The Foundations of Signal Integrity

Paul G. Huray

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Preface

This book marries the principles of solid-state physics with the mathematics of time-retarded solutions to Maxwell's equations. It includes the quantum mechanical nature of magnetism in thermal equilibrium with materials to explain how electromagnetic waves propagate in solid materials and across boundaries between dielectrics and insulators. The text uses electromagnetic scattering analysis to show how electromagnetic fields induce electric and magnetic multipoles in "good" conductors and how that process leads to delay, attenuation, and dispersion of signals in transmission lines. The text explains the basis for boundary conditions used with the vector forms of Maxwell's equations to describe analytic problems that can be solved by the first and second Born approximation for real-world applications through successive approximations of

- perfect flat boundaries to boundaries with nanometer deviations,
- perfect electric conductors to materials with finite conductivity, and
- inclusions of multiple impurities in otherwise homogeneous media.

Finally, the text gives examples of how system-level printed circuit board (PCB) geometries can use these principles to numerically simulate solutions for very complex systems.

This book is intended to be a foundation for the discipline of electricity and magnetism upon which measurements, simulations, and "rules-of-thumb" are built through the rigorous application of Maxwell's equations. Assumptions are stated when they are employed, and the set of steps known as the Born approximations is used to show the relative magnitude of neglected terms. In that sense, this is intended to be a book that takes carefully applied theory to practice. It is written in the language of an electrical engineer rather than a mathematician or physicist and is intended to support engineering practice.*

PROBLEMS ADDRESSED

As bit rates of computers have increased into the tens of gigahertz, scientists and engineers have recognized that a less-than-rigorous knowledge of electromagnetic

* Textbooks that support design practices are *Advanced Signal Integrity for High-Speed Digital Designs* by Stephen H. Hall and Howard L. Heck (John Wiley & Sons, 2009); *High-Speed Digital System Design* by Stephen H. Hall, Garrett W. Hall, and James A. McCall (John Wiley & Sons, 2000); and *High-Speed Signal Propagation: Advanced Black Magic* by Howard Johnson and Martin Graham (Prentice Hall, 2003).

field propagation can yield incomplete or even contradictory concepts about attenuation, phase, and dispersion of received electric signals that represent information. Many books present concepts of electricity and magnetism via models of transmitted power in terms of low-frequency harmonic potentials and currents that then yield "rules-of-thumb" that are extended to higher frequencies by modifying the definition of resistance, capacitance, or inductance. Simulation codes often neglect the relatively slow propagation of electromagnetic fields in conductors when solving for propagation of those quantities in a dielectric medium. On physically large circuit boards, the propagation speed of electric signals requires dozens or even hundreds of bits of information to be "on their way" from a transmitter to a receiver, so that timing budgets require picoseconds precision. Some solutions are made by using quasistatic (or other) approximations that are forgotten when applied to situations that violate those assumptions; for numerical simulation software, the assumptions are often not even stated. Most engineering models that are chosen to represent "real-world" transmission lines, vias, or packages make simplifying assumptions that cannot be justified based on the complexity of microscopic examination. Power losses on printed circuit boards are so large at high frequencies that signal-to-noise ratio is unacceptable to preserve targeted bit error rates or to recommend new procedures or processes for fabrication needed for higher speed applications. In short, many intuitive concepts that are learned in undergraduate courses for simple transverse electromagnetic (TEM) field propagations simply do not carry over into the real world of conducting boundaries when employing microwave frequencies is tried.

Most existing texts on signal integrity do **not** provide a **foundational basis** of signal integrity principles based on the propagation of electromagnetic fields but base explanations on traditional circuit theory parameter (resistance, inductance, conductance, capacitance—RLGC) models with plausibility arguments that are comforting to the intuition. However, some of these plausible explanations lead to incorrect pictures of behavior of currents, which cause conundrums for the students. These texts do not explain how electron charge and currents physically distribute themselves in space and time for a complex transmission line that includes "good" conductors and "complex dielectrics." The nonrigorous solutions can also lead students to causal contradictions, conduction electrons that travel faster than the speed of light, and nonsense phrases like currents that "rush-over" imperfections or "crowd" at discontinuous surfaces.

FEATURES OF THE BOOK

Causal electric and magnetic field quantities are color coordinated throughout the book. For example, electric charge density, electric field intensity, electric flux density, scalar electric potential, and vector electric potential, versus current density, magnetic field intensity, magnetic flux density, scalar magnetic potential, and vector magnetic potential are consistently identified, along with the symbols that pertain to those quantities in equations and vector lines that correspond in figures. It is revealing to see that time derivatives of those quantities (e.g., dq/dt) change their causal character and that it is equivalent to state that electric charge *causes* electric field intensity (current *causes* magnetic field intensity) or vice versa. Electric and magnetic field intensity is shown *inside* conductors in the quasistatic approximation, and an analysis of how they move with time is shown to yield dynamic properties that cause them to be conservative (close on themselves).

By using colors, Maxwell's equations are seen to be even more beautifully symmetric than in their black-and-white formats.

RECOGNITION

The author owes a debt of gratitude to Dr. Yinchao Chen of the Electrical Engineering Department at the University of South Carolina, Columbia. Dr. Chen has published articles with the author and has had many discussions on the techniques and meaning of the solutions to Maxwell's equations and their applications. Other USC professors who contributed to the physical and chemical understanding of PCB materials were Michael Myrick of the Chemistry Department and Richard Webb of the Physics Department.

Huray, Chen, and three Signal Integrity engineers (Brian Knotts, Hao Li, and Richard Mellitz) from the Intel Corporation (Columbia, SC) created the first graduate Signal Integrity program in 2003, which has since produced more than 80 practicing Signal Integrity engineers, many of whom read and corrected early drafts of this text.

Huray conducts industrial research on a part-time basis with the Intel Corporation in the area of high-speed electromagnetic signals. In this work, he has had the privilege to work closely with Richard Mellitz and Stephen Hall, on applications of electromagnetism for practical use. It was their penetrating questions that prompted many of the explanations in this text. Another Intel employee, Dan Hua, provided a sequence of exchanged articles on the evaluation of scattering and absorption in the language of vector spherical harmonics; it was through these discussions that the sections on absorption by small good conducting spheres arose. Gary Brist taught the author (and many of his graduate students) about the process of manufacturing PCB stack-ups and stimulated many of the questions that are sprinkled throughout the book. Anusha Moonshiram and Chaitanya Sreerema conducted many of the high-frequency vector network analyzer (VNA) measurements in this text. Femi Oluwafemi conducted many of the numerical simulations on phase analysis to identify time-dependent fields inside good conductors and provided many of the final comparisons to the VNA data. Guy Barnes and Paul Hamilton provided the Fabry–Perot measurements of permittivity. Brandon Gore helped work on magnetic losses, and David Aerne assisted the analysis of spherical composition profiles and near-neighbor interference effects. Peng Ye was a sounding board for arguments about the analytical analysis associated with electromagnetic field dynamics. Kevin Slattery introduced the author to near-field scanning electromagnetic probes and helped direct the work of two USC graduate students, Jason Ramage and Christy Madden Jones, whose work on proof of Snell's law at microwave frequencies and absorption by impurities appears in the text. Intel engineers such as Howard Heck, Richard Kunze, Ted Ballou, Steve Krooswyk, Matt Hendrick, David Blakenbeckler, and Johnny Gibson passed through USC during the writing of this text to present lectures to the author's Signal Integrity classes and to build richness into the intellectual atmosphere. Mark Fitzmaurice was always ready to help make the Signal Integrity program at USC a success through his support for measurement equipment, student internships, and common sense.

Many USC undergraduate and graduate students contributed to the testing and writing of this book. Steven Pytel worked with the author on scanning electron microscope SEM and analysis measurements at the Oak Ridge National Laboratory in Oak Ridge, TN, and, while working for Intel, was the sounding board for many of the arguments presented here. After receiving his PhD, he became an employee of the Ansoft Corporation, Pittsburg, PA, where he became an applications engineer for Signal Integrity tools. He is primarily responsible for the material in Chapter 8 on numerical simulations. Ken Young helped with editing, Fisayo Adepetun provided assistance with figures, and David London supported Web pages for testing and transmittal of the chapters. Tom McDonough gave lectures to the Signal Integrity classes on the use of Synopsys Corporation, Boston, MA HSPICE software and helped in the analysis of ceramic capacitor fields.

John Fatcheric of the Oak Mitsui Corporation, Camden, SC, assisted the presentation on copper surface production. Bob Helsby, Charles Banyon, and Zol Cendes of the Ansoft Corporation supported the use of forefront numerical solutions to Maxwell's equations. James Rautio of Sonnet Software, Syracuse, NY, assisted on the history of Maxwell and the use of his portrait. Mike Resso of Agilent Corporation, Santa Rosa, CA, supported a joint Intel–Agilent VNA donation. Lee Riedinger, Harry M. Meyer III, Larry Walker, and Marc Garland of the Oak Ridge National Laboratory assisted in making qualitative and quantitative measurements of PCB components by SEM and Auger analysis. José E. Rayas Sánchez of ITESO, Guadalajara, Mexico, James Gover of Kettering University, Flint, MI, and John David Jackson of UC-Berkeley and LBL, Berkeley, CA, provided discussions on Maxwell's interpretations and Signal Integrity of high-speed circuits.

This book is dedicated to the author's lifelong partner:

Susan Lyons Huray

Intent of the Book

The Foundations of Signal Integrity is intended to be a text for a one-semester course in Signal Integrity, under the assumption that the students have a solid foundation in the development and solution techniques of Maxwell's equations. A preliminary text by the author[1] presents that information at a relatively complete level, but it is recognized that students may have had other textbooks for that material. This book presents equations, words, and figures in a consistent, color-coded format so that students can see the relationship between variables of a common type or color. Generally, other textbooks will have used either the symmetric or the asymmetric form of Maxwell's equations as defined below but may have used other symbols for the variables, and they will not generally be color-coded. This section thus presents the form of Maxwell's equations used in *The Foundations of Signal Integrity* with enough introduction that the text may be used by itself.

The Foundations of Signal Integrity concentrates on the solutions to Maxwell's equations in a variety of media and with a variety of boundary conditions. Here, techniques that show how to obtain analytic solutions to Maxwell's equations for ideal materials and boundary conditions are presented. These solutions are then used as a benchmark for the student to solve "real world" problems via computational techniques; first confirming that a computational technique gives the same answer as the analytic solution for an ideal problem.

This information is presented to 21st-century students* in the hope that they will consider mathematical and physical concepts as *integral*. The student is challenged not to accept uncertainty but to be honest within him- or herself in appreciating and understanding the derivations of the electromagnetic giants. After the mathematical solution has been obtained, we hope the student will ask, "What are these equations telling me?" and "How could I use this in some other application?" Perhaps the student will delve even deeper to ask, "What are the physical phenomenon that cause fields to exist, to move, to reflect or to transmit through materials?" With such an armada of knowledge, the student can take these electromagnetic concepts to further applications and to further "stand on the shoulders of giants"†

* One reader from the Physics Web poll that rated Maxwell's equations as the most beautiful equations ever derived recalled how he learned Maxwell's equations during his second year as an undergraduate student. "I still vividly remember the day I was introduced to Maxwell's equations in vector notation," he wrote. That these four equations should describe so much was extraordinary ... For the first time, I understood what people meant when they talked about elegance and beauty in mathematics or physics. It was spine-tingling and a turning point in my undergraduate career."

† The quote "If I have seen farther than others, it is because I have stood on the shoulders of giants" was attributed to Sir Isaac Newton because it appeared in a letter he wrote to Robert Hooke in 1675, but it was also used by an 11th-century monk named John of Salisbury, and there is evidence he may have read it in an older text while studying with Abelard in France.

(perhaps for monetary gain). Sometimes, open-ended questions are asked so that the student questions the giants or questions his or her own set of learned models.

In *Maxwell's Equations*, the justification for using the symmetric form of the equations given in the following table was developed.

Symmetric Form of Maxwell's Equations

Differential form	Integral form	Name
$\vec{\nabla} \times \vec{E} = -\vec{J} - \partial\vec{B}/\partial t$	$\oint_C \vec{E} \cdot d\vec{l} = -I - \iint_S (\partial\vec{B}/\partial t) \cdot d\vec{s}$	Faraday's law
$\vec{\nabla} \times \vec{H} = \vec{J} + \partial\vec{D}/\partial t$	$\oint_C \vec{H} \cdot d\vec{l} = I + \iint_S (\partial\vec{D}/\partial t) \cdot d\vec{s}$	Ampere's law
$\vec{\nabla} \cdot \vec{D} = \rho_V$	$\oiint_S \vec{D} \cdot d\vec{s} = Q$	Gauss's law for electric charge
$\vec{\nabla} \cdot \vec{B} = \rho_V$	$\oiint_S \vec{B} \cdot d\vec{s} = Q$	Gauss's law for magnetic charge

The symmetric form of Maxwell's equations represents the vector field quantities:

$\vec{E}$ = Electric field intensity (Volts/meter).
$\vec{H}$ = Magnetic field intensity (Ampere/meter)
$\vec{D}$ = Electric flux density (Coulombs/meter^2)
$\vec{B}$ = Magnetic flux density (Weber/meter^2 or Tesla)
ρ_V = Electric charge density (Coulomb/meter^3)
ρ_V = Magnetic charge density (Weber/meter^3)
$\vec{J}$ = Magnetic current density (Volts/meter^2)
$\vec{J}$ = Electric current density (Ampere/meter^2)

with the units of the new field quantities in SI units shown in parenthesis.

The equation of continuity was developed for both electric and magnetic charge density by using conservation of charge to write the symmetric forms[2]:

$$\vec{\nabla} \cdot \vec{J} = -\partial \rho_V / \partial t$$

$$\vec{\nabla} \cdot \vec{J} = -\partial \rho_V / \partial t$$

Based on the symmetric equations, we can see that, in a *magnetic charge-free* region of space, $\vec{B}$ is solenoidal ($\vec{\nabla} \cdot \vec{B} = 0$), and, because the divergence of the curl of *any* vector field is identically zero, we can thus assume that $\vec{B}$ may be written in terms of another vector field, $\vec{A}$, called the *magnetic vector potential*:

$$\vec{B} = \vec{\nabla} \times \vec{A}.$$

In a *magnetic current-free* region of space, the symmetric equations are the same as the asymmetric equations most physicists use as Maxwell's equations.

In an *electric charge-free* region of space, $\vec{D}$ is solenoidal ($\vec{\nabla} \cdot \vec{D} = 0$), and we can assume that $\vec{D}$ may be written in terms of another vector field, $\vec{A}$, called the *electric vector potential:*

$$\vec{D} = \vec{\nabla} \times \vec{A}$$

For charge and current density-free space ($\rho_V = 0$, $\rho_V = 0$, $\vec{J} = 0$ and $\vec{J} = 0$), a **unique** definition of the vector fields, $\vec{A}$ and $\vec{A}$, may be specified through additional restrictions ($\vec{\nabla} \times \vec{E} = -\partial\vec{B}/\partial t$) and ($\vec{\nabla} \times \vec{H} = \partial\vec{D}/\partial t$), so we can write

$$\vec{\nabla} \times \vec{E} = -\partial\left(\vec{\nabla} \times \vec{A}\right)/\partial t \quad \text{or} \quad \vec{\nabla} \times \left(\vec{E} + \partial\vec{A}/\partial t\right) = 0$$

$$\vec{\nabla} \times \vec{H} = -\partial\left(\vec{\nabla} \times \vec{A}\right)/\partial t \quad \text{or} \quad \vec{\nabla} \times \left(\vec{H} + \partial\vec{A}/\partial t\right) = 0$$

One can also show that $\vec{\nabla} \times (-\vec{\nabla}V) = 0$ for *any* scalar field.[3] Thus, because the curl of the vector field shown in parentheses above is zero, then that field can be written as the negative gradient of another scalar field that is successively called the *electric scalar potential*, V, and the *magnetic scalar potential*, V, with

$$\vec{E} + \partial\vec{A}/\partial t = -\vec{\nabla}V \quad \text{or} \quad \vec{E} = -\vec{\nabla}V - \partial\vec{A}/\partial t$$

$$\vec{H} + \partial\vec{A}/\partial t = -\vec{\nabla}V \quad \text{or} \quad \vec{H} = -\vec{\nabla}V - \partial\vec{A}/\partial t$$

We can see from the first of these equations that the electric field intensity, $\vec{E}$, can be written in terms of the electric scalar potential, V, and the time derivative of the magnetic vector potential, $\vec{A}$. As long as these scalar and vector potentials are unique, the electric field intensity produced by them will also be unique. *Note:* In the special case of *static* (time independent) fields and potentials, $\partial\vec{A}/\partial t = 0$, and $\partial\vec{A}/\partial t = 0$ the electric and magnetic field intensities reduce to $\vec{E} = -\vec{\nabla}V$ and $\vec{H} = -\vec{\nabla}V$ as Maxwell originally proposed.

For homogeneous media in time-varying fields ($\vec{B} = \mu\vec{H}$ and $\vec{D} = \varepsilon\vec{E}$), the symmetric forms yield $\vec{\nabla} \times \vec{B} = \mu\vec{J} + \mu\varepsilon\, \partial\vec{E}/\partial t$ or $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu\vec{J} + \mu\varepsilon\, \partial\vec{E}/\partial t$ or $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \mu\vec{J} + \mu\varepsilon\, \partial(-\vec{\nabla}V - \partial\vec{A}/\partial t)/\partial t$, and using identity $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2\vec{A}$

$$\vec{\nabla}\left(\vec{\nabla} \cdot \vec{A}\right) - \vec{\nabla}^2\vec{A} = \mu\vec{J} - \vec{\nabla}\left(\mu\varepsilon\,\partial V/\partial t\right) - \mu\varepsilon\,\partial^2\vec{A}/\partial t^2 \quad \text{or}$$

$$\vec{\nabla}^2\vec{A} - \mu\varepsilon\partial^2\vec{A}/\partial t^2 = -\mu\vec{J} + \vec{\nabla}\left(\vec{\nabla} \cdot \vec{A} + \mu\varepsilon\,\partial V/\partial t\right).$$

Likewise, the symmetric form $\vec{\nabla} \times \vec{E} = -\vec{J} - \partial\vec{B}/\partial t$ or $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\varepsilon\vec{J} - \varepsilon\partial\vec{B}/\partial t$ or $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \varepsilon\vec{J} + \mu\varepsilon\, \partial(-\vec{\nabla}V - \partial\vec{A}/\partial t)/\partial t$ and using identity $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2\vec{A}$

$$\vec{\nabla}\left(\vec{\nabla} \cdot \vec{A}\right) - \vec{\nabla}^2\vec{A} = \varepsilon\vec{J} - \vec{\nabla}\left(\mu\varepsilon\,\partial V/\partial t\right) - \mu\varepsilon\,\partial^2\vec{A}/\partial t^2 \quad \text{or}$$

$$\vec{\nabla}^2\vec{A} - \mu\varepsilon\partial^2\vec{A}/\partial t^2 = -\varepsilon\vec{J} + \vec{\nabla}\left(\vec{\nabla} \cdot \vec{A} + \mu\varepsilon\partial V/\partial t\right)$$

Now, the definition of a **unique** vector field $\vec{A}$ or $\vec{A}$ requires an additional restriction or gauge. One way to provide this restriction (gauge) is to specify their divergence. Lorenz used the now-called Lorenz gauge to write[4]

$$\vec{\nabla}\cdot\vec{A}+\mu\varepsilon\partial V/\partial t=0$$
$$\vec{\nabla}\cdot\vec{A}+\mu\varepsilon\partial V/\partial t=0$$

From a mathematical solutions perspective, that choice is convenient because it requires $\vec{A}$ and $\vec{A}$ to satisfy second-order, linear, inhomogeneous partial differential equations (PDEs):

$$\vec{\nabla}^2\vec{A}-\mu\varepsilon\partial^2\vec{A}/\partial t^2=-\mu\vec{J}$$
$$\vec{\nabla}^2\vec{A}-\mu\varepsilon\partial^2\vec{A}/\partial t^2=-\varepsilon\vec{J},$$

which are called the inhomogeneous wave equation for the *magnetic vector potential* and the inhomogeneous wave equation for the *electric vector potential*. To solve these equations for $\vec{A}$ or $\vec{A}$, the current density, $\vec{J}$ or $\vec{J}$, is needed.

A corresponding wave equation for the electric scalar potential can be found by using Gauss's law $\vec{\nabla}\cdot\vec{D}=\rho_V$ and $\vec{\nabla}\cdot\vec{E}=\rho_V/\varepsilon \Rightarrow \vec{\nabla}\cdot(\vec{\nabla}V+\partial\vec{A}/\partial t)=-\rho_V/\varepsilon$, which leads to $\vec{\nabla}^2V+\partial(\vec{\nabla}\cdot\vec{A})/\partial t=-\rho_V/\varepsilon$, and, by using the Lorenz gauge ($\vec{\nabla}\cdot\vec{A}+\mu\varepsilon\partial V/\partial t = 0$), we see that the *electric scalar potential*, V, also satisfies the inhomogeneous wave equation

$$\vec{\nabla}^2V-\mu\varepsilon\partial^2V/\partial t^2=-\rho_V/\varepsilon$$

This equation needs only ρ_V to solve for the electric scalar potential, V.

Likewise, a corresponding wave equation for the magnetic scalar potential can be found by using Gauss's law $\vec{\nabla}\cdot\vec{B}=\rho_V$ and $\vec{\nabla}\cdot\vec{H}=\mu\rho_V \Rightarrow \vec{\nabla}\cdot(\vec{\nabla}V+\partial\vec{A}/\partial t)=-\mu\rho_V$ or $\vec{\nabla}^2V+\partial(\vec{\nabla}\cdot\vec{A})/\partial t=-\mu\rho_V$, and, by using the Lorenz gauge ($\vec{\nabla}\cdot\vec{A}+\mu\varepsilon\,\partial V/\partial t=0$), we see that the *magnetic scalar potential*, V, also satisfies the inhomogeneous wave equation

$$\vec{\nabla}^2V-\mu\varepsilon\partial^2V/\partial t^2=-\mu\rho_V$$

This equation needs only ρ_V to solve for the magnetic scalar potential, V.

Symmetric Form Conclusion

With a prior knowledge of ρ_V , ρ_V, $\vec{J}$, and $\vec{J}$, we can separate the x, y, and z components of the wave equations and solve for V and V and each component of $\vec{A}$ and $\vec{A}$ independently of the others. All four of these equations are of in the form of the same inhomogeneous wave equation and are independent of one another. Thus, given the electric charge density, the magnetic charge density, the vector electric current density, and the vector magnetic current density, we can solve the inhomogeneous wave equation (subject to boundary conditions specified by a particular application) to find the potentials V, V, $\vec{A}$, and $\vec{A}$ from which we can then find all of the components of the electric field intensity and magnetic field intensity. The inhomogeneous wave equations for V, V, $\vec{A}$, and $\vec{A}$ form a set of four equations equivalent in all respects to the symmetric Maxwell's equations (subject to the restriction of the Lorenz gauge). However, unlike Maxwell's equations, these four inhomogeneous PDEs are independent of one another so they are often easier to solve.

NOTE Using the electric vector potential and the magnetic vector potential results in electric and magnetic fields that originate from $\vec{B} = \vec{\nabla} \times \vec{A}$, $\vec{D} = \vec{\nabla} \times \vec{A}$, $\vec{E} = -\vec{\nabla}V - \partial\vec{A}/\partial t$, and $\vec{H} = -\vec{\nabla}V - \partial\vec{A}/\partial t$. The resulting electric and magnetic field intensity is the vector sum as a result of both potentials: $\vec{E}_{total} = -\vec{\nabla}V - \partial\vec{A}/\partial t + \vec{\nabla} \times \vec{A}/\varepsilon$ and $\vec{H}_{total} = -\vec{\nabla}V - \partial\vec{A}/\partial t + \vec{\nabla} \times \vec{A}/\mu$.

Engineers sometimes use electric vector potential and magnetic vector potential to develop solutions because they are easier to find via the inhomogeneous wave equations with boundary conditions. The solutions can be chosen to have boundary conditions so that one part of the solution yields a transverse electromagnetic (TEM), transverse electric (TE), or transverse magnetic (TM) solution in a particular coordinate system. However, this approximation is poor when considering fields in the microscopic near-field regime so that the two-vector potential technique will not suffice for the analysis of crystal field effects or fields internal to atoms or molecules.

The physics community usually assumes that there is no such thing as magnetic charge density or magnetic current density so that $\rho_V = 0$ and $\vec{J} = 0$. In this formalism, Maxwell's equations are equivalent to their asymmetric form shown below. Because we will often evaluate near-fields, the asymmetric form of Maxwell's equations will be used in this book to find solutions to applied problems in Signal Integrity.

Asymmetric Form of Maxwell's Equations[‡,5]

Differential form	Integral form
$\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$	$\oint_C \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$
$\vec{\nabla} \times \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$	$\oint_C \vec{H} \cdot d\vec{l} = I + \int_S \dfrac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$
$\vec{\nabla} \cdot \vec{D} = \rho_V$	$\oint_C \vec{D} \cdot d\vec{s} = Q$
$\vec{\nabla} \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{s} = 0$

For the *special case* of *source-free* problems (i.e., $\rho_V = 0$ and $\vec{J} = 0$), we can see that both the symmetric and asymmetric forms of Maxwell's equations reduce to:

‡ Oliver Heaviside reformulated Maxwell's equations (originally in quaternion format) to this asymmetric vector form.

Maxwell's Equations for *Source-Free* Problems

Differential form	Integral form	Name of law
$\vec{\nabla} \times \vec{E} = -\partial\vec{B}/\partial t$	$\oint_C \vec{E}\cdot d\vec{l} = -\iint_S (\partial\vec{B}/\partial t)\cdot d\vec{s}$	Faraday's law
$\vec{\nabla} \times \vec{H} = \partial\vec{D}/\partial t$	$\oint_C \vec{H}\cdot d\vec{l} = \iint_S (\partial\vec{D}/\partial t)\cdot d\vec{s}$	Ampere's law
$\vec{\nabla} \cdot \vec{D} = 0$	$\oint_S \vec{D}\cdot d\vec{s} = 0$	Gauss's law
$\vec{\nabla} \cdot \vec{B} = 0$	$\oint_S \vec{B}\cdot d\vec{s} = 0$	No isolated magnetic charge

So if we take the curl of Faraday's law, $\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\vec{\nabla} \times \partial\vec{B}/\partial t$ or $\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2\vec{E} = -\mu\partial(\vec{\nabla} \times \vec{H})/\partial t$ and substitute Gauss's law ($\vec{\nabla} \cdot \vec{E} = 0$) and Ampere's Law, we see

$$\vec{\nabla}^2\vec{E} - \mu\varepsilon\partial^2\vec{E}/\partial t^2 = 0$$

Likewise, taking the curl of Ampere's law,

$$\vec{\nabla} \times \vec{\nabla} \times \vec{H} = \vec{\nabla} \times \partial\vec{D}/\partial t \text{ or } \vec{\nabla}\left(\vec{\nabla}\cdot\vec{H}\right) - \vec{\nabla}^2\vec{H} = \varepsilon\partial\left(\vec{\nabla}\times\vec{D}\right)/\partial t$$

and using ($\vec{\nabla} \cdot \vec{H} = 0$) with Faraday's law, we see

$$\vec{\nabla}^2\vec{H} - \mu\varepsilon\partial^2\vec{H}/\partial t^2 = 0$$

Asymmetric Form Conclusion

In *source-free space*, V, all of the components of $\vec{A}$, all of the components of $\vec{V}$, and all of the components of $\vec{H}$ satisfy the homogeneous wave equation, and we will label $\mu\varepsilon = 1/u_p^2$ and $\mu_0\varepsilon_0 = 1/c^2$.

TIME-RETARDED SOLUTIONS TO MAXWELL'S EQUATIONS

The solution of the inhomogeneous wave equation is a linear combination of the general solution to the homogeneous equation (with coefficients determined by boundary conditions) plus a particular solution of the inhomogeneous wave equation. For the equations above,

$$\vec{\nabla}^2\psi - \mu\varepsilon\partial^2\psi/\partial t^2 = f(\vec{x}, t)$$

where $\quad f(\vec{x}, t) = -\rho(\vec{x}, t)/\varepsilon \quad$ when $\psi(\vec{x}, t) = V(\vec{x}, t)$ and

$$f(\vec{x}, t) = -\mu J_i(\vec{x}, t) \quad \text{when } \psi(\vec{x}, t) = A_i(\vec{x}, t)$$

for each of the i components of the *magnetic vector potential* in Cartesian coordinates.

Any technique that provides *a* solution of the inhomogeneous part provides *the* solution because the particular solution is unique. Some authors (e.g., Matthews and Walker) use an informed guess technique, and others (e.g., Jackson) use a formal Green's function technique to obtain an answer. Using the latter Green's function technique for time-varying fields, we can find the solution for an inhomogeneous PDE by first taking its Fourier transform with respect to the variable t.

In 1824, George Green claimed that, if we solve the equation $(\vec{\nabla}^2 - \mu\varepsilon\, \partial^2/\partial t^2)\, G(\vec{x}, t; \vec{x}', t') = \delta(\vec{x} - \vec{x}')\delta(t - t')$, then (in infinite space with no boundary surfaces) the solution will be

$$\psi(\vec{x},t) = \int\iiint G(\vec{x},t;\vec{x}',t')\, f(\vec{x}',t')\, d^3x'dt'$$

To solve the differential equation with delta functions on the right-hand side, we can insert the four-dimensional Fourier transform of the Green's function, $g(\vec{k}, \omega)$, on the left-hand side of the equation and the four-dimensional delta function representation on the right-hand side of the equation as follows:

$$G(\vec{x},t;\vec{x}',t') = \iiint d^3k \int d\omega\, g\left(\vec{k},\omega\right) e^{j\vec{k}\cdot(\vec{x}-\vec{x}')} e^{-j\omega(t-t')}$$

$$\delta(\vec{x}-\vec{x}')\,\delta(t-t') = 1\big/(2\pi)^4 \iiint d^3k \int d\omega e^{j\vec{k}\cdot(\vec{x}-\vec{x}')} e^{-j\omega(t-t')}$$

The result is a simple algebraic equation:

$$g\left(\vec{k},\omega\right) = \left[1\big/(2\pi)^4\right]\left(k^2 - \mu\varepsilon\omega^2\right)^{-1} = \left[1\big/(2\pi)^4\right]\left(k^2 - \omega^2/c^2\right)^{-1}$$

and the answer is

$$G(\vec{x},t;\vec{x}',t') = (-1/4\pi|\vec{x}-\vec{x}'|)\,\delta((t-t') - |\vec{x}-\vec{x}'|/c)$$

This Green's function is called the *Retarded Green's function* because it exhibits causal behavior associated with the propagation of a wave source to a response location; that is, an effect observed at a point $\vec{x}$ as a result of a source at a point $\vec{x}'$ and time t' will not occur until the wave has had time to propagate the distance $|\vec{x} - \vec{x}'|$, traveling at speed $c = 1\big/\sqrt{\mu\varepsilon}$.

Finally, we can use the Green's function to find the solution to the inhomogeneous wave equation in the absence of boundary conditions as

$$\psi(\vec{x},t) = -\int\iiint \frac{\delta((t-t') - |\vec{x}-\vec{x}'|/c)}{4\pi|\vec{x}-\vec{x}'|} f(\vec{x}',t')\, d^3x'dt'$$

The integration over dt' can be performed to yield the "retarded potential solution"

$$\psi(\vec{x},t) = -\iiint \frac{[f(\vec{x}',t')]_{retarded}}{4\pi|\vec{x}-\vec{x}'|} d^3x'$$

The electric potential due to an electric charge distribution, ρ_V, over a volume V' is then

$$V(R,t) = (1/4\pi\varepsilon)\iiint_{V'} \frac{\rho_V(t - R/c)}{R} d^3x'$$

called the retarded electric scalar potential, which indicates that the scalar potential at (R,t) depends on the value of electric charge at an earlier time $(t - R/c)$.

Similarly, we can obtain the retarded magnetic vector potential

$$\vec{A}(R,t) = (\mu/4\pi)\iiint_{V'} \frac{\vec{J}(t - R/c)}{R} d^3x'$$

The time-retarded electric field intensity and magnetic field intensity are then found from

$$\vec{E} = -\vec{\nabla}V - \partial\vec{A}/\partial t$$

and

$$\vec{H} = -\vec{\nabla}V - \partial\vec{A}/\partial t$$

Time-retarded information is often neglected in applications problems involving microscopic distances of μm because time delay at the speed of light in a vacuum, $c = \sqrt{1/\mu_0\varepsilon_0}$, is considered to be negligible over those distances. We have shown[6] that time-retarded potentials at microscopic distances in a dielectric medium 2 with $c_2 = \sqrt{1/\mu_2\varepsilon_2}$ are also negligible for ordinary values of permittivity and permeability. However, when electromagnetic waves propagate in a conductor with conductivity, σ, their phase velocity decreases to $u_p = c/\sqrt{\sigma/2\omega\varepsilon_0}$, and the time delay, even over a one-micrometer distance, can be substantial for *good* conductors at some frequencies.

We will see that time-retarded effects influence signals propagating in mixed media that include conductors. Those signals will be measurably delayed, attenuated, and dispersed as determined by the solutions to Maxwell's equations in propagating media with conducting boundaries, and this will affect our ability to produce information signals with integrity (signals that transmit information between two points reliably). In these applications, we will see that Maxwell's equations form the foundations of *Signal Integrity*.

ENDNOTES

1. Paul G. Huray, *Maxwell's Equations* (Hoboken, NJ: John Wiley & Sons, 2009).
2. Ibid., 7.120 and 7.121.
3. Ibid., Chapter 3.
4. L. V. Lorenz, "Eichtransformationen, und die Invarianz der Felder unter solchen Transformationen nennt man Eichinvarianz," *Phil. Mag. Series* 4, no. 34 (1867): 287–301.
5. James Clerk Maxwell, "A Dynamical Theory of the Electromagnetic Field," *Philosophical Transactions of the Royal Society of London* 155 (1865): 459–512.
6. Huray, *Maxwell's Equations*, Chapter 7.

Chapter 1

Plane Electromagnetic Waves

LEARNING OBJECTIVES

- Develop and understand the spatial and temporal relationships between electric and magnetic fields for propagating waves
- Relate the spatial and temporal relationships between electric and magnetic fields for polarized waves
- Use dielectric, magnetic, and conduction properties of a medium to modify plane wave field properties
- Use the relative velocity between a source and receiver to find the relativistically accurate frequency shift (Doppler Shift) of harmonic E&M waves
- Recognize the difference between group and phase velocity and relate them to the transmission of power and transfer of momentum
- Describe the properties of plane waves that are incident on a boundary between two media with differing permittivity, permeability, and conductivity
- Show how E&M pulses attenuate and disperse in common transmission materials such as copper, glass, and liquids

INTRODUCTION

In the development of the solutions to Maxwell's equations (see Intent of the Book), we have used the scalar electric potential, $V(x, y, z, t)$, the magnetic vector potential, $\vec{A}(x, y, z, t)$, and the Lorenz gauge to uncouple the differential equations and to write an equivalent pair of inhomogeneous partial differential equations (PDEs) for V and $\vec{A}$:

$$\vec{\nabla}^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho_V}{\varepsilon} \tag{1.1a}$$

$$\vec{\nabla}^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J} \tag{1.1b}$$

The Foundations of Signal Integrity, by Paul G. Huray

We have found that these PDEs can be solved independently to find a particular solution in terms of the time-harmonic source electric charge density, $\rho(x, y, z, t) = \rho_s(\vec{x})e^{j\omega t}$, and the source current density, $\vec{J}(x, y, z, t) = \vec{J}_s(\vec{x})e^{j\omega t}$, as

$$V(\vec{x}, \vec{x}', t) = \frac{1}{4\pi\varepsilon}\iiint_{V'} \frac{\rho_S(\vec{x}')e^{-jk|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} d^3x' e^{j\omega t} \tag{1.2a}$$

$$\vec{A}(\vec{x}, \vec{x}', t) = \frac{\mu}{4\pi}\iiint_{V'} \frac{\vec{J}_S(\vec{x}')e^{-jk|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} d^3x' e^{j\omega t} \tag{1.2b}$$

The most general form of the solution is then a linear combination of the general solutions to the homogeneous PDEs (Equation 1.1 in which $\rho = 0$ and $\vec{J} = 0$) and Equation 1.2. Knowing the relationship between electric field $\vec{E}(\vec{x}, t) = \vec{E}_S(\vec{x})e^{j\omega t}$ and magnetic field, $\vec{H}(\vec{x}, t) = \vec{H}_S(\vec{x})e^{j\omega t}$ and the scalar electric and magnetic vector potentials, we then develop an understanding of the behavior of those fields in a homogeneous material medium with electric permittivity, ε, electric conductivity, σ, and magnetic permeability, μ (where $\vec{B} = \mu\vec{H}$ and $\vec{D} = \varepsilon\vec{E}$):

$$\vec{H}_S = \frac{1}{\mu}\vec{\nabla}\times\vec{A}_S \tag{1.3a}$$

$$\vec{E}_S = -\vec{\nabla}V_S - j\omega\vec{A}_S \tag{1.3b}$$

These solutions satisfy the time-harmonic form of Maxwell's equations

$$\vec{\nabla}\times\vec{E}_S = -j\omega\mu\vec{H}_S \tag{1.4a}$$

$$\vec{\nabla}\times\vec{H}_S = \vec{J}_S + j\omega\varepsilon\vec{E}_S \tag{1.4b}$$

$$\vec{\nabla}\cdot\vec{E}_S = \frac{\rho_S}{\varepsilon} \tag{1.4c}$$

$$\vec{\nabla}\cdot\vec{H}_S = 0 \tag{1.4d}$$

so we are free to use these relationships where they are convenient. For example, if we use Equation 1.3a to find $\vec{H}_S$ in source-free space, we may use Equation 1.4b (in the absence of current density, $\vec{J}_S$) to find $\vec{E}_S$ without having to find V_s.

1.1 PROPAGATING PLANE WAVES

We begin by considering the propagation of a magnetic vector potential in a source-free region of space:

$$\vec{A}(\vec{x}, t) = \vec{A}_S(\vec{x})e^{j\omega t} = A_z^+(x, y)e^{-j(k_z z-\omega t)}\hat{a}_z + A_z^-(x, y)e^{j(k_z z+\omega t)}\hat{a}_z, \tag{1.5}$$

which is a linear combination of the two independent solutions to the homogeneous PDE 1.1b. Here, we have expressed the plane wave in terms of its motion along the

z-axis because we are at liberty to orient the Cartesian coordinates in a direction of our choice. By incrementing the time t in this expression from t' to $t' + dt$, we can follow a point of constant phase, $(k_z z - \omega t)$ = constant, to see that the first term represents the propagation of a wave in the z-direction (along the positive z-axis), with speed $u_p = dz/dt = \omega/k_z = 1/\sqrt{\mu\varepsilon}$ (also called the phase velocity). The second term in Equation 1.5 represents the propagation of a wave along the negative z-axis with the same phase velocity. To simplify our understanding of the wave propagation and the relative position of the resulting electric and magnetic fields, we will assume that the boundary conditions require the coefficient of the second term to be zero; that is, we will consider only propagation in the positive z-direction. Such a field might, for example, be created by current sources in a region of space in which the electric current density is forced by boundary conditions to have a component only in the z-direction.

Relative Directions and Magnitudes of $\vec{E}$ and $\vec{H}$

For the special case with $A_z^-(x, y) = 0$, we can use Equation 1.3a to see that

$$\vec{H}_S^+ = \frac{1}{\mu}\vec{\nabla}\times\vec{A}_S^+ = \frac{1}{\mu}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & 0 & A_z^+(x, y)e^{-jk_z z} \end{vmatrix}$$

$$= \frac{1}{\mu}\frac{\partial A_z^+}{\partial y}e^{-jk_z z}\hat{a}_x - \frac{1}{\mu}\frac{\partial A_z^+}{\partial x}e^{-jk_z z}\hat{a}_y \tag{1.6a}$$

We can also use Equation 1.4b to see that

$$\vec{E}_S^+ = \frac{1}{j\omega\varepsilon}\vec{\nabla}\times\vec{H}_S^+ = \frac{1}{j\omega\varepsilon\mu}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \dfrac{\partial A_z^+}{\partial y}e^{-jk_z z} & -\dfrac{\partial A_z^+}{\partial x}e^{-jk_z z} & 0 \end{vmatrix}$$

$$= \frac{1}{j\omega\varepsilon\mu}\frac{\partial^2\left(A_z^+ e^{-jk_z z}\right)}{\partial x\partial z}\hat{a}_x + \frac{1}{j\omega\varepsilon\mu}\frac{\partial^2\left(A_z^+ e^{-jk_z z}\right)}{\partial y\partial z}\hat{a}_y$$

$$= \frac{-k_z}{\omega\varepsilon\mu}\frac{\partial A_z^+}{\partial x}e^{-jk_z z}\hat{a}_x + \frac{-k_z}{\omega\varepsilon\mu}\frac{\partial A_z^+}{\partial y}e^{-jk_z z}\hat{a}_y \tag{1.6b}$$

We may now see that

$$\vec{H}_S^+ \cdot \vec{A}_S^+ = 0 \tag{1.7a}$$

$$\vec{E}_S^+ \cdot \vec{A}_S^+ = 0 \tag{1.7b}$$

$$\vec{H}_S^+ \cdot \vec{E}_S^+ = 0 \tag{1.7c}$$

Conclusion

In this special case, the propagating electric field intensity waves, magnetic field intensity waves, and magnetic vector potential waves are all orthogonal to one another. We call such propagating waves transverse electric (TE^z) and transverse magnetic (TM^z) because they are moving in the z-direction, in phase with the magnetic vector potential. When both TE and TM waves occur in the same propagation (as they do here), the waves are transverse electromagnetic and labeled TEM^z waves.

Relative Magnitudes

We can also use the relationship $k_z = \omega\sqrt{\mu\varepsilon}$ to compare the components of the electric and magnetic field intensity for TEM^z waves as

$$\frac{E^+_{S,x}}{H^+_{S,y}} = \sqrt{\frac{\mu}{\varepsilon}} = Z^+_W = \eta \tag{1.8a}$$

$$-\frac{E^+_{S,y}}{H^+_{S,x}} = \sqrt{\frac{\mu}{\varepsilon}} = Z^+_W = \eta \tag{1.8b}$$

The quantity η is called the ***intrinsic impedance*** of the medium because it is a function only of the permeability and permittivity of the medium. Some texts call this ratio, Z_W, which they call the ***wave impedance***, to remind us that the ratio of an electric field intensity and magnetic field intensity has units of ohms. Thus, this quantity is a measure of the impedance of the medium; the ratio is labeled Z_0 in the case of waves propagating in a vacuum. In air or a vacuum, $\varepsilon = \varepsilon_0 \approx (1/36\pi) \times 10^{-9}$ F/m or (s/Ωm) and $\mu = \mu_0 = 4\pi \times 10^{-7}$H/m or (Ωs/m) so $\eta = Z_0 \approx 120\pi\Omega = 377\Omega$. This is called the ***intrinsic impedance of free space***.

Physical Meaning of the Propagating Wave Equations

Equations 1.6 give us the relative vector directions, phase, and magnitude of $\vec{E}$ and $\vec{H}$ relative to the magnetic vector potential, $\vec{A}$. Without some knowledge of how $\vec{A}$ varies with x and y, we cannot take the partial derivatives. However, the x-direction is just as arbitrary as the z-direction, which we choose to be in the direction of propagation of $\vec{A}$. We can therefore choose the x-direction to be in the direction of the electric field intensity vector, in which case, we write

$$\vec{E}^+(\vec{x}, t) = E^+_0 e^{-j(k_z z - \omega t)}\hat{a}_x \tag{1.9a}$$

$$\vec{H}^+(\vec{x}, t) = \left(E^+_0/\eta\right) e^{-j(k_z z - \omega t)}\hat{a}_y \tag{1.9b}$$

Here, we have chosen the component of $\vec{H}$ to satisfy the ratio condition required by Equation 1.8a.

Assuming the coefficient in 1.9a is a real number, let us now diagram the propagating waves for the real part of the functions 1.9:

$$\mathrm{Re}\left[\vec{E}^{+}(\vec{x},t)\right] = E_0^{+}\cos(k_z z - \omega t)\,\hat{a}_x \tag{1.10a}$$

$$\mathrm{Re}\left[\vec{H}^{+}(\vec{x},t)\right] = \left(E_0^{+}/\eta\right)\cos(k_z z - \omega t)\,\hat{a}_y \tag{1.10b}$$

A graph of these functions is shown in Figure 1.1 at time $t = 0$.

In Figure 1.1, we see that, at time $t = 0$, both the electric field intensity and the magnetic field intensity are distributed under a cosine curve envelope in space with a wavelength $\lambda = 2\pi/k_z$ and both envelopes are propagating along the positive z-axis with velocity $u_p = \lambda f = 1/\sqrt{\mu\varepsilon}$.

In this figure, the x-axis direction has been chosen to lie in the direction of the electric field, and Equations 1.7 thus require that the magnetic field must lie in the y-direction. We may use the right-hand rule to see that $\vec{E} \times \vec{H}$ lies in the direction of $\vec{A}$ (the z-direction) at every point in space. Furthermore, the electric field intensity and the magnetic field intensity remain in phase with one another (both are a maximum at the same point in space and both are zero at the same point). For later values of time, both continue to point in their respective x- and y-directions so we say that they are linearly polarized. Finally, we note that the magnitude of the magnetic field envelope $H_0^{+} = E_0^{+}/\eta$, where E_0^{+} is the magnitude of the electric field intensity envelope and $\eta = \sqrt{\mu/\varepsilon}$ is the intrinsic impedance of the medium in which the wave is propagating.

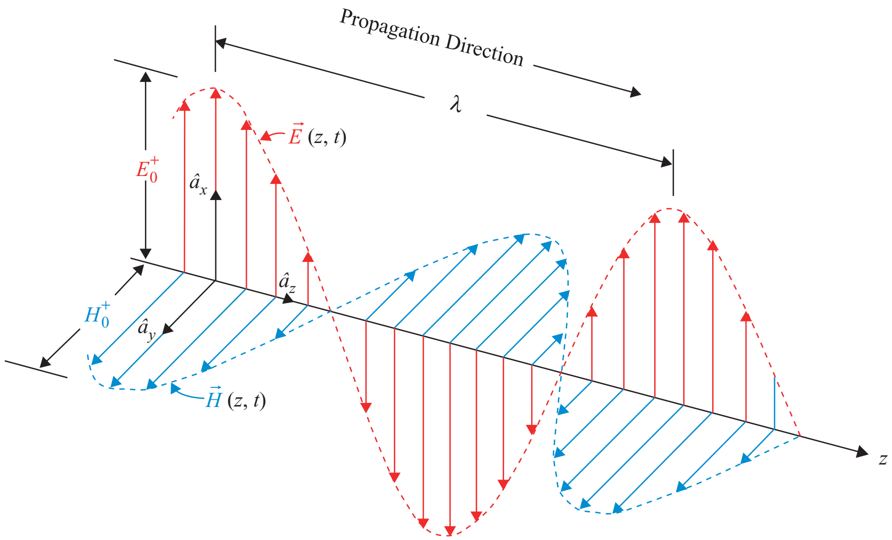

Figure 1.1 Plot of the real parts of the electric and magnetic field intensity as a function of position z, at time $t = 0$ when the x-axis is chosen to lie in the direction of the electric field intensity vector.

NOTE Some texts prefer to graph the magnetic flux density $\vec{B} = \mu\vec{H}$ rather than the magnetic field intensity

$$B_0^+ = \mu\sqrt{\frac{\varepsilon}{\mu}}E_0^+ = \sqrt{\mu\varepsilon}\,E_0^+ = \frac{E_0^+}{u_P} \tag{1.11}$$

because, in the ***special case*** when the propagating medium (e.g., air) has the same permeability and permittivity of free space, $B_0^+ = E_0^+/c$, where c is the speed of light in a vacuum, 2.99792458×10^8 m/s. When the electric field intensity of an electromagnetic wave remains in the same direction as it propagates in a medium, it is said to be linearly polarized. Of course, the relations above show that the magnetic field intensity associated with the wave is also linearly polarized.

1.2 POLARIZED PLANE WAVES

An observer located along the z-axis at a position of maximum electric field (i.e., at position $z = n\lambda$ with n = an integer at $t = 0$) looking back in the $-z$ direction (as shown in Figure 1.2a) would see the electric and magnetic field intensity, as shown in Figure 1.2b.

As a function of time, an observer at $z = n\lambda$ would measure the electric field intensity to be a maximum (in the x-direction) at time $t = 0$, as shown in Figure 1.2b, then observe it to decrease to zero by time $t = (1/4)(\lambda/c)$, then observe it to further decrease to its maximum negative value by time $t = (1/2)(\lambda/c)$, then increase back to zero by $t = (3/4)(\lambda/c)$, then increase back to its maximum positive value by $t = \lambda/c$, and so forth in a cosinusoidal manner with time. The magnetic field intensity

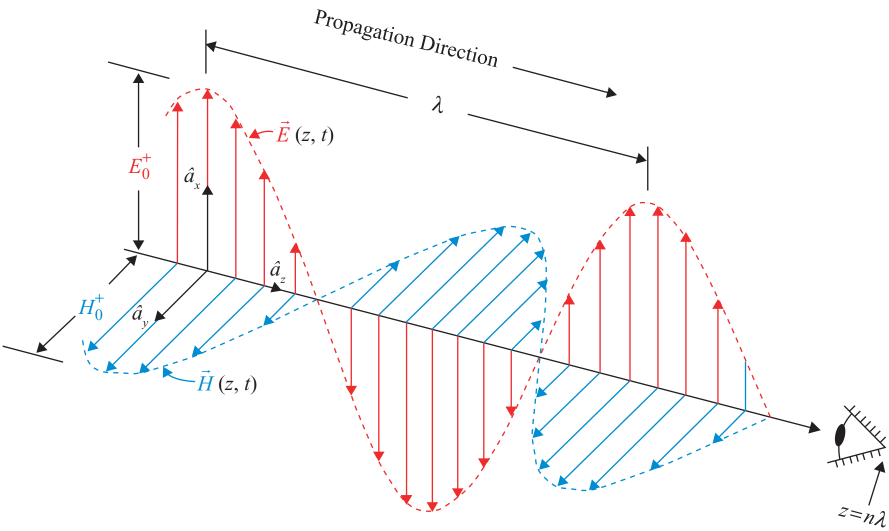

Figure 1.2 (a) Observer at $z = n\lambda$ (n = integer);

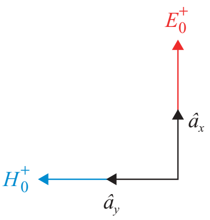

Figure 1.2 (b) electric and magnetic field intensity components observed at time $t = 0$.

would be behaving in a similar manner except it would occur only in the y-direction, and its amplitude would be $H_0^+ = E_0^+/\eta$.

More General Case

If we express the field intensity in the general case (not choosing the x-axis to lie in the direction of the electric field intensity), Equations 1.6a and 1.6b specify their components:

$$\vec{E}_S^+(z) = \frac{-k_z}{\omega\varepsilon\mu}\frac{\partial A_z^+}{\partial x}e^{-jk_z z}\hat{a}_x + \frac{-k_z}{\omega\varepsilon\mu}\frac{\partial A_z^+}{\partial y}e^{-jk_z z}\hat{a}_y$$
$$\vec{E}^+(z,t) = E_{0,x}^+ e^{-j(k_z z-\omega t)}\hat{a}_x + E_{0,y}^+ e^{-j(k_z z-\omega t)}\hat{a}_y \tag{1.12a}$$

$$\vec{H}_S^+(z) = \frac{1}{\mu}\frac{\partial A_z^+}{\partial y}e^{-jk_z z}\hat{a}_x - \frac{1}{\mu}\frac{\partial A_z^+}{\partial x}e^{-jk_z z}\hat{a}_y$$
$$\vec{H}^+(z,t) = H_{0,x}^+ e^{-j(k_z z-\omega t)}\hat{a}_x + H_{0,y}^+ e^{-j(k_z z-\omega t)}\hat{a}_y, \tag{1.12b}$$

where the components of $\vec{E}$ and $\vec{H}$ obey the relations 1.8a and 1.8b, $E_{0,x}^+ = H_{0,y}^+ = \eta$, and $E_{0,y}^+/H_{0,x}^+ = -\eta$. In this case, we can draw the electric field measured by the observer at position $z = n\lambda$ (n = integer) at time $t = 0$ to be that shown in Figure 1.3.

As seen from a point $z = n\lambda$ on the z-axis, the two components of electric field would add vectorally to form a resultant vector $\vec{E}_{0,R}^+$ whose components would vary with time cosinusoidally. Thus, $\vec{E}_{0,R}^+$ would be seen as a linearly polarized field at angle

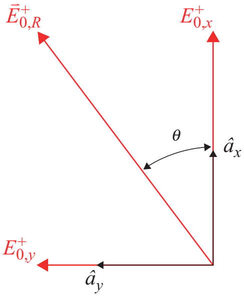

Figure 1.3 Components of the electric field intensity observed at time $t = 0$ (components of the magnetic field intensity are orthogonal to these components but are not shown).

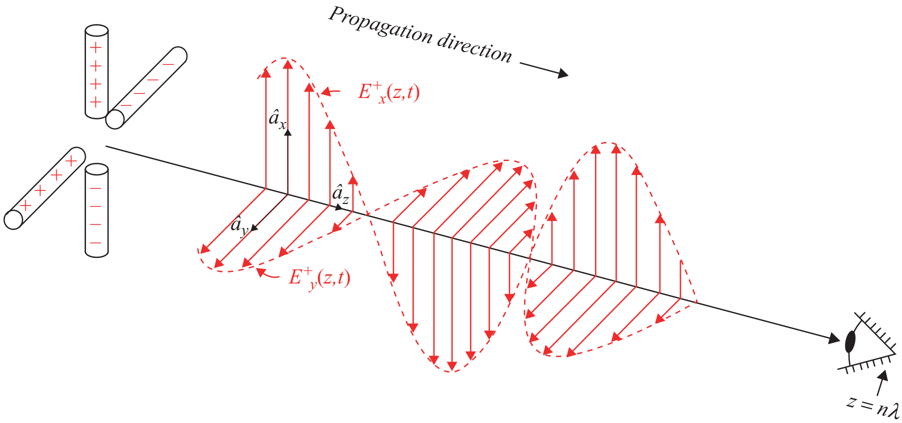

Figure 1.4 Two electric field intensities produced by orthogonal dipole antennas operating at the same frequency and with the same phase.

$$\theta = \tan^{-1}\left(E^+_{0,y} / E^+_{0,x}\right) \tag{1.13}$$

with respect to the x-axis. We would say that the two components of the electric field are in ***space quadrature*** with one another. While both of the measured components change with time in a $\cos\omega t$ manner, the angle θ remains constant so the resultant polarized electric field oscillates in amplitude with the same orientation with respect to the x-axis.

A simple way to picture the resultant of two components is to picture them as originating from two orthogonal sources such as the two dipole antennas shown in Figure 1.4.

Even More General Case

If the two dipole antennas that create the two space quadrature polarized electric field intensities are displaced from one another along the z-axis by an amount $z = \lambda/4$, as shown in Figure 1.5 and are driven at the same frequency and in the same phase, the resulting electric field intensities will be *displaced from one another in phase* by one quarter of a cycle. As seen by the observer at $z = n\lambda$, the second electric field intensity (oriented in the y-direction) will be delayed in time from the first (oriented in the x-direction) by $t = (\pi/2)/\omega$.

The equivalent equation for the observed electric fields at point z is

$$\mathrm{Re}\left[\vec{E}^+_S(z,t)\right] = \mathrm{Re}\left[E^+_{0,x} e^{-j(k_z z - \omega t)}\hat{a}_x + E^+_{0,y} e^{-j\left(k_z z - \omega t - \frac{\pi}{2}\right)}\hat{a}_y\right] \quad \text{or} \tag{1.14}$$

$$\begin{aligned}\mathrm{Re}\left[\vec{E}^+_S(z,t)\right] &= E^+_{0,x}\cos(k_z z - \omega t)\hat{a}_x + E^+_{0,y}\cos\left(k_z z - \omega t + \frac{\pi}{2}\right)\hat{a}_y \\ &= E^+_{0,x}\cos(k_z z - \omega t)\hat{a}_x - E^+_{0,y}\sin(k_z z - \omega t)\hat{a}_y\end{aligned} \tag{1.15}$$