



Modeling and Simulation of Turbulent Flows

Roland Schiestel

ISTE

 **WILEY**

This page intentionally left blank

Modeling and Simulation of Turbulent Flows

This page intentionally left blank

Modeling and Simulation of Turbulent Flows

Roland Schiestel

ISTE

 **WILEY**

First published in France in 2006 by Hermes Science/Lavoisier entitled "Méthodes de modélisation et de simulation des écoulements turbulents"

First published in Great Britain and the United States in 2008 by ISTE Ltd and John Wiley & Sons, Inc.

Apart from any fair dealing for the purposes of research or private study, or criticism or review, as permitted under the Copyright, Designs and Patents Act 1988, this publication may only be reproduced, stored or transmitted, in any form or by any means, with the prior permission in writing of the publishers, or in the case of reprographic reproduction in accordance with the terms and licenses issued by the CLA. Enquiries concerning reproduction outside these terms should be sent to the publishers at the undermentioned address:

ISTE Ltd
6 Fitzroy Square
London W1T 5DX
UK

www.iste.co.uk

John Wiley & Sons, Inc.
111 River Street
Hoboken, NJ 07030
USA

www.wiley.com

© ISTE Ltd, 2008

© LAVOISIER, 2006

The rights of Roland Schiestel to be identified as the author of this work have been asserted by him in accordance with the Copyright, Designs and Patents Act 1988.

Library of Congress Cataloging-in-Publication Data

Schiestel, Roland.

[Méthodes de modélisation et de simulation des écoulements turbulents. English]

Modeling and simulation of turbulent flows / Roland Schiestel.

p. cm.

Includes bibliographical references and index.

ISBN: 978-1-84821-001-1

1. Turbulence--Mathematical models. I. Title.

TA357.5.T87S3713 2008

532'.0527015118--dc22

2007028098

British Library Cataloguing-in-Publication Data

A CIP record for this book is available from the British Library

ISBN: 978-1-84821-001-1

Printed and bound in Great Britain by Antony Rowe Ltd, Chippenham, Wiltshire.

Front cover picture (IRPHE): Turbulence kinetic energy levels in the unsteady flow between rotating disks, using Reynolds stress transport modeling (see [RAN 04]).



Table of Contents

Foreword	xi
Preface	xv
Acknowledgements	xix
Introduction	xxi
Chapter 1. Fundamentals in Statistical Modeling:	
Basic Physical Concepts	1
1.1. The nature of turbulence	1
1.2. The various approaches to turbulence	8
1.3. Homogenous and isotropic turbulence (HIT).	18
1.4. Kolmogorov hypotheses and the local isotropy theory	34
1.5. One point closures.	42
1.6. Functional description of turbulence	52
1.7. Turbulent diffusion and Lagrangian description	54
1.8. Two-dimensional turbulence	56
Chapter 2. Turbulence Transport Equations for an Incompressible Fluid	59
2.1. General transport equations	59
2.2. Equations specific to the main types of turbulent flows.	69
Chapter 3. Mathematical Tools	79
3.1. Tensors	79
3.2. Euclidian space in curvilinear coordinates, tensor fields	85
3.3. Orthogonal curvilinear coordinates.	88
3.4. Conformal transformation	92

3.5. Invariants	94
3.6. Representation of tensorial functions	101
3.7. Fourier transform in the fluctuating field	110
3.8. Wavelet transform	114
Chapter 4. Methodology for One Point Closures	115
4.1. Order of magnitude estimate of terms in the turbulence transport equations	116
4.2. Application to the momentum equations, and the k and ε equations	119
4.3. Derivation of closure hypotheses	120
4.4. The formalist approach: Lumley's invariant modeling	121
4.5. Examples of application	126
4.6. Realizability problem	131
4.7. Objectivity and material indifference	146
4.8. Diffusive correlations	149
4.9. Probability densities and stochastic models	152
4.10. Intermittency	156
4.11. Practicing with the development tools	158
Chapter 5. Homogenous Anisotropic Turbulence	159
5.1. The Craya equation	159
5.2. One-dimensional spectral properties in homogenous turbulent shear flows	163
5.3. Rapid part of pressure correlations in the rapid distortion of isotropic turbulence	164
5.4. Spectral models	166
5.5. Turbulence associated to a passive scalar	166
5.6. One point correlation equations in HAT	167
5.7. Examples of anisotropic homogenous turbulent flows	167
5.8. Rapid distortion theory for an homogenous turbulent flow	173
5.9. Additional information on linear solutions	177
5.10. Interdependency between differing closure levels: the spectral integral approach	178
Chapter 6. Modeling of the Reynolds Stress Transport Equations	183
6.1. The Reynolds stress transport equations and their trace	183
6.2. Modeling viscous dissipation terms	187
6.3. Modeling turbulent diffusion terms	188
6.4. Pressure-strain correlations	192
6.5. Determination of numerical constants	208
6.6. The realizability of the basic models	212

Chapter 7. Turbulence Scales	217
7.1. The turbulent kinetic energy dissipation rate equation	218
7.2. Modeling of diffusive terms	220
7.3. Modeling of source and sink terms	221
7.4. Determination of numerical constants	226
7.5. Corrective changes introduced on the dissipation equation	228
7.6. Reconsidering the ε equation: an asymptotic behavior with finite energy?	230
7.7. Tensorial volumes	232
7.8. Case of generation of turbulence injected at a fixed wavenumber	234
7.9. Modeling the dissipation tensor	234
Chapter 8. Advanced Closures: New Directions in Second Order Modeling	241
8.1. A new generation of second order models	242
8.2. Constraints related to the invariance properties with respect to the frame of reference	252
8.3. Other methods of approach for the pressure-strain correlations	254
8.4. Elimination of topographical parameters	257
8.5. Models based on the renormalization group (RNG models)	259
8.6. Memory effects	260
8.7. Pressure-velocity correlations	261
8.8. Internal variable models, structural models	262
Chapter 9. Modeling the Turbulent Flux Evolution Equations for a Passive Scalar	269
9.1. Evolution equations of the turbulent fluxes of a passive scalar	269
9.2. Order of magnitude of terms	271
9.3. Modeling dissipative terms	272
9.4. Modeling the turbulent diffusion terms	272
9.5. Modeling the pressure-passive scalar gradient correlations	274
9.6. Determination of numerical constants	278
9.7. New generation of modeling	284
Chapter 10. The Passive Scalar Variance and its Dissipation Rate	285
10.1. Transport equation for the variance of a passive scalar	285
10.2. Modeling the turbulent diffusion terms	286
10.3. Modeling the dissipation rate	287
10.4. Equation for the dissipation rate of the passive scalar variance	288
10.5. New directions of research	290

Chapter 11. Simplified Closures: Two and Three Transport Equation Models	293
11.1. The k - R_{12} - ε model for turbulent thin shear flows	293
11.2. Two equation models	295
11.3. Algebraic modeling of the Reynolds stresses and the turbulent fluxes of a passive scalar	313
11.4. Non-linear models	317
11.5. Explicit algebraic models	323
Chapter 12. Simplified Closures: Zero and One Transport Equation Models	331
12.1. One equation models	332
12.2. Zero equation models	337
Chapter 13. Treatment of Low Reynolds Number Turbulence	347
13.1. Reynolds stress equations	348
13.2. Equation for the dissipation rate	349
13.3. The k - R_{12} - ε model for wall flows	351
13.4. Modification of the turbulent fluxes in low intensity turbulence	353
13.5. Lower order models	355
13.6. Advanced modeling	363
13.7. Transition and laminarization	384
Chapter 14. Wall Treatment: Methods and Problems	385
14.1. The turbulent flow near a wall	385
14.2. Wall functions	388
14.3. Simple models for the viscous sublayer	398
14.4. Models using several transport equations for the viscous sublayer	403
14.5. New directions in the wall function formulation	403
Chapter 15. Influence of Archimedean Forces	407
15.1. Transport equations of turbulence in the Boussinesq approximation	407
15.2. Influence of buoyancy terms in the pressure-strain correlations	411
15.3. Influence of buoyancy forces on the pressure-temperature gradient correlations	412
15.4. Influence of buoyancy forces on the turbulence length scales or the dissipation rate	414
15.5. Two-dimensional horizontal flows in the presence of buoyancy forces	415
15.6. Algebraic modeling	416
15.7. Simplified models	419
15.8. Advanced models of the new generation	421

Chapter 16. Notes on the Problems Posed by the Study of Complex Flows	423
16.1. Curvature effect	424
16.2. Secondary motions.	428
16.3. Rotation effects.	430
16.4. Examples of complex turbulent flows for which the traditional one point closures fail	432
16.5. More on the Navier-Stokes equations in a relative frame of reference	433
16.6. Algebraic modeling of turbulence submitted to rotation	437
16.7. Implicit effects of rotation on the turbulent field	444
16.8. Rotating turbulence in the presence of active thermal effects	450
16.9. Coherent structures and modeling.	452
16.10. Laminar/turbulent interface, free boundaries	452
Chapter 17. Variable Density Turbulent Flows	457
17.1. Averaging.	458
17.2. Transport equations	459
17.3. Reynolds stress transport modeling in the framework of mass weighted averaging.	464
17.4. Dissipation rate equation	466
17.5. Turbulent heat flux equations	467
17.6. Equation for the variance of temperature fluctuations.	468
17.7. Two equation models and simplified models	469
17.8. Approach in non-weighted variables	470
17.9. Continuity.	471
17.10. Statistical equations and modeling.	472
17.11. Dissipation rate equation	475
17.12. Other approaches	475
17.13. Note on compressed turbulence	477
Chapter 18. Multiple Scale Models	481
18.1. Intuitive approach	487
18.2. Foundations of the method	493
18.3. Practical formulations and extensions	516
18.4. Other multiple scale models: models using spectral weighted integration	536
Chapter 19. Large Eddy Simulations	539
19.1. The filters	542
19.2. The filtered Navier-Stokes equations.	546
19.3. Subgrid-scale modeling.	551
19.4. Some remarks on the numerical methods	559
19.5. Simulation of homogenous flows	560
19.6. Simulation of non-homogenous turbulent wall flows	562

19.7. Estimate of subgrid-scale energy	566
19.8. Variable filters	567
19.9. Advanced subgrid-scale models	568
19.10. Flows undergoing laminar-turbulent transition.	579
19.11. Other transport equation models.	580
19.12. Approximate deconvolution methods	581
19.13. Simulations based on POD or on wavelets.	584
19.14. Hybrid methods.	586
Chapter 20. Synopsis on Numerical Methods	601
20.1. Numerical techniques	602
20.2. Plates.	604
Exercises	645
Bibliography	661
Nomenclature	715
Index	719

Foreword

When, rather more than 10 years ago, Roland Schiestel sent me the manuscript for a new book on turbulent flows that he had written, I was delighted to see that, while rigorous in the development of traditional approaches to turbulence, these were used to serve the main theme of the work, namely the modeling of turbulence in a form suitable for use in CFD solvers. His invitation to write a preface was gladly accepted and the words I wrote then perhaps still merit repeating:

The fluid mechanics of the world we live in is overwhelmingly dominated by that chaotic, unsteady motion called turbulent flow. Whether it be the flow of air and water in the natural environment or the man-managed interior environment, heat, momentum and mass exchange is brought about by large-scale, irregular eddying motions rather than by molecular diffusion and the design of virtually all types of thermo-fluids equipment: pipes, boilers, compressors, turbines, IC engines, condensers, etc. are variously designed to cope with or exploit the fact that the fluids passing through or around them are in turbulent motion.

This is such a commonplace observation that the reader may feel it hardly deserves mention. Yet, if – instead of using our eyes to view the world about us – we formed our view of the nature of fluid motion by reading fluid mechanics textbooks, what a different impression would be gained! From such a study we would understand that for a great many problems fluid viscosity is an irrelevancy, in most others the flow remains perfectly laminar while, to handle that rather inconvenient (and apparently unimportant) state called “turbulence” we refer to tediously compiled experimental correlations.

This distorted view of the relative importance of different strands of engineering fluid mechanics underlines the extent to which academics base the syllabus of their courses on what they know, rather than on what is relevant. That, I suppose, is as inescapable a fact of life as turbulent flow itself.

At the research level, the computation of turbulent flows has long been a subject receiving greater attention than its scant coverage in textbooks would lead us to expect. Now the rapid growth of software companies marketing commercial CFD packages (coupled with a corresponding growth of users of such software) has helped bring home the need for more – and more systematic – instruction on the internal workings of these black boxes. The aspect of CFD software where questions most often arise and where, through the absence of textbooks, they are least easily handled is on turbulence modeling. There is, manifestly, a need for a comprehensive textbook treatment of engineering turbulence modeling, perhaps particularly one written by an active contributor to the continuing advance of the subject.

The above was the scene, as I perceived it, in the early 1990s when the first edition of Roland Schiestel's book appeared to warm reviews. Over those intervening years, of course, the world of turbulence modeling has moved on, and with the first edition and then an enlarged second edition sold out, the author and publisher have concluded that the time has come for a new edition. The fact that the earlier editions *had* sold out was a good indication that the book was meeting a real need and that the structure and philosophy should remain intact – as it *has*. For example, the book rightly focuses on second-moment closure for it is only at this level that the subject can be developed formally as a branch of mathematical physics (having adopted that starting point, simpler levels of closure naturally emerge as particularly limiting cases that are applicable under increasingly restricted circumstances). Moreover, without recourse to modeling the unknown processes in the second-moment equations, an examination of the exact generation terms explains, at least qualitatively, so many of the paradoxes of turbulent shear flows. For example: why turbulent mixing typically results in twice as much heat flow at *right angles* to the mean temperature gradient as along it; why a secondary strain associated with streamline curvature whose magnitude is only 2% of the primary strain produces a 25% modification in the turbulent shear stress; or why, in orthogonal mode rotation, a relatively weak Coriolis force augments shear stress on the pressure surface by 10% whereas further increase in the rotation rate produces no further augmentation.

Thus, the fabric and style of the very successful original version are retained in this new edition. Among the several new additions, is the inclusion of new approaches to the economical handling of the near-wall region where “wall functions” are normally adopted to escape the crippling cost of a fine-scale resolution of the sublayer and buffer region. The usual log-law based wall functions had such a narrow range of applicability that alternative strategies were sorely needed. These are now included in a new presentation of this material. The number of references has also increased by some 30%, the great majority of which are to works appearing in the last five years. Thus, this new edition continues to serve admirably both those in industrial CFD needing to understand the physical basis of

their software as well as those engaged in or about to start their research in turbulence modeling.

In the foreword to the original edition I had written: “turbulence modeling is still seen by many as a black art founded on bad physics and capable of producing computed flow patterns in accord with measurements only by the arbitrary, case-by-case adjustment of a sackful of empirical constants and other less reputable fudges”. That perception is, happily, much less commonly found today. In France, Roland Schiestel’s previous editions have been a major contributor to the better-founded appreciation of turbulence modeling by the scientific and industrial communities. May this new edition continue the good work!

Brian E. LAUNDER
Manchester

This page intentionally left blank

Preface

The present book was originally developed for a postgraduate course on Modeling and Simulation of Turbulent Flows taught by the author at Aix-Marseille University for a number of years. A first edition in French appeared in 1993 at Hermes publishers, with new expanded editions in 1998 and 2006. This last French edition was the basis for the present English edition.

Although there exists extensive scientific literature that deals with turbulent flows and their numerical modeling, the information is generally disseminated among numerous papers in specialized international publications. The aim of the present book is to give an introductory, synthetic presentation of numerical modeling and simulation methods for turbulent flows from its basic foundations. It is primarily intended for potential users of numerical models, postgraduate students at university, as well as researchers and practicing engineers interested in the practical calculation of turbulent flows. Some technical details have been marked by a vertical line in the left margin. These may be of interest to some readers but can be skipped if desired. The book gives the physical foundations of the modeling methods and leads the reader to a point where he can implement and make use of the model equations in practical applications with a clear knowledge of the underlying physics and then go deeper into more advanced techniques. Having in mind the actual numerical solution on the computer, some recapitulative tables on numerical methods are grouped together in the last chapter. We shall not claim to or aim at exhaustiveness by any means, considering the numerous works that have been achieved in the field of turbulence modeling, but rather we shall try to follow a rational pathway through the multiform landscape of turbulence modeling, with an emphasis laid upon basic concepts and methods of approach.

In spite of the great variety of experimental studies on the structure of turbulent flows, the fundamental mechanisms in turbulence phenomena still remain

incompletely elucidated and even nowadays many problems remain open, sometimes enigmatic.

However, most fluid flows encountered in the domain of industrial practice are likely to be turbulent and many phenomena, such as heat or mass transfer, are so intimately linked to the fluid motion that their study requires prior calculation of the turbulent flow. Thus, the numerical prediction of turbulent flows is of primary importance for numerous practical applications (industrial, environment, etc.).

The proposed presentation relies on several traditional basic concepts for fluid turbulence phenomenology, and in particular the Kolmogorov theory. The methodology of one point closures is developed all the way to its application to second order moment transport models. Lower order models will then be presented as simplified approaches deduced from second order closures, even if the historical order is the reverse. The impact of spectral theories is also essential, in particular on the notions of spectral equilibrium or on linear and non-linear interactions. These spectral theories, although mainly representative of homogenous turbulence, allow a more refined description of turbulence interaction mechanisms.

Among existing theories and models, preference is given to methods that lead to actual numerical predictions of turbulent flows. In this way, and besides one point and two point statistical closures, the book also addresses large eddy simulation methods that have been developed and increasingly used since the advent of supercomputers. This standpoint has led us to discard detailed presentation of analytical theories and every purely theoretical approach not leading to practical prediction methods.

Compared to the deductive reasoning prevailing in exact sciences, the method of approach used in turbulence modeling may be surprising because of its empiricism. However, the modeling approach is not specific to turbulence, it is also used in many domains in physics. According to M. Dode¹ there are two ways to study natural phenomena: the method of exact science and the model method. The method of exact science such as in thermodynamics, mechanics, optics, electromagnetism, is based on very few fundamental principles, the value of which is considered absolute in the field of the science under consideration. These fundamental principles or postulates have an experimental origin and have been discovered by induction. All scientific laws are then deduced from these first premises by applying mathematics and the rigor of these laws is absolute within the domain in which the postulates are assumed.

¹ Translated from M. DODE, *Le deuxième principe*, Sédes Ed., Paris, 1965.

On the other hand, the model method tries to interpret the phenomena and to represent their mechanisms through a picture. This is the case for the atomic theory which proposed a mechanical model for the structure of an atom. In the model method, we try to imagine a mechanism whose details remain hidden and that would be capable of giving an interpretation of observed facts. Once this mechanism is defined, we try to draw all the possible conclusions from it. The general character of the proposed model is then recognized according to the value of the predictions it allows, predictions that exact sciences are not able to provide. The method of exact science and the model method are not conflictive, they are complementary. The further our knowledge advances, the further the physical models move away from direct experimentation and become more and more abstract until they become pure “model equations”.

According to this line of thought, a turbulence model is thus composed of “model equations”. They describe a phenomenon, which is not actual turbulence but which is sufficiently close to it for representing a useful simplified picture. The accuracy of the model and its ability to represent the properties of a turbulent flow, are directly dependent on our knowledge of the physics of the phenomena that it has been possible to build in the equations.

The model is an almost quantified summary of our present knowledge on fluid turbulence. According to this point of view, mathematical models are perfectible and indeed are improving everyday, enriched by new concepts inspired by experimental results or numerical simulations or progress achieved in theoretical approaches.

The present book gives a prominent position to formalism and equations, thus allowing a rational approach to models in connection with underlying physical concepts and intuition. Beyond the description of existing models, the aim is to show how to develop mathematical models and their elaboration process. In this prospect, equations, which are the very language of science, allow us both to condense the physical concepts into a mathematical scheme and to provide a predictive tool. They call to mind this “bijection” between the physical world and mathematical formalism toward which theories and models are tending.

Recent years have seen remarkable developments in turbulence modeling towards more and more advanced formulae that are the fruit of much research carried on in this domain by various research teams around the world. Effort has been concentrated in particular on the realizability properties allowing us to deal with “extreme” states of the turbulence field (high anisotropies and also compressibility effects, reorganization by rotation, etc.) and also on a more refined description of the underlying physics aiming at a wider universality of description. These efforts towards model development, along with the emergence of new concepts, are currently ongoing, largely boosted by the quick expansion of new

measurement techniques such as laser velocimetry and especially by the generalized use of direct numerical turbulence simulations that provide numerical databases for calibrating and testing turbulence models. Direct numerical simulations, in spite of their limitations in Reynolds number, allow us not only to consider types of flows or physical parameter values that are impossible to obtain experimentally but also to test the closure hypotheses directly. Thus, the different turbulent phenomena being more easily differentiated and studied separately, this results in a more detailed physical description that goes beyond a mere mimicry of the observed behavior of a turbulent flow. We can say that the development of new techniques of approach including direct numerical simulations of turbulence, far from substituting for modeling, have been on the contrary a catalyst for progression. We can then observe, not only an improvement of turbulence models but also a larger variety of model types (non-linear models, transport equations for new quantities, etc.) arising. Second order closures remain however a preferred reference level of closure allowing both extended potentials in physical description and an efficient numerical solution of the equations. From the user point of view, the methods of approach of turbulent flows are very varied, ranging from one point closure models with a limited number of equations up to advanced transport modeling and large eddy simulations that can be considered as hybrid methods between modeling and simulation. All these methods must be considered as more complementary than competitive and the choice of a particular method will be mainly guided by the type of problem to be solved and by the type of answers that are expected.

Acknowledgements

I would like, first of all, to thank the French Center for Scientific Research “Centre National de la Recherche Scientifique” who allowed me, within the framework of my researcher position at CNRS, to develop research work in turbulence modeling.

Through the undertaking of the present book on turbulence modeling (since its first edition in French in 1993), I would also like to thank all the people who had a creative influence on my professional activities in scientific research, in particular Professor Jean GOSSE (former director of the ENSEM in Nancy, and then holder of the Chair of Thermics at the CNAM in Paris) who, as early as 1971-1978 introduced me to the fascinating world of turbulence and its applications, Professor Brian E. LAUNDER (University of Manchester, UK) for the very fruitful scientific collaborations on the development of new models initiated at the beginning of my career, and also Professor Geneviève COMTE-BELLOT (École Centrale de Lyon, France) whose points of view have been very invaluable to me.

May I recall the memory of the late Professor Alexandre FAVRE (former director and founder of the IMST, “Institut de Mécanique Statistique de la Turbulence” in Marseille) who welcomed me into his laboratory in 1978 and whose school of thought had a great impact on our approach to turbulence. I am also very grateful to MM. Régis DUMAS (former director of the IMST) who granted me his large knowledge on physics and the structure of turbulent flows and Louis FULACHIER (former director of the IMST) whose collaborations on the experimental aspect provided a useful basis in model development.

In addition, I would like to thank Professor Gouri DHATT (then at the Compiègne University) and Mr. François BOUTTES (Compiègne University) for their enlightened advice on the conception of the book for its first edition in 1993, Mr. Patrick BONTOUX (research director and colleague at CNRS) for his support given in the first French edition which has served as a basis for subsequent editions, and also Mr. Bruno CHAOUAT, senior scientist at ONERA in Paris for his participation in the rereading of the French and English editions.

Introduction

We shall consider, among the numerous approaches of turbulence in fluids, only those which lead to effective prediction methods of turbulent shear flows by numerical calculation, methods which allow the study of real flow situations. In the absence of general predictive theory of turbulent phenomena, practical approaches resort to modeling and simulation methods. These various approaches have different levels of complexity, ranging from statistical modeling based on a hypothesis of turbulence viscosity, then more advanced modeling involving numerous transport equations, up to turbulent large eddy numerical simulations. These various methods are complementary rather than competitive, they correspond to different levels of description, each of them having its own performances and limitations. Thus, among the large variety of turbulence models and possible approaches, the practical user will often be led to make a choice which will be generally dictated by the type of physical problem to be solved and by the type of answers expected (the characteristics of the flow which are looked for). From this perspective, we shall try, in the present introduction, to point out some guiding lines, which will be like an Ariadne's thread through a varied and multiform landscape.

The full description of a random field with probability densities is very complex, therefore the usual methods only consider the first moments on single or several points. The statistical method, using mean value operators, leads to the closure problem of the system equations. This system of equations is closed by introducing modeling hypotheses. Generally, two approaches can be distinguished in the statistical method. They are the one point closures based on a single point moment hierarchy and the two point closures or spectral models based on a two point moment hierarchy or their Fourier transforms which are spectral tensors. The first method has been largely used for the calculation of non-homogenous turbulent shear flows encountered in real situations, the second method is usually limited to homogenous isotropic or anisotropic turbulence.

With the passing years, many turbulence models have been proposed and developed, allowing us to predict in each case the mean velocity distributions, mean temperature or concentration in a turbulent flow. The early empirical theories with very simple formulations are generally based on a turbulence viscosity coefficient or on a mixing length scheme. These types of models and their various improved forms have been widely used and have allowed us, in spite of their theoretical limitations, to reproduce fairly well the measurements obtained by the experimental approach, in particular in the turbulent boundary layers. However, each of these empirical theories has its own formulation which is valid only inside a very limited domain and therefore different schemes must be used for the calculation of confined turbulent flows in pipes and channels, in jets, in wakes and in boundary layers, etc.

Bringing into play more and more elaborate hypotheses describing the behavior of the turbulent fluid has then led to the development of more complex schemes allowing us, in addition to the prediction of mean flow, also to predict the characteristics of the turbulent field. These are the transport equation models, the most popular of which was the energy-dissipation model, the so-called “ $k-\varepsilon$ ” model. The following step, in the direction of increasing complexity, is the development of second order models. The aim of this research on advanced turbulence models composed of an increasing number of transport equations is the development of more universal schemes that would be able to solve a varied group of practical flows in different configurations without having to revise the calibration of numerical constants. We should point out, however, that the domain of application of a particular turbulence model always remains limited and that every progress towards universality is sometimes gained to the detriment of accuracy if we consider a particular flow application. If transport equation models are now widely used in industrial practice, in particular the energy-dissipation model, second order advanced closures still remain under development and progress is being achieved every day.

Transport equation models allow us to deal with the turbulent interaction phenomena more explicitly by assigning a physical interpretation to each unknown statistical correlation to be modeled. They are thus the privileged framework for introducing extra phenomena in interaction with turbulence such as the influence of gravity forces, rotation effects, thermal effects, etc.

The basis of this statistical approach has been codified in the invariant modeling method of J. Lumley. This methodology is presented in the first chapters.

These basic models, with or without transport equations, were generally initially introduced for incompressible fluid and fully developed turbulence. Extensions were then made to account for the effect of the turbulence Reynolds number, in particular close to a wall, often after empirical additions. The methods used for compressible

flows are still very often mere adaptations of models developed for incompressible flows.

Traditional one point closures are based on an implicit hypothesis of single scale description. This hypothesis can be incorrect in turbulent flows with high departures from equilibrium. This is the case when the turbulent flow is subjected to a force varying strongly in space or in time or when energy is fed at particular wavenumbers or frequencies in the spectrum. In these types of flow, multiscale models have been developed in order to account for some spectral information in a simplified way.

One point models require the introduction of hypotheses, often numerous, linking unknown correlations to known quantities. The modeler is guided for this by his physical intuition of the acting turbulent mechanisms, dimensional analysis, fundamental theoretical concepts, comparing experimental results, etc. These hypotheses involve numerical constants that are determined against key experiments that are relatively simple and well known experimentally (grid turbulence, wall boundary layer turbulence, pure strain, homogenous shear, etc.). A turbulence model can then be viewed as a quantified summary of our present knowledge on turbulence that is synthesized as an equation set. The validity of these schemes is subsequently validated or invalidated after confrontation between numerical predictions and experimental results for various turbulent flows. It is thus through its consequences that a turbulence model finds its ultimate justification. The domain of application of the model will be, of course, more or less limited depending on the generality level of the hypotheses that have been retained.

The increase of memory size and speed of scientific computers has more recently allowed the development of simulation methods. If direct numerical solution of the Navier-Stokes equations for turbulent regimes, due to the extent of computational means that are necessary, remains limited to fundamental problems in turbulence at relatively low Reynolds numbers, the numerical simulation of turbulent large eddies now allows us to tackle real situations. This method allows the simulation by a three-dimensional and unsteady calculation of the motion of the larger turbulent eddies whereas fine grained structures of a size smaller than the calculation grid step size and whose characteristics are more universal, are modeled. The aim is thus to simulate realizations of turbulent flows that provide a detailed picture and description of the turbulent structures generated by the calculation and their evolution. In large eddy simulations, these eddies are not exactly real turbulence because of the use of small scale modeling but its statistical properties are expected to be preserved. The various methods of simulation are useful for deeper insights into the study turbulent flows because they allow us to generate large-scale fluctuating fields that can be analyzed using statistical post-processing exactly in the same way as the experimentalist proceeds with laboratory measurements. Numerical simulations can thus provide statistical correlations that would not be attainable by

measurement and thus give the possibility of direct testing of the closure hypotheses. Also, numerical simulations allow us to tackle types of flows or to consider physical parameter values that would not be possible experimentally. The drawback of direct numerical simulations or refined large eddy simulations is their Reynolds number limitations. Another application of simulation is the study of flows in which one point statistical models do not work or are not able to give the desired information, this is the case for complex or pathological flows with unsteady or irregular large-scale evolutions. Numerical simulations may also be useful for the production of databases used to tune simpler models, in particular for turbulent quantities that cannot be measured. We will note, however, that in some sense, numerical simulation methods by calculating more and modeling less, bypass the turbulence problem rather than solving it, the true original turbulence problem remaining statistical in essence.

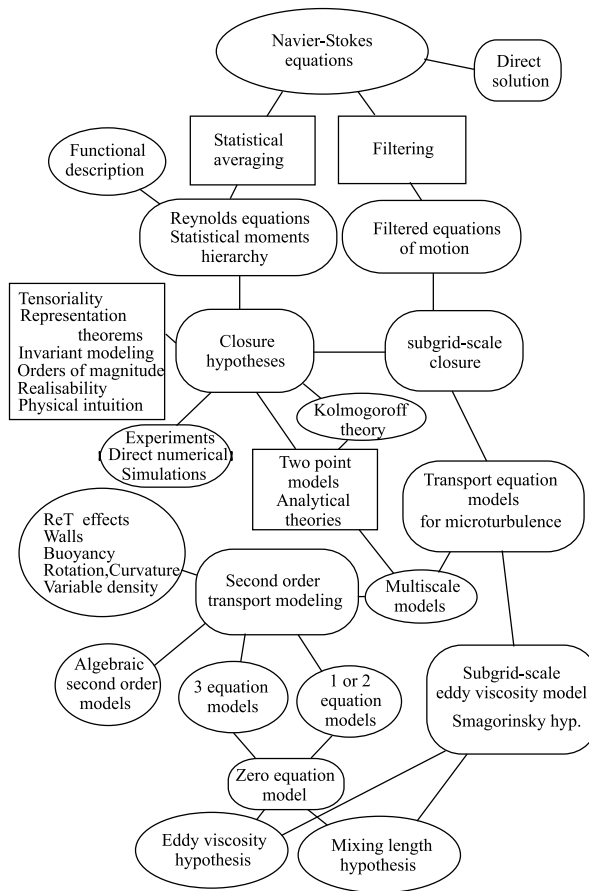


Figure i-1. Implication diagram

Chapter 1

Fundamentals in Statistical Modeling: Basic Physical Concepts

1.1. The nature of turbulence

Turbulence is a property of the flow and not a property of the fluid itself. In flows that are originally laminar, the turbulent regime arises from instabilities that develop as the Reynolds number increases. There is no precise definition of turbulence in fluids, nor does there exist any general theory of turbulence. Turbulence is thus characterized by several observable properties that we shall make clear in the following. If we accept the usually retained hypothesis that the detailed motion of the turbulent fluid is governed by the Navier-Stokes equations, then the fundamental equations of turbulence can be considered as known and thus they can form the basis of any tentative statistical theory to describe the turbulence field.

1.1.1. *Observable properties of turbulent flows*

The chaotic character of turbulent fluctuations appears as a direct consequence of non-linear terms present in the Navier-Stokes equations. These non-linearities are apparent through several important consequences which, considering the absence of any precise definition of turbulence, will serve as characteristic properties.

1.1.1.1. *Irregular signal in space and time*

Physical quantities such as velocity and pressure vary in an apparently random way (cf. Figure 1.1). Let us note that organized or periodic fluctuations are not part

of the turbulent agitation, that is the case for example for pulsed flows in which we have to remove the periodic component in order to get the true turbulent signal.

1.1.1.2. *Rotational flow*

The presence of countless swirling eddies conveys the fact that a turbulent flow is highly rotational. The turbulent motion thus presents strong fluctuations in the curl of velocity. The non-linearities control the interactions between these eddies of differing size. Let us note, in this connection, that an acoustic field, even random, is not turbulent at all because it is irrotational.

1.1.1.3. *High diffusivity*

A turbulent field diffuses any transportable quantity, such as temperature or a dye, but also momentum, rapidly. In reality, turbulent diffusion is due to convective terms at the fluctuating level. A traced fluid particle marked by a dye is then distorting, branching out and progressively fraying (cf. Figure 1.2). Diffusive shapes are described by fractal dimension ensembles ([VIL 99]).

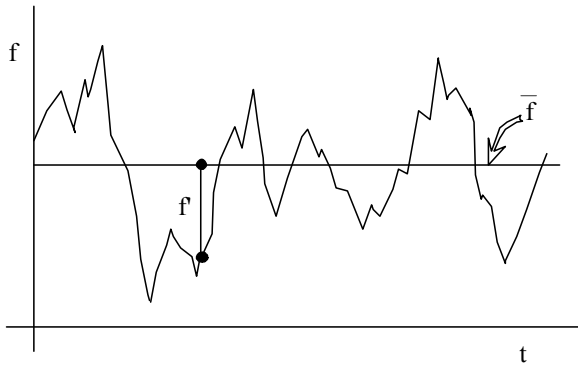


Figure 1.1. *Turbulent signal*

1.1.1.4. *A three-dimensional phenomenon*

Fluctuating turbulent motions are always three-dimensional and unsteady. Let us mention here and now that a so-called “two-dimensional” turbulence also exists and can be found in very special situations. Its mechanisms are very different from those prevailing in three-dimensional turbulence.

1.1.1.5. *Unpredictable character of trajectories*

The detailed properties of a turbulent flow present an extreme sensitivity to initial and boundary conditions. This behavior is apparent if we consider a tiny

deviation in the initial conditions, we then observe that the two flows become rapidly very different from each other if we look at its instantaneous detailed description. This unpredictable character of the detailed fluid particle trajectories on sufficiently long time intervals corresponds to a loss of the memory of initial conditions. This is the unpredictability phenomenon. It explains, for example, the difficulties encountered in long term weather prediction. The unpredictability problem has been studied numerically by Chollet J.P. and Métais O. [CHO 89B].

Some properties of turbulence however remain reproducible, such as statistical properties, mean values and spectral distributions.

The progressive loss of memory of a turbulent flow which forgets, after some elapsed time, the detail of fluctuations in the initial conditions some way justifies the statistical approach to turbulence since, to some extent, the detail of initial conditions can be ignored. In this connection, it is possible to distinguish newly created turbulence which still retain the memory of the conditions in which it was created (much more difficult to study) from fully developed turbulence which has lost the memory of initial conditions (and which can be studied relatively more easily because it is subject to universal laws).

1.1.1.6. *Coexistence of eddies of very different scales*

There exists a whole cascade of eddies of smaller and smaller scales, created by non-linear processes due to inertial terms in the equations of motion.

1.1.1.7. *Dissipation*

Turbulence cannot be sustained by itself, it needs an energy supply. This source of energy can have various origins, the most usual is shear or strain of the mean flow, but the origin can also come from external forces such as Archimedean forces. If turbulence is deprived of any generation process, it decays progressively. Turbulence is dissipative. The mechanism of viscous dissipation of turbulence is related to the existence of strong gradients of the instantaneous velocity field. The instantaneous strain rates indeed become very high inside the smallest eddies and the degradation of the turbulent kinetic energy into heat is thus very strong.

Although there is “turbulence” in plasmas, we shall consider here only the macroscopic approach which supposes that the molecular scales are largely smaller than the scales of the smallest turbulent eddies, which justifies the method of continuous media and the Navier-Stokes equations in particular.

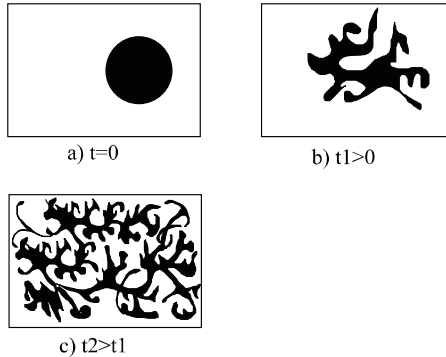


Figure 1.2. Evolution of a volume marked by a dye under the effect of turbulent diffusion (after Monin and Yaglom, 1971)

1.1.2. Traditional point of view on turbulence

Turbulence is an eddying motion which, at high Reynolds numbers usually reached in practical flows, shows a wide spectrum spreading over a significant range of eddy scales and a corresponding spectrum in frequency. The turbulent motion, always rotational, can be perceived as a muddle of swirling eddies whose rotational curl vectors are directed randomly in space and strongly unsteady.

The largest eddies, which are associated with the low frequency range of the energy spectrum, are determined by the boundary conditions of the flow. Their length scale is comparable to the order of magnitude of the whole domain occupied by the flow itself. The smallest eddies, associated with high frequencies in the spectrum, are determined by viscous forces. The energy spectrum width, i.e. the scale difference between the largest and the smallest eddies, increases with the Reynolds number. Momentum and heat transfer are mainly due to large-scale motions which contribute to statistical correlations $\overline{u_i u_j}$ and $\overline{u_i \gamma}$ where u_i represents the velocity components and γ a transported scalar.

Thus, it is mainly these large eddies that must be considered for determining $\overline{u_i u_j}$ and $\overline{u_i \gamma}$ in turbulence models: the velocity and length scales introduced in the usual turbulence models are basically macroscales.

The large eddies interact with the mean flow (because their characteristic scales have the same order of magnitude), they extract kinetic energy from the mean flow and supply this energy to the large-scale agitations.