



PRINCIPLES AND MODERN APPLICATIONS OF MASS TRANSFER OPERATIONS

Second Edition

Jaime Benítez



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*A Jaime por ser un hijo tan especial;
tu sonrisa angelical es todo lo que necesito para ser feliz.*

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Preface to the Second Edition

The idea for the first edition of this book was born out of my experience teaching a course on mass-transfer operations at the Chemical Engineering Department of the University of Puerto Rico during the previous 25 years. This course is the third in a three-course unit operations sequence. The first course covers momentum transfer (fluid mechanics), and the second course covers heat transfer. Besides these two courses, another prerequisite of the mass-transfer course is a two-semester sequence of chemical engineering thermodynamics.

I decided to write a textbook for a first course on mass-transfer operations with a level of presentation that was easy to follow by the reader, but with enough depth of coverage to guarantee that students using the book will, upon successful completion of the course, be able to specify preliminary designs of the most common mass-transfer equipment (such as absorbers, strippers, distillation columns, liquid extractors, etc.). I decided also to incorporate, from the very beginning of the book, the use of Mathcad, a computational tool that is, in my opinion, very helpful and friendly. The first edition of this book was the result of that effort.

Part of my objective was achieved, as evidenced by the following excerpt from a very thorough review of the first edition of my book, written by Professor Mark J. McCready, a well-known expert in chemical engineering education: "If the topics that are needed for a given course are included in this text, I would expect the educational experience to go smoothly for both student and instructor. I think that students will like this book, because the explanations are clear, the level of difficulty is appropriate, and the examples and included data give the book very much of a 'handbook' flavor. Instructors will find that, overall, the topics are presented in a logical order and the discussion makes sense; there are many examples and lots of homework problems" (McCready, M. J., *AIChE J.*, Vol. 49, No. 1, January 2003).

"Each major section of the book has learning objectives which certainly benefit the students and perhaps the instructor. A key feature of the book, which separates it from the other texts mentioned above, is the incorporation of Mathcad for both example problems and homework questions. A library of Mathcad programs for solving the Maxwell-Stefan equations, packed column calculations, sieve-tray design, binary distillation problems by McCabe-Thiele method, and multistage crosscurrent extraction is given in the appendices. These programs enable students to obtain useful solutions with less effort, as well as allow them to explore the different variables or parameters. The wide availability, low cost, and ease of use of Mathcad allow it to be the modern equivalent of 'back of the envelope' calculations, which can be refined, if necessary, using full-scale process simulators" (McCready, 2003).

However, the same reviewer also points out some limitations of the book. One of the main objectives of this second edition is to remedy those shortcomings of the first edition to make it more attractive as a textbook to a broader audience. Another important objective of the second edition is to incorporate material related to mass transfer-phenomena in biological systems. Many chemical engineering

departments all over the world are changing their names and curricula to include the area of biochemical engineering in their offerings. The second edition includes pertinent examples such as convection and diffusion of oxygen through the body's circulatory system, bio-artificial kidneys, separation of sugars by chromatography, and purification of monoclonal antibodies by affinity adsorption.

As with the first edition, the first four chapters of the book present a basic framework for analysis that is applicable to most mass-transfer operations. Chapters 5 to 7 apply this common methodology to the analysis and design of some of the most popular types of mass-transfer operations. Chapter 5 covers gas adsorption and stripping; Chapter 6 covers distillation; and Chapter 7 covers liquid extraction. Chapter 8, new to the second edition, covers humidification operations in general, and detailed design of packed cooling towers specifically. These operations—in particular, cooling towers—are very common in industry. Also, from the didactic point of view, their analysis and design involve simultaneous mass- and heat-transfer considerations. Therefore, the reader is exposed in detail to the similarities and differences between these two transport phenomena. Chapter 9, also new, covers mass-transfer processes using barriers (membranes) and solid sorption agents (adsorption, ion exchange, and chromatography).

In response to suggestions by Professor McCready and other reviewers, some other revisions and additions to the second edition are:

- In Chapter 1, the Maxwell-Stefan equations (augmented by the steady-state continuity equation for each component) are solved numerically using a combination of a Runge-Kutta-based differential equation solver (*Rkfixed*) and an algebraic equation solver (*Given-Find*), both included in Mathcad. This methodology is much more flexible than the one presented in the first edition (orthogonal collocation), and its theoretical justification is well within the scope of the mathematical background required for a first course in mass-transfer operations.
- Chapter 1 includes a section on diffusion in solids.
- Chapter 2 includes a section on boundary-layer theory and an example on simultaneous mass and heat transfer during air humidification.
- Chapter 6 includes a section on multistage batch distillation.

I wish to acknowledge gratefully the contribution of the University of Puerto Rico at Mayagüez to this project. My students in the course INQU 4002 reviewed the material in the book, found quite a few errors, and gave excellent suggestions on ways to improve its content and presentation. My students are my source of motivation; they make all the effort to prepare this book worthwhile!

Jaime Benítez
Mayagüez, Puerto Rico

Preface to the First Edition

The importance of the mass-transfer operations in chemical processes is profound. There is scarcely any industrial process that does not require a preliminary purification of raw materials or final separation of products. This is the realm of mass-transfer operations. Frequently, the major part of the cost of a process is that for the separations accomplished in the mass-transfer operations, a good reason for process engineers and designers to master this subject. The mass-transfer operations are largely the responsibility of chemical engineers, but increasingly practitioners of other engineering disciplines are finding them necessary for their work. This is especially true for those engaged in environmental engineering, where separation processes predominate.

My objective in writing this book is to provide a means to teach undergraduate chemical engineering students the basic principles of mass transfer and to apply these principles, aided by modern computational tools, to the design of equipment used in separation processes. The idea for it was born out of my experiences during the last 25 years teaching mass-transfer operations courses at the University of Puerto Rico.

The material treated in the book can be covered in a one-semester course. Chapters are divided into sections with clearly stated objectives at the beginning. Numerous detailed examples follow each brief section of text. Abundant end-of-chapter problems are included, and problem degree of difficulty is clearly labeled for each. Most of the problems are accompanied by their answers. Computer solution is emphasized, both in the examples and in the end-of-chapter problems. The book uses mostly SI units, which virtually eliminates the tedious task of unit conversions and makes it “readable” to the international scientific and technical community.

Following the lead of other authors in the chemical engineering field and related technical disciplines, I decided to incorporate the use of Mathcad into this book. Most readers will probably have a working knowledge of Mathcad. (Even if they don't, my experience is that the basic knowledge needed to begin using Mathcad effectively can be easily taught in a two-hour workshop.) The use of Mathcad simplifies mass-transfer calculations to a point that it allows the instructor and the student to readily try many different combinations of the design variables, a vital experience for the amateur designer.

The Mathcad environment can be used as a sophisticated scientific calculator, can be easily programmed to perform a complicated sequence of calculations (for example, to check the design of a sieve-plate column for flooding, pressure drop, entrainment, weeping, and calculating Murphree plate efficiencies), can be used to plot results, and as a word processor to neatly present homework problems. Mathcad can perform calculations using a variety of unit systems, and will give a warning signal when calculations that are not dimensionally consistent are tried. This is a most

powerful didactic tool, since dimensional consistency in calculations is one of the most fundamental concepts in chemical engineering education.

The first four chapters of the book present a basic framework of analysis that is applicable to any mass-transfer operation. Chapters 5 to 7 apply this common methodology to the analysis and design of the most popular types of mass-transfer operations. Chapter 5 covers gas absorption and stripping, chapter 6 distillation columns, and chapter 7 liquid extraction. This choice is somewhat arbitrary, and based on my own perception of the relevance of these operations. However, application of the general framework of analysis developed in the first four chapters should allow the reader to master, with relative ease, the peculiarities of any other type of mass-transfer operation.

I wish to acknowledge gratefully the contribution of the University of Puerto Rico at Mayagüez to this project. My students in the course INQU 4002 reviewed the material presented in the book, found quite a few errors, and gave excellent suggestions on ways to improve it. My special gratitude goes to Teresa, my wife, and my four children who were always around lifting my spirits during the long, arduous hours of work devoted to this volume. They make it all worthwhile!

Jaime Benítez
Mayagüez, Puerto Rico

Nomenclature

LATIN LETTERS

A	absorption factor; dimensionless.
A	mass flow rate of species A; kg/s.
A_a	active area of a sieve tray; m ² .
A_d	area taken by the downspout in a sieve tray; m ² .
A_h	area taken by the perforations on a sieve tray; m ² .
A_M	membrane area; m ² .
A_n	net cross-section area between trays inside a tray column; m ² .
A_t	total cross-section area, m ² .
a	mass-transfer surface area per unit volume; m ⁻¹ .
a_h	hydraulic, or effective, specific surface area of packing; m ⁻¹ .
B	mass flow rate of species B; kg/s.
B_0	viscous flow parameter; m ² .
c	total molar concentration; moles/m ³ .
c_i, C_i	molar concentration of species i ; moles/m ³ .
C	total number of components in multicomponent distillation.
C_p	specific heat at constant pressure; J/kg-K.
C_S	humid heat; J/kg-K.
C_D	drag coefficient; dimensionless.
Da	Damkohler number for first-order reaction; dimensionless.
\mathcal{D}_{ij}	Maxwell-Stefan diffusivity for pair i - j ; m ² /s.
D_{ij}	Fick diffusivity or diffusion coefficient for pair i - j ; m ² /s.
$D_{K,i}$	Knudsen diffusivity for component i ; m ² /s.
d_e	equivalent diameter; m.
d_i	driving force for mass diffusion of species i ; m ⁻¹ .
d_i	inside diameter; m.
d_o	outside diameter; m.
d_o	perforation diameter in a sieve plate; m.
d_p	particle size; m.
d_{vs}	Sauter mean drop diameter defined in equation (7-48); m.
DM	dimensional matrix.
D	tube diameter; m.
D	distillate flow rate; moles/s.
E	fractional entrainment; liquid mass flow rate/gas mass flow rate.
E	extract mass flow rate, kg/s.
E_m	mechanical efficiency of a motor-fan system; dimensionless.
E_o	Eotvos number defined in equation (7-53); dimensionless.

EF	extraction factor defined in equation (7-19); dimensionless.
E_{ME}	Murphree stage efficiency in terms of extract composition; dimensionless.
E_{MG}	Murphree gas-phase tray efficiency; dimensionless.
E_{MGE}	Murphree gas-phase tray efficiency corrected for entrainment.
E_O	overall tray efficiency of a cascade; equilibrium trays/real trays.
E_{OG}	point gas-phase tray efficiency; dimensionless.
f_{12}	proportionality coefficient in equation (1-21).
f	friction factor; dimensionless.
f	fractional approach to flooding velocity; dimensionless.
f_{ext}	fractional extraction; dimensionless.
F	mass-transfer coefficient; mol/m ² -s.
F	molar flow rate of the feed to a distillation column; mol/s.
F	mass flow rate of the feed to a liquid extraction process; kg/s.
F_p	packing factor; ft ⁻¹ .
$FR_{i,D}$	fractional recovery of component i in the distillate; dimensionless.
$FR_{i,W}$	fractional recovery of component i in the residue; dimensionless.
Fr_L	liquid Froude number; dimensionless.
Ga	Galileo number; dimensionless.
G_M	superficial molar velocity; mol/m ² -s.
G_{Mx}	superficial liquid-phase molar velocity; mol/m ² -s.
G_{My}	superficial gas-phase molar velocity; mol/m ² -s.
G_x	superficial liquid-mass velocity; kg/m ² -s.
G_y	superficial gas-mass velocity; kg/m ² -s.
Gr_D	Grashof number for mass transfer; dimensionless.
Gr_H	Grashof number for heat transfer; dimensionless.
Gz	Graetz number; dimensionless.
g	acceleration due to gravity; 9.8 m/s ² .
g_c	dimensional conversion factor; 1 kg-m/N-s ² .
H	Henry's law constant; atm, kPa, Pa.
H	molar enthalpy; J/mol.
H	height of mixing vessel; m.
H'	enthalpy of gas-vapor mixture; J/kg.
HETS	height equivalent to a theoretical stage in staged liquid extraction columns; m.
HK	heavy-key component in multicomponent distillation.
ΔH_s	heat of solution; J/mol of solution.
H_{iL}	height of a liquid-phase transfer unit; m.
H_{iG}	height of a gas-phase transfer unit; m.
H_{iOG}	overall height of a gas-phase transfer unit; m.
H_{iOL}	overall height of a liquid-phase transfer unit; m.
h	convective heat-transfer coefficient, W/m ² -K.
h_d	dry-tray head loss; cm of liquid.
h_l	equivalent head of clear liquid on tray; cm of liquid.

h_L	specific liquid holdup; m^3 holdup/ m^3 packed bed.
h_t	total head loss/tray; cm of liquid.
h_w	weir height; m.
h_σ	head loss due to surface tension; cm of liquid.
$h_{2\phi}$	height of two-phase region on a tray; m.
i	number of dimensionless groups needed to describe a situation.
j_D	Chilton-Colburn j -factor for mass transfer; dimensionless.
j_H	Chilton-Colburn j -factor for heat transfer; dimensionless.
\mathbf{j}_i	mass diffusion flux of species i with respect to the mass-average velocity; $kg/m^2\text{-s}$.
\mathbf{J}_i	molar diffusion flux of species i with respect to the molar-average velocity; $mol/m^2\text{-s}$.
J_0	Bessel function of the first kind and order zero; dimensionless.
J_1	Bessel function of the first kind and order one; dimensionless.
K	distribution coefficient; dimensionless.
K	Krogh diffusion coefficient; $cm^3 O_2/cm\text{-s-torr}$.
K	parameter in Langmuir adsorption isotherm; Pa^{-1} .
K_{AB}	molar selectivity parameter in ion exchange; dimensionless.
K_W	wall factor in Billet-Schultes pressure-drop correlations; dimensionless.
k	thermal conductivity, $W/m\text{-K}$.
k_c	convective mass-transfer coefficient for diffusion of A through stagnant B in dilute gas-phase solution with driving force in terms of molar concentrations; m/s .
k_c'	convective mass-transfer coefficient for equimolar counterdiffusion in gas-phase solution with driving force in terms of molar concentrations; m/s .
k_G	convective mass-transfer coefficient for diffusion of A through stagnant B in dilute gas-phase solution with driving force in terms of partial pressure; $mol/m^2\text{-s-Pa}$.
K_G	overall convective mass-transfer coefficient for diffusion of A through stagnant B in dilute solutions with driving force in terms of partial pressures; $mol/m^2\text{-s-Pa}$.
k_G'	convective mass-transfer coefficient for equimolar counterdiffusion in gas-phase solution with driving force in terms of partial pressure; $mol/m^2\text{-s-Pa}$.
k_L	convective mass-transfer coefficient for diffusion of A through stagnant B in dilute liquid-phase solution with driving force in terms of molar concentrations; m/s .
k_L'	convective mass-transfer coefficient for equimolar counterdiffusion in liquid-phase solution with driving force in terms of molar concentrations; m/s .
Kn	Knudsen number, dimensionless.
k_r	reaction rate constant; $mol/m^2\text{-s-mole fraction}$.

K_r	restrictive factor for diffusion of liquids in porous solids; dimensionless.
k_x	convective mass-transfer coefficient for diffusion of A through stagnant B in dilute liquid-phase solution with driving force in terms of mole fractions; mol/m ² -s.
K_x	overall convective mass-transfer coefficient for diffusion of A through stagnant B in dilute solutions with driving force in terms of liquid-phase molar fractions; mol/m ² -s.
k_x'	convective mass-transfer coefficient for equimolar counterdiffusion in liquid-phase solution with driving force in terms of mole fractions; mol/m ² -s.
k_y	convective mass-transfer coefficient for diffusion of A through stagnant B in dilute gas-phase solution with driving force in terms of mole fractions; mol/m ² -s.
K_y	overall convective mass-transfer coefficient for diffusion of A through stagnant B in dilute solutions with driving force in terms of gas-phase molar fractions; mol/m ² -s.
k_y'	convective mass-transfer coefficient for equimolar counterdiffusion in gas-phase solution with driving force in terms of mole fractions; mol/m ² -s.
L	characteristic length, m.
L	molar flow rate of the L -phase; mol/s.
L	length of settling vessel; m.
LK	light-key component in multicomponent distillation.
L_S	molar flow rate of the nondiffusing solvent in the L -phase; mol/s.
L'	mass flow rate of the L -phase; kg/s.
L_S'	mass flow rate of the nondiffusing solvent in the L -phase; kg/s.
L_e	entrainment mass flow rate, kg/s.
L_w	weir length; m.
l	characteristic length, m.
l	tray thickness; m.
l_M	membrane thickness; m.
Le	Lewis number; dimensionless.
M_i	molecular weight of species i .
M_0	oxygen demand; cm ³ O ₂ /cm ³ -min.
MTZ	width of the mass-transfer zone in fixed-bed adsorption; m.
m	amount of mass; kg.
m	slope of the equilibrium distribution curve; dimensionless.
\mathbf{n}	total mass flux with respect to fixed coordinates; kg/m ² -s.
\mathbf{n}_i	mass flux of species i with respect to fixed coordinates; kg/m ² -s.
n	number of variables significant to dimensional analysis of a given problem.
n	rate of mass transfer from the dispersed to the continuous phase in liquid extraction; kg/s.
n	number of species in a mixture.

N	total molar flux with respect to fixed coordinates; mol/m ² -s.
N_i	molar flux of species i with respect to fixed coordinates; mol/m ² -s.
N	number of equilibrium stages in a cascade; dimensionless.
N_E	mass of B/(mass of A + mass of C) in the extract liquids.
N_R	number of stages in rectifying section; dimensionless.
N_R	mass of B/(mass of A + mass of C) in the raffinate liquids.
N_S	number of stages in stripping section; dimensionless.
N_{tL}	number of liquid-phase transfer units; dimensionless.
N_{tG}	number of gas-phase transfer units; dimensionless.
N_{tOD}	overall number of dispersed-phase transfer units; dimensionless.
N_{tOG}	overall number of gas-phase transfer units; dimensionless.
N_{tOL}	overall number of liquid-phase transfer units; dimensionless.
Nu	Nusselt number; dimensionless.
O_t	molar oxygen concentration in the air leaving an aeration tank; percent.
O_{eff}	oxygen transfer efficiency; mass of oxygen absorbed by water/total mass of oxygen supplied.
p'	pitch, distance between centers of perforations in a sieve plate; m.
p_i	partial pressure of species i ; atm, Pa, kPa, bar.
$p_{B,M}$	logarithmic mean partial pressure of component B; atm, Pa, kPa, bar.
P	total pressure; atm, Pa, kPa, bar.
P	permeate flow through a membrane; mol/s.
P	impeller power; kW.
P_c	critical pressure, Pa, kPa, bar.
Pe_D	Peclet number for mass transfer.
Pe_H	Peclet number for heat transfer.
P_i	vapor pressure of species i ; atm, Pa, kPa, bar.
Po	power number defined in equation (7-37); dimensionless.
Pr	Prandtl number; dimensionless.
Q	volumetric flow rate; m ³ /s.
Q	net rate of heating; J/s.
Q	membrane permeance; m/s.
q	membrane permeability; barrer, m ² /s.
q	parameter defined by equation (6-27); dimensionless.
q_m	parameter in Langmuir adsorption isotherm; g/g.
r	rank of the dimensional matrix, DM ; dimensionless.
r_A	solute particle radius; m.
R	radius; m.
R	ideal gas constant; J/mol-K.
R	reflux ratio; moles of reflux/moles of distillate.
R	raffinate mass flow rate; kg/s.
R_A	volumetric rate of formation of A; moles per unit volume per unit time.
R_m	retentate flow in a membrane; mol/s.
Re	Reynolds number; dimensionless.

R_i	volumetric rate of formation of component i ; mol/m ³ -s.
S	surface area, cross-sectional area; m ² .
S	stripping factor, reciprocal of absorption factor (A); dimensionless.
S	mass flow rate of the solvent entering a liquid extraction process; kg/s.
Sc	Schmidt number; dimensionless.
Sh	Sherwood number; dimensionless.
SR	salt rejection; dimensionless.
St_D	Stanton number for mass transfer; dimensionless.
St_H	Stanton number for heat transfer; dimensionless.
t	tray spacing; m.
t	time; s, hr.
t_b	breakthrough time in fixed-bed adsorption; s.
t_{res}	residence time; min.
T	temperature; K.
T_{as}	adiabatic saturation temperature; K.
T_b	normal boiling point temperature; K.
T_c	critical temperature, K.
T_w	wet-bulb temperature; K.
u	fluid velocity past a stationary flat plate, parallel to the surface; m/s.
v	mass-average velocity for multicomponent mixture; m/s.
v_i	velocity of species i ; m/s.
v_t	terminal velocity of a particle; m/s.
V	molar-average velocity for multicomponent mixture; m/s.
V	volume; m ³ .
V	molar flow rate of the V -phase; moles/s.
V_S	molar flow rate of the nondiffusing solvent in the V -phase; moles/s.
V'	mass flow rate of the V -phase; kg/s.
V_S'	mass flow rate of the nondiffusing solvent in the V -phase; kg/s.
V_A	molar volume of a solute as liquid at its normal boiling point; cm ³ /mol.
V_B	boilup ratio; moles of boilup/moles of residue.
V_b	molar volume of a substance as liquid at its normal boiling point; cm ³ /mol.
V_c	critical volume; cm ³ /mol.
w	mass-flow rate; kg/s.
W	work per unit mass; J/kg.
W	molar flow rate of the residue from a distillation column; mol/s.
We	Weber number defined in equation (7-49); dimensionless.
x_i	mole fraction of species i in either liquid or solid phase.
x_i	mass fraction of species i in raffinate (liquid extraction).
$x_{B,M}$	logarithmic mean mole fraction of component B in liquid or solid phase.
x	rectangular coordinate.
x'	mass of C/mass of A in raffinate liquids.
X	mole ratio in phase L ; moles of A/mole of A-free L .

X	flow parameter; dimensionless.
X	parameter in Gilliland's correlation, see equation (6-87); dimensionless.
X	mass of C/(mass of A + mass of C) in the raffinate liquids.
X'	mass ratio in phase L ; kg of A/kg of A-free L .
y	rectangular coordinate.
y'	mass of C/mass of B in extract liquids.
$y_{B,M}$	logarithmic mean mole fraction of component B in gas phase.
y_i	mole fraction of species i in the gas phase.
y_i	mass fraction of species i in extract (liquid extraction).
Y	mole ratio in phase V ; moles of A/mole of A-free V .
Y	pressure-drop parameter defined in equation (4-6); dimensionless.
Y	parameter in Gilliland's correlation, see equation (6-86); dimensionless.
Y	mass of C/(mass of A + mass of C) in the extract liquids.
Y	molal absolute humidity; mol A/mol B.
Y'	absolute humidity; kg A/kg B
Y'	mass ratio in phase V ; kg of A/kg of A-free V .
z	rectangular coordinate.
z_i	average mole fraction of component i in a solution or multiphase mixture.
Z	total height; m.
Z_c	compressibility factor at critical conditions; dimensionless.
Z_R	total height of the rectifying section of a packed fractionator; m.
Z_S	total height of the stripping section of a packed fractionator; m.

GREEK LETTERS

α	thermal diffusivity; m^2/s .
α	relative volatility; dimensionless.
α_m, α_{AB}	membrane separation factor; dimensionless.
β	volume coefficient of thermal expansion; K^{-1} .
Γ	matrix of thermodynamic factors with elements defined by equation (1-32).
Γ	concentration polarization factor; dimensionless.
γ_i	activity coefficient of species i in solution.
δ	length of the diffusion path; m.
δ	velocity boundary-layer thickness; m.
δ_{ij}	Kronecker delta; 1 if $i = k$, 0 otherwise.
Δ_R	difference in flow rate, equation (7-12); kg/s.
ϵ	porosity or void fraction; dimensionless.
ϵ_{AB}	Lennard-Jones parameter; erg.
θ	membrane cut; moles of permeate/mole of feed.
κ	Boltzmann constant; 1.38×10^{-16} erg/K.
κ	constant in equation (4-46), defined in equation (4-47); dimensionless.
λ_i	molar latent heat of vaporization of component i ; J/mol.
λ	similar to the stripping factor, S , in equations (4-56) to (4-61).

λ	mean free path in gases; m.
μ_i	chemical potential of species i ; J/mol.
μ_B	solvent viscosity; cP.
ν	momentum diffusivity, or kinematic viscosity; m^2/s .
ν_i	stoichiometric number of species i .
ξ	reduced inverse viscosity in Lucas method; $(\mu\text{P})^{-1}$.
π	constant; 3.1416
π	Pi groups in dimensional analysis.
π	osmotic pressure; Pa.
ρ	mass density; kg/m^3 .
ρ_i	mass density of species i ; kg/m^3 .
σ_{AB}	Lennard-Jones parameter; \AA .
σ	surface tension, dyn/cm, N/m.
τ	shear stress; N/m^2 .
τ	pore-path tortuosity; dimensionless.
Φ_B	association factor of solvent B; dimensionless.
ϕ	packing fraction in hollow-fiber membrane module; dimensionless.
ϕ	root of equation (6-82); dimensionless.
ϕ_e	effective relative froth density; height of clear liquid/froth height.
ϕ_C	fractional holdup of the continuous liquid phase.
ϕ_D	fractional holdup of the dispersed liquid phase.
ϕ_G	specific gas holdup; m^3 holdup/ m^3 total volume.
ω_i	mass fraction of species i .
Ω_D	diffusion collision integral; dimensionless.
Ω	impeller rate of rotation; rpm.
Ψ	stream function; m^2/s .
Ψ_A	molar flux fraction of component A; dimensionless.
Ψ_0	dry-packing resistance coefficient in Billet-Schultes pressure-drop correlations; dimensionless.