developed by the two co-editors of this book through their collaborative research effort of several years that included their doctoral students in the United States and France. The co-editors have patented this technology; the worldwide patent has been published with international number W0 2004/044790 A1. They have also published a book chapter and a number of research papers on DPSM in scientific journals and conference proceedings. However, no book had been written on this subject hitherto. This book is the first book on this technique; it describes the theory of DPSM in detail and covers its applications in ultrasonic, magnetic, electrostatic, and electromagnetic problems in engineering.

The book chapter on DPSM that is available in the current literature covers only one application—the ultrasonic modeling. Its other applications, such as for solving magnetic, electrostatic, and electromagnetic problems in engineering, have been published mostly in research papers over the past few years. For the convenience of the users, the detailed theory of DPSM and its applications in different engineering fields are published here in one book. The engineers and scientists can read this book to acquire a unified knowledge on DPSM.

In Chapter 1, the authors present the basic principles of the method. First, the modeling of a transducer or a sensor in a free space is presented, starting from an example of U-shaped magnetic sensor with high-permeability core. Then, the method is extended to the problem geometry with one interface, assuming only one reflection at the interface. This is illustrated through some examples extracted from ultrasonic transducer modeling. In this chapter different DPSM source configurations are presented that include controlled-space radiation or controlled radiation source points and triplets.

In Chapter 2, the advanced theory of DPSM is discussed: Multilayer, multisensor problems and the problem of scattering by inclusions of arbitrary shape are discussed. Some sample solutions are presented from the electric and ultrasonic fields that validate the theory presented in this chapter.

In Chapter 3, the DPSM theory relevant to the ultrasonic problem modeling in the fluid medium is presented, starting with the basic equations of ultrasonic problems. This chapter is written in such a manner that if a person wants to know the ultrasonic applications only and is not necessarily interested in the general theory of DPSM, he/she can read this chapter skipping the first two and still understand the materials. However, while developing the theory and going through the example problems, several references to Chapters 1 and 2 are made, and the readers can go back to the first two chapters and read the specific sections of those chapters referred to in Chapter 3.

Ultrasonic modeling in solid materials is much more complex. In the fluid medium, only the fluid pressure and the particle velocity normal to an interface is of interest. Boundary and interface conditions are defined only on these two parameters, and only one kind of wave, the compressional wave or P-wave, can be present in a fluid medium. On the contrary, in a solid medium both compressional (P) and

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shear (S) waves can be present, and across an interface three components of displacement and three components of stress must be continuous. Thus, the number of continuity conditions across an interface increases from two (for fluid) to six (for solid). Similarly, the number of boundary conditions increases from one (for fluid) to three (for solid) and mode conversion of waves from P to S and vice versa at the interface and boundary can occur for solids. For all these reasons the DPSM modeling of ultrasonic problems in the solid medium is much more complex. This problem is discussed in Chapter 4. Solutions of several example problems involving solid specimens are presented in this chapter along with the relevant theory.

In Chapter 5, basic magnetic problems are presented: current elements and current loops, current loops above a semi-infinite conductive target, current loops above a semi-infinite magnetic target, current loops and magnetic cores, current loop circling a magnetic core, and current loop exciting a U-shaped magnetic core. These points are illustrated through some examples, and a comparison with mastered modeling techniques (finite elements) is presented. In this chapter, DPSM elemental current sources are introduced and validated for generating magnetic field in a given sub-space, under some particular assumptions such as infinite conductivity for the targets.

In Chapter 6, magnetodynamic and electromagnetic modeling is presented. As discussed Chapter 5, DPSM sources are elemental current sources but radiating now in a given medium with finite conductivity, permeability, and permittivity. So, instead of using simplified Green’s functions, we now use the general formulation, which helps to solve complex problems such as eddy current problems, illustrated in this chapter by some examples of different media in which an incident field is created by a winding driven by an AC current, for a wide range of frequencies (100 Hz–3 Ghz). Then, the DPSM electromagnetic formulation allows, without heavy complexity, to obtain relevant modeling for problems of detection and Nondestructive Evaluation, such as crack detection in conductive materials with eddy current techniques, particularly used in industry. Some examples of this ability are given in Chapter 10, with modeling of small cracks in carbon composite plates. It is interesting to point out the problem in dependent universal nature of the DPSM, in which the boundary and interface conditions are fulfilled, in various applications, by taking care of the continuity of the potential and its first derivative along the normal direction to the interface. For electromagnetic problems, we may note that the potential becomes a vector and the number of continuity conditions across an interface increases from two (for magnetostatic or electrostatic problems in which the potential is a scalar) to six, like in the ultrasonic problems solved in Chapter 4. DPSM automatically adapts the number of point sources required on each side of the interface to the corresponding number of equations in order to satisfy the boundary conditions in all cases.

In Chapter 7, basic electrostatic problems are discussed. Through an introduction based on parallel-plate structures, the DPSM technique using electrostatic points are applied to the modeling of standard parallel-plate air-gap capacitors.

In Chapter 8, advanced electrostatic problems considering dielectric media or more complex structures with multiple electrodes and masking issues are studied with DPSM. MEMS (Micro-electro-mechanical systems) tunable capacitors are used to demonstrate that this method can solve engineering problems of interest.
Chapters 9 and 10 are devoted to Nondestructive evaluation (NDE) electromagnetic applications, mainly in the aerospace domain. Then, advanced electromagnetic techniques and devices for industrial applications are presented: modeling one emitter, modeling multiple emitters, and applications to electromagnetic NDE, with some examples illustrated by the principle of hybrid electromagnetic probes for NDE of carbon–fiber plates. These chapters also present a specific sensor for structural health monitoring: the HELP-Layer, with emphasis on the DPSM modeling of the detection of a crack, in Chapter 10.

Chapter 11 gives the user’s Manual for the DPSM-based computer program for those readers who are interested in running the computer code.

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BASIC THEORY OF DISTRIBUTED POINT SOURCE METHOD (DPSM) AND ITS APPLICATION TO SOME SIMPLE PROBLEMS

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1.1 INTRODUCTION AND HISTORICAL DEVELOPMENT OF DPSM

In this chapter, the historical evolution of distributed point source method (DPSM) and its basic principles are presented. First, the magnetic field generated by a magnetic transducer/sensor in a free space is obtained. A U-shaped magnetic sensor with high-permeability core is first modeled (Placko and Kundu, 2001). Then, the method is extended to problem geometries with one interface (Placko et al., 2001, 2002). The source of the field is denoted as “transducer” or “sensor,” and the interface between two media is sometimes called “target.” Observation points that are not necessarily on the interface are also called “target points.” Figure 1.1 shows the relative orientations of the transducer and interface. The interface or target can be an infinite plane or it can have a finite dimension, acting as a finite scatterer. Only one reflection by the target surface is first considered. The method is illustrated through some examples from electromagnetic and ultrasonic applications. In this chapter, different DPSM source
configurations are considered including controlled-space radiation (CSR) sources and triplets, as it can be seen in the patent (Placko et al., 2002).

DPSM modeling is based on a spatial distribution of point sources and can be applied to both two-dimensional (2D) and three-dimensional (3D) problem geometries. Mostly, 3D modeling is presented in this book for magnetic, acoustic, electrostatic, and electromagnetic field problems. In the DPSM modeling technique the transducer surface and interface are replaced by a distribution of point sources, as shown in Figure 1.2a. One layer of sources is introduced near the transducer and a second layer near the interface. Point sources that model the transducer are called the “active” sources and those near the interface are called the “passive” sources. It should be noted here that a transducer generates a field and an interface alters that field by introducing reflected, transmitted, and scattered fields. If the interface is removed, the active point sources should still be present. However, if the active sources are turned off, then the passive point sources must be turned off as well because in the absence of active sources, the passive sources do not exist. Active and passive point sources can be distributed very close to the transducer face and interface, respectively, as shown in Figure 1.2a or away from them as shown in Figure 1.2b. It is also not

![Figure 1.1 Problem geometry with interface.](image)

![Figure 1.2 Synthesizing the field by placing point sources: (a) close to the sensor and interface, (b) away from the sensor and interface.](image)
necessary for the layer of point sources to be parallel to the surface (transducer or interface) that is modeled by these sources. Strengths of the point sources are adjusted such that the boundary conditions on the transducer surface and continuity conditions across the interface are satisfied. This can be achieved by inverting some matrices. By adjusting the point source strengths, the total field can be correctly modeled by different layers of point sources placed in different orientations. Naturally, for different orientations of the point sources, individual source strength vectors should be different. The total field is computed by adding fields generated by all active and passive point sources. Note that unlike the boundary element or finite element techniques, in this formulation the discretization of the problem boundary or of the problem domain is not necessary.

Like other numerical modeling schemes, accuracy of the computation depends on the number of point sources considered. This process of introducing a number of point sources can be called “mesh generation.” In this chapter, we study the effect of the spacing between two neighboring point sources on the accuracy of the field computation and the optimum spacing for accurate numerical computation. It is shown here that for accurately modeling acoustic fields, the spacing between two neighboring point sources should be less than the acoustic wavelength (in fact, as we will see later, this condition has to be fulfilled for all kinds of waves, but the proof is given for the acoustic wave modeling). This restriction can be relaxed if we are interested in computing the field far away from the point source locations. For example, if one is interested in computing the field generated by a circular sensor of finite dimension in a homogeneous medium, the point source spacing must be a fraction of the wavelength if one is interested in computing the field accurately adjacent to the transducer face. However, at a larger distance the field can be computed accurately by considering fewer point sources of higher strength although it will not give good results near the transducer. Flat transducers or sensors with circular and rectangular cross-sections as well as point-focused concave transducers are modeled accurately by taking appropriate source spacing and are presented in this chapter.

Figures 1.3–1.5 show the steps of DPSM evolution, improvements in elemental source modeling, and different problems that have been solved so far by this technique (Placko, 1984, 1990; Placko and Kundu, 2001, 2004; Placko et al., 1985, 1989, 2002; Ahmad et al., 2003, 2005; Dufour and Placko, 1996; Lee et al., 2002; Lemistre and Placko, 2004; Banerjee et al., 2006).

1.2 BASIC PRINCIPLES OF DPSM MODELING

1.2.1 The fundamental idea

In this subsection, we first describe the basic principle of this method, which is based on the idea of using multiple point sources distributed over the active part of a sensor or an interface. Active sources synthesize the transducer-generated signals in
**BASIC THEORY OF DISTRIBUTED POINT SOURCE METHOD (DPSM)**

**Figure 1.3** DPSM birth and evolution.

**Figure 1.4** DPSM source improvement.
a homogeneous medium, whereas the passive point sources distributed along the interface generate signals to model the reflection and transmission fields. For a finite interface the passive sources also model the scattered field. Because the distributed point sources model the total field, we call this method the “distributed point source method” or DPSM. It should be mentioned here that this technique is based on the analytical solutions of basic point source problems. Therefore, it can be considered as a semi-analytical technique for solving sensor problems that include magnetic, ultrasonic, and electrostatic sensors. For example, it is possible to compute the magnetic field emitted by the open magnetic core of an eddy current sensor, or acoustic pressure in front of an ultrasonic transducer without discretizing the space by a large number of 3D finite elements. Magnetic and ultrasonic sensor examples are presented in this chapter to illustrate the method because these problems have some interesting properties as discussed later. It should be noted here that for a magnetic sensor, the magnetic potential remains constant on the sensor surface and the magnetic flux varies from point to point, whereas for the acoustic sensor in a fluid, the particle velocity remains constant on the sensor surface and the acoustic pressure varies. It requires an additional matrix inversion in the magnetic field modeling, which is not necessary for the acoustic field modeling.

An elemental point source is shown in Figure 1.6. In a nonconductive medium, it involves both scalar potential and vector field, the field being proportional to the gradient of the potential. Each source is surrounded by a surface (“bubble”) on which
the boundary conditions are applied. Because the boundary conditions are specified on the sensor surface for active point sources and on the interface surface for the passive point sources, the bubble surface should touch those surfaces such that the transducer surface or interface are tangents to the surface. Therefore, the point sources at the centers of the bubbles cannot be located on the transducer surface or on the interface. Reason for this restriction will be discussed later.

1.2.1.1 Basic equations The basic principle of the DPSM is illustrated in Figure 1.7. The implementation of the model simply requires the replacement of the active surface of the transducer by an array of point sources, so that the initial
complexity associated with a complex finite shape of the transducer is changed into a superposition of elementary point source problems. One way of replacing the surface by an array of point sources is discussed below.

The active surface of the transducer is discretized into a finite number of elemental surfaces $dS$, a point source is placed at the centroid of every elemental surface. The source strength and the radiation area of the sources are controlled. Unlike ordinary point sources, the sources used in DPSM do not necessarily radiate energy in all directions. For this reason these sources can be called CSR sources. For example, a source can be defined to radiate only in the bottom or top half space, or right or left half space (see Fig. 1.8).

In the generic derivation, symbols $\theta$ and $\varphi$ are used to represent different parameters for different engineering problems as described below. For magnetic sensors, $\theta$ and $\varphi$ represent the scalar magnetic potential $\Theta$ and the flux $\Phi/\mu_0$ of the magnetic induction ($\mathbf{H}$), respectively. For ultrasonic transducers, $\theta$ and $\varphi$ represent the acoustic pressure $P$ and the flux $\Phi$ of the particle displacement ($\mathbf{x}$), respectively. Note that the particle velocity $v = \frac{dx}{dt}$. For electrostatic systems, $\theta$ and $\varphi$ represent the scalar magnetic potential $V$ and the flux $Q/e_0$ of the electric field ($\mathbf{E}$). The interaction function that relates the field generated by the unit source (such as the elemental charge for electrostatic problems) to $\theta$ is denoted by $f$. Table 1.1 shows the fundamental equations in different fields of engineering. It should be mentioned here that it is possible to obtain similar equivalent equations for problems from other fields of engineering such as thermal problems, for example. Nevertheless, it will be shown later that for electromagnetic waves the situation is slightly different because the sources in this case are elemental vectors of current, and in addition, the potential is often a vector and not a scalar, due to eddy currents generated in conductive media.

It is interesting to note that the energy (or the power) radiated by such a system is the product of a scalar quantity and the flux of a vector (or the time derivative of the flux, for power). Let us denote the scalar quantity by $\theta_k$ and the flux emitted by the point source $k$ by $\varphi_k$. Figure 1.9 shows how the total field at a given point is computed by adding fields generated by all the point sources. It also shows that because of
rotating symmetry, the elemental surface $dS$ can be changed into a hemispherical surface $dS$ with radius $r_S$.

$1.2.1.2$ Boundary conditions One needs to introduce the boundary conditions before solving the problem. For computing the values of the flux $\Phi$ for $N$ sources, one needs $N$ number of equations. These equations are obtained by introducing rotating symmetry, the elemental surface $dS$ can be changed into a hemispherical surface $dS$ with radius $r_S$.

TABLE 1.1 Some physical values in DPSM modeling

<table>
<thead>
<tr>
<th>Surface power</th>
<th>Surface energy</th>
<th>Function $f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrostatic $\vec{E} = -\nabla \phi(V)$ $\int\int \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$ $\frac{dQ_0}{dt} \times V = \frac{dD}{dt} \times V$ $\frac{1}{2} Q_0 V$ $\frac{e^{-i(k_r r)}}{2\pi \varepsilon_0 \cdot r}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Magnetostatic $\vec{H} = -\nabla \psi(\Theta)$ $\int\int \vec{H} \cdot d\vec{S} = \frac{\Phi}{\mu_0}$ $\frac{d\Phi}{dt} \times \Theta = \frac{dB}{dt} \times \Theta$ $\frac{1}{2} \Phi_0 \Theta$ $\frac{e^{-i(k_r r)}}{2\pi \mu_0 \cdot r}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ultrasonic $\vec{F} = k \cdot \vec{v} = \rho \frac{d\vec{v}}{dt}$ $\int\int \vec{\Phi} \cdot d\vec{S} = \Phi$ $\frac{dx}{dt} \times P$ $\frac{1}{2} \Phi P$ $\frac{-i\omega \rho v_0 e^{i(k_r r)}}{2\pi r}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ r \text{ is the distance of the point source, } K_f \text{ is the wave number; for electrostatic systems, } \theta \text{ is the scalar magnetic potential } V, \phi \text{ is the flux } Q/\varepsilon_0 \text{ of the electric field } (E); \text{ for magnetic sensors, } \theta \text{ is the scalar magnetic potential } \Theta, \phi \text{ is the } \Phi/\mu_0 \text{ of the the magnetic induction } (H); \text{ for ultrasonic problems, } \rho \text{ is the fluid density, } P \text{ is the pressure, } x \text{ is the particle displacement, } \Phi \text{ is the flux of particle displacement, } v_0 \text{ is the transducer velocity, } \omega \text{ is the signal frequency.} \]

Figure 1.9 Equivalent surface discretization.