ADVANCED DESIGN TECHNIQUES
AND REALIZATIONS OF
MICROWAVE AND RF FILTERS

PIERRE JARRY
JACQUES BENEAT
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Being asked to review the manuscript of Advanced Design Techniques and Realizations of Microwave and RF filters was an honor. The title truly represents the book’s focus and its contents.

Filters are the most important passive components used in RF and microwave subsystems and instruments to obtain a precise frequency response. In the early years of filter development, significant progress was made in waveguide and planar TEM filters. During the past two decades, filter technology has advanced in the area of emerging applications for both military and commercial markets. Several major developmental categories in filter technology are included: performance improvement, development of CAD tools, full-wave analysis, new structures and configurations, and advanced materials and associated technologies. Advanced materials/technologies such as high-temperature superconductor substrates, micromanufacturing, multilayer monolithic, low-temperature co-fired ceramic, and liquid-crystal polymer are commonly used in the development of advanced filters. Some recent applications of filters include dual-band communication, such as wireless local area networks and ultrawideband communication and imaging.

This book treats the subject to meet the needs for advanced filter design based on planar and waveguide structures that can satisfy the ever-increasing demand for design accuracy, reliability, fast development times, and cost-effective solutions. The topics discussed include analyses, design, modeling, fabrication, and practical considerations for both ladder and bridged filters. Modern design techniques are discussed for a wide variety of microwave filters, including comprehensive analyses and modeling of structures. These topics are self-contained, with practical aspects addressed in detail. Extensive design information in the form of equations, tables, graphs, and solved examples are included. To aid in solving filter-related design problems from specifications to realization of the end-product, the book provides a unique integration of theory and practical aspects of filters. Simple design equations and numerous practical examples are included which simplify the concepts of advanced filter design. With emphasis on theory,
design, and practical aspects geared toward day-to-day applications, the book is suitable for students, teachers, scientists, and practicing engineers.

Overall, the book is well balanced and includes exhaustive treatment of relevant topics important to a filter designer. I congratulate the authors on an outstanding book that I am confident will be very well received in the RF and microwave community.

Dr. Inder Bahl

Roanoke, Virginia
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Microwave and RF filters play an important role in communication systems, and due to the proliferation of radar, satellite, and mobile wireless systems there is a need for design methods that can satisfy the ever-increasing demand for accuracy, reliability, and fast development times. This book, which provides modern design techniques for a wide variety of microwave filters operating over the frequency range 1 to 35 GHz, has grown out of the authors’ own research and teaching and thus can present a unity of methodology and style, essential for a smooth reading.

The book is intended for researchers and for radio-frequency (RF) and microwave engineers, but is also suitable for an advanced graduate course in the subject area. Furthermore, it is possible to choose material from the book to supplement traditional courses in microwave filter design. The fundamental principles that can be applied to the synthesis and design of microwave filters are first recalled in a concise manner. Each of the 10 design chapters provides a complete analysis and modeling of the microwave structure used for filtering, as well as the design methodology. We hope that this will provide researchers with a set of approaches that can be used for current and future microwave filter designs. We also emphasize the practical nature of the subject by summarizing the design steps and giving numerous examples of filter realizations and measured responses so that RF and microwave engineers can have an appreciation of each filter in view of their needs. This approach, we believe, has produced a coherent, practical, and real-life treatment of the subject. The book is therefore theoretical but also experimental, with over 20 microwave filter realizations.

The book is divided into four parts. In Part I comprising the first four chapters, fundamental concepts and equations for microwave filter analysis and design are provided. Chapter 1 covers definitions and examples of the scattering and ABCD parameters of two-port systems. Classical elements used in microwave filter design, such as impedance and admittance inverters, are reviewed. The bisection theorem, which is often very useful to simplify the synthesis of microwave
filters, is presented. Chapter 2 summarizes filter approximations and synthesis. Several functions, such as the Butterworth, Chebyshev, elliptic, and pseudoelliptic, are given for amplitude-oriented filters. The Bessel and Rhodes equidistant linear-phase functions are provided for phase-oriented filters. The general synthesis method for some of these functions is discussed in terms of lumped ladder realizations. The chapter ends with useful properties and equations related to realizations based on impedance and admittance inverters. In Chapter 3 we recall the fundamental equations for waveguides, striplines, suspended substrate striplines, and distributed circuits. The general design approaches used for the filters presented in the book are given in Chapter 4. The filters are regrouped in terms of minimum or non-minimum-phase microwave filters. The latter being regrouped according to the low-pass symmetry of their magnitude response around 0 Hz, referred to as non-minimum-phase symmetrical or asymmetrical microwave filters.

Part II consists of Chapters 5 through 8, corresponding to the analysis and design of four minimum-phase filters. In Chapter 5 we describe the analysis and modeling of capacitive gap filters implemented using suspended substrate stripline. A straightforward design technique is presented and actual realizations and measured responses are given for narrow- and wideband filters in the range 8 to 16 GHz. The chapter concludes with preliminary results showing that this technique can be extended to the design of millimeter-wave filters. An example of a 35-GHz suspended substrate stripline filter is given. It becomes clear, however, that more rigorous electromagnetic analysis and treatment of problems will be needed to improve the performance of filters at these frequencies. In Chapter 6 we present a design technique for evanescent-mode waveguide filters, which can have either rectangular or circular cross sections and include a number of dielectric inserts. These filters can be smaller than traditional waveguide filters, since the cutoff frequency of the guide can be increased even if this means that the fundamental mode will be evanescent in part of the structure.

Several realizations showing the reliability of the technique are given at 14 GHz. In an attempt to reduce the form factor of these filters, the design of folded evanescent mode waveguide filters is introduced. Two realizations with different angles of curvature around 8.4 and 14.6 GHz are described. In Chapter 7 we present the design of interdigital filters, structures modeled using the method of graphs rather than the admittance matrix approach. The design is based on the use of a low-pass prototype circuit, due to the periodic nature in frequency of interdigital line segments. Two realizations around 1 and 2 GHz made using suspended substrate stripline are shown. In Chapter 8 an exact design procedure for combline filters is provided. The technique is applied to a filter at 1.2 GHz using suspended stripline.

Part III which includes Chapters 9 and 10, provides two techniques for designing non-minimum-phase symmetrical response filters. In the case of symmetrical responses, the bisection theorem can be used to reduce the complexity of the design method. The filters presented in this part are concerned primarily with phase characteristics. The non-minimum-phase condition leads to additional
degrees of freedom that can be used to shape the frequency response of the filter. However, this also requires microwave structures that can accommodate the non-minimum-phase condition, and the design techniques are usually more complex. Chapter 9 makes use of the additional degrees of freedom to design filters that have good group delay characteristics. The filters are implemented using a generalized interdigital structure. A realization at 2.7 GHz based on interdigital bars is shown to give a group delay variation of 2 ns in the passband. In Chapter 10 we present very narrowband (e.g., 1%) circular monomode TE_{011} cavity filters. A method for optimizing the group delay is introduced. A monomode filter design and realization is given at 14.5 GHz. These filters are also suited for temperature variations. The stability of filter responses from $-10$ to $+20^\circ\text{C}$ is demonstrated.

Part IV groups Chapters 11 through 14 and deals with non-minimum-phase asymmetrical response filters. The filters in this chapter present generalized Chebyshev or pseudoelliptic bandpass responses with a given number of transmission zeros. In the case of asymmetrical responses, the lowpass prototype is imaginary, and neither the bisection theorem nor traditional frequency transform techniques can be used. The microwave structures must also be able to generate zeros of transmission. In Chapter 11 we describe the design technique for capacitive-gap coupled line filters that have generalized Chebyshev asymmetrical responses. A third-order realization in suspended stripline at 10 GHz shows an asymmetrical response with a zero of transmission at a frequency above the passband. Chapter 12 deals with a state-of-the-art family of in-line dual-mode rectangular structures using resonant TE_{102} and TE_{301} modes. Two realizations are given for frequencies around 12 GHz. In Chapter 13 we describe a technique used to design cylindrical dual-mode cavity filters with asymmetrical responses. Good amplitude characteristics are obtained using the TE_{113} resonant mode. Several realizations using a dual-mode cavity are given around 8.5 GHz. The responses show that various combinations of the zeros of transmission can be obtained.

In Chapter 14 we introduce a new concept for the design of non-minimum-phase microwave filters. The filters are made of basic rectangular waveguide building blocks, and the filters can be designed by using a powerful optimization algorithm called the genetic algorithm. Use of this method makes it possible to design rectangular multimode cavity filters with generalized Chebyshev responses. The realization of a fourth-order filter at 14 GHz with two zeros of transmission using one building block (TE_{100}, TM_{120}, and TM_{210} modes), and a seventh-order filter at 20 GHz with four zeros of transmission using two building blocks are given. Due to the simplicity of the building blocks, these filters are also easy to manufacture.

The plan of the book is summarized in Figure P1. It shows that we begin in Part I with fundamental concepts and equations useful for designing microwave filters. The reader can stop after Part II which provides a synthesis of minimum-phase microwave filters. The reader could also go directly to more advanced filters known as non-minimum-phase filters. Part III provides the design techniques
used for non-minimum-phase filters with symmetrical responses, and Part IV covers approaches for designing non-minimum-phase filters with asymmetrical responses. The difference between the latter two types of filters depends on the low-pass network being considered (real or imaginary).

We would like to acknowledge the contributions of our past and present research students whose collaboration has resulted in much of the material in the book. In particular, we would like to mention Professor Humberto Abdalla Junior and Professor Horacio Tertuliano from Brazil, Professor G. Tanné and Associate Professors J.F. Favenne and F. Le Pennec from France, Professor C. Djoub from Ivory Coast, Engineer M. Lyakoubi from Canada, and Engineers N. Bouteille, CL. Guichaoua, E. Hanna, M. Lecouve, D. Lo Line Tong and O. Roquebrun from France.

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The work resulted in 2 patents with ESA, 3 patents with ALCATEL SPACE and approximately 16 contracts with the different agencies and companies.

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Pierre Jarry
Jacques Beneat
PART I

MICROWAVE FILTER FUNDAMENTALS
1

SCATTERING PARAMETERS AND
ABCD MATRICES

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1.1 INTRODUCTION

In this chapter we recall the most important characterization techniques used in
the design of microwave filters [1.1]. These consist of the scattering parameters,
which are often based on electromagnetic analysis of the microwave structures,
and the ABCD parameters, which are useful to make the link with two-port
systems and have been studied exhaustively over the years. Several examples
are presented to better understand the relations between the two formalisms. The
bisection (or Bartlett) theorem is also reviewed and proves to be very useful in
the case of symmetrical networks.
1.2 SCATTERING MATRIX OF A TWO-PORT SYSTEM

1.2.1 Definitions

The scattering matrix [1.2] of a two-port system provides relations between the input and output reflected waves $b_1$ and $b_2$ and the input and output incident waves $a_1$ and $a_2$ when the structure is to be connected to a source resistance $R_G$ and a load resistance $R_L$, as depicted in Figure 1.1. The notion of waves rather than voltages and currents is better suited for microwave structures.

For a two-port system, the equations relating the incident and reflected waves and the $S$ parameters are given by

\[
\begin{align*}
  b_1 &= S_{11}a_1 + S_{12}a_2 \\
  b_2 &= S_{21}a_1 + S_{22}a_2
\end{align*}
\]

These equations can be summarized in the matrix form \((b) = (S)(a)\), where

\[
\begin{pmatrix}
  b_1 \\
  b_2
\end{pmatrix} =
\begin{pmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
  a_1 \\
  a_2
\end{pmatrix}
\]

The parameter $S_{11}$, called the input reflection coefficient, can be computed by setting the output incident wave $a_2$ to zero and taking the ratio of the input reflected wave over the input incident wave:

\[
S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}
\]

The output incident wave $a_2$ is set to zero by connecting the output of the system to the reference resistor $R_L$. The parameter $S_{11}$ provides a measure of how much of the input incident wave does not reach the output of the system and is reflected back at the input. For microwave filters, ideally, $S_{11}$ should be equal to zero in the passband of the filter.

The parameter $S_{21}$, called the forward transmission coefficient, can be computed by setting the output incident wave $a_2$ to zero and taking the ratio of the output reflected wave over the input incident wave:

\[
S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}
\]

Figure 1.1 Notation used in defining the scattering matrix of a two-port system.
The output incident wave $a_2$ is set to zero by connecting the output to the reference resistor $R_L$. The parameter $S_{21}$ provides a measure of how much of the input incident wave reaches the output of the system. For microwave filters, ideally, $S_{21}$ should be equal to 1 in the passband of the filter.

The parameter $S_{22}$, called the reflection coefficient at the output of the system, can be computed by setting the input incident wave $a_1$ to zero and taking the ratio of the output reflected wave over the output incident wave:

$$S_{22} = \frac{b_2}{a_2} \mid a_1 = 0$$

The input incident wave $a_1$ is set to zero by connecting the input of the system to the reference resistor $R_G$. As in the case of $S_{11}$, it is desirable that $S_{22}$ be kept close to zero in the passband of the filter. $S_{11}$ and $S_{22}$ provide a measure of how well the system impedances are matched to the reference terminations.

The parameter $S_{12}$, called the reverse transmission coefficient, can be computed by setting the input incident wave $a_1$ to zero and taking the ratio of the input reflected wave over the output incident wave:

$$S_{12} = \frac{b_1}{a_2} \mid a_1 = 0$$

The input incident wave $a_1$ is set to zero by connecting the input of the system to the reference resistor $R_G$. The parameter $S_{12}$ provides a measure of how much of an incident wave set at the output of the system would reach the input. Due to symmetries in the system, $S_{21}$ and $S_{12}$ can have similar values. Since there are no generators at the output of the system, an output incident wave could appear due to a poor $S_{22}$.

The scattering parameters can be illustrated using a graph, as shown in Figure 1.2. The graph shows that part of the incident wave $a_1$ results in a reflected wave $b_1$ through the parameter $S_{11}$, and in a transmitted wave $b_1$ through the parameter $S_{21}$. Similar descriptions can be given for $a_2$ and the parameters $S_{22}$ and $S_{12}$. It is always important to remember that the $S$-parameter values are associated with a given set of termination values. Changing the termination values will change the $S$-parameter values.

![Figure 1.2](image-url)  
*Figure 1.2* Graph of a two-port scattering matrix.
1.2.2 Computing the S Parameters

A common example of a scattering matrix in microwave is that of a waveguide of length $l_0$ and characteristic impedance $Z_0$, as shown in Figure 1.3. When the structure is to be connected to a source and load resistance equal to the characteristic impedance of the waveguide, the scattering matrix is given by

$$(S) = \begin{pmatrix}
0 & e^{-j\beta l_0} \\
e^{-j\beta l_0} & 0
\end{pmatrix}$$

where $j\beta$ is the propagation function of a given mode above the cutoff frequency of the waveguide. This matrix tells us that the structure will be perfectly matched to the terminations since $S_{11}$ and $S_{22}$ are equal to zero. It also tells us that $b_1$, the wave transmitted, will simply be a delayed version of $a_1$, the incident wave, since the forward transmission coefficient, $S_{21}$, has a magnitude of 1 and a linear phase of $-\beta l_0$, and the longer the length, the longer the delay. Since we cannot differentiate one end of a waveguide from the other, we would have similar results if connecting the source to the output and the load to the input (e.g., $S_{12} = S_{21}$).

As will be seen, microwave structures will at times have discontinuities that result in the apparition of “scattered” and unwanted electromagnetic fields. For these cases, matching the electromagnetic fields on each side of the discontinuity will provide relations that can be used for defining the scattering parameters of the discontinuity. In that case, the scattering parameters will be defined directly from electromagnetic wave equations. It should be noted, however, that scattering parameters are not limited to microwave structures and electromagnetic field equations.

The incident and reflected waves can be expressed in terms of voltages and currents, as shown in Figure 1.1.

$$a_1 = \frac{V_1 + R_G I_1}{2\sqrt{R_G}} \quad a_2 = \frac{V_2 + R_L I_2}{2\sqrt{R_L}}$$

$$b_1 = \frac{V_1 - R_G I_1}{2\sqrt{R_G}} \quad b_2 = \frac{V_2 - R_L I_2}{2\sqrt{R_L}}$$

Figure 1.3  Waveguide of length $l_0$ and characteristic impedance $Z_0$. 
Figure 1.4  Defining the scattering parameters of a resistive two-port system.

For example, the scattering parameters of the resistive two-port system in Figure 1.4 can be defined from these voltages and currents.

The input reflection coefficient $S_{11}$ is defined from the input incident and reflected waves when the system is connected to the reference resistor $R_L$, as shown in Figure 1.5. Also shown in the figure, the system connected to resistor $R_L$ can be modeled as $Z_{in}$, an input impedance of the system. In this case, $V_1 = Z_{in}I_1$ and $S_{11}$ is given by

$$S_{11} = \frac{V_1 - R_GI_1}{V_1 + R_GI_1} = \frac{Z_{in}I_1 - R_GI_1}{Z_{in}I_1 + R_GI_1} = \frac{Z_{in} - R_G}{Z_{in} + R_G}$$

This gives $Z_{in} = R_1 + R_2||(R_1 + R_L)$ for the input impedance of the system. For $S_{11}$ to be equal to zero, the input impedance $Z_{in}$ should be equal to the source resistor $R_G$.

The forward transmission coefficient $S_{21}$ is defined from the input incident wave and output reflected wave when the system is connected to the reference resistor $R_L$, as shown in Figure 1.6. Replacing the incident and reflected waves by their voltage and current expressions, $S_{21}$ is given by

$$S_{21} = \sqrt{\frac{R_G \frac{V_2 - R_LI_2}{R_L}}{V_1 + R_GI_1}}$$

Also from the computations of $S_{11}$, $V_1 = Z_{in}I_1$ when the system is connected to $R_L$. From Figure 1.6 it is also seen that $V_2 = -R_LI_2$. Therefore, $S_{21}$ will be

Figure 1.5  Defining the input reflection coefficient $S_{11}$. 
Figure 1.6  Defining the forward transmission coefficient $S_{21}$.

given by

$$S_{21} = \sqrt{\frac{R_G}{R_L}} \frac{V_2 - (-V_2)}{V_1 + (R_G/Z_{in})V_1} = 2 \sqrt{\frac{R_G}{R_L}} \frac{1}{1 + R_G/Z_{in}} \frac{V_2}{V_1}$$

Note that in the case where $Z_{in} = R_G$ (input impedance matching) and $R_G = R_L$ (similar source and load terminations), the forward coefficient reduces to

$$S_{21} = \frac{V_2}{V_1}$$

In Figure 1.6,

$$V_2 = \frac{R_L}{R_L + R_1} V_A \quad \text{and} \quad V_A = \frac{R}{R + R_1} V_1$$

where $R = R_2 || (R_1 + R_L)$, so that

$$V_2 = \frac{R_L}{R_L + R_1} \frac{R}{R + R_1} V_1$$

and a general expression for $S_{21}$ is given by

$$S_{21} = 2 \sqrt{\frac{R_G}{R_L}} \frac{1}{1 + R_G/Z_{in}} \frac{R_L}{R_L + R_1} \frac{R}{R + R_1}$$

The $S_{22}$ and $S_{12}$ parameters can be defined using a similar process, where the input is now connected to the reference resistor $R_G$. In the resistive example above, the $S$ parameters are independent of frequency since the impedances of the resistors are independent of frequency. However, the results can be used to define the $S$ parameters of a more general case, as shown in Figure 1.7. The input reflection coefficient $S_{11}$ will now be a function of the impedances of the system and therefore depend on the frequency of application through the Laplace variable $s$:

$$S_{11}(s) = \frac{Z_{in}(s) - R_G}{Z_{in}(s) + R_G}$$