FINITE ELEMENT ANALYSIS OF ANTENNAS AND ARRAYS

Jian-Ming Jin
University of Illinois

Douglas J. Riley
Northrop Grumman Corporation

IEEE

IEEE PRESS

WILEY

A JOHN WILEY & SONS, INC., PUBLICATION

FINITE ELEMENT ANALYSIS OF ANTENNAS AND ARRAYS

FINITE ELEMENT ANALYSIS OF ANTENNAS AND ARRAYS

Jian-Ming Jin
University of Illinois

Douglas J. Riley
Northrop Grumman Corporation

IEEE

IEEE PRESS

WILEY

A JOHN WILEY & SONS, INC., PUBLICATION

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in
preparing this book, they make no representations or warranties with respect to the accuracy or
completeness of the contents of this book and specifically disclaim any implied warranties of
merchantability or fitness for a particular purpose. No warranty may be created or extended by sales
representatives or written sales materials. The advice and strategies contained herein may not be suitable
for your situation. You should consult with a professional where appropriate. Neither the publisher nor
author shall be liable for any loss of profit or any other commercial damages, including but not limited to
special, incidental, consequential, or other damages.

For general information on our other products and services or for technical support, please contact our
Customer Care Department within the United States at (800) 762-2974, outside the United States at (317)
572-3993 or fax (317) 572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may
not be available in electronic formats. For more information about Wiley products, visit our web site at
www.wiley.com.

CONTENTS

PREFACE

The idea to write this book came about purely by accident. Neither of the authors even considered this prior to 2005. In the summer of 2005, the authors were invited by Dr. Constantine Balanis to contribute a chapter on "Finite Element Analysis and Modeling of Antennas" for his *Modern Antenna Handbook*. During the preparation of this book chapter, the authors discovered two interesting facts. First, the work by their two research groups combined had touched nearly all aspects of the finite element analysis of antennas, ranging from frequency- and time-domain analysis of narrow- and broadband antennas, to the modeling of antenna feeds and dispersive media, and to the simulation of phased-array antennas and finite arrays. Second, the amount of material collected was by far more than what can be included in a book chapter. The idea of expanding the book chapter into a full book was mentioned, but eventually dropped after submission of the book chapter in the summer of 2006. In May 2007 the authors jointly offered a short course on "Finite Element Analysis of Complex Antennas and Arrays" for the Electromagnetic Code Consortium's Annual Conference. This short course provided an opportunity for the authors to organize the course material into various topics, which resulted in over 200 viewgraphs and formed the framework of this book.

The book is organized into 12 chapters. In Chapter 1 we describe the need for numerical simulation in the design of complex antennas and arrays and compare the finite element method with other numerical techniques from the viewpoint of antenna analysis. Chapter 2 covers the formulations of finite element analysis of antennas in the frequency and time domains. Chapter 3 deals with the problem of mesh truncation in the finite element analysis of antennas. In Chapter 4 we describe a hybrid technique that combines the finite element time-domain method with the finite-difference time-domain method. Chapter 5 deals with the critical problem of modeling antenna feeds for radiation analysis and plane-wave excitation for scattering analysis. In Chapter 6 we consider the finite element modeling of complex structures and circuit components that are widely used in antenna designs. In Chapter 7 we present a variety of antenna simulation examples to demonstrate the capability and versatility of the finite element method. Chapter 8 covers the analysis of axisymmetric antennas using a two-dimensional finite element method. The modeling of infinitely large phased arrays is the topic of Chapter 9, which covers both the frequency- and time-domain analyses. In Chapter 10 we present various domain-decomposition techniques for the analysis of large finite arrays, which is one of the most challenging problems for computational electromagnetics. Chapter 11 deals with another challenging problem, which is modeling of antenna–platform interactions by combining the finite element

method with other numerical or asymptotic techniques. Finally, in Chapter 12 we discuss various numerical and practical considerations in the finite element analysis of antennas and arrays.

In the writing of this book, special care was exercised to avoid significant overlap with existing books on the finite element method for electromagnetic analysis, including a book by the first author. The writing assumed that the reader has basic knowledge of antennas, electromagnetics, and the finite element method. For the frequency-domain analysis, the time convention $e^{j\omega t}$ is used and suppressed throughout. References are listed at the end of each chapter. All the numerical examples were obtained using our in-house-developed computer codes and whenever possible they were verified using published data, experimental data, or results computed using commercial software.

The book is the result of a close collaboration between the two authors—both have worked on every chapter. Each author takes responsibility for the accuracy of the material he wrote. The first author (J.M.J.) was mainly responsible for writing Chapters 1 to 3, 5, and 7 to 11. The second author (D.J.R.) was mainly responsible for writing Chapters 4, 6, and 12 and Sections 9.2.3, 10.2.2, and 11.1.2, and he also contributed to the writing of Chapter 7 and Sections 2.3, 5.1.2, and 5.2.3. The authors would appreciate having any errors brought to their attention.

This book can be used as a research reference by graduate students and researchers in the fields of computational electromagnetics and antennas and as a general reference by practicing antenna designers and engineers. It can also be used as a textbook for a graduate course on numerical analysis and modeling of electromagnetic fields. The techniques presented in the book can be extended to deal with problems in other electromagnetics-, RF-, microwave-, and optics-related technical fields.

Acknowledgments

JIAN-MING JIN
DOUGLAS J. RILEY

1 Introduction

In recent years, antenna technologies have received heightened interest because of their importance in wireless communication, remote sensing, space exploration, defense, electronic warfare, and many other electronic systems. Quantitative antenna analysis is critical to the design and optimization of antennas, especially complex antennas that are not easily designed by intuitive approaches. In a typical antenna analysis, the goal is to find the radiated field and input impedance. In the case of multiple antennas, such as antenna arrays, it is also important to quantify the mutual coupling between antennas, which can be characterized by either a mutual impedance matrix or a scattering matrix. The calculation of radiated fields, input impedances, and scattering matrices requires solving Maxwell's equations subject to certain boundary conditions determined by antenna configurations. Unfortunately, Maxwell's equations can be solved analytically only for a very few idealized antenna geometries. For example, when a linear antenna can be approximated as an infinitesimally short current element or a finite wire with a known current distribution, its radiated field can be calculated analytically. When a biconical antenna is assumed to extend to infinity, its radiated field and input impedance can also be obtained analytically. Without an approximation, antennas cannot be analyzed analytically primarily because of their structural configurations. Whereas a variety of approximate analytical techniques have been developed for relatively simple antennas, accurate and complete analysis of complex antennas, especially antenna arrays, can be accomplished only through a numerical method that solves Maxwell's equations numerically with the aid of high-speed computers.

1.1 NUMERICAL SIMULATION OF ANTENNAS

Computational electromagnetics deals with the art and science of solving Maxwell's equations numerically or with the numerical simulation of electromagnetic fields. It has become an indispensable tool for antenna analysis because of the predictive power of Maxwell's equations: If these equations are solved correctly, the solution can predict experimental outcomes and design performances. Because of their high predictive power and capability of dealing with complex structures, numerical simulation tools can support a wide variety of engineering applications, such as designing antennas analytically and predicting the impact of platforms on antenna performance, and

address more complex applications, including calibration of antenna systems, estimating co-site interference of multiple-antenna systems on a platform, and predicting scattering from low-observable antenna installations.

In addition to the capability of analyzing complex antennas, numerical simulation has four more distinctive advantages over traditional antenna design by experiment. The first advantage is low cost. When an antenna can be designed, analyzed, and optimized on a computer, its design cost is reduced significantly compared to that of constructing a prototype physically and measuring it in an anechoic chamber. The second advantage is the short design cycle. It typically takes far less time to simulate an antenna on a computer than to actually build one and measure it in a laboratory. The third advantage is the full exploration of the design space. Because of the low cost and short design cycle, the designer can evaluate a large variety of design parameters systematically to come up with an optimal design through numerical simulation, which is simply impossible with laboratory experiments. The last but not the least advantage of numerical simulation is the enormous amount of physical insight it provides. With a numerical solution to Maxwell's equations, the designer can now use a computer visualization tool to "see" the current flow on an antenna and field distributions around the antenna. Such a capability is extremely useful because it can help to pinpoint the source of design deficiency, such as the source of mutual coupling between antennas and the source of interference for antennas mounted on a platform. All these advantages become much more pronounced when dealing with more complex antennas involving many design parameters. Indeed, in many cases numerical simulation coupled with an appropriate set of validating measurements is the best practical solution to an antenna design problem.

Unfortunately, the great advantages of numerical simulation are also accompanied by a series of challenges. The main challenge is due to improper use of a numerical simulation, such as insufficient discretization and use of a method outside its bounds. Such improper use would yield either a poor or a completely erroneous design while wasting time and resources. Therefore, it is very important to understand the basic principles, solution technologies, and applicability and capabilities of numerical methods behind the numerical simulation tools. Such knowledge can not only reduce the possibility of improper use of a method, but also help in choosing from a suite of tools the technique best suited for a specific problem, thus increasing the designer's productivity.

1.2 FINITE ELEMENT ANALYSIS VERSUS OTHER NUMERICAL METHODS

Among a variety of numerical simulation tools in computational electromagnetics that provide a complete solution to Maxwell's equations, many are based on the method of moments, the finite-difference time-domain method, and the finite element method. Other methods, such as the transmission-line method and the finite integration technique, can be identified as either a variation or an equivalent of one of the first three.

Among the three major numerical techniques, the method of moments [1–6] has the longest history for antenna analysis. The method is based on the formulation of integral equations in terms of Green's functions as the fundamental solution to Maxwell's equations. Early development of the moment method for antenna analysis is natural because certain traditional antennas, such as dipoles and monopoles, can be represented by wires and thus require only a one-dimensional discretization for a numerical solution by the moment method. Furthermore, the Sommerfeld radiation condition, which has to be satisfied by an antenna's radiated fields, is built into the moment-method formulation automatically through the use of an appropriate Green's function; therefore, it requires no special treatment. The moment method is ideally suited for modeling metallic antennas because by using a surface integral equation, the computational domain is confined to the metallic surfaces. It is also highly efficient for antennas consisting of layered substrates, such as microstrip patch antennas, and for antennas comprising bulk homogeneous dielectrics, such as dielectric resonator antennas, because for these cases, the effect of the dielectrics can either be accounted for by a special Green's function or be modeled by equivalent electric and magnetic surface currents. However, the capability of the moment method is challenged when one attempts to model complex antennas designed with complex materials that may be anisotropic and inhomogeneous. Moreover, because of the use of Green's functions, the moment method generates a fully populated matrix whose computation and solution are associated with a high degree of computational complexity. Therefore, the traditional moment method becomes very time consuming and memory intensive for the analysis of large antennas, especially array-type antennas, which are often modeled with millions of unknowns. Fortunately, this challenge has largely been alleviated by the development of a variety of fast solvers, such as the fast multipole method, the adaptive integral method, and other fast fourier transform (FFT)–based methods [7–10]. Despite its drawbacks, the distinctive advantages of the moment method (mainly a surface-only discretization for a three-dimensional problem), coupled with the development of fast solvers, make the method a powerful tool and a preferred choice for the analysis of metallic antennas and antennas mounted on a metallic platform.

The finite-difference time-domain method [11–13], invented in the mid-1960s, solves Maxwell's equations discretized on a rectangular grid directly in the time domain. The method can easily handle material anisotropy and inhomogeneity and has become very powerful and increasingly popular because of its simplicity in formulation, implementation, and grid generation. It is also highly efficient because it does not involve any matrix solutions, and through the Fourier transform it yields a broadband solution with one time-domain calculation. As a method that solves partial differential equations directly, the finite-difference time-domain method requires a grid discretization of a three-dimensional volume to compute the fields in the volume. Since the solution region extends to infinity in an antenna radiation problem, the volume must be truncated and treated specially so that the truncated volume still mimics the original open environment. This was a major limiting factor affecting the accuracy and use of the method for many years; however, this difficulty has been removed successfully with the development of perfectly matched layers for grid

truncation [14]. The remaining major challenge for the finite-difference time-domain method is the accurate modeling of complex geometrical structures, especially very fine structures whose sizes are on the order of a few hundredths or even thousandths of a wavelength, by using a rectangular grid. Although the geometrical modeling accuracy can be improved by the use of conformal grids or subgridding techniques, the resulting numerical schemes become either more complicated or much less efficient because in such a case one has to reduce the time-step size to maintain the stability of the numerical solution. Nevertheless, since most antenna geometries can be modeled accurately with a sufficiently fine grid, the finite-difference time-domain method will remain a powerful and popular choice for the modeling of antennas with complex structures and embedded in complex materials.

Compared to the method of moments and the finite-difference time-domain method, the finite element method [15–22] is not as mature and popular for antenna analysis because its formulation is more complicated than that of the finite-difference time-domain method and its use requires sophisticated volumetric mesh generation. However, the finite element method has an unmatched capability for modeling both complex structures and materials. By using unstructured meshes with curvilinear triangular and tetrahedral elements, the method can accurately model curved surfaces, fine structures, and artificial engineered materials. Since the finite element method in the time domain can be formulated to be unconditionally stable, the time-step size does not have to be reduced even for problems containing very small finite elements. This unconditional stability is critical to the analysis of complicated antenna applications such as the one illustrated in Figure 1.1. Although the finite element method requires solving a large matrix equation, the associated matrix is very sparse and
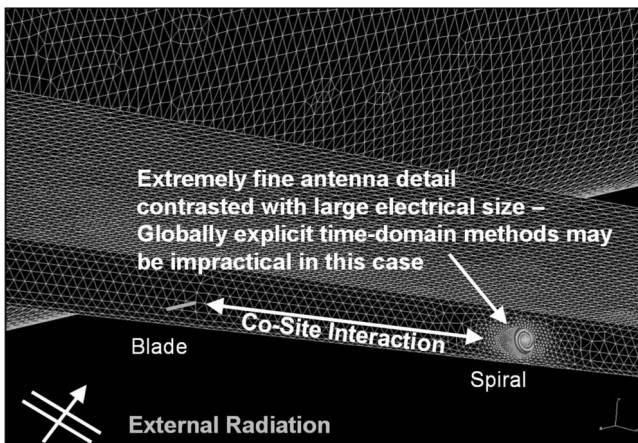
Figure 1.1   Example showing very small finite elements to model fine structures on a large object. Such a problem is challenging for explicit, conditionally stable time-domain methods and can be better handled by either implicit, unconditionally stable time- or frequency-domain techniques. (*See insert for color representation of figure.*)

often symmetrical, and its solution can be obtained efficiently by using advanced sparse solvers. Furthermore, the finite element method is well suited for parallel computation through the use of a variety of domain-decomposition algorithms. Like the finite-difference time-domain method, the finite element method solves partial differential equations directly without using Green's functions. As such, it requires the discretization of the three-dimensional space that surrounds the antenna to be analyzed and the truncation of this open space to make the solution domain finite. Proper treatment of the mesh truncation has been one of the major research subjects for the finite element analysis of antenna problems, and a variety of highly effective techniques have now been developed. The remaining major obstacle that has made the finite element method a less popular choice is the necessity for complicated mesh generation. However, this situation is changing quickly because of tremendous ongoing activities in the development of highly robust mesh generators.

From the discussions above, it can be seen clearly that the three methods have unique strengths and shortcomings. No single method is superior to the other two for every application. The moment method models free space accurately and requires only a surface discretization; thus, it is an attractive choice for modeling large metallic surfaces and homogeneous objects. The finite-difference time-domain method does not require a solution of a matrix equation and thus is highly efficient. Its implementation of perfectly matched layers for grid truncation has been well developed and is highly robust. For the finite element method to be competitive with these two methods, it must absorb their strengths into its formulation to compensate for its deficiencies. For example, the finite element method can be combined with the moment method such that the exterior open space and the antenna platform can be modeled accurately using the moment method, and the finite element method can then focus on the modeling of complex antenna structures. The finite element method can also be combined with the finite-difference time-domain method, with the latter being used to model the surrounding free space and any other homogeneous regions to fully exploit its high efficiency and its robust implementation of perfectly matched layers. These ideas lead to the development of various hybrid techniques, which are much more powerful than their individual components. These hybrid techniques should not simply "bundle" different methods together; rather, they should be formulated based on well-established electromagnetic and mathematical principles, they should be error controllable, and it should be possible to improve their accuracy in a systematic manner. Two such hybrid techniques are covered in this book; one combines the finite element method and the moment method, and the other combines the finite element method and the finite-difference time-domain method.

1.3   FREQUENCY- VERSUS TIME-DOMAIN SIMULATIONS

Since Maxwell's equations can be cast in both the time and frequency domains, a numerical solution to an electromagnetic problem can be sought in either the time or the frequency domain. In principle, it is sufficient to seek a solution in only one domain because the solution in the other domain can always be obtained using

the Fourier transform. However, since the solution processes in the two domains are different, the two solutions possess different strengths. For example, when a frequency-domain numerical method is employed to solve Maxwell's equations, we have to solve a system of linear equations (matrix equation) for each frequency. However, for a general electromagnetic problem, the system matrix is independent of the excitation. Once this matrix is inverted or factorized, it becomes trivial to find a solution for a new excitation. This feature makes the frequency-domain method ideally suited for scattering analysis, where one is often interested in scattering due to plane waves from many incident directions, and perhaps less attractive for antenna analysis, where the number of different excitations is usually small and the response over many frequencies is typically required. On the other hand, when a time-domain numerical method is adopted to solve Maxwell's equations, we have to seek a solution by time marching for each excitation. Once the solution in the time domain is obtained, we can find the solution over a wide band of frequency using the Fourier transform. However, the entire solution process must be repeated for a new excitation. Therefore, the time-domain method is ideally suited for antenna analysis, where one is often interested in a solution over a broad frequency band for one or a few excitations, and becomes less efficient for scattering analysis because it requires many solutions to many excitations. Because of its importance to antenna analysis, we have devoted much effort in this book to time-domain techniques.

Although the discussions and the conclusion above are true in a general sense, we have to consider many factors when we evaluate and choose a specific solution method. For example, when a frequency-domain method is equipped with a fast solver and a robust frequency interpolation algorithm, it can become as efficient as, or even more efficient than, a time-domain technique even for the broadband analysis of antennas. Besides the use of a fast solver, a frequency-domain method has three additional unique capabilities. The first is the ability to use different mesh densities at different frequencies. This allows the use of a much coarser mesh at a lower frequency, which can speed up the simulation greatly. In contrast, the mesh density in a time-domain solution has to be determined based on the highest frequency of interest. The second capability is the ease of performing parallel computations for a broadband simulation. One need only assign different processors to carry out computations at different frequencies. This embarrassingly simple parallelization requires no interprocessor communications and hence is highly efficient. The third capability, which is perhaps also the most important, is that in the frequency domain a large discretized electromagnetic problem can be represented by a reduced-order model that contains only a few degrees of freedom. For example, the property of an antenna, which is originally characterized by a matrix having an order of a few thousands, can be represented accurately by a much smaller matrix having an order of a few tens. This feature allows the development of special techniques that can handle very large antenna arrays which originally have to be modeled with millions or even billions of degrees of freedom. The development of such a technique in the time domain is, however, not as straightforward. Therefore, because of these enhancements, the frequency-domain methods will remain important simulation tools for antenna analysis.

The truly unique strength of time-domain methods is their capability to model nonlinear components, devices, and media in an antenna system, similar to the nonlinear circuits treated in Ref. 23. This capability will become more important in the future with the development of advanced antenna systems that integrate active devices such as sources directly into antenna radiating elements. Simulation of such antenna systems in the frequency domain by means of harmonic balancing is cumbersome and very time consuming. Although the time-domain methods discussed in the book can be employed to model nonlinear antenna problems, this topic is discussed only briefly, in the context of nonlinear lumped-circuit components.

1.4   BRIEF REVIEW OF PAST WORK

Since Silvester [24] introduced the finite element method into the field of microwave engineering and electromagnetics in 1969, a tremendous amount of research has been carried out to develop the method for the analysis of electrostatic, magnetostatic, and electrodynamic problems. Most early applications dealt with problems within a bounded region, such as waveguide problems. In 1974, Mei developed a technique that combined the finite element method with a wavefunction expansion to deal with open-region electromagnetic problems such as antenna and scattering analysis [25]. In 1982, Marin developed an alternative method to deal with open-region scattering problems, which combined the finite element method with a boundary integral equation [26]. This work can be considered an extension of early formulations [27,28] for static fields. These developments enabled the application of the finite element method to open-region electromagnetic problems.

An important breakthrough in the finite element analysis of vector electromagnetic field problems occurred in the 1980s with the development of edge-based vector elements [29–31]. These new elements accurately model the nature of the electric and magnetic fields and eliminate many of the challenges associated with traditional node-based scalar elements that were used in the early finite element formulations. Since the development of vector elements, the finite element method has become a very powerful numerical technique for the analysis of three-dimensional electromagnetic fields. Although much research has been carried out and published on the finite element method for electromagnetic analysis, most of it focused on bounded field and open-region scattering problems. The subject of the finite element analysis of antennas has not received as much attention as it deserves. In the following text we review briefly the development and application of the finite element method for antenna analysis.

Application of the finite element method for the analysis and design of various antennas dates back to the 1970s, when Mei developed the first accurate approach that enabled the finite element method to deal with unbounded open-region problems [25]. The method was applied to axisymmetric antennas. For many years, the finite element method was limited to simplified two-dimensional and axisymmetric models of antennas [32–35] because of the difficulty of using the node-based elements to model vector electromagnetic fields, with the exception that Ref. 34 contained

an example of calculating the field radiated by an electric current element in free space using edge-based elements. The first full-wave three-dimensional finite element analysis of realistic antennas appeared in the early 1990s [36], where the finite element method was coupled with a boundary integral equation to simulate cavity-backed microstrip antennas on a ground plane. A simple probe feed model was developed to excite antennas, and both radiation patterns and input impedances were calculated and compared with measured data. Thereafter, a variety of finite element–based numerical techniques have been developed for the analysis and simulation of various antennas and antenna arrays.

Most notably, the finite element–based numerical techniques have been developed to analyze infinitely periodic array antennas [37–40] and finite array antennas [41–48]. For the analysis of infinite array antennas, boundary integral equations were developed based on the Floquet theorem to accurately model the radiation condition, and periodic boundary conditions were formulated to confine the analysis to a single unit cell. For the analysis of large finite array antennas, novel domain-decomposition schemes were proposed that exploit the geometric repetition in the array configuration to make the analysis possible. The finite element method has also been used for the analysis of complex horn antennas [49–54] and dielectric lens antennas and radomes [55–58]. For these analyses, the axisymmetric feature of the antenna geometry, except for the excitation, can be utilized to reduce the computational domain from a three-dimensional volume to a two-dimensional slice by expanding the fields in terms of Fourier modes. The finite element method has been found to be ideally suited for modeling conformal antennas, such as cavity-backed aperture, slot, and patch antennas [59–66], because the finite element analysis can be confined to the cavity region, which contains complex antenna geometries, leaving the aperture field to be handled by a boundary integral equation. The excellent material modeling capability of the finite element method enabled the analysis of antennas residing on complex materials, such as those designed with ferrite and chiral substrates [67–69]. Combined with the moment method and a high-frequency asymptotic technique, the finite element method has been employed to analyze antennas mounted on a finite platform [70–77]. In this type of analysis, the finite element method is used to model antennas, and the effect of platforms is modeled either by the moment method based on a surface integral equation or by a high-frequency asymptotic technique such as physical optics, the geometrical theory of diffraction, the uniform theory of diffraction, and the shooting- and bouncing-ray method.

Most of the analyses discussed above were carried out in the frequency domain. To perform a frequency sweep analysis, a model-order reduction technique has been proposed [78,79], which was based on the asymptotic waveform evaluation technique originally developed for circuit analysis. Recently, the finite element method has been developed for antenna analysis directly in the time domain [80–93]. As discussed earlier, such a time-domain analysis is highly efficient for the characterization of broadband responses and is capable of modeling nonlinear materials and devices. In these works, the computational domain was truncated by perfectly matched layers implemented either directly in the finite element method [82,85] or in combination with the finite-difference time-domain method [81,83,84]. Accurate feed models have

been developed to provide an excitation to antennas and to extract input impedances or *S*-parameters [85]. Novel domain-decomposition schemes have been developed for the analysis of large antennas and finite arrays [86–88]. A highly effective approach based on field transformation has been proposed for the analysis of infinitely periodic antenna arrays using time-domain finite element formulations [89–91]. Preliminary studies have also been conducted on incorporating a distributed feed network into the finite element modeling of antenna arrays [92] and on the simulation of antennas installed on a platform by combining the time-domain finite element method with a fast solution of a time-domain surface integral equation for induced currents on the platform [93].

1.5 OVERVIEW OF THE BOOK

The objective of this book is to present the basic formulations and discuss all the technical aspects in the finite element analysis of complex antennas and arrays. The remaining 11 chapters of the book are organized as follows.

In Chapter 2 we describe the formulations of the finite element analysis of antennas in the frequency and time domains. In this description, emphasis is placed on the basic principle of the finite element method instead of its numerical implementation. The modeling of complex anisotropic, dispersive, and lossy materials in the time-domain finite element analysis is discussed in detail. Techniques for solving finite element equations and the use of higher-order curvilinear finite elements are also addressed briefly.

Chapter 3 deals with the fundamental challenge in the partial differential equation–based numerical analysis of open-region electromagnetic radiation and scattering problems, which is the truncation of the infinite solution space into a finite-sized computational domain. The truncation techniques covered include first- and second-order absorbing boundary conditions, various perfectly matched layers, and free-space and half-space boundary integral equations. Their formulation and implementation in the frequency and time domains are discussed in detail.

In Chapter 4 we describe a stable formulation that combines the finite element time-domain method with the highly efficient finite-difference time-domain method. An immediate benefit of this combination is to use the well-established finite-difference time-domain implementation of perfectly matched layers for the truncation of computational domains. Certain equivalence between the finite-difference time-domain and finite element time-domain methods, which provides a theoretical foundation for the stable interface formulation, is illustrated and an accurate near-to-far-field transformation is described.

Chapter 5 deals with another critical challenge specific to the finite element analysis of antennas: the modeling of antenna feeds for radiation analysis and plane-wave excitation for scattering analysis. Described are simplified feed models such as current probes and voltage gaps, as well as a more accurate modeling of waveguide feeds using a waveguide port boundary condition. For scattering analysis, we describe the total- and scattered-field formulations and a novel total- and scattered-field

decomposition approach. Far-field computation and near-field visualization are also addressed briefly.

In Chapter 6 we consider the finite element modeling of complex structures and circuit components, such as thin-material layers and sheets, thin wires and slots, lumped-circuit components, and feeding networks. Such modeling is important for practical applications since these types of structures and circuit components are used widely in antenna designs. The chapter includes many practical examples and, in particular, an example of predicting electromagnetic coupling into an electronic subsystem.

Chapter 7 covers a variety of antenna simulation examples, including two scattering examples, to demonstrate the capability and versatility of the finite element method based on Chapters 2 through 6. The narrowband examples include monopole and microstrip patch antennas, and the broadband examples include the horn, spiral, sinuous, Vivaldi, and Vlasov antennas. Whenever possible, the simulation results are validated or verified with published data, experimental data, or results computed using commercial software.

In Chapter 8 we describe the finite element analysis of axisymmetric antennas in conjunction with absorbing boundary conditions, perfectly matched layers, and boundary integral equations. The analysis exploits the rotational symmetry of the problem by expanding fields and excitations in terms of Fourier modes, which reduces the original three-dimensional problem to a two-dimensional problem where the simulation can be carried out more efficiently using a two-dimensional finite element method.

The modeling of infinitely large phased arrays is the topic of Chapter 9, which covers both the frequency- and time-domain analyses. The implementation of periodic boundary conditions, the formulation of mesh truncation techniques specific to this class of problems, and the modeling of general complex materials are discussed in detail. The use of an infinite phased-array solution to approximate a corresponding finite array is also addressed as a fast practical solution to a very complicated problem.

In Chapter 10 we treat one of the more challenging applications in the numerical simulation of antennas and perhaps in the entire field of computational electromagnetics: analysis of large finite arrays. Two major numerical techniques are presented to deal with this problem. One is based on the finite element tearing and interconnecting algorithm in the frequency domain, and the other is based on domain decomposition strategies in the time domain. Both approaches effectively exploit the geometrical repetitions of a finite array to make the problem tractable.

Chapter 11 deals with another highly challenging problem for antenna analysis: modeling of antenna–platform interactions. Two approaches are described. One approach is based on an accurate simultaneously coupled analysis, which simulates the entire problem numerically in one stage. The other approach is first to apply the finite element method to the antenna and its nearby structure and compute the near field, then calculate the far-field radiation according to Huygens' principle using either a numerical method based on surface integral equations or a high-frequency asymptotic technique based on ray tracing.

Finally, in Chapter 12 we discuss various numerical and practical considerations in the application of finite element analysis to antennas and arrays, such as selection of the most suitable analysis tool and solver for a specific problem, finite element discretization and corresponding numerical convergence, fast frequency sweep based on sampled frequency solutions, the application of domain decomposition and parallel computing, and the verification and validation of numerical predictions.

REFERENCES

1. R. F. Harrington, *Field Computation by Moment Methods*. New York: Macmillan, 1968; reprinted by IEEE Press, 1993.

2. R. Mittra, Ed., *Computer Techniques for Electromagnetics*. Elmsford, NY: Permagon, 1973.

3. R. C. Hansen, Ed., *Moment Methods in Antennas and Scattering*. Norwood, MA: Artech House, 1990.

4. J. J. H. Wang, *Generalized Moment Methods in Electromagnetics*. New York: Wiley, 1991.

5. E. K. Miller, L. Medgyesi-Mitschang, and E. H. Newman, Eds., *Computational Electromagnetics: Frequency-Domain Method of Moments*. New York: IEEE Press, 1992.

6. A. F. Peterson, S. L. Ray, and R. Mittra, *Computational Methods for Electromagnetics*. New York: IEEE Press, 1998.

7. N. N. Bojarski, "*k*-Space formulation of the electromagnetic scattering problem," Tech. Rep. AFAL-TR-71-75, Air Force Avionics Laboratory, Mar. 1971.

8. R. Coifman, V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: a pedestrian prescription," *IEEE Antennas Propagat. Mag.*, vol. 35, pp. 7–12, June 1993.

9. E. Bleszynski, M. Bleszynski, and T. Jaroszewicz, "AIM: adaptive integral method for solving large-scale electromagnetic scattering and radiation problems," *Radio Sci.*, vol. 31, pp. 1225–1251, Sept.–Oct. 1996.

10. W. C. Chew, J. M. Jin, E. Michielssen, and J. M. Song, Eds., *Fast and Efficient Algorithms in Computational Electromagnetics*. Norwood, MA: Artech House, 2001.

11. K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antennas Propagat.*, vol. 14, pp. 302–307, May 1966.

12. K. S. Kunz and R. J. Luebbers, *The Finite Difference Time Domain Method for Electromagnetics*. Boca Raton, FL: CRC Press, 1994.

13. A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite Difference Time Domain Method*, 3rd ed. Norwood, MA: Artech House, 2005.

14. J.-P. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Phys.*, vol. 114, no. 2, pp. 185–200, 1994.

15. P. P. Silvester and R. L. Ferrari, *Finite Elements for Electrical Engineers*, 3rd ed. Cambridge, UK: Cambridge University Press, 1996.

16. J.-M. Jin, *The Finite Element Method in Electromagnetics*, 2nd ed. Hoboken, NJ: Wiley, 2002.

17. P. P. Silvester and G. Pelosi, Eds., *Finite Elements for Wave Electromagnetics*. New York: IEEE Press, 1994.

18. T. Itoh, G. Pelosi, and P. P. Silvester, Eds., *Finite Element Software for Microwave Engineering*. New York: Wiley, 1996.

19. J. L. Volakis, A. Chatterjee, and L. C. Kempel, *Finite Element Method for Electromagnetics: Antennas, Microwave Circuits and Scattering Applications*. New York: IEEE Press, 1998.

20. M. Salazar-Palma, T. K. Sarkar, L. E. Garcia-Castillo, T. Roy, and A. R. Djordjevic, *Iterative and Self-Adaptive Finite Elements in Electromagnetic Modeling*. Norwood, MA: Artech House, 1998.

21. A. Bossavit, *Computational Electromagnetism: Variational Formulations, Complementarity, Edge Elements*. San Diego, CA: Academic Press, 1998.

22. Y. Zhu and A. C. Cangellaris, *Multigrid Finite Element Methods for Electromagnetic Field Modeling*. New York: IEEE Press, 2006.

23. S.-H. Chang, R. Coccioli, Y. Qian, and T. Itoh, "Global finite-element time-domain analysis of active nonlinear microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 12, pp. 2410–2416, Dec. 1999.

24. P. P. Silvester, "Finite element solution of homogeneous waveguide problems," *Alta Freq.*, vol. 38, pp. 313–317, May 1969.

25. K. K. Mei, "Unimoment method of solving antenna and scattering problems," *IEEE Trans. Antennas Propagat.*, vol. 22, pp. 760–766, Nov. 1974.

26. S. P. Marin, "Computing scattering amplitudes for arbitrary cylinders under incident plane waves," *IEEE Trans. Antennas Propagat.*, vol. 30, pp. 1045–1049, Nov. 1982.

27. P. P. Silvester and M. S. Hsieh, "Finite-element solution of 2-dimensional exterior field problems," *IEE Proc. H*, vol. 118, pp. 1743–1747, Dec. 1971.

28. B. H. McDonald and A. Wexler, "Finite-element solution of unbounded field problems," *IEEE Trans. Microwave Theory Tech.*, vol. 20, pp. 841–847, Dec. 1972.

29. J. C. Nedelec, "Mixed finite elements in R^3," *Numer. Math.*, vol. 35, pp. 315–341, 1980.

30. A. Bossavit and J. C. Verite, "A mixed FEM–BIEM method to solve 3-D eddy current problems," *IEEE Trans. Magn.*, vol. 18, pp. 431–435, Mar. 1982.

31. M. L. Barton and Z. J. Cendes, "New vector finite elements for three-dimensional magnetic field computation," *J. Appl. Phys.*, vol. 61, pp. 3919–3921, Apr. 1987.

32. T. Orikasa, S. Washisu, T. Honma, and I. Fukai, "Finite element method for unbounded field problems and application to two-dimensional taper," *Int. J. Numer. Methods Eng.*, vol. 19, pp. 157–168, 1983.

33. J. D'Angelo, M. J. Povinelli, and M. A. Palmo, "Hybrid finite element/boundary element analysis of a strip line notch array," *IEEE AP-S Int. Symp. Dig.*, vol. 3, pp. 1126–1129, 1988.

34. J. D'Angelo and I. D. Mayergoyz, "Finite element methods for the solution of RF radiation and scattering problems," *Electromagnetics*, vol. 10, pp. 177–199, 1990.

35. H. Ali and G. Costache, "Finite-element time-domain analysis of axisymmetrical radiators," *IEEE Trans. Antennas Propagat.*, vol. 42, no. 2, pp. 272–275, Feb. 1994.

36. J. M. Jin and J. L. Volakis, "A hybrid finite element method for scattering and radiation by microstrip patch antennas and arrays residing in a cavity," *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 1598–1604, Nov. 1991.

37. J. M. Jin and J. L. Volakis, "Scattering and radiation analysis of three-dimensional cavity arrays via a hybrid finite element method," *IEEE Trans. Antennas Propagat.*, vol. 41, pp. 1580–1586, Nov. 1993.

38. D. T. McGrath and V. P. Pyati, "Phased array antenna analysis with the hybrid finite element method," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 1625–1630, Dec. 1994.

39. E. W. Lucas and T. P. Fontana, "A 3-D hybrid finite element/boundary element method for the unified radiation and scattering analysis of general infinite periodic arrays," *IEEE Trans. Antennas Propagat.*, vol. 43, no. 2, pp. 145–153, Feb. 1995.

40. Z. Lou and J. M. Jin, "Finite element analysis of phased array antennas," *Microwave Opt. Tech. Lett.*, vol. 40, no. 6, pp. 490–496, Mar. 2004.

41. R. Kindt, K. Sertel, E. Topsakal, and J. L. Volakis, "Array decomposition method for the accurate analysis of finite arrays," *IEEE Trans. Antennas Propagat.*, vol. 51, pp. 1364–1372, June 2003.

42. M. N. Vouvakis, S.-C. Lee, K. Zhao, and J.-F. Lee, "A symmetric FEM-IE formulation with a single-level IE-QR algorithm for solving electromagnetic radiation and scattering problems," *IEEE Trans. Antennas Propagat.*, vol. 52, no. 11, pp. 3060–3070, Nov. 2004.

43. S.-C. Lee, M. N. Vouvakis, and J.-F. Lee, "A non-overlapping domain decomposition method with non-matching grids for modeling large finite antenna arrays," *J. Comput. Phys.*, vol. 203, pp. 1–21, Feb. 2005.

44. J. Rubio, M. A. Gonzalez, and J. Zapata, "Generalized-scattering-matrix analysis of a class of finite arrays of coupled antennas by using 3-D FEM and spherical mode expansion," *IEEE Trans. Antennas Propagat.*, vol. 53, no. 3, pp. 1133–1144, Mar. 2005.

45. Y. J. Li and J. M. Jin, "A vector dual–primal finite element tearing and interconnecting method for solving 3-D large-scale electromagnetic problems," *IEEE Trans. Antennas Propagat.*, vol. 54, no. 10, pp. 3000–3009, Oct. 2006.

46. K. Zhao, V. Rawat, S.-C. Lee, and J.-F. Lee, "A domain decomposition method with non-conformal meshes for finite periodic and semi-periodic structures," *IEEE Trans. Antennas Propagat.*, vol. 55, no. 9, pp. 2559–2570, Sept. 2007.

47. Y. J. Li and J. M. Jin, "A new dual–primal domain decomposition approach for finite element simulation of 3D large-scale electromagnetic problems," *IEEE Trans. Antennas Propagat.*, vol. 55, no. 10, pp. 2803–2810, Oct. 2007.

48. C. Smith, M. Little, B. Porter, and M. N. Vouvakis, "Analysis of co-planar phased array coupling using finite element domain decomposition," *IEEE AP-S Int. Symp. Dig.*, pp. 3524–3527, 2007.

49. G. C. Chinn, L. W. Epp, and D. J. Hoppe, "A hybrid finite-element method for axisymmetric waveguide-fed horns," *IEEE Trans. Antennas Propagat.*, vol. 44, no. 3, pp. 280–285, Mar. 1996.

50. C. Zuffada, T. Cwik, and V. Jamnejad, "Modeling radiation with an efficient hybrid finite-element integral-equation waveguide mode-matching technique," *IEEE Trans. Antennas Propagat.*, vol. 45, no. 1, pp. 34–39, Jan. 1997.

51. D. T. McGrath and C. E. Baum, "Scanning and impedance properties of TEM horn arrays for transient radiation," *IEEE Trans. Antennas Propagat.*, vol. 47, no. 3, pp. 469–473, Mar. 1999.

52. A. D. Greenwood and J. M. Jin, "Finite element analysis of complex axisymmetric radiating structures," *IEEE Trans. Antennas Propagat.*, vol. 47, no. 8, pp. 1260–1266, Aug. 1999.

53. J. M. Gil, J. Monge, J. Rubio, and J. Zapata, "A CAD-oriented method to analyze and design radiating structures based on bodies of revolution by using finite elements and generalized scattering matrix," *IEEE Trans. Antennas Propagat.*, vol. 54, no. 3, pp. 899–907, Mar. 2006.

54. G. G. Gentili, P. Bolli, R. Nesti, G. Pelosi, and L. Toso, "High-order FEM mode matching analysis of circular horns with rotationally symmetric dielectrics," *IEEE Trans. Antennas Propagat.*, vol. 55, no. 10, pp. 2915–2918, Oct. 2007.

55. A. D. Greenwood and J. M. Jin, "A field picture of wave propagation in inhomogeneous dielectric lenses," *IEEE Antennas Propagat. Mag.*, vol. 41, no. 5, pp. 9–18, Oct. 1999.

56. C. S. Liang, D. A. Streater, J. M. Jin, E. Dunn, and T. Rozendal, "A quantitative study of Luneberg lens reflectors," *IEEE Antennas Propagat. Mag.*, vol. 47, no. 2, pp. 30–42, Apr. 2005.

57. R. K. Gordon and R. Mittra, "Finite element analysis of axisymmetric radomes," *IEEE Trans. Antennas Propagat.*, vol. 41, no. 7, pp. 975–981, July 1993.

58. E. Dunn, J. K. Byun, E. Branch, and J. M. Jin, "Numerical simulation of BOR scattering and radiation using a higher-order FEM," *IEEE Trans. Antennas Propagat.*, vol. 54, no. 3, pp. 945–952, Mar. 2006.

59. L. C. Kempel, J. L. Volakis, and R. J. Sliva, "Radiation by cavity-backed antennas on a circular cylinder," *IEE Proc. Microwaves Antennas Propagat.*, vol. 142, no. 3, pp. 233–239, June 1995.

60. G. E. Antilla and N. G. Alexopoulos, "Radiation and scattering from complex 3D curvilinear geometries using a hybrid finite element–integral equation method," *IEEE AP-S Int. Symp. Dig.*, pp. 1758–1761, 1992.

61. M. A. Gonzalez de Aza, J. A. Encinar, J. Zapata, and M. Lambea, "Full-wave analysis of cavity-backed and probe-fed microstrip patch arrays by a hybrid mode-matching generalized scattering matrix and finite-element method," *IEEE Trans. Antennas Propagat.*, vol. 46, no. 2, pp. 234–242, Feb. 1998.

62. T. Ozdemir, J. L. Volakis, and M. W. Nurnberger, "Analysis of thin multioctave cavity-backed slot spiral antennas," *IEE Proc. Microwaves Antennas Propagat.*, vol. 146, pp. 447–454, Dec. 1999.

63. C. A. Macon, L. C. Kempel, and S. W. Schneider, "Radiation and scattering by complex conformal antennas on a circular cylinder," *Adv. Comput. Math.*, vol. 16, pp. 191–209, 2002.

64. M. N. Vouvakis, C. A. Balanis, C. Birtcher, and A. C. Polycarpou, "Multilayer effects on cavity-backed slot antennas," *IEEE Trans. Antennas Propagat.*, vol. 52, no. 3, pp. 880–887, Mar. 2004.

65. C. A. Macon, L. C. Kempel, S. W. Schneider, and K. D. Trott, "Modeling conformal antennas on metallic prolate spheroid surfaces using a hybrid finite element method," *IEEE Trans. Antennas Propagat.*, vol. 52, no. 3, pp. 750–758, Mar. 2004.

66. K. Mao, J. K. Byun, and J. M. Jin, "Enhancing the modeling capability of the FE-BI method for simulation of cavity-backed antennas and arrays," *Electromagnetics*, vol. 26, no. 7, pp. 503–515, Oct. 2006.

67. A. C. Polycarpou and C. A. Balanis, "Finite-element investigation of scan performance characteristics of probe-fed phased arrays on magnetized ferrite substrates," *IEEE AP-S Int. Symp. Dig.*, vol. 1, pp. 666–669, July 1999.