Puzzles
for Programmers and Pros

Dennis E. Shasha

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About the Author

Dennis Shasha is a professor of computer science at the Courant Institute of New York University, where he works with biologists on pattern discovery for microarrays, combinatorial design, and network inference; with physicists, musicians, and financial people on algorithms for time series; and on database applications in untrusted environments. Other areas of interest include database tuning as well as tree and graph matching.

Because he likes to type, he has written five books of puzzles before this one, a biography about great computer scientists, and technical books about database tuning, biological pattern recognition, and time series. In addition, he has co-written more than fifty journal papers, sixty conference papers, and seven patents. For fun, he writes the monthly puzzle column for the *Scientific American* website and sometimes for *Dr. Dobb's Journal*.

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Acknowledgments

My warmest thanks to the readers of my puzzles. They span every continent except Antarctica and they reveal their intelligence with every communication. Some readers have become good friends, such as that gifted teacher and mathematician Andy Liu and the irrepressibly creative architect Mike Whittaker. When a puzzle that I have invented talks back and twists itself into a version that I can't answer, I call in my puzzle brain trust. If I were faced with a math/computational problem of great difficulty, I would huddle with these gifted individuals for a week and we'd solve it.

Many of these puzzles have benefited greatly from the editorial suggestions of *Scientific American's* John Rennie and Jon Erickson and Deirdre Blake of *Dr. Dobb's*. Editors are the first clarity test that an author has of the written version of a puzzle. These editors have surgically exposed any obscurities they have found in my writing. I often pose the oral version of a puzzle to a certain young man named Tyler. Besides solving them (sometimes with the help of a hint or two), he makes it clear when he considers them "lame" (bad) or "ill" (cool). Only the ill ones see print.

Gary Zamchick has graced some of the puzzles with his irreverent cartoons. He has the remarkable ability to capture the essence of a puzzle with just a few pen strokes. This book's Part III, "Faithful Foes," requires Dr. Ecco and you to navigate a labyrinth of tunnels and treachery. Karen Shasha has managed, in her photographs, to portray the spirit of that quest exactly as I have envisioned it.

My acquisitions editor, Carol Long; my development editor, Sara Shlaer; the book's copy editor, Mildred Sanchez; and the book's designer, LeAndra Hosier, have put up with my demands for ever higher visual appeal and supported the demands of the text. If this book is a winner, they are in large part responsible.

Contents

Introduction

"My interview at a company well known for its hard and puzzling questions was a joke after this class."
—Boris, a graduate of my puzzle class at NYU

Some people, like me, love puzzles. Others feel they must study puzzles to succeed at job interviews. I've written this book for both. You'll find here some kick-ass puzzles, but I also take you on a kind of tutorial tour of problem solving techniques to help you face new puzzle challenges. Oh, and then there's the possibility of a prize if you solve the super-hard puzzles at the end.

Puzzles as an interviewing tool have many detractors. Criticism often comes down to the implausibility of a puzzle scenario in which, say, a perfectly logical person is mute and refuses to write. Now, I admit to having written such puzzles, but most of my puzzles come from real problems (e.g., occasionally failing hardware being modeled by occasional liars). In my own research, I try to abstract the problem at hand to a puzzle, in the hopes that I can understand the fundamental issues and take care of the window dressing later. It works pretty well. So, to me, puzzles, especially the right kinds of puzzles, provide a tunnel into scientific and engineering insight.

So why do I write puzzles for other people? First, because they're fun. Second, because they exercise the brain in a useful way. In the puzzle course that Boris talks about in the epigraph, students write programs every week. The programs compete (each having two minutes to run) and the winner gets a Kit Kat bar. I lecture little, offering the techniques you'll find in Part II. By the end of the course, the students find themselves incomparably better at solving the kind of application problems the real world throws at them—problems that algorithms professors will call "intractable" but that nevertheless have to be solved. I can't tell you exactly what brings about the transformation in the students, but it happens.

Most of the puzzles in Part I of this book come from my columns in *Scientific American* and *Dr. Dobb's Journal*, where many readers have given me important and imaginative feedback. That feedback, or sometimes the puzzle itself, has suggested new variants, so you will find more to think about, even if you've already encountered the puzzles in those magazines.

Many times, when I have no idea how to solve a puzzle (and this happens for puzzles I invent, too), I begin with an initial attempt scribbled on paper. It's usually wrong, but sometimes it leads to a better approach. You'll find room to scribble at the end of each puzzle statement in Part I.

Solving puzzles demands a mindset, a vulnerable openness at the beginning followed by a rigorous drive to find a solution—spiritually akin to the technique discussed in Part II called "simulated annealing." Mine is rarely the only possible mindset. When other puzzlists have shared better ideas with me, I present their approaches to you.

Each short chapter in Part I, "Mind Games," presents a puzzle. The solutions to those puzzles appear at the end of Part I. Part II, "The Secret of the Puzzle," shows how to solve several classes of puzzles, both by hand and by computer. You'll find ways to solve elimination puzzles like Sudoku, scheduling puzzles, mathematical word puzzles, and probabilistic puzzles. The approach is unapologetically algorithmic. I believe in using every tool at your disposal. Part III, "Faithful Foes," asks you to solve a mystery involving codes, bank accounts, and geography. You'll accompany a certain mathematical detective named Dr. Ecco and some of his friends. (You may have met them before.) Solve the mystery and you have a chance to win a prize.

Have fun and good luck.

Contest Info

To enter the contest, send your solutions to all the puzzles that appear in Part III to:

`shasha@courant.nyu.edu`

The solutions must be submitted in Microsoft Word or PDF format to receive consideration. Entries submitted after August 31, 2008, will not be considered. The author, Dennis E. Shasha, is the sole judge of all entries and will select up to ten entries from among the correct responses. Winners will receive a Wrox T-shirt and baseball cap (or equivalent merchandise), three Wrox books of the winner's choosing, and a Certificate of Omniheurism. To receive the prizes, entrants must supply a street mailing address (no P.O. boxes) along with their entry. Offer void where prohibited by law. Allow six to eight weeks for delivery. Wiley is not responsible for lost, illegible, or incomplete entries. Employees of Wiley Publishing are not eligible to enter.

The cryptograms that appear in Part III of the book are available in Microsoft Word format for download at `www.wrox.com`. Once at the site, simply use the Search box to locate the book's title or ISBN (978-0-470-12168-9) and click the Download link on the book's detail page to obtain the cryptograms.

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We also invite you to offer variations on any of the puzzles in the book, or alternative solutions, in the P2P forums at `p2p.wrox.com`. The forums are a Web-based system for you to post messages relating to Wrox books and related technologies and interact with other readers and technology users. The forums offer a subscription feature to e-mail you topics of interest of your choosing when new posts are made to the forums. Wrox authors, editors, other industry experts, and your fellow readers are present on these forums.

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Part I

Mind Games

If you can solve these puzzles, you're not management material. You're too good.

Note: *This Screaming Head icon denotes an especially difficult puzzle.*

COMPETITION

We can't all be winners.

Sweet Tooth

Two children, perhaps similar to ones you know, love cake and mathematics. For this reason, Jeremy convinces Marie to play the following game on two identical rectangular cakes chef Martine has prepared for them.

Jeremy will cut the first cake into two pieces, perhaps evenly, perhaps not. After seeing the cut, Marie will decide whether she will choose first or allow Jeremy to do so. If she goes first, she will take the larger piece. If she goes second, she can assume that Jeremy will take the larger piece.

Next, Jeremy will cut the second cake into two pieces (remember that one of the pieces can be vanishingly small if he so chooses). If Marie had chosen first for the first cake, then Jeremy gets to take the larger piece of the second cake. If Marie had chosen second for the first cake, then she gets to take the larger piece of the second cake.

Warm-Up

P^3 Assuming each child will strive to get the most total cake possible, what is an optimal strategy for Jeremy?

> ***Hint:*** *Before you look at the solution: Assume that Jeremy divides the first cake into fractions f and 1-f where f is at least 1/2. Then explore the consequences if Marie chooses to take the piece of fraction f or if she goes second, so gets the piece having fraction 1-f.*

Solution to Warm-Up

Following the hint, Marie reasons as follows: If she takes the fraction f piece, then Jeremy will take the entire second cake (he'll cut the smallest crumb from the second cake and then will take the large first piece, leaving Marie only the crumb). So, Marie will get exactly f and Jeremy will get (1-f) + 1. If Marie takes the smaller piece of the first cake (fraction 1-f), Jeremy will do best if he divides the second cake in half. This gives Marie (1-f) + 1/2. Jeremy follows this reasoning, so realizes that the best he can do is to make f = (1-f) + 1/2. That is 2f = 1 1/2 or f = 3/4. If Marie takes the first piece of the first cake, then Jeremy will get 1/4 of the first cake and all of the second cake. If Marie takes the second piece of the first cake, then Jeremy will get 3/4 of the first cake and 1/2 of the second cake. In both cases, Marie gets a total of 3/4 of a cake and Jeremy 1 1/4. Note that if Jeremy cuts the first cake such that the larger fraction is less than 3/4, Marie will get more than 1/4 of the first cake by going second and will still get 1/2 of the second cake, thus increasing her cake amount beyond 3/4. By contrast, if Jeremy cuts the first cake such that the larger fraction is more than 3/4, Marie will just take that larger piece, again increasing her cake amount beyond 3/4.

I have discussed the warm-up at great length, because a week later a harder challenge has arisen. Chef Martine has made three new identical rectangular cakes. Jeremy and Marie both eye them greedily.

They decide on the following rules. Again, Jeremy will cut. But this time, Marie gets the first choice twice and Jeremy only once. That is, Jeremy cuts the first cake. Marie decides whether she wants to choose first or second for that cake. Next, Jeremy cuts the second cake; again Marie decides whether she wants to choose first or second. Same for the third cake. The only caveat is that Marie must allow Jeremy to choose first at least once.

1. How does Jeremy maximize the amount of cake he gets, given these rules? How much will he get?
2. Suppose there are seven cakes and Marie gets the first choice for six of the seven cakes. Who has the advantage? By how much?
3. Is there any way to ensure that each child receives the same amount of cake, assuming Jeremy always cuts?

P^3

Byzantine Bettors

In the popular imagination, Byzantium is famous for the incessant intrigues and plotting that supposedly took place among the courtiers of the palace. Apparently, the Byzantine court in no way deserved this reputation. Modern historical research suggests that Byzantium achieved a remarkable stability and harmony by the standards of the first millennium. Reputations die hard, however, and this puzzle concerns a game show inspired by the imagined intrigues of Byzantium. We call it Byzantine Bettors.

Each bet works as follows. There are some number of "advisors" and you. One advisor will write either 1 or 0 on a piece of paper, show it to the other advisors but not to you, and put that paper face down in front of you. Each advisor will tell you what the value is. They are all very good actors, so no obvious tick or facial expression will reveal to you who is telling the truth. The amount you can wager on each bet is anything from 0 to the total amount you have.

Warm-Up

Suppose there are four advisors, two of whom always tell the truth, though you don't know which ones. You get three even money bets. You start with $100. How much can you guarantee to win?

Solution to Warm-Up

If all four or three out of four agree on the first bet, then bet the maximum. There must be at least one truth teller in that group. If they are two and two, then bet nothing. After the first bet, you will know who the truth tellers are and will be able to bet the maximum each time. Thus, at least two times you will be able to bet all you have and be sure to win. This will give you a total of $400 at the end.

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1. Now suppose there are only three advisors and only one of them always tells the truth. Again, you can make three even money bets. You start with $100. How much can you guarantee to win?

The game has just become a little longer but also a lot more nasty. You have four bets, but there is no longer a truth teller, but merely a "partial truth teller." That advisor is not guaranteed to tell the truth all the time, but must do so at least three out of four times. Further, the advisors can actually substitute what's on the paper for a worse result for you once they hear your bet. However, they cannot change the result if so doing would eliminate the possibility that at least one of them is a partial truth-telling advisor.

2. What can you guarantee to win in four bets, with four advisors, three of whom can lie at will, and one who must tell the truth at least three out of four times?

3. Under the reliability conditions of one partial truth teller who tells the truth four out of five times and three liars at will, but assuming you have five bets, can you guarantee to end with at least $150?

A Touch of Luck

The movie *Intacto* posits a world in which luck is a more or less permanent quality of a person. Lucky people are lucky at the gambling table and they are lucky at traffic intersections. But such people have to avoid physical contact with a certain group of people who know how to steal their luck by touch. Finding lucky people is the quest of one of the movie's protagonists. In one case he tests the luck of possible recruits by having them run blindfolded through a forest. The winner is the one who gets to the destination first. Many others crash into one of the trees.

We will try a physically gentler game. In its general form, there are N people and B chances to bet. Every player knows the value of both N and B. Each person starts with an initial (not necessarily equal) wealth in units.

Each bet is an even money bet depending on the flip of a shared fair coin. So, if you bet an amount x and win, your wealth increases by x more. Otherwise you lose the x wagered.

Before each flip, each person bets an amount of his/her choosing (from 0 to any amount he/she has at that point) on either heads or tails.

The winner is the person having the greatest number of units after all B bets are done. If two people tie, then nobody wins. The units are worthless after the betting is done. So, a player receives a reward if and only if he or she is the absolute winner.

Warm-Up

Bob and Alice have the same number of units and Bob must state his bet first. Suppose there is just one remaining flip of the shared coin. What is Alice's chance of winning?

Solution to Warm-Up

If Bob bets x on heads, then Alice could bet $x+1$ on heads and Alice will win on heads and will lose on tails. Alternatively, Alice could bet nothing and then Alice will win on tails. Either way, Alice has a probability of 1/2 to win.

Warm-Up 2

Alice and Bob are the only players again. Alice has more units than Bob. There are five more coin flips to do. If Bob must state his bet before Alice on every flip, then how can Alice maximize her chance to win?

Solution to Warm-Up 2

Alice can win the game every time. For each flip, Alice simply copies Bob's bet. Suppose Bob bets b on heads. Alice bets b on heads too. Whether the shared coin lands on heads or tails, Alice will end up ahead of Bob.

Now here are some challenges for you.

1. Bob, Carol, and Alice play. Alice has 51 units, whereas Bob and Carol have 50 each. Bob must state his bet first, then Carol, and then Alice. Bob and Carol collude to share the reward if either wins. How can Bob and Carol maximize the probability that at least one of them will win if there is just one coin flip left?
2. Does the result change if Alice must state her bet first?
3. Suppose Bob has 51 units, and Alice 50. There are two coin flips left. Bob bets first for the penultimate flip. Alice bets first for the last flip. Does Bob have more than a 1/2 chance of winning? If so, how much more?
4. Suppose Bob has 51 units and Alice 50. Again, there are two coin flips left. This time, Alice bets first for the penultimate flip. Bob bets first for the last flip. Does Bob have more than a 1/2 chance of winning? If so, how much more?
5. Suppose Bob has 51 units and Alice 50. Again, there are two coin flips left. Again, Alice states her bet first in the penultimate round and Bob states his bet first in the final one. This time, Bob announces that he will bet 20 in the penultimate round, though he will wait to see Alice's bet before saying whether he will bet on heads or tails. Can Alice arrange to have more than a 1/2 chance to win?

Like joining a ballet company, winning the Olympics, or getting ahead in a narrow hierarchy of power, the more competition there is for fewer spots, the more risk one must take. Let's see whether you agree.

6. Bob, Alice, Rino, and Juliana have 100 units each and there are two flips left. Each person is out to win — no coalitions. Bob and Alice have bet 100 on heads. Rino has bet 100 on tails. Juliana must now state her bet, knowing that she will state her bet first on the last flip. What are her chances of winning if she bets 90 on either heads or tails? What should she bet?