Modeling, Performance Analysis and Control of Robot Manipulators

Edited by
Etienne Dombre
Wisama Khalil

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Chapter 1

Modeling and Identification of Serial Robots

1.1. Introduction

The design and control of robots require certain mathematical models, such as:

– transformation models between the operational space (in which the position of the end-effector is defined) and the joint space (in which the configuration of the robot is defined). The following is distinguished:

- direct and inverse geometric models giving the location of the end-effector (or the tool) in terms of the joint coordinates of the mechanism and vice versa,

- direct and inverse kinematic models giving the velocity of the end-effector in terms of the joint velocities and vice versa,

– dynamic models giving the relations between the torques or forces of the actuators, and the positions, velocities and accelerations of the joints.

This chapter presents some methods to establish these models. It will also deal with identifying the parameters appearing in these models. We will limit the discussion to simple open structures. For complex structure robots, i.e. tree or closed structures, we refer the reader to [KHA 02].

Chapter written by Wisama KHALIL and Etienne DOMBRE.

Mathematical development is based on (4 × 4) homogenous transformation matrices. The homogenous matrix ${}^{i}\mathbf{T}_{j}$ representing the transformation from frame R_i to frame R_j is defined as:

$$ {}^{i}\mathbf{T}_{j} = \begin{bmatrix} & {}^{i}\mathbf{R}_{j} & & {}^{i}\mathbf{P}_{j} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^{i}\mathbf{s}_{j} & {}^{i}\mathbf{n}_{j} & {}^{i}\mathbf{a}_{j} & {}^{i}\mathbf{P}_{j} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [1.1] $$

where ${}^{i}\mathbf{s}_{j}$, ${}^{i}\mathbf{n}_{j}$ and ${}^{i}\mathbf{a}_{j}$ of the orientation matrix ${}^{i}\mathbf{R}_{j}$ indicate the unit vectors along the axes $\mathbf{x}_j$, $\mathbf{y}_j$ and $\mathbf{z}_j$ of the frame R_j expressed in the frame R_i; and where ${}^{i}\mathbf{P}_{j}$ is the vector expressing the origin of the frame R_j in the frame R_i.

1.2. Geometric modeling

1.2.1. *Geometric description*

A systematic and automatic modeling of robots requires an appropriate method for the description of their morphology. Several methods and notations have been proposed [DEN 55], [SHE 71], [REN 75], [KHA 76], [BOR 79], [CRA 86]. The most widely used one is that of Denavit-Hartenberg [DEN 55]. However, this method, developed for simple open structures, presents ambiguities when it is applied to closed or tree-structured robots. Hence, we recommend the notation of Khalil and Kleinfinger which enables the unified description of complex and serial structures of articulated mechanical systems [KHA 86].

A simple open structure consists of n+1 links noted C_0, …, C_n and of n joints. Link C_0 indicates the robot base and link C_n, the link carrying the end-effector. Joint j connects link C_j to link C_{j-1} (Figure 1.1). The method of description is based on the following rules and conventions:

– the links are assumed to be perfectly rigid. They are connected by revolute or prismatic joints considered as being ideal (no mechanical clearance, no elasticity);

– the frame R_j is fixed to link C_j;

– axis $\mathbf{z}_j$ is along the axis of joint j;

– axis $\mathbf{x}_j$ is along the common perpendicular with axes $\mathbf{z}_j$ and $\mathbf{z}_{j+1}$. If axes $\mathbf{z}_j$ and $\mathbf{z}_{j+1}$ are parallel or collinear, the choice of $\mathbf{x}_j$ is not unique: considerations of symmetry or simplicity lead to a reasonable choice.

The transformation matrix from the frame R_{j-1} to the frame R_j is expressed in terms of the following four geometric parameters:

– α_j: angle between axes $\mathbf{z}_{j-1}$ and $\mathbf{z}_j$ corresponding to a rotation about $\mathbf{x}_{j-1}$;

– d_j: distance between $\mathbf{z}_{j-1}$ and $\mathbf{z}_j$ along $\mathbf{x}_{j-1}$;

– θ_j: angle between axes $\mathbf{x}_{j-1}$ and $\mathbf{x}_j$ corresponding to a rotation about $\mathbf{z}_j$;

– r_j: distance between $\mathbf{x}_{j-1}$ and $\mathbf{x}_j$ along $\mathbf{z}_j$.

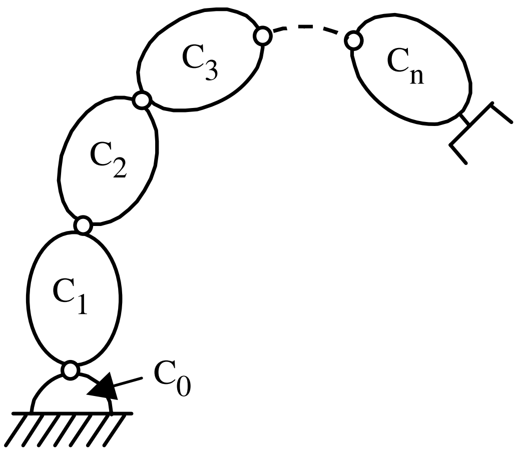


Figure 1.1. *A simple open structure robot*

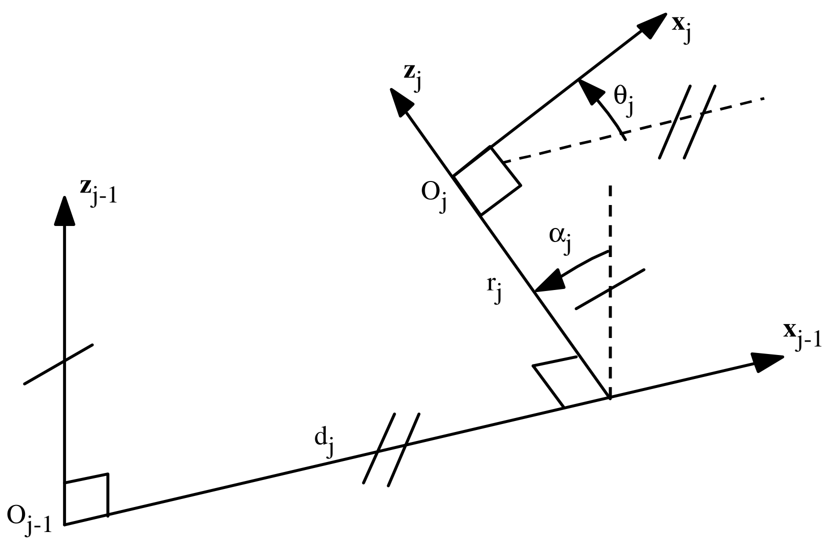


Figure 1.2. *Geometric parameters in the case of a simple open structure*

The joint coordinate q_j associated to the j^{th} joint is either θ_j or r_j, depending on whether this joint is revolute or prismatic. It can be expressed by the relation:

$$q_j = \bar{\sigma}_j \theta_j + \sigma_j r_j \qquad [1.2]$$

with:

– $\sigma_j = 0$ if the joint is revolute;

– $\sigma_j = 1$ if the joint is prismatic;

– $\bar{\sigma}_j = 1 - \sigma_j$.

The transformation matrix defining the frame R_j in the frame R_{j-1} is obtained from Figure 1.2 by:

$$^{j-1}\mathbf{T}_j = \mathbf{Rot}(\mathbf{x}, \alpha_j)\ \mathbf{Trans}(\mathbf{x}, d_j)\ \mathbf{Rot}(\mathbf{z}, \theta_j)\ \mathbf{Trans}(\mathbf{z}, r_j)$$

$$= \begin{bmatrix} C\theta_j & -S\theta_j & 0 & d_j \\ C\alpha_j S\theta_j & C\alpha_j C\theta_j & -S\alpha_j & -r_j S\alpha_j \\ S\alpha_j S\theta_j & S\alpha_j C\theta_j & C\alpha_j & r_j C\alpha_j \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad [1.3]$$

where **Rot**(**u**, α) and **Trans**(**u**, d) are (4×4) homogenous matrices representing, respectively, a rotation α about the axis **u** and a translation d along **u.**

NOTES.

– for the definition of the reference frame R_0, the simplest choice consists of taking R_0 aligned with the frame R_1 when $q_1 = 0$, which indicates that $\mathbf{z}_0$ is along $\mathbf{z}_1$ and $O_0 \equiv O_1$ when joint 1 is revolute, and $\mathbf{z}_0$ is along $\mathbf{z}_1$ and $\mathbf{x}_0$ is parallel to $\mathbf{x}_1$ when joint 1 is prismatic. This choice renders the parameters α_1 and d_1 zero;

– likewise, the axis $\mathbf{x}_n$ of the frame R_n is taken collinear to $\mathbf{x}_{n-1}$ when $q_n = 0$. This choice makes r_n (or θ_n) zero when $\sigma_n = 1$ (or = 0 respectively);

– for a prismatic joint, the axis $\mathbf{z}_j$ is parallel to the axis of the joint; it can be placed in such a way that d_j or d_{j+1} is zero;

– when $\mathbf{z}_j$ is parallel to $\mathbf{z}_{j+1}$, the axis $\mathbf{x}_j$ is placed in such a way that r_j or r_{j+1} is zero;

– in practice, the vector of joint variables $\mathbf{q}$ is given by:

$$\mathbf{q} = \mathbf{K_c}\, \mathbf{q_c} + \mathbf{q_0}$$

where $\mathbf{q_0}$ represents an offset, $\mathbf{q_c}$ are encoder variables and $\mathbf{K_c}$ is a constant matrix.

EXAMPLE 1.1.– description of the structure of the Stäubli RX-90 robot (Figure 1.3). The robot shoulder is of an RRR anthropomorphic type and the wrist consists of three intersecting revolute axes, equivalent to a spherical joint. From a methodological point of view, firstly the axes $\mathbf{z}_j$ are placed on the joint axes and the axes $\mathbf{x}_j$ are placed according to the rules previously set. Next, the geometric parameters of the robot are determined. The link frames are shown in Figure 1.3 and the geometric parameters are given in Table 1.1.

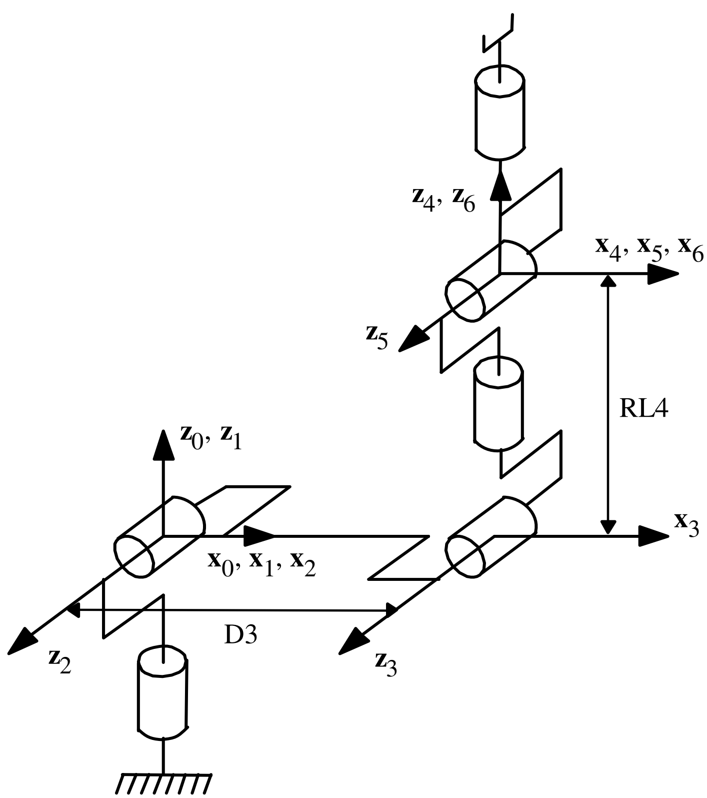


Figure 1.3. *Link frames for the Stäubli RX-90 robot*

j	σ_j	α_j	d_j	θ_j	r_j
1	0	0	0	θ_1	0
2	0	$\pi/2$	0	θ_2	0
3	0	0	D3	θ_3	0
4	0	$-\pi/2$	0	θ_4	RL4
5	0	$\pi/2$	0	θ_5	0
6	0	$-\pi/2$	0	θ_6	0

Table 1.1. *Geometric parameters for the Stäubli RX-90 robot*

1.2.2. *Direct geometric model*

The direct geometric model (DGM) represents the relations calculating the operational coordinates, giving the location of the end-effector, in terms of the joint coordinates. In the case of a simple open chain, it can be represented by the transformation matrix ${}^0\mathbf{T}_n$:

$$ {}^0\mathbf{T}_n = {}^0\mathbf{T}_1(q_1)\ {}^1\mathbf{T}_2(q_2) \ldots {}^{n-1}\mathbf{T}_n(q_n) \qquad [1.4] $$

The direct geometric model of the robot may also be represented by the relation:

$$ \mathbf{X} = \mathbf{f}(\mathbf{q}) \qquad [1.5] $$

q being the vector of joint coordinates such that:

$$ \mathbf{q} = [q_1 \ \ q_2 \ \ldots \ q_n]^T \qquad [1.6] $$

The operational coordinates are defined by:

$$ \mathbf{X} = [x_1 \ \ x_2 \ \ldots \ x_m]^T \qquad [1.7] $$

There are several possibilities to define the vector **X**. For example, with the help of the elements of matrix ${}^{0}\mathbf{T}_{n}$:

$$\mathbf{X} = [P_x \;\; P_y \;\; P_z \;\; s_x \;\; s_y \;\; s_z \;\; n_x \;\; n_y \;\; n_z \;\; a_x \;\; a_y \;\; a_z]^T \qquad [1.8]$$

or otherwise, knowing that **s = nxa**

$$\mathbf{X} = [P_x \;\; P_y \;\; P_z \;\; n_x \;\; n_y \;\; n_z \;\; a_x \;\; a_y \;\; a_z]^T \qquad [1.9]$$

For the orientation, other representations are currently used such as Euler angles, Roll-Pitch-Yaw angles or Quaternions. We can easily derive direction cosines **s**, **n**, **a** from any one of these representations and vice versa [KHA 02].

EXAMPLE 1.2. – direct geometric model for the Stäubli RX-90 robot (Figure 1.3). According to Table 1.1, the relation [1.3] can be used to write the basic transformation matrices ${}^{j-1}\mathbf{T}_{j}$. The product of these matrices gives ${}^{0}\mathbf{T}_{6}$ that has as components:

$$s_x = C1(C23(C4C5C6 - S4S6) - S23S5C6) - S1(S4C5C6 + C4S6)$$
$$s_y = S1(C23(C4C5C6 - S4S6) - S23S5C6) + C1(S4C5C6 + C4S6)$$
$$s_z = S23(C4C5C6 - S4S6) + C23S5C6$$
$$n_x = C1(-C23\,(C4C5S6 + S4C6) + S23S5S6) + S1(S4C5S6 - C4C6)$$
$$n_y = S1(-C23\,(C4C5S6 + S4C6) + S23S5S6) - C1(S4C5S6 - C4C6)$$
$$n_z = -S23(C4C5S6 + S4C6) - C23S5S6$$
$$a_x = -C1(C23C4S5 + S23C5) + S1S4S5$$
$$a_y = -S1(C23C4S5 + S23C5) - C1S4S5$$
$$a_z = -S23C4S5 + C23C5$$
$$P_x = -C1(S23\,RL4 - C2D3)$$
$$P_y = -S1(S23\,RL4 - C2D3)$$
$$P_z = C23\,RL4 + S2D3$$

with C23=cos $(\theta_2 + \theta_3)$ and S23 = sin $(\theta_2 + \theta_3)$.

1.2.3. *Inverse geometric model*

We saw that the direct geometric model of a robot calculates the operational coordinates giving the location of the end-effector in terms of joint coordinates. The inverse problem consists of calculating the joint coordinates corresponding to a given location of the end-effector. When it exists, the explicit form which gives all possible solutions (there is rarely uniqueness of solution) constitutes what we call the inverse geometric model (IGM). We can distinguish three methods for the calculation of IGM:

– Paul's method [PAU 81], which deals with each robot separately and is suitable for most of the industrial robots;

– Pieper's method [PIE 68], which makes it possible to solve the problem for the robots with six degrees of freedom having three revolute joints with intersecting axes or three prismatic joints;

– the general Raghavan and Roth's method [RAG 90] giving the general solution for robots with six joints using at most a 16-degree polynomial.

Whenever calculating an explicit form of the inverse geometric model is not possible, we can calculate a particular solution through numeric procedures [PIE 68], [WHI 69], [FOU 80], [FEA 83], [WOL 84], [GOL 85] [SCI 86].

In this chapter, we present Paul's method; Pieper's method, and Raghavan and Roth's method are detailed in [KHA 02].

1.2.3.1. *Stating the problem*

Let ${}^{f}\mathbf{T}_{E}{}^{d}$ be the homogenous transformation matrix representing the desired location of the end-effector frame R_E with respect to the world frame R_f. In general cases, ${}^{f}\mathbf{T}_{E}{}^{d}$ can be expressed in the following form:

$$ {}^{f}\mathbf{T}_{E}{}^{d} = \mathbf{Z}\ {}^{0}\mathbf{T}_{n}(\mathbf{q})\ \mathbf{E} \qquad [1.10] $$

where (see Figure 1.4):

– $\mathbf{Z}$ is the transformation matrix defining the location of the robot frame R_0 in the world reference frame R_f;

– ${}^{0}\mathbf{T}_{n}$ is the transformation matrix of the terminal link frame R_n with respect to frame R_0 in terms of the joint coordinates $\mathbf{q}$;

– $\mathbf{E}$ is the transformation matrix defining the end-effector frame R_E in the terminal frame R_n.

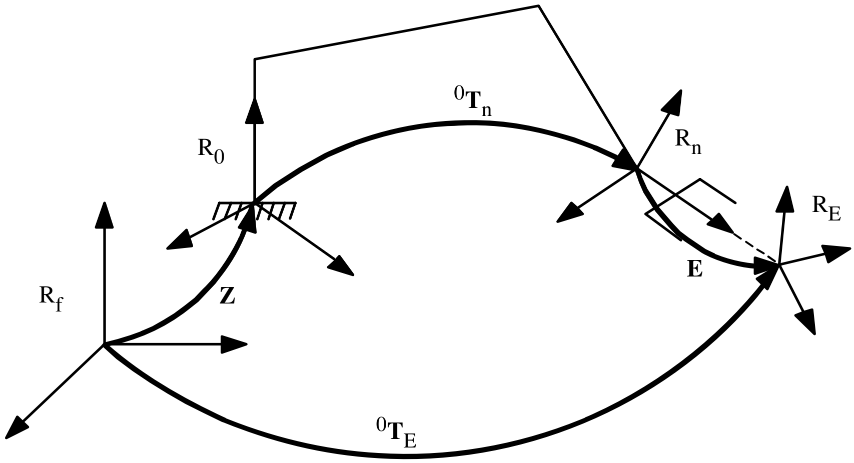


Figure 1.4. *Transformations between the end-effector frame and the world reference frame*

When n ≥ 6, we can write the following relation by grouping on the right hand side all known terms:

$$^0\mathbf{T}_n(\mathbf{q}) = \mathbf{Z}^{-1}\ {}^f\mathbf{T}_E{}^d\ \mathbf{E}^{-1} \qquad [1.11]$$

When n < 6, the robot's operational space is less than six. It is not possible to place the end-effector frame R_E in an arbitrary location $R_E{}^d$ describing the task, except when the frames R_E and $R_E{}^d$ are conditioned in a particular way in order to compensate for the insufficient number of degrees of freedom. Practically, instead of bringing frame R_E onto frame $R_E{}^d$, we will seek to only place some elements of the end-effector (points, straight lines).

In the calculation of IGM, three cases can be distinguished:

a) no solution when the desired location is outside of the accessible zone of the robot. It is limited by the number of degrees of freedom of the robot, the joint limits and the dimension of the links;

b) infinite number of solutions when:

– the robot is redundant with respect to the task,

– the robot is in some singular configuration;

c) a finite number of solutions expressed by a set of vectors $\{\mathbf{q}^1, \ldots, \mathbf{q}^r\}$. A robot is said to be solvable [PIE 68], [ROT 76] when it is possible to calculate all the configurations making it possible to reach a given location. Nowadays, all serial manipulators having up to six degrees of freedom and which are not redundant may be considered as solvable. The number of solutions depends on the structure of the robot and is at most equal to 16.

1.2.3.2. *Principle of Paul's method*

Let us consider a robot whose homogenous transformation matrix has the following form:

$$^{0}\mathbf{T}_{n} = {}^{0}\mathbf{T}_{1}(q_1)\ {}^{1}\mathbf{T}_{2}(q_2) \ldots {}^{n-1}\mathbf{T}_{n}(q_n) \qquad [1.12]$$

Let $\mathbf{U}_0$ be the desired location, such that:

$$U_0 = \begin{bmatrix} s_x & n_x & a_x & P_x \\ s_y & n_y & a_y & P_y \\ s_z & n_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad [1.13]$$

We seek to solve the following system of equations:

$$\mathbf{U}_0 = {}^{0}\mathbf{T}_{1}(q_1)\ {}^{1}\mathbf{T}_{2}(q_2) \ldots {}^{n-1}\mathbf{T}_{n}(q_n) \qquad [1.14]$$

Paul's method consists of successively pre-multiplying the two sides of equation [1.14] by the matrices ${}^{j}\mathbf{T}_{j-1}$ for j = 1, …, n–1, operations which make it possible to isolate and identify one after another of the joint coordinates.

For example, in the case of a six degrees of freedom robot, the procedure is as follows:

– left multiplication of both sides of equation [1.14] by ${}^{1}\mathbf{T}_{0}$:

$${}^{1}\mathbf{T}_{0}\ \mathbf{U}_0 = {}^{1}\mathbf{T}_{2}\ {}^{2}\mathbf{T}_{3}\ {}^{3}\mathbf{T}_{4}\ {}^{4}\mathbf{T}_{5}\ {}^{5}\mathbf{T}_{6} \qquad [1.15]$$

The right hand side is a function of the variables q_2, …, q_6. The left hand side is only a function of the variable q_1;

– term-to-term identification of the two sides of equation [1.15]. We obtain a system of one or two equations function of q_1 only, whose structure belongs to a particular type amongst a dozen of possible types;

– left multiplication of both sides of equation [1.15] by ${}^{2}\mathbf{T}_1$ and calculation of q_2.

The succession of equations enabling the calculation of all q_j is the following:

$$\begin{aligned}
&\mathbf{U}_0 = {}^{0}\mathbf{T}_1\,{}^{1}\mathbf{T}_2\,{}^{2}\mathbf{T}_3\,{}^{3}\mathbf{T}_4\,{}^{4}\mathbf{T}_5\,{}^{5}\mathbf{T}_6\\
&{}^{1}\mathbf{T}_0\,\mathbf{U}_0 = {}^{1}\mathbf{T}_2\,{}^{2}\mathbf{T}_3\,{}^{3}\mathbf{T}_4\,{}^{4}\mathbf{T}_5\,{}^{5}\mathbf{T}_6\\
&{}^{2}\mathbf{T}_1\,\mathbf{U}_1 = {}^{2}\mathbf{T}_3\,{}^{3}\mathbf{T}_4\,{}^{4}\mathbf{T}_5\,{}^{5}\mathbf{T}_6\\
&{}^{3}\mathbf{T}2\,\mathbf{U}_2 = {}^{3}\mathbf{T}_4\,{}^{4}\mathbf{T}_5\,{}^{5}\mathbf{T}_6\\
&{}^{4}\mathbf{T}_3\,\mathbf{U}_3 = {}^{4}\mathbf{T}_5\,{}^{5}\mathbf{T}_6\\
&{}^{5}\mathbf{T}_4\,\mathbf{U}_4 = {}^{5}\mathbf{T}_6
\end{aligned} \qquad [1.16]$$

with:

$$\mathbf{U}_{j+1} = {}^{j+1}\mathbf{T}_6 = {}^{j+1}\mathbf{T}_j\,\mathbf{U}_j \text{ for } j = 0, ..., 4 \qquad [1.17]$$

The use of this method for a large number of industrial robots has shown that only a few types of equations are encountered, and that their solutions are relatively simple.

NOTES.

1) When a robot has more than six degrees of freedom, the system to be solved contains more unknowns than parameters describing the task: it lacks (n–6) relations. Two strategies are possible:

– the first strategy consists of setting arbitrarily (n–6) joint variables. In this case we deal with a problem with six degrees of freedom. The choice of these joints results from the task's specifications and from the structure of the robot;

– the second strategy consists of introducing (n–6) supplementary relations describing the redundancy, like for example in [HOL 84] for robots with seven degrees of freedom.

2) A robot with less than six degrees of freedom cannot place its end-effector at arbitrary positions and orientations. Thus, it is not possible to bring the end-effector frame R_E onto another desired frame R_E^d except if certain elements of ${}^{0}\mathbf{T}_E^d$ are imposed in a way that compensates for the insufficient number of degrees of freedom. Otherwise, we have to reduce the number of equations by considering only certain elements (points or axes) of the frames R_E and R_E^d.

EXAMPLE 1.3.– inverse geometric model of the Stäubli RX-90 robot. After performing all the calculations, we obtain the following solutions:

$$\begin{cases} \theta_1 = \text{atan2}(P_y, P_x) \\ \theta'_1 = \theta_1 + \pi \end{cases}$$

$$\theta_2 = \text{atan2}(S2, C2)$$

with:

$$\begin{cases} C2 = \dfrac{YZ - \varepsilon X\sqrt{X^2 + Y^2 - Z^2}}{X^2 + Y^2} \\ S2 = \dfrac{XZ - \varepsilon Y\sqrt{X^2 + Y^2 - Z^2}}{X^2 + Y^2} \end{cases} \quad \text{with } \varepsilon = \pm 1$$

$$X = -2P_z D3$$
$$Y = -2\, B1D3$$
$$Z = (RL4)^2 - (D3)^2 - (P_z)^2 - (B1)^2$$
$$B1 = P_x\, C1 + P_y\, S1$$

$$\theta_3 = \text{atan2}\left(\frac{-P_z S2 - B1C2 + D3}{RL4}, \frac{-B1S2 + P_z C2}{RL4}\right)$$

$$\begin{cases} \theta_4 = \text{atan2}[S1\, a_x - C1\, a_y, -C23(C1\, a_x + S1\, a_y) - S23\, a_z] \\ \theta'_4 = \theta_4 + \pi \end{cases}$$

$$\theta_5 = \text{atan2}(S5, C5)$$

with:

$$S5 = -C4\, [C23\, (C1\, a_x + S1\, a_y) + S23 a_z] + S4\, (S1\, a_x - C1\, a_y)$$
$$C5 = -S23\, (C1\, a_x + S1\, a_y) + C23\, a_z$$

$$\theta_6 = \text{atan2}(S6, C6)$$

with:

$$S6 = -C4\, (S1\, s_x - C1\, s_y) - S4\, [C23\, (C1\, s_x + S1\, s_y) + S23\, s_z]$$
$$C6 = -C4\, (S1\, n_x - C1\, n_y) - S4\, [C23\, (C1\, n_x + S1\, n_y) + S23\, n_z]$$

NOTES.

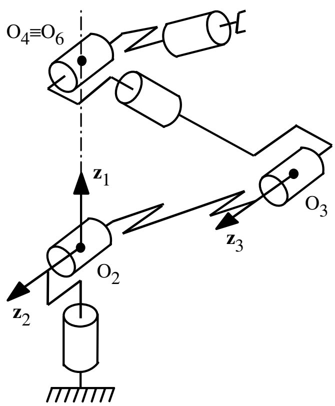
1) Singular positions:

i) when $P_x = P_y = 0$, which corresponds to S23RL4 – C2D3 = 0, the point O_4 is on the axis $\mathbf{z}_0$ (Figure 1.5a). The two arguments used for calculating θ_1 are zero and consequently θ_1 is not determined. We can give any value to θ_1, generally the value of the current position, or, according to optimization criteria, such as maximizing the distance from the mechanical limits of the joints. This means that we can always find a solution, but a small change of the desired position might call for a significant variation of θ_1, which may be impossible to carry out considering the velocity and acceleration limits of the actuators,

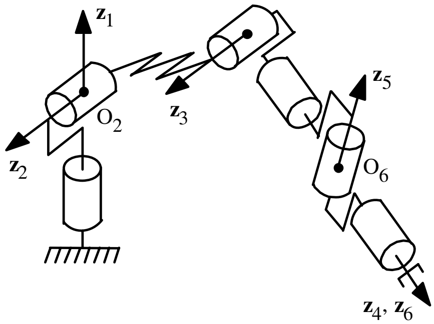
ii) when $C23(C1a_x + S1a_y) + S23a_z = 0$ and $S1a_x - C1a_y = 0$, the two arguments of the atan2 function used for the calculation of θ_4 are zero and hence the function is not determined. This configuration happens when axes 4 and 6 are aligned ($C\theta_5 = \pm 1$) and it is the sum ($\theta_4 \pm \theta_6$) which can be obtained (see Figure 1.5b). We can give to θ_4 its current value, then we calculate θ_6 according to this value. We can also calculate the values of θ_4 and θ_6, which move joints 4 and 6 away from their limits,

iii) a third singular position occurring when C3 = 0 will be highlighted along with the kinematic model. This singularity does not pose any problem for the inverse geometric model (see Figure 1.5c).

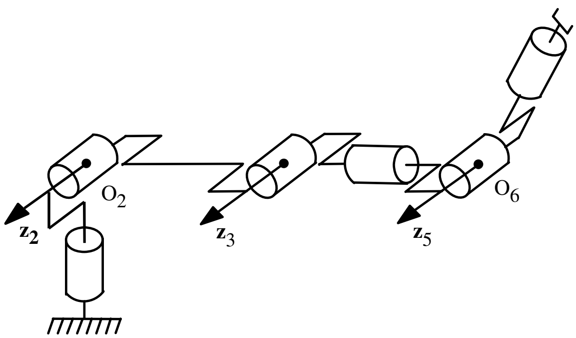
2) Number of solutions: apart from singularities, the Stäubli RX-90 robot has eight theoretical configurations for the IGM (product of the number of possible solutions on each axis). Some of these configurations may not be accessible due to their joint limits.



a) Singularity of the shoulder ($P_x = P_y = 0$ and $S23RL4 - C2D3 = 0$)



b) Singularity of the wrist (S5 = 0)



c) Singularity of the elbow (C3 = 0)

Figure 1.5. *Singular positions of the Stäubli RX-90 robot*

1.3. Kinematic modeling

1.3.1. *Direct kinematic model*

The direct kinematic model of a robot gives the velocities of the operational coordinates $\dot{\mathbf{X}}$ in terms of the joint velocities $\dot{\mathbf{q}}$. We write:

$$\dot{\mathbf{X}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \qquad [1.18]$$