FIBER OPTIC ESSENTIALS

K. Thyagarajan
Ajoy Ghatak

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To Raji and Gopa

CONTENTS

PREFACE

The dramatic reduction in transmission loss of optical fibers coupled with very important developments in the area of light sources and detectors have resulted in phenomenal growth of the fiber optic industry during the last 35 years or so. Indeed, the birth of optical fiber communication systems coincided with the fabrication of low-loss optical fibers and the operation of room-temperature semiconductor lasers in 1970. Since then, scientific and technological growth in this field has been phenomenal. Although the major applications of optical fibers have been in the area of telecommunications, many new areas, such as fiber optic sensors, fiber optic devices and components, and integrated optics, have witnessed immense growth.

As with any technological development, the field of fiber optics has progressed through a number of ideas based on sound mathematical and physical principles. For a thorough understanding of these, one needs to go through a good amount of mathematical rigor and analysis, which is carried out in undergraduate and graduate curricula. At the same time there are a sizable number of engineering and technical professionals, technical managers, and inquisitive students of other disciplines who are interested in having a basic understanding of various aspects of fiber optics either to satisfy their curiosity or to help them in their professions. For these professionals a book describing the most important aspects of fiber optics without too much mathematics, based purely on physical reasoning and explanations, should be very welcome. A book taking the reader from the basics to the current state of development in fiber optics does not seem to exist, and the present book aims to fill that gap.

The book begins with a basic discussion of light waves and the phenomena of refraction and reflection. The next set of chapters introduces the reader to the field of fiber optics, discussing different types of fibers used in communication systems, including dispersion-compensating fibers. In later chapters we discuss recent developments, such as fiber Bragg gratings, fiber amplifiers, fiber lasers, nonlinear fiber optics, and fiber optic sensors. Examples and comparison with everyday experience are provided wherever feasible to help readers understanding by relation to known facts. The book is interspersed with numerous diagrams for ease of visualization of some of the concepts.

The mathematical details are kept to a bare minimum in the hope of providing easy reading and understanding of some of the most important technological developments of the twentieth century, which are penetrating more and more deeply into our society and helping to make our lives a bit easier.

We are very grateful to all our colleagues and students at IIT Delhi for numerous stimulating discussions and academic collaborations. One of the authors (A.G.) is grateful to Disha Academy of Research and Education, Raipur for supporting this endeavor.

K. THYAGARAJAN
AJOY GHATAK

New Delhi

UNITS AND ABBREVIATIONS

1 Å (1 angstrom)	one-tenth of a billionth of a meter (= 10^{-10} m)
1 nm (1 nanometer)	one-billionth of a meter (= 10^{-9} m)
1 μm (1 micrometer)	one-millionth of a meter (= 10^{-6} m)
1 cm (1 centimeter)	one-hundredth of a meter (= 10^{-2} m)
1 mm (1 millimeter)	one-thousandth of a meter (= 10^{-3} m)
1 km (1 kilometer)	1000 meters (= 10^3 m)
speed of light in vacuum, *c*	300 million kilometers per second (= 3×10^8 m/s)
1 fs (1 femtosecond)	one-millionth of a billionth of a second (= 10^{-15} s)
1 ps (1 picosecond)	one-thousandth of a billionth of a second (= 10^{-12} s)
1 ns (1 nanosecond)	one-billionth of a second (= 10^{-9} s)
1 μs (1 microsecond)	one-millionth of a second (= 10^{-6} s)
1 ms (1 millisecond)	one-thousandth of a second (= 10^{-3} s)
1 kHz (1 kilohsertz)	1000 vibrations per second (= 10^3 Hz)
1 MHz (1 megahertz)	1 million vibrations per second (= 10^6 Hz)
1 GHz (1 gigahertz)	1 billion vibrations per second (= 10^9 Hz)
1 THz (1 terahertz)	1000 billion vibrations per second (= 10^{12} Hz)
1 nW (1 nanowatt)	one-billionth of a watt (= 10^{-9} W)
1 μW (1 microwatt)	one-millionth of a watt (= 10^{-6} W)
1 mW (1 milliwatt)	one-thousandth of a watt (= 10^{-3} W)
1 kW (1 kilowatt)	1000 watts (= 10^3 W)
1 MW (1 megawatt)	1 million watts (= 10^6 W)
3 dB loss	power loss by a factor of 2
10 dB loss	power loss by a factor of 10
20 dB loss	power loss by a factor of 100
30 dB loss	power loss by a factor of 1000
3 dB gain	power amplification by a factor of 2
10 dB gain	power amplification by a factor of 10
20 dB gain	power amplification by a factor of 100
30 dB gain	power amplification by a factor of 1000
1 kb/s	1000 bits per second (= 10^3 bits per second)
1 Mb/s	1 million bits per second (= 10^6 bits per second)
1 Gb/s	1 billion bits per second (= 10^9 bits per second)
1 Tb/s	1000 billion bits per second (= 10^{12} bits per second)
0 dBm	1 mW
−30 dBm	1 μW
+30 dBm	1 W

AM	amplitude modulation
APD	avalanche photo diode
ASE	amplified spontaneous emission
AWG	arrayed waveguide grating
BER	bit error rate
BW	bandwidth
CSF	conventional single-mode fiber
CW	continuous wave
CWDM	coarse wavelength-division multiplexing
dB	decibel
DBR	distributed Bragg reflector
DCF	dispersion-compensating fiber
DFB	distributed-feedback
DMD	differential mode delay
DSF	dispersion-shifted fiber
DWDM	dense wavelength-division multiplexing
EDFA	erbium-doped fiber amplifier
FBG	fiber Bragg grating
FM	frequency modulation
FOG	fiber optic gyroscope
FSO	free-space optics
FTTH	fiber to the home
FWM	four-wave mixing
ITU	International Telecommunication Union
LD	laser diode
LEAF	large effective area fiber
LED	light-emitting diode
LPG	long-period grating
MCVD	modified chemical vapor deposition
MZ	Mach–Zehnder
NA	numerical aperture
NEP	noise equivalent power
NF	noise figure
NRZ	non return to zero
NZDSF	nonzero dispersion-shifted fiber
OOK	on–off keying
OSNR	optical signal-to-noise ratio
OTDR	optical time-domain reflectometer
PCM	pulse-code modulation
PIN	p(doped)–intrinsic–n(doped)
PMD	polarization mode dispersion
RFA	Raman fiber amplifier
RZ	return to zero
SC	supercontinuum

SDH	synchronous digital hierarchy
SMF	single-mode fiber
SNR	signal-to-noise ratio
SOA	semiconductor optical amplifier
SONET	synchronous optical network
SPM	self-phase modulation
TDM	time-division multiplexing
TIR	total internal reflection
VCSEL	vertical cavity surface-emitting laser
XPM	cross-phase modulation
WDM	wavelength-division multiplexing

CHAPTER 1

Introduction

Optics today is responsible for many revolutions in science and technology. This has been brought about primarily by the invention of the laser in 1960 and subsequent development in realizing the extremely wide variety of lasers. One of the most interesting applications of lasers with a direct impact on our lives has been in communications. Use of electromagnetic waves in communication is quite old, and development of the laser gave communication engineers a source of electromagnetic waves of extremely high frequency compared to microwaves and millimeter waves. The development of low-loss optical fibers led to an explosion in the application of lasers in communication, and today we are able to communicate almost instantaneously between any two points on the globe. The backbone network providing this capability is based on optical fibers crisscrossing the Earth: under the seas, over land, and across mountains. Today, more than 10 terabits of information can be transmitted per second through one hair-thin optical fiber. This amount of information is equivalent to simultaneous transmission of about 150 million telephone calls—certainly one of the most important technological achievements of the twentieth century. We may also mention that in 1961, within one year of the demonstration of the first laser by Theodore Maiman, Elias Snitzer fabricated the first fiber laser, which is now finding extremely important applications in many diverse areas: from defense to sensor physics.

Since fiber optic communication systems are playing very important roles in our lives, an introduction to these topics, with a minimum amount of mathematics, should give many interested readers a glimpse of the developments that have taken place and that continue to take place. In Chapter 2 we introduce the reader to light waves and their characteristics and in Chapter 3 explain how it is possible to use light waves to carry information. Chapters 4 to 8 deal with various characteristics of the optical fiber relevant for applications in communication and sensing. The erbium-doped fiber amplifier has revolutionized high-speed communication; this is discussed in Chapter 9, where we also discuss fiber lasers, which have found extremely important industrial applications. Chapter 10 covers Raman fiber amplifiers, which are playing increasingly important roles in optical communication systems. In Chapter 11 we describe fiber Bragg grating, which is indeed a very beautiful device with numerous practical

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applications. In Chapter 12 we discuss some important fiber optic components, which are an integral part of many devices used in fiber optic communication systems.

When the light power within an optical fiber becomes substantial, the properties of the fiber change due to the high intensity of the light beam. Such an effect, called a nonlinear effect and discussed in Chapter 13, plays a very important role in the area of communication. There is also considerable research and development (R & D) effort to utilize such effects for signal processing of optical signals without converting them into electronic signals. Such an application should be very interesting when the speed of communications that use light waves goes up even further as electronic circuits become limited due to the extremely fast response required. Fiber optic sensors, discussed in Chapter 14, form another very important application of optical fibers, and some of the sensors discussed are already finding commercial applications. They are expected to outperform many conventional sensors in niche applications and there is a great deal of research effort in this direction.

In this book we introduce and explain various concepts and effects based on physical principles and examples while keeping the mathematical details to a minimum. The book should serve as an introduction to the field of fiber optics, one of the most important technological revolutions of the twentieth century. If it can stimulate the reader to further reading in this exciting field and help him or her follow developments as they are taking place, with applications in newer areas, it will have served its purpose.

CHAPTER 2

Light Waves

2.1 INTRODUCTION

What is light? That is indeed a very difficult question to answer. To quote Richard Feynman: "Newton thought that light was made up of particles, but then it was discovered that it behaves like a wave. Later, however (in the beginning of the twentieth century), it was found that light did indeed sometimes behave like a particle. . . . So it really behaves like neither." However, all phenomena discussed in this book can be explained very satisfactorily by assuming the wave nature of light. Now the obvious question is: What is a wave? A *wave* is propagation of disturbance. When we drop a small stone in a calm pool of water, a circular pattern spreads out from the point of impact (Fig. 2.1).[1] The impact of the stone creates a disturbance that propagates outward. In this propagation, the water molecules do not move outward with the wave; instead, they move in nearly circular orbits about an equilibrium position. Once the disturbance has passed a certain region, every drop of water is left at its original position. This fact can easily be verified by placing a small piece of wood on the surface of water. As the wave passes, the piece of wood comes back to its original position. Further, with time, the circular ripples spread out; that is, the disturbance (which is confined to particular region at a given time) produces a similar disturbance at a neighboring point slightly later, with the pattern of disturbance remaining roughly the same. Such a propagation of disturbances (without any translation of the medium in the direction of propagation) is termed a *wave*. Also, the wave carries energy; in this case the energy is in the form of the kinetic energy of water molecules. There are many different types of waves: sound waves, light waves, radio waves, and so on, and all waves are characterized by properties such as wavelength and frequency.

2.2 WAVELENGTH AND FREQUENCY

We next consider the propagation of a transverse wave on a string. Imagine that you are holding one end of a string, with the other end being held tightly by another

[1] Water waves emanating from a point source are shown very nicely at the Web site http://www.colorado.edu/physics/2000/waves_particles/waves.html.

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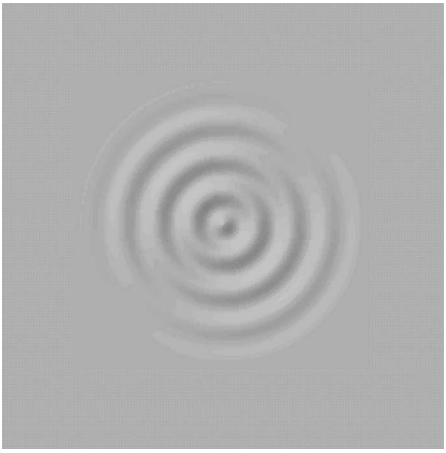

FIGURE 2.1 Water waves spreading out from a point source. (Adapted from http://www.colorado.edu/physics/2000/waves_particles/waves.html.)

person so that the string does not sag. If we move the end of the string in a periodic up-and-down motion ν times per second, we generate a wave propagating in the $+x$ direction. Such a wave can be described by the equation (Fig. 2.2)

$$y(x, t) = a \sin(\omega t - kx) \tag{2.1}$$

where a and ω ($=2\pi\nu$) represent the amplitude and angular frequency of the wave, respectively; further,

$$\lambda = \frac{2\pi}{k} \tag{2.2}$$

represents the wavelength associated with the wave. Since the displacement (which is along the y direction) is at right angles to the direction of propagation of the wave, we have what is known as a *transverse wave*. Now, if we take a snapshot of the string at $t = 0$ and at a slightly later time Δt, the snapshots will look like those shown in Fig. 2.2*a*; the figure shows that the disturbances have identical shapes except for the fact that one is displaced from the other by a distance $v\Delta t$, where v represents the speed of the disturbance. Such a propagation of a disturbance without a change in form is characteristic of a wave. Now, at $x = 0$, we have

$$y(x = 0, t) = a \sin \omega t \tag{2.3}$$

Fig. 2.2*b*, and each point on the string vibrates with the same frequency ν, and therefore if T represents the time taken to complete one vibration, it is simply the inverse of the frequency:

$$T = \frac{1}{\nu} \tag{2.4}$$

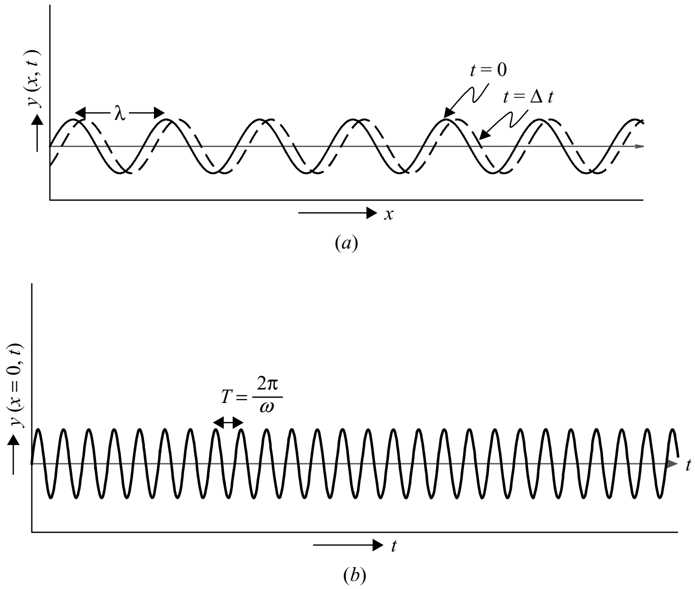

FIGURE 2.2 (a) Displacement of a string at $t = 0$ and at $t = \Delta t$, respectively, when a sinusoidal wave is propagating in the $+x$ direction; (b) time variation of the displacement at $x = 0$ when a sinusoidal wave is propagating in the $+x$ direction. At $x = \Delta x$, the time variation of the displacement will be slightly displaced to the right.

It is interesting to note that each point of the string moves up and down with the same frequency ν as that of our hand, and the work we do in generating the wave is carried by the wave, which is felt by the person holding the other end of the string. Indeed, all waves carry energy.

Referring back to Fig. 2.2a, we note that the two curves are the snapshots of the string at two instants of time. It can be seen from the figure that at a particular instant, any two points separated by a distance λ (or multiples of it) have identical displacements. This distance is known as the *wavelength* of the wave. Further, the shape of the string at the instant Δt is identical to its shape at $t = 0$, except for the fact that the entire disturbance has traveled through a certain distance. If v represents the speed of the wave, this distance is simple $v\ \Delta t$. Indeed, in one period (i.e., in time T) the wave travels a distance equal to λ. Thus, the wavelength of the wave is nothing but the product of the velocity and time period of the wave:

$$\lambda = vT \tag{2.5}$$

which implies that the velocity of the wave is the product of the wavelength and the frequency of the wave:

$$v = \nu\lambda \tag{2.6}$$

Unlike the waves on a string, which are *mechanical waves*, light waves are characterized by changing electric and magnetic fields and are referred to as *electromagnetic waves*. In the case of light waves, a changing magnetic field produces a time- and space-varying electric field, and the changing electric field in turn produces a time- and space-varying magnetic field; this results in the propagation of the electromagnetic wave even in free space. The electric and magnetic fields associated with a light wave can be described by the equations:

$$\mathbf{E} = \hat{\mathbf{y}}\, E_0 \cos(\omega t - kx) \tag{2.7}$$

$$\mathbf{H} = \hat{\mathbf{z}}\, H_0 \cos(\omega t - kx) \tag{2.8}$$

where E_0 represents the amplitude of the electric field (which is in the y direction) and H_0 represents the amplitude of the magnetic field (which is in the z direction); $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ are unit vectors along the y and z directions, respectively. Equation (2.7) describes a y-polarized electromagnetic wave propagating in the x direction. Further, $\omega/k = v$ is the velocity of the electromagnetic waves, and in free space $v = c \approx 3 \times 10^8$ m/s. In contrast, sound waves need a medium to propagate since they are formed by mechanical strains produced in the medium in which they propagate.

For propagation along the x direction one could also have an electromagnetic wave whose electric field points along the the z direction while the magnetic field points along the $-y$ direction. The electric field of this wave is perpendicular to the electric field given by Eq. (2.7) and represents a z-polarized wave. The y- and z-polarized waves are the two polarization states of the light wave that can propagate along the x direction.

The intensity of the light wave, which represents the amount of energy crossing a unit area perpendicular to the direction of propagation in a unit time. The intensity I and the peak electric field E_0 of an electromagnetic wave are related to each other through the equation:

$$I = \frac{n}{2c\mu_0} E_0^2 \tag{2.9}$$

where n is the refractive index of the medium through which the wave is propagating and μ_0 is a constant with the value $4\pi \times 10^{-7}$ SI units.

As an example, we can consider a light beam with a cross-sectional diameter of 2 mm propagating through free space. If the power carried by the beam is 1 W, the intensity of the field is 3×10^5 W/m^2, and the electric field associated with this wave would be about 15,500 V/m.

We mention here that a low-powered ($\approx$2 mW) diffraction-limited laser beam incident on the eye gets focused on a very small spot and can produce an intensity of about 10^8 W/m^2 at the retina; this could indeed damage the retina. On the other hand, when we look at a 20-W bulb at a distance of about 5 m from the eye, the eye produces an image of the bulb on the retina, and this would produce an intensity of only about 10 W/m^2 on the retina of the eye. Thus, whereas it is quite safe to look at a 20-W bulb, it is very dangerous to look directly into a 2-mW laser beam. Indeed, because a laser beam can be focused to very narrow areas, it has found important applications in such areas as eye surgery and laser cutting.

It is of interest here to note that if we look directly at the sun, the power density in the image formed is about 30 kW/m^2. This follows from the fact that on Earth, about 1.35 kW of solar energy is incident (normally) on an area of 1 m^2. Thus, the energy entering the eye is about 4 mW. Since the sun subtends about 0.5° on Earth, the radius of the image of the sun (on the retina) is about 2×10^{-4} m. Therefore, if we are looking directly at the sun, the power density in the image formed is about 30 kW/m^2. The corresponding electric field is about 4700 V/m. *Never look into the sun; your retina would be damaged: not only because of the high intensities but also because of the high level of ultraviolet light in sunlight.*

Lasers can generate extremely high powers, and since they can also be focused to very small areas, it is possible to generate extremely high intensity values. At currently achievable intensities such as 10^{21} W/m^2, the electric fields are so high that electrons can get accelerated to relativistic velocities (velocities approaching that of light), leading to very interesting effects. Apart from scientific investigations of extreme conditions, continuous-wave lasers having power levels of about 10^5 W, and pulsed lasers having a total energy of about 50,000 J have many applications (e.g., welding, cutting, laser fusion, Star Wars).

The wave represented by Eq. (2.7) represents a monochromatic wave since it has only one frequency component, represented by ω. We shall see in Chapter 3 that when a wave of the type represented by Eq. (2.7) is modulated in amplitude or frequency according to a signal to be transmitted, this process leads to a wave which then contains many frequency components. In a light pulse the amplitude of the electric field varies with time (Fig. 2.3), and such a field has many frequency components. The frequency spectrum of the pulse is related inversely to the pulse width in time. Thus, a shorter pulse would have a broader spectrum, and conversely, a broader pulse would have a narrower spectrum. The spectrum occupied by a pulse is an important feature and finally determines the information capacity of the fiber optic system.

There exists a wide and continuous variation in the frequency (and wavelength) of electromagnetic waves. The electromagnetic spectrum is shown in Fig. 2.4. Radio waves correspond to wavelength in the range 10 to 1000 m, whereas the wavelength of x-rays are in the region of angstroms (1 Å $= 10^{-10}$ m). The ranges of the wavelengths of various types of electromagnetic waves are shown in Fig. 2.4, and as can be seen, the visible region (0.4 μm $< \lambda <$ 0.7 μm) occupies a very small portion of the spectrum. Although the range noted above represents the visible range for humans, there are animals and insects whose sensation can extend to regions not visible to humans. For example, pit vipers can sense infrared radiation (heat radiation), and bees are sensitive to ultraviolet radiation, which helps them locate sources of honey. Special cameras that convert infrared radiation to visible light help humans to see objects even in the dark.

The methods of production of various types of electromagnetic waves are different; for example, x-rays are usually produced by the sudden stopping or deflection of electrons, whereas radio waves may be produced by oscillating charges on an antenna.

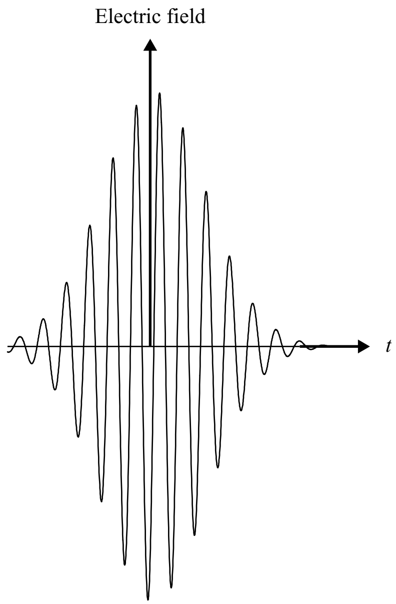

FIGURE 2.3 Optical pulse; the oscillatory portion is due to the high frequency of the pulse, and the envelope is the pulse shape.

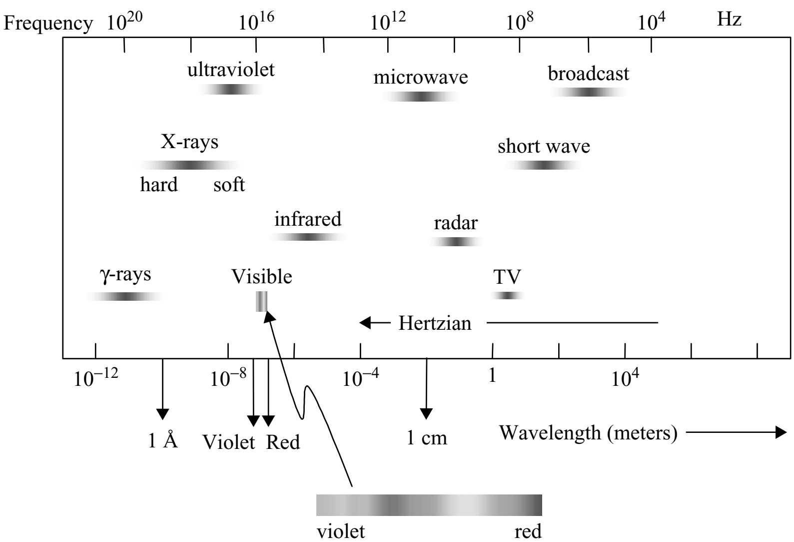

FIGURE 2.4 Electromagnetic spectrum.

However, all electromagnetic waves propagate with the same speed in vacuum, and this speed is denoted by c and is equal to 299,792.458 km/s. This value is usually approximated by 300,000 km/s. Thus, whether it is ultraviolet light or infrared light or radio waves, they all travel with an identical velocity in vacuum.

Knowing the wavelength and the velocity, one can calculate the corresponding frequencies. Thus, yellow light corresponding to a wavelength of 600 nm would have a frequency of 500,000 GHz, where 1 GHz (1 gigahertz) = 10^9 Hz (=1 billion vibrations per second), so the frequency is 0.5 million GHz (i.e., the electric and magnetic fields oscillate 5 hundred thousand billion times per second!). Compare this with audible sound waves at, say, a frequency of 5 kilohertz, where the vibrations take place only 5000 times per second. On the other hand, for $\lambda = 30$ m (shortwave radio broadcast), the corresponding frequency is 10 megahertz (i.e., oscillations take place 10 million vibrations per second).

According to the theory of relativity, the highest velocity that any wave or object can have is the velocity of light in free space. This velocity is so high that in 1 second, light can travel about 7.5 times around the Earth, and it takes only about 8 minutes for light from the sun to reach us. Similarly, radio signals from the probe that has landed recently on Titan (one of the moons of Saturn) will take about 1.2 hours to reach the radio station on Earth. If we look at a star that is, say, 10 light-years away (i.e., light takes 10 years to reach us from that star), the light that reaches us right now from the star started its journey 10 years ago, and what we are witnessing right now happened 10 years ago!

2.3 REFRACTIVE INDEX

Light waves travel at a slightly slower speed when propagating through a medium such as glass or water. The ratio of the speed of light in vacuum to that in the medium, known as the *refractive index* of the medium, is usually denoted by the symbol n:

$$n = \frac{c}{v} \tag{2.10}$$

where c ($\approx 3 \times 10^8$ m/s) is the speed of light in free space and v represents the velocity of light in that medium. For example,

$$n \approx \begin{cases} 1.5 & \text{for glass} \\ \frac{4}{3} & \text{for water} \end{cases}$$

Thus, in glass, the speed of light $\approx$200,000 km/s, and in water, the speed of light $\approx$225,000 km/s.

When a ray of light is incident at the interface of two media (e.g., air and glass), it undergoes partial reflection and partial refraction as shown in Fig. 2.5*a*. The dotted line represents the normal to the surface. The angles ϕ_1, ϕ_2, and ϕ_r represent the angles that the incident ray, refracted ray, and reflected ray make with the normal.

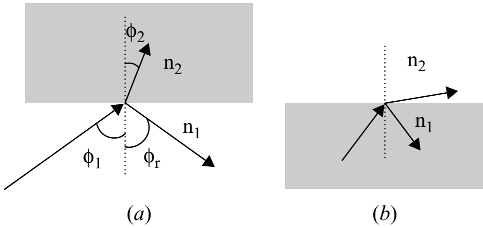

FIGURE 2.5 (*a*) Ray of light incident on a denser medium ($n_2 > n_1$); (*b*) ray incident on a rarer medium ($n_2 < n_1$).

According to *Snell's law*,

$$n_1 \sin \phi_1 = n_2 \sin \phi_2 \quad \text{and} \quad \phi_r = \phi_1 \tag{2.11}$$

Further, the incident ray, reflected ray, and the refracted ray lie in the same plane. In Fig. 2.5*a*, since $n_2 > n_1$, we must have (from Snell's law) $\phi_2 < \phi_1$ (i.e., the ray will bend towards the normal). On the other hand, if a ray is incident at the interface of a rarer medium ($n_2 < n_1$), the ray will bend away from the normal as shown in Fig. 2.5*b*.

Example 2.1 For the air–glass interface, $n_1 = 1.0$, $n_2 = 1.5$ and if $\phi_1 = 45°$, then $\phi_2 \simeq 28°$ (Fig. 2.6*a*). Similarly, for the air–water interface, $n_1 = 1.0$, $n_2 = 1.33$ and if $\phi_1 = 45°$, then $\phi_2 \simeq 32°$ (Fig. 2.6*b*).

The path of rays is reversible; that is, if a light ray (passing through water) is incident on air, the ray will bend away from the normal. Figure 2.7 shows exactly the reverse of the situation in Fig. 2.5*b*, where the ray is incident from water and refracts into air. It is because of this refraction that when we look at a fish (which is inside the

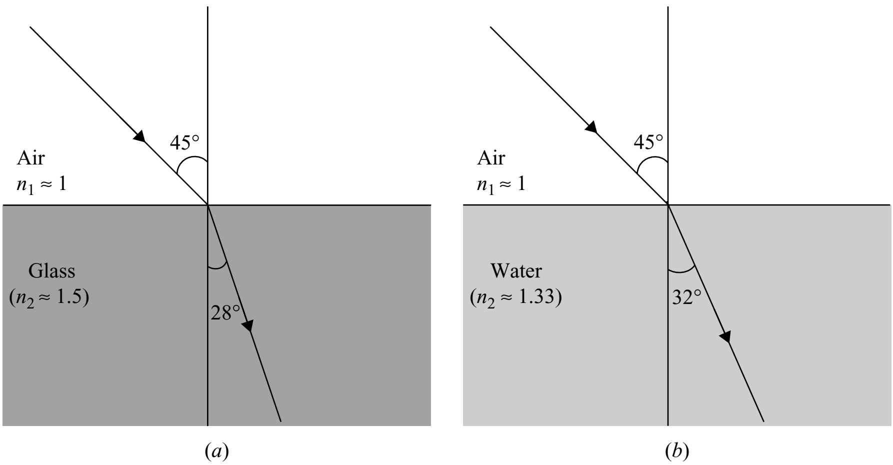

FIGURE 2.6 For a ray incident on a denser medium ($n_2 > n_1$), the ray bends toward the normal and the angle of refraction is less than the angle of incidence: (*a*) for the air–glass interface, for $\phi_1 = 45°$, $\phi_2 \approx 28$; (*b*) for the air–water interface, for $\phi_1 = 45°$, $\phi_2 \approx 32°$.