THE ELLIPSE

A HISTORICAL AND MATHEMATICAL JOURNEY

Arthur Mazer

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PREFACE

Two of my passions are history and math. Historians often consider mathematics separate or at best tangential to their own discipline, while, by contrast, historians snuggle up with philosophy and the arts in an intimate embrace. Try this experiment: go to the library and randomly select a history book on ancient Greece. The book will describe the geopolitical landscape in which Greek culture emerged, the incessant feuding between the city states, the wars with Persia, the Peloponnesian war, and the Macedonian conquest. Also included in the book will be a section on the influential Greek philosophers and philosophical schools. And equally likely is an analysis of the artwork that provides a reflection of the times. Most likely, there is no reference to mathematical and scientific achievements, and the rare book that does mention mathematics and science is very stingy in its offerings. The reader is left to conclude that philosophical ideals are the drivers of historical change, the evolution of which can be seen in the arts. Mathematical and scientific achievements are mere outcomes of the philosophical drivers and not worth mentioning in a book on history.

There is of course the opposite argument in which one exchanges the positions of the mathematician with that of the philosophers. That is, mathematics and science are the drivers of historical evolution and in Darwinian fashion philosophies and political entities that promote scientific excellence flourish, while those that do not fade away. This latter argument provides the perspective for this book.

The seventeenth century was the bridge between the sixteenth century's counterreformation and the eighteenth century's enlightenment. It was the mathematicians who built that bridge as their efforts to settle the geocentric versus heliocentric debate over the universal order resulted in Newton's and Leibniz' invention of calculus along with Newton's laws of motion. The mathematicians concluded the debate with their demonstration that the planets revolve around the sun along elliptic pathways. In a broader context, the outcome of the argument was a scientific breakthrough that altered European philosophies so that their nations could utilize their newly found scientific prowess. *The Ellipse* relates the story from the beginnings of the geocentric versus heliocentric debate to its conclusion.

The impact of the debate is sufficient to warrant a retelling of the story. But this is not only a story of tremendous political, philosophical, and not to mention scientific and mathematical consequences, it is also one heck of a story that rivals any Hollywood production. Were we not taken in by Humphrey Bogart and Katherine Hepburn's dedication to a seemingly impossible mission in *The African Queen*? Johannes Kepler launched himself on a mission impossible that he pursued with fierce dedication as it consumed 8 years of his life. Were we not enthralled by Abigail Breslin as her fresh honesty disarmed the pretentious organizers of the Sunshine Pageant in *Little Miss Sunshine*? In the face of the Inquisition as they condemned Bruno to death at the pyre, Bruno exposed the hypocrisy of his sentencers stating, "You give this sentence with more fear than I receive it." Such are the elements of this story that it is not only significant but also compelling.

Those somewhat familiar with this story might launch a protest. Given its centrality to man's development, this story has been picked over by many outstanding individuals. The result is that there are already many fine accessible books on the topic, such as Arthur Koestler's *The Sleepwalkers*. What does *The Ellipse* offer? There are two offerings. First, the premise above that mathematics and science are the drivers of historical evolution directs the historical narrative. There is a true exchange of the roles of philosophers and mathematicians from what is evident in the standard historical literature. As with standard history books, this book describes the geopolitical environment. But philosophers are given a scant role, while mathematicians assume the center stage. Second, this is predominantly a math book with a specific objective. The objective is to take the reader through all of the mathematics necessary to derive the ellipse as the shape of a planet's path about the sun. The historical narrative accompanies the mathematics providing background music.

Throughout the book, the ellipse remains the goal, but it receives little attention until the very last mathematical section. Most of the book sets the stage, and the mathematical props of geometry, algebra, trigonometry, and calculus are put in place. Presenting these topics allows for the participation of a wide audience. Basic topics are available for those who may not as of yet had an introduction to one or more of the foundational subjects. And for those who have allowed their mathematical knowledge to dissipate due to lack of practice over several years, a review of the topics allows for a reacquaintance. Finally, for those who are well versed and find the exercise of deriving the ellipse trivial, enjoy the accompanying narrative.

Apart from devoting quite a few pages to history, the presentation is unconventional in several respects. The style is informal with a focus on intuition as opposed to concrete proof. Additionally, the book includes topics that are not covered in a standard curriculum, that is, fractals, four-dimensional spheres, and constructing a pentagon. (I particularly want to provide supplementary material to teachers having students with a keen interest in mathematics.) Finally, I include linear algebra as a part of the chapter that addresses high school algebra. Normally, this material follows calculus. Nevertheless, calculus is not a prerequisite for linear algebra, and by keeping the presentation at an appropriate level, the ideas are accessible to a high school student. Once this tool is available, the scope of problems that one can address expands into new dimensions, literally.

There are prefaces in which the author claims their writing experience was filled with only joy and that the words came so naturally that the book nearly wrote itself. I am jealous for my experience has certainly been different. There were joyous moments, but difficulties visited me as well. The challenge of maintaining technical soundness within an informal writing style blanketed the project from its inception to the final word. Setting a balance between storytelling and mathematics has been equally confounding, as has been determining the information that I should park in these two zones. Fortunately, I have had the advice of many a good-natured friend to assist me with these challenges. I would like to acknowledge my high school geometry teacher, Joseph Triebsch, who first introduced me to Euclid and advised me to address

the above-mentioned challenges head on. Others who have assisted include Alejandro Aceves, Ted Gooley, David Halpern, and Tudor Ratiu. Their willingness to take time from their quite busy schedules and provide honest feedback is greatly appreciated. Should the reader judge that I have not adequately met the aforementioned challenges, it is not due to my not having been forewarned and equally not due to a lack of alternative approaches as suggested by my friends. The project did allow me to get in touch with old friends, all of whom I have not been in contact with for many years. This experience was filled with only joy and more than compensated for the difficulties that surfaced during the writing.

I must also acknowledge my family, Lijuan, Julius, and Amelia, for putting up with me. For over a year around the dinner table, they were absolutely cheery while listening to my discourses on *The Ellipse*. I still cannot discern whether they actually enjoyed my hijacking of the normal family conversation time and conversion of it to lecture sessions or were just indulging their clueless old man. Either way, I am lucky and in their debt.

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CHAPTER 1

INTRODUCTION

My first teaching job did not start out too smoothly. I would feverishly spend my evenings preparing material that I thought would excite the students. Then the next day I would watch the expression on my students' faces as they sat through my lecture. Their expressions were similar to that on my Uncle Moe's face when he once recalled an experience on the Bataan Death March. How could the lectures that I painstakingly prepared with the hope of instilling excitement have been as tortuous as the Bataan Death March? To find an answer to this question, I went to the source. I asked the students what was going wrong. After 16 years, with the exception of one suggestion, I have vague recollection of the students' feedback. After 16 years, with the exception of one individual, I cannot remember the faces behind any of the suggestions. Concerning the one individual, not only do I have clarity concerning her face and suggestion, but I also have perfect recollection of my response.

The individual suggested that I deliver the lectures in storylike fashion and have a story behind the mathematics that was being taught. My response that I kept to myself was "you have got to be joking." My feeling was that mathematics was the story; the story cannot be changed to something else to accommodate someone's lack of appreciation for the subject. This was one suggestion that I did not oblige. And while for the most part the other students responded positively to the changes that I did make, this student sat through the entire semester with her tortured expression intact.

It is difficult to recall the specifics of something that was said over 16 years ago, the contents of a normal conversation remain in the past while we move on. Despite my reaction, there must have been some meaning that resonated and continued doing so, otherwise I would have long ago forgotten the conversation. Now I see the student's suggestion as brilliant and right on target. By not taking her suggestion, I blew the chance to get more students excited by mathematics through compelling and human stories that are at the heart of mathematics. At the time, I just did not have the vision to see what she was getting at. After 16 years, I have once more given it some thought and this book is the resulting vision. This is a mathematical story and a true one at that.

The story follows man's pursuit of the ellipse. The ellipse is the shape of a planet's path as it orbits the sun. The ellipse is special because it is a demonstration of man's successful efforts to describe his natural environment using mathematics and this mathematical revelation paved the pathway from the Counter Reformation

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to the Enlightenment. Man pursued the ellipse in a dogged manner as if a mission to find it had been seeded into his genetic code. Through wars, enlightened times, book burnings, religious persecution, imprisonment, vanquished empires, centuries of ignorance, more wars, plagues, fear of being ridiculed, the Renaissance, the Reformation, the Counter Reformation, excommunication, witchcraft trials, the Inquisition, more wars, and more plagues, man leaf by leaf nurtured a mathematical beanstalk toward the ellipse. This book examines the development and fabric of the beanstalk. It describes the creation of geometry, algebra, trigonometry, and finally calculus, all targeted toward the ellipse.

What are the ingredients that make up a good story? *Heroes*: They are in this story as the book presents a glimpse of the lives of several mathematicians from Aristarchus to Leibniz who made significant contributions to the beanstalk. *Villains*: The story of men threatened by progress and doing their best to thwart—being central to the story of the ellipse. *Struggles*: The problem of planetary motion is sufficiently vexing to assure some mathematical difficulty, and as the previous paragraph indicates, additional struggles result from a tormented history. *Dedication*: The dedicated effort of the contributors is at once admirable and inspiring. *Uncertainty*: While the book reconstructs mathematical history with the certainty that man arrives at the ellipse, many contributors had absolutely no premonition of where their contributions would lead. This uncertainty is germane to our story. *Character flaws*: Our heroes were not perfect and their mistakes are part of the story. *Tragedy*: Getting speared in the back while contemplating geometry, a victim of one's own insecurity, a burning at the stake as a victim of the Inquisition—these are a small sampling of personal tragedies that unfold as we follow the ellipse. *Triumph*: After a tortuous path, this story triumphantly ends at the ellipse. What else is in a good story? I dare not get explicit, but it is in there.

With such a great story, one would think that someone had told it before. Indeed, the story has been told; the most comprehensive historical presentation is Arthur Koestler's distinguished book, *The Sleepwalkers*. In addition, there are history books and excellent biographies of the main contributors, mathematical history books, and books covering the various mathematical topics that are contained in this book. So what is different about this book? Simply put, the history books only address the history, the math books only address the math, and the mathematical history books only address the mathematical history. This book is a math book covering the topics of geometry, algebra, trigonometry, and calculus which contains a historical narrative that sets the context for the mathematical developments. Following my belief that separating the disciplines of the history of mathematics and science from general history is an unnatural amputation, the narrative weaves the mathematical history into the broader history of the times while focusing along the main thread of uncovering the ellipse.

There is a final category of book that readers of this book may be interested in, popular books that explain mathematical and scientific theory—books explaining general relativity, quantum mechanics, chaos theory, and string theory abound for those without the requisite mathematical background. Of necessity, the core is missing in these books, the mathematics. Just as love binds two humans in true intimacy, mathematics binds the theorist with evidence. It is difficult to have a true appreciation of the theory without the mathematics, which is unfortunate because it keeps the general public at a distance from theory. This book takes the reader through all the mathematical developments needed to uncover the ellipse, and the reader will become truly intimate with the theory. The book delves into the subjects of geometry, algebra, trigonometry, and calculus, and once the mathematical machinery is finally assembled, we stock the ellipse.

Mathematicians are explorers. They follow their imagination into new territory and map out their findings. Then their discoveries become gateways for other mathematicians who can push the path into further unexplored territories. Unlike the great sea-going explorers of the fifteenth and sixteenth centuries who were exploring the surface of a finite earth, the domain of the mathematician is infinite. The subject will never be exhausted, mathematical knowledge will continue to expand, and the beanstalk will keep growing. However, like the great explorers of the fifteenth and sixteenth centuries, mathematical journeys may target a specific objective (akin to Magellan's circumnavigation of the world) or the consequences of mathematical journeys may be fully unrelated to their intentions (akin to Columbus' accidental discovery of a new continent). We can even go one step further; it is possible that some mathematical journeys have no intent whatsoever other than to amuse the journeying mathematician.

This book presents mathematics as a journey. There is the intended pathway toward the ellipse and there are sojourns along bifurcating branches of the beanstalk that are unrelated to the ellipse. The journey passes through the normal high school curriculum and calculus. By placing all the subject matter together, it is possible to demonstrate relations between what are normally taught as separate disciplines. For example, the area of an ellipse, a geometric concept, is finally arrived at only after developing concepts in linear algebra and trigonometry; the approach highlights the interplay of all the disciplines toward an applied problem. In addition, setting the objective of uncovering the ellipse motivates the mathematics. For example, studies of motion motivate the presentation of calculus and the fundamental theorem of calculus is presented as a statement of the relation between displacement and velocity. The sojourns with no apparent relation to the ellipse are undertaken solely because they are irresistible.

The book allows you as a reader to plot your own course in accordance with your own purpose. Readers with excellent proficiency in calculus will certainly plot their way through the book differently from those who may be a little out of touch with their high school mathematics and calculus. And those entirely unfamiliar with one or more of the subjects will plot another course altogether. The first section of Chapter 2 hosts the main narrative and tells the story of man's pursuit of the ellipse beginning with Aristarchus, the first known heliocentrist, and ending with Newton's successful unveiling nearly two millennia after Aristarchus. Each subsequent chapter begins with a narrative that is pertinent to the mathematical material in the chapter. By and large the mathematical material is included for one or more of the following reasons. The material is necessary to understand the topic of the chapter and will be used in subsequent chapters, or the material presents concrete examples of relevant concepts, or I have just indulged my own fancy and included material that I find fun. Sections containing material that falls solely within the last category are clearly marked as excursions and may be skipped without compromising your understanding of the remaining material. As for the remaining material, plot your course in accordance with your own purpose. You may grasp the high-level concepts and move on, or for those who want to go through the nitty-gritty, it is in there. Enjoy your journey.

CHAPTER 2

THE TRAIL: STARTING OUT

2.1 A STICKY MATTER

CLASSMATE: *Be careful. Take such stands in the classroom only. If you speak like that in public, you could be called a heretic.*

KEPLER: *My beliefs are my beliefs. I will make no secret of them.*

Kepler and Galileo lived during a time of transition. The church had lost much of its authority during the Reformation and answered with the Counter Reformation in an attempt to recover its former position. There were several factors contributing to the Reformation: nationalism, taxation, and a wayward clergy. The central method of the Counter Reformation was that that the church had honed over its 1000-year reign of power, fear.

For centuries the church could afford its excesses. Its position as the sole interpreter of scriptures allowed it to control human activity with the threat of eternal damnation. The message was simple and not subtle—follow the church's dogma toward eternal salvation or suffer unimaginable consequences, not only for the short period of your life on earth, but for eternity. And the church proffered vivid descriptions of what the consequences would be so that the unimaginable became images that were seared into the minds of medieval Europe. Demons thrusting pitch forks into screaming victims, deformed beasts pursuing their victims without mercy, and rings of fire forever scorching its victims—these images of hell had been painted in medieval churches across Europe. Through fear, the church stifled intellectual development throughout the Dark Ages.

The church maintained its monopoly as the sole interpreter of scriptures through two methods. First, the predominant avenue to an education was through church seminaries or church-sponsored universities; there were few independent secular educational institutions. Second, Latin, which was only taught in the seminaries and universities, was the language of the Bible. There were no translations into local languages, so the majority of Europeans could only rely upon the church's interpretation. In 1439, Johannes Gutenberg invented a simple device that would challenge the church's monopoly on intellectual activity, the printing press. Soon the Bible would be printed and distributed in local languages and the masses would be free to read and interpret scriptures for themselves. The Reformation was born, and after recovering its

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footing, the church responded; it launched the Counter Reformation and unleashed the Inquisition.

The result was chaos as Protestants responded to the Counter Reformation with war. The Germanic states that comprised the Holy Roman Empire launched a revolt against the church-supported Habsburg dynasty. This spawned the Thirty Years' War between Christians. Each side required discipline from their followers. In Italy, the church accepted no challenges to its authority and enforced its dogma with the Inquisition. Protestants followed suit, enforcing discipline the only way that they knew how, with fear. Those who did not agree with the dogma of the leading Protestant clergymen were excommunicated. It was in this environment that the Lutheran Kepler and the Catholic Galileo initiated modern science and mathematics, and it was in this environment that both were punished for their remarkable accomplishments.

Is the sun fixed, with the earth and its sister planets revolving about the sun, or is the earth fixed with all that is in heaven revolving about the earth? This seems to be an innocent question, certainly not a question that would lead to censorship, excommunication, imprisonment, torture, and execution on the pyre, with all of these indignities sponsored by an institution claiming to show humanity the way to salvation. And yet, the quest to answer this seemingly innocent question catalyzed all of these responses within the church. In those times, the church was far more politically consumed than the present-day church and political motives engendered these ugly responses. On the scientific side, the quest to determine the path of the planets catalyzed the development of calculus and brought science and mathematics into the modern era. This chapter follows the history of the quest in a narrative that addresses both political and scientific dimensions. The mathematics presented later in the book follows the narrative.

While there are many potential points to begin this story, we choose to begin with Aristarchus (310–230 B.C.), a Greek astronomer and mathematician from Samos. Aristarchus is the first individual known to have proposed heliocentricity based upon geometric analysis. The analysis contained two components: a method for calculating the relative size of the sun and a proposition that distance explains the fixed path of the stars from the perspective of a moving earth. This latter proposition explicitly addresses what is known as the parallax problem. Detractors of heliocentrism state that the stars would not daily appear in the same position as the earth revolves around the sun if the earth were to do so. In short order, their argument goes, the stars do appear in the same position, so the earth must be stationary. Aristarchus retorts that even though the earth moves, the stars appear fixed because the distance between the earth and stars is orders of magnitude greater than the comparatively small distances that the earth moves. With this argument, Aristarchus confronts man with the scale of the universe and how little we are within it, not a very popular notion.

Aristarchus makes another scaling argument, this one a bit more quantitative with his estimate of the relative size of the sun. This estimate demonstrates Aristarchus' grasp of geometry while at the same time illustrating the limitations of the instruments used to take astronomical observations. The geometric argument is flawless, providing a correct equation, but the measurement of an angle required by the equation is far off base. Placing his poor measurement into the formula, Aristarchus calculated that the diameter of the sun was about 20 times that of the earth, whereas the sun's actual diameter is on the order of 300 times that of the earth's. Nevertheless, Aristarchus is the first to propose that the sun is the significantly larger body, likewise not a popular notion. Aristarchus' work in which he proposes the heliocentric model did not survive. We can only conjecture that he found it more reasonable that the smaller body should orbit the larger body, not vice versa.

The prefix *geo*, which finds great use in the English language, has its origins in the Greek word *ge*, meaning "earth." A slight permutation of geo yields *ego*, which is the Latin word for *I*. The heliocentric universe that Aristarchus proposed was much less ego friendly than the geocentric universe that had been accepted since Aristotle. The earth lies precisely in the middle of Aristotle's universe and it is dominant everything else, much lighter than the earth, revolves around the earth in perfect circles. Given man's collective ego, not even the finest snake oil salesman in history could have sold Aristarchus' view in Aristarchus' time. The church's response to the heliocentric view nearly 1800 years later echoes a response by a contemporary of Aristarchus, Cleanthes. Cleanthes was so affronted by Aristarchus that he wrote a treatise entitled *Against Aristarchus* in which he states that "it was the duty of Greeks to indict Aristarchus of Samos on the charge of impropriety." The charge of impropriety is eerily similar to the charge of heresy that the church would accuse adherents of heliocentrism of at a later time.

Despite Cleanthes' appeals, there is no evidence of court action against Aristarchus. In fact, Aristarchus' proposal was firmly rooted in Greek tradition, one that respects not only knowledge but also the quest for knowledge. Chaos often begets intellectual activity and the percolating cauldron that was Greece fits this pattern. Prior to Alexander, there was not an empire or even a monolithic civilization known as Greece. On the contrary, Greece was a constellation of city states, each with its own distinct culture. Some were ruled by tyrants and others by assembly. Some stressed military values, while others stressed arts and learning. What held them together as distinctively Greek was a common polytheistic religion, a common language, and geographic proximity. Another element binding the Greeks was the common threat of Persia, which would place them in temporary alliance. More often than not, when external threats diminished, the Greek states would war with one another.

It was not until Philip of Macedonia united the Greeks that a national entity emerged. When Philip's son, Alexander, assumed power and established his empire, it was the Athenian culture of arts and knowledge that he exported and transplanted. This culture had an unusual tolerance for individual expression, one that resonated well with the indigenous inhabitants of the lands that Alexander conquered. The Athenian theater tradition demonstrates the high esteem that Athenians held for the right of self-expression, provided of course that you were a citizen as opposed to a woman, foreigner, or slave. (The latter category was not an insignificant portion of the population; at one time slaves comprised 30% of the Athenian population.)

The Athenians delighted in theater. At the festival of Dionysus, there was a tradition of sponsoring four playwrights to showcase their work. A playwright whose work was selected for sponsorship received much prestige and the competition to be selected was fierce. Even in the midst of war, the Athenians would celebrate the festival of Dionysus. During the Peloponnesian War, which was poorly managed by

Athenian politicians and drained the city's economy, the playwright Aristophanes lampooned the Athenian leadership in his comedy *Lysistrata* (414 B.C.). In the play, the women of both Athens and Sparta, Athens' nemesis, unite to bring an end to the senseless war. Their plan is simple; they would deprive their respective males of carnal pleasure by going on a sex strike. The men are unable to withstand this denial and are driven to a peace treaty. There would be no other political entity until the advent of modern democracy that would allow its citizens to openly mock the policies of its leadership, particularly when the entity is at war.

The Greek valued knowledge and learning centers were established throughout the lands conquered by Alexander. These learning centers inherited the Athenian respect for self-expression. Foremost among all learning centers was the university at Alexandria; the city established by and named for the famous conqueror is in Egypt. The debt of mathematics to the university at Alexandria cannot be understated. Shortly after Aristarchus, Euclid (circa 300 B.C.) wrote his incomparable work *The Elements*.

The Elements would be the standard text for mathematics training throughout the Middle East and Europe over the next 2000 years. At the age of 40, Abraham Lincoln undertook the study of *The Elements* to exercise his mind and much of our modern-day high school mathematics curriculum draws from Euclid's *The Elements*. It is *The Elements* that cements the axiomatic deductive process that lies at the core of mathematics. *The Elements* begins with a set of definitions that are used throughout the book. Axioms follow the definitions. Afterward, the text branches out in a tree of propositions that engender yet more propositions, but all are derived from *The Elements'* axiomatic roots.

Aside from Euclid, there were others of tremendous intellectual stature associated with the university at Alexandria who made lasting contributions to mathematics. Central to the quest of an understanding of the juxtaposition of the stars, sun, planets, and earth are Archimedes (287–212 B.C.), Apollonius (262–190 B.C.), and Ptolemy (83–168 A.D.). All of these outstanding mathematicians and scientists were thoroughly educated in Euclid's *Elements* and it permeates their work. All made significant contributions beyond *The Elements*. All were dead wrong in their assessment of Aristarchus' thesis.

The story predates the university at Alexandria. Aristotle had posited the commonly held view of the universe's structure. According to Aristotle, the stars, planets, sun, and moon float above the earth as these entities are lighter than earth and they circulate about the earth through an invisible medium that he coined the ether. The earth itself, being the heaviest of all objects, is immovable, fixed within the ether. As with many scientific theses that Aristotle posited, this was more of a product of fanciful imagination than an actual scientific investigation. And as with many of Aristotle's physical theses, it has been thoroughly discredited. Despite the fact that he wrote a significant amount on topics that he knew nothing about, Aristotle's scientifically vacant musings became dogma over a 2000-year span. The idolization of Aristotle would not happen under the Greeks; indeed, while Greek philosophers may have frequently cited Aristotle, the Greek scientists give little mention of him.

Aside from his theory on the structure of the universe, another of Aristotle's theories has significance in the development of calculus. That is his incorrect view concerning the motion of falling objects. Aristotle's view is that a heavy object falls faster than a light object. An argument against Aristotle's theory accessible to contemporary Greeks was that two objects could be combined into one by fusing them together; the combined object would not gain speed in its descent from the separate objects. Indeed, just tie a rope around two objects. If Aristotle is correct, the new object joined by the rope will fall faster than the separate objects, but it does not.

Because of Aristotle's prominence in history, it is worthwhile to examine his role as a scientist. Aristotle is frequently credited with the development of the scientific method, subjecting a hypothesis to rigorous testing. If this is the case, Aristotle never applied the scientific method to his views on the dimensions of the universe, the composition of the universe, or the motion of falling objects. Concerning the dimensions and composition of the universe, he had neither the knowledge nor the means to subject his views to testing. Concerning the motion of falling objects, an experiment whereby objects of different shapes and weights are dropped repeatedly from a fixed height allowing the scientist to observe which, if any, objects fall at greater speed could have easily been performed. Such an experiment would have shown Aristotle's hypothesis to be incorrect. But Aristotle never performed the experiment. As with his views on the dimension and composition of the universe, Aristotle proposed them without evidence.

By contrast, Aristarchus, a near contemporary of Aristotle's, only 70 years younger, performed a careful geometric analysis and then subjected his analysis to experimental measurement. The evidence he gave for his correct conclusion that the sun is larger than the earth was accepted and endorsed by the most capable of the Greek scientists, Archimedes. We have little record of the more personal aspects of his life, and yet personal stories about Archimedes have become a part of mathematical folklore, a tribute to his well-deserved legendary status. The mathematical achievements of this man are staggering. The originality of his work and the scope of subjects that he investigated placed him years ahead of his contemporaries and science would not catch up for another 1800 years. Archimedes left for posterity 12 works that we know of. He was a consummate problem solver developing brilliant methods. Archimedes did not formalize his solutions into theory. But the theory is recognizable and the depth of it is amazing. We briefly describe three works.

In *On Floating Bodies*, Archimedes determines stable configurations of floating bodies. A stable configuration is one that does not drift away from its equilibrium position under a disturbance. As an example, a pendulum with its weight directly above the pendulum's pivot is in equilibrium but not stable since the weight will swing downward upon being disturbed. The pendulum with its weight directly below the pivot is in equilibrium and stable. Archimedes examined equilibrium configurations for bodies floating in water and then determined which were stable, that is, which would not flip over in the presence of a disturbance. Archimedes correctly asserts that the stable configuration of a floating object is the one with the lowest energy level. He then proceeds to apply this principle to nontrivial shapes and determines their stable configuration. The physical assertion of the stable configuration does not explicitly use the term *energy* for the concept of energy had not yet been discovered. Rather, Archimedes poses his work in terms of centers of gravity. The very thought of attempting a stability analysis at this stage of intellectual development demonstrates his creativity and daring. One would not find comparable works for another

1900 years, well after the advent of calculus. And arguably, one could cite this work to justify Archimedes as the father of integral calculus because he develops methods of integral calculus to calculate centers of gravity for nontrivial shapes.

Again, Archimedes demonstrates his pioneering efforts in physics and calculus in his work *On Spirals*. In this work, Archimedes calculates the velocity of an object moving along a prescribed spiral pathway, relates the velocity to lengths of arcs, and determines the area that the particle sweeps out between the spiral and a coordinate axis. With this work, one could cite Archimedes as the father of differential calculus and even go one step further. While he does not formalize it, Archimedes uses the fundamental theorem of calculus to relate velocity to length and area.

Archimedes judges his work by a different standard than those who review it with a historical perspective. The work that Archimedes was most proud of is *On the Sphere and Cylinder*, where he determines the formula for the volume of a sphere. Perhaps this is because it is such a difficult problem and Archimedes is a consummate problem solver, rightfully proud of his problem-solving capacity. The method is exceptionally original and once again demonstrates Archimedes' comfort with calculus. Indeed, Archimedes also relates the volume of the sphere to the surface area, giving another example of the application of the fundamental theorem of calculus.

It is not too difficult to imagine a historical scenario in which scholars following Archimedes' works formalize his problem-solving methods into theory and develop calculus long before Newton. But this is not how history happened. On the scientific side, Archimedes was far ahead of his time. The number system that Archimedes used did not have the counterpart of a zero, making calculations tedious and difficult to follow. Even more, algebra had not yet been formalized, and the many complex algebraic manipulations that Archimedes executes are difficult to communicate in the geometric language of Archimedes' times. In addition, formalization of Archimedes' results requires the general concept of a function as well as the concept of a coordinate axis system, neither of which was developed in Europe until Rene Descartes in the seventeenth century. Archimedes' works do not receive the attention that they merit. Perhaps, sadly, they served no other purpose than to dazzle modern historians by the stunning capacity of this man to be not only centuries but nearly two millennia ahead of his time. Then again, perhaps, as we shall see, they played a more inspiring role.

One further work of Archimedes requires our attention for it is germane to the topic of this book. In *The Sandreckoner*, Archimedes supports the geocentric vision of the universe and opposes Aristarchus. Indeed, it is from Archimedes' response to Aristarchus that we are aware of Aristarchus' works; as previously noted, the original works of Aristarchus in which he expounds his heliocentric theory have been lost. It is a pity that Archimedes placed his opposition to Aristarchus in writing. Certainly, Archimedes carried significant authority in the intellectual world. His refutation may well have influenced others to turn away from Aristarchus.

The historical route to calculus and the structure of the universe bypasses Archimedes and flows through a contemporary, Apollonius. Apollonius was a lecturer and researcher at Alexandria. Apollonius' initial impact upon the search for the structure of the universe was to point in the wrong direction with a very persuasive finger. He applied his ingenuity toward correcting a flaw in the geocentric universe. The stars follow a daily circular motion and maintain their relationship with each other. But the Greeks identified objects with curious behavior and assigned the Greek word planetoid, meaning wanderer, to these bodies. Aristotle's model does not give a convincing description of planetary motion.

The planets float among the stars, shifting their position. A first attempt to describe this drifting behavior was to hypothesize that they follow their circular orbits around the earth at different speeds than the stars. This appears to be almost correct, but there are anomalies. In particular, the planets appear to not be moving at a constant speed and the planets shift their direction on occasion. How could one account for this observation? Apollonius salvages the Aristotelian view with a proposition that each planet moves in a small circle that rotates about a larger circle. Picture a giant circular arch across the sky and a planet being carried across by an invisible wheel that rolls over the arch. As the wheel moves along the dominant archway, its motion creates a secondary circular path known as an epicycle. The planet follows these composite motions. With the correct diameters for the larger arch and smaller wheel, as well as correct speeds around each circle, the anomalies could be explained. This is what Apollonius had in mind, but he did not carry out the calculations in detail.

Apollonius is best known for his studies of conic sections. These are curves that result from the intersection of a plane with a cone. The intersection generates a circle, ellipse, hyperbola, or parabola dependent upon the manner in which the intersection occurs. Apollonius was not the first Greek to investigate conic sections, but he presents the most thorough accounting of their properties. Apollonius also calculated the value of pi (so did Archimedes), providing a necessary constant for finding the lengths and areas of both circles and ellipses.

In addition, in his investigations of conics, Apollonius determines the tangent line to the surface of the curves, the objective of differential calculus. This places Apollonius as perhaps, not the father of differential calculus, but certainly an inspirer. Apollonius' interest is purely abstract as he does not show applications for any of his works. He could not imagine that the ellipse would supplant his own epicycles as the correct description of planetary motion, but that would be a millennium and a smattering of centuries away.

The weight of authority from both Archimedes and Apollonius was enough to crush further investigations into a heliocentric system. Nobody from Alexandria followed the direction indicated by Aristarchus. A Babylonian, Seleucus, took up the cause of Aristarchus two centuries later (circa 190 B.C.) and presented additional arguments in favor of a heliocentric system. However, Seleucus' works received little attention. It is noteworthy that Seleucus had adopted Greek culture although he was not racially Greek. The acceptance of Greek culture by inhabitants of the lands ruled by the Greeks was common and many of the intellectuals in the Greek universities were not in fact Greek.

Ptolemy (83–161 A.D.) was another great ancient scientist who received a Greek education but was not Greek. Ptolemy endowed the West with the longitudinal and latitudinal coordinate system that is used to locate points on the earth's surface; this achievement cements Ptolemy's reputation in the West as the father of geography.

His works were widely read and quoted for centuries, particularly by mapmakers and navigators. Ptolemy was also a fine mathematician who developed extremely accurate trigonometric tables and in doing so demonstrated the level of sophistication in the area of trigonometry during his time.

Central to our story, Ptolemy turned his attention to astronomy and calculated the specifics of motion along the larger and smaller wheels as proposed by Apollonius. Unfortunately, he found that no matter how he chose dimensions and speeds, he could not precisely replicate the motion of the planets as measured and logged at the astronomical observatory in Alexandria. The stubbornness and ingenuity of mankind in pursuing what is dead wrong are mind boggling. Ptolemy responded by introducing two devices, the eccentric and the equant. Ptolemy ever so slightly displaced the earth from the center of the universe by assigning the center of a planet's orbit to a point that is distant from the earth. Another device was to introduce an equant for each planet. The planet's angular velocity about its equant remains constant but as the equant is not centered on the earth, the planet's speed about the earth is not uniform. Ptolemy's universe is like an organ with its rotating gears moving in precision to produce a harmonic outcome. As the observations are not quite so harmonic, take a monkey wrench to the machinery and make corrections. One point of note is that Ptolemy's mastery of trigonometry was essential to carry out the calculations. And over the centuries the calculations became unwieldy. By the time of Copernicus, the Ptolemaic universe consisted of 40 epicycles.

While at first glance this does not look good for Ptolemy, let us look at his approach in hindsight from a different perspective. In the early nineteenth century, the mathematician Fourier proposed a solution to a difficult set of equations known as the heat equations. His solution was an infinite composition of trigonometric functions. This caused much controversy because it was unknown if an infinite combination of functions had any meaning. As always, controversy in mathematics begets progress and Fourier was vindicated; indeed, one can construct a meaningful solution using an infinite composition of trigonometric functions. Ptolemy, with his wheel on a wheel on a wheel construction of the pathways of planets, was the first individual to attempt this. So even though he was dead wrong about the universe, he was years ahead of his time in creating functional series expansions.

Returning to our story, it is in the Ptolemaic universe that Western thought stagnates for 1500 years. No further contributions are made toward the question of the structure of the universe and planetary motions. Western science is frozen in Aristotelian musings, while Aristarchus, Seleucus, and the far-reaching works of Archimedes are ignored. What forces are responsible for these sad circumstances? The force most immediately responsible for the decline in the pace of scientific progress is the Romans. Their march to power did not bode well for the sciences; a Roman soldier killed Archimedes. The historian Polybius (200–118 B.C.) recorded the story of Archimedes' death and it has become folklore that is retold in nearly every text that mentions Archimedes. So here it is.

It is believed that Archimedes was educated at Alexandria. While in Egypt, Archimedes demonstrated his mechanical creativity for there he invented the Archimedean screw, a water-pumping device that is still used today. Archimedes' gift with mechanics would be integral to his legendary status. Archimedes did not remain in Egypt but left to return to his native city of Syracuse, one of a series of cities that the Greeks had established throughout the Mediterranean prior to the days of Alexander.

During the life of Archimedes, a century after Alexander's death, Rome was establishing itself as a power within the Mediterranean. Another contender, Carthage, battled with Rome for supremacy throughout the Punic Wars. Syracuse was situated in between these two powers, trying to ally itself with the unknown victor. At first Syracuse lined up behind Rome. Then under Hannibal the Carthaginians gained the upper hand; Hannibal led an army that included warrior elephants across the Alps and defeated the Romans. At this point, Syracuse as well as other Sicilian cities realigned themselves with Carthage. This set the stage for Rome to besiege the cities of Sicily when Rome reestablished itself as the stronger power.

At the time Syracuse was a city state under the leadership of King Hiero. Hiero instructed Archimedes to draw up a defense plan against the Roman assault. In this task, Archimedes exhibited the same brilliance that he did in his scholarly works. He had the advantage of a well-protected city as the city walls had been well placed. Facing the sea, the wall abutted the Mediterranean. The remainder of the wall enclosed the city along a roughly semicircular path. The semicircular wall was predominantly built upon cliffs with few places of safe access.

Archimedes applied his mechanical ingenuity, which was evident when he invented the Archimedean screw. Archimedes designed catapults of various sizes, large, mid, and small sized, optimally tuned for different ranges. Midsized and smaller sized catapults allowed for quicker reloading as they held lighter projectiles. Aside from the range-specific catapults, Archimedes built several cranelike structures that could be used to hoist large and heavy objects. The hoisting mechanism consisted of a beam that rotated on a pivot. A heavy object was hooked and connected to one end of the beam by a rope. A counterweight on the other side of the beam hoisted the object using leverage. Archimedes designed a hooking mechanism that has been likened to a claw for it could grapple objects that would then be hoisted by the cranes. Finally, Archimedes designed short-range weapons that could fire multiple darts with a single shot, a sort of ancient machine gun.

Surveying the land and recognizing that there were few places where the wall was accessible, Marcellus, the commanding Roman general, at first decided to attack Syracuse by sea. His plan was to breach the walls with a series of ladders carried and secured by specially designed boats. The Romans had used this technique on cities similar to Syracuse and were confident of easily breaching the city walls by reason of overwhelming manpower. Once within the city walls, Roman soldiers with spears would operate according to standard procedure: pillage, burn, kill, and rape at will.

On the day of the attack, the Romans approached Syracuse by sea and Archimedes' long-range catapults bombarded Marcellus' navy. The vessels that survived the initial barrage were then targets for Archimedes' mid- and short-range catapults. If a boat reached the city wall, Archimedes' claw grappled the vessel and hoisted the vessel into a dangling vertical position, whereupon the vessel was released and smashed into the rocky seashore. All the while, the Roman soldiers were targets for shorter range weapons. The result was a resounding victory for the Greeks.

This was discomfiting to Marcellus and Marcellus decided to attack by land. Unfortunately for Marcellus, his analysis that initially caused him to forgo a land assault proved to be correct. Archimedes lined up his firepower in a few areas where the wall was approachable. The firepower was concentrated, overwhelming, and accurate as the Roman army had to pass through the same series of long-range, midrange, and short-range weaponry that earlier greeted their navy. A barrage of iron and stone projectiles as well as arrows greeted the Roman soldiers who came close to the wall. In addition, the claw grappled individual soldiers and dropped them into the rocks below. The overall picture for the Romans was very demoralizing.

Marcellus became prudent; he decided to forgo an assault and dug in for a siege embargoing the city and starving it into submission. This was not the preferred approach since it pinned down many Roman resources that could otherwise be used in further conquest. Furthermore, one never knew how long the supplies of the besieged city would last and the uncertainty of the duration of the siege, 6 months, 1 year, 2 years, or longer, would not be good for one's career. However, Marcellus saw no other choice and he set up camp.

Two and a half years into the siege, the embargo had taken its toll and the Syracusans resolve weakened. Marcellus launched an attack along with an order to spare Archimedes. The assault was successful, but the order to spare Archimedes was not obeyed. One legend has it that Archimedes was steeped in concentration, observing geometric figures that he had drawn in a sandbox, when the attack came. A Roman soldier entered his home and Archimedes snapped, "Don't disturb my circles." Not knowing the man was Archimedes, the soldier executed him by impaling Archimedes with a spear. Marcellus honored Archimedes by burying him in accordance with Archimedes' wishes. A sphere and a cylinder were engraved on the tombstone to commemorate Archimedes' most proud discovery of the formula for the volume of a sphere.

If Archimedes had been at the university, perhaps he could have developed a school of followers who would further develop his ideas. Instead, he did the next best thing. He sent copies of his works to Alexandria and most likely these were shared with other universities. His works were revolutionary and difficult to comprehend. The wealth of material that could have been formalized into theory and text was not given the attention it merited. Eventually, mathematics would catch up with Archimedes, but that would not be for another millennium and a smattering of centuries.

The Romans cobbled an empire together through fear. Local leaders understood that they would submit to Roman authority or suffer retribution. And the Romans were not subtle about the form that the retribution would take. Indeed, the standard infantry operating procedure described above, burn, pillage, kill, and rape, was meant as an intentional warning to those who might waiver in their commitment to Rome.

In the early days of the Roman Empire, as long as there was willing submission to Roman authority and taxes were paid, local leaders could maintain their autonomy. Under this arrangement, local religious and cultural traditions continued. If the culture had a tradition of learning, as in the case of Alexandria, it could locally support a university. Ptolemy lived under Roman rule and the university at Alexandria continued until 415 A.D. But learning and science were never germane to Roman culture. Indeed, on the cultural front, the Romans offered next to nothing over the 500-year span of their empire. The Roman Empire was an exercise in power. Whereas the Greeks bequeathed the university system along with a formidable accumulation of knowledge, the Romans bequeathed their profane and unenlightened credo of power for the sake of power. Within this cultural vacuum, Romans adopted the Christian religion.

Upon adopting Christianity in 312, the Romans assumed a less charitable attitude toward their territories. They reversed their previous tolerance for local traditions, and in 391 the Roman emperor Theodosius prohibited the practice of any religion other than Christianity. The prohibition sparked significant unrest as local populations were unwilling to forgo their traditional ways. The Roman response to this unrest was harsh. What occurred next at Alexandria was emblematic of what was happening across the empire.

The university's library was within the city's pagan temple complex, a place where the Alexandrian pagan population sought sanctuary from marauding Roman Christians. The library housed the most complete collection of literature within the Roman Empire. The library consisted of scrolls that were painstakingly transcribed one letter at a time by a professional cadre of scribes. At its largest, the library is rumored to have contained 500,000 scrolls. This was before its original burning by Mark Antony (48 B.C.), an accident that was brought on by Roman soldiers under Antony's command. During the battle between Marc Antony and Cleopatra's rival to the throne, her brother Ptolemy, the library was a victim which in modern-day jargon would be described as collateral damage. Mark Antony made it up to Cleopatra by pilfering 200,000 scrolls from another library, at Pergamum; unfortunately for Pergamum, Mark Antony did not find such an enchanting spirit as Cleopatra there. Theodosius was not a patron of knowledge or beauty. His decree of 391 instigated christians mobs to destroy pagan institutions. The library at Alexandria and the scrolls were not spared. While dedicated teachers tried to continue lecturing at the university, the staff eventually disbanded due to harassment through prosecution on the charges of heresy. In 415, the university's last remaining scholar-lecturer, a woman by the name of Hypatia, was accused of teaching heretical philosophies, assassinated, and cremated.

The mix of politics with faith has been an explosive combination that has turned disastrous throughout history. Christianity had moved into the cultural vacuum of the Roman Empire and with the fall of the Roman Empire the church would assume an even larger political role. Throughout its life at the center of European politics, the church would attempt to reconcile its religious inheritance, the message of salvation, with its political inheritance, power for the sake of power. All too often, the message of salvation became subservient to politics.

The church was a principal author of the post-Roman political order. The church struck a bargain with influential chieftains throughout Europe, and the church would legitimize their authority, conferring recognition of divine authority for a ruling class of nobility. In exchange the nobility would accept the Christian religion, furnish the church with lands and material support for cathedrals and monasteries through taxation, and allow the church to establish its own separate governing authority upon church-owned lands. In constructing this political edifice, the church had no compunction about sacrificing its moral authority. It explicitly accepted the indentured servitude of a landless class to the nobility by legitimizing the institute of serfdom, an institute rife with abuses that are contrary to Christian philosophy. Heredity perpetuated the arrangement as an individual's social status was conferred at birth. Through its political dealings, the church had become politically dominant, and more so than the Romans for from the fall of the Romans until the Reformation the reign of the church lasted 800 years.

Mirroring the social fixed point that the church in league with the ruling class had established, the church also established spiritual and philosophical fixed points to create a triangle of stability. Spiritually, the church offered the inspirational teachings of the Gospels, but intellectually the church inherited little direction. In the thirteenth century, the Europeans rediscovered Aristotle, which created a stir among a previously intellectually comatose population. Thomas Aquinas was the best known promoter of Aristotelian reason. The initial church response was predictable. Aristotle was summarily rejected by Pope John I, who in no uncertain terms condemned Aristotelian philosophy. But the church could not keep intellectual curiosity at bay and the church adapted. By accepting Aristotle's scientific principles and reinterpreting his political and ethical philosophies, the church coopted Aristotle and completed the final fixed point for their triangle of stability. What was it about Aristotelian science as opposed to the far-reaching ideas of Archimedes and Aristarchus that the church found attractive? Archimedes and Aristarchus had profound ideas that required further investigation. An institution that wishes to maintain the status quo does not want individuals following their own inquiry and certainly does not promote such a philosophy. Alternatively, Aristotle offered a comprehensible knowledge of everything. For church leadership, Aristotle was simple and intuitive and stifled independent inquiry. In one word, this was perfect. The effect on mathematics and science is known to all; there was little European progress in either mathematics or science under church domination.

Throughout the Middle Ages, the church maintained a grip on intellectual activity through its influence on university education. The university evolved to meet church requirements for an educated clergy that could address complex theological issues as well as administrative and financial issues. An additional impetus was the needs of a growing mercantile class who had educational requirements that the church could not fulfill. Both of these sectors worked together to meet their corresponding educational requirements.

Student-funded universities developed as the mercantile class sought knowledge beyond theological concerns. In the twelfth century, citizens would seek out a knowledgeable individual to teach a topic of interest in exchange for pay. At first, the process was ad hoc, and there was no formal administration or location. A natural place to hold classes was the old monastery schools that had been established during the Gregorian reforms, so the universities evolved out of the church's infrastructure. The process expanded both administratively and physically. Buildings were rented for the purpose of holding classes and later were owned as universities became their own entities.