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Pre-Algebra Essentials

FOR DUMMIES®

Learn to:

- **Work with and convert fractions, decimals, and percents**
- **Solve for variables in algebraic expressions**
- **Get the right answer when solving basic math problems**

Mark Zegarelli
Author, Basic Math & Pre-Algebra Workbook For Dummies

Pre-Algebra Essentials FOR DUMMIES®

by Mark Zegarelli
with Krista Fanning

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About the Authors

Mark Zegarelli is the author of *Logic For Dummies* (Wiley) plus three *For Dummies* books on pre-algebra and Calculus II. He holds degrees in both English and math from Rutgers University. Mark lives in Long Branch, New Jersey, and San Francisco, California.

Krista Fanning writes and edits textbooks and supplementary materials for several publishing houses. As a former elementary school teacher, she has a passion for education and details. In her publishing career, she has been involved in the production of over 50 titles. She enjoys spending time with her family and stalking her local library for the newest mysteries and thrillers.

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Some of the people who helped bring this book to market include the following:

Acquisitions, Editorial, and Media Development

Senior Project Editor: Tim Gallan

Acquisitions Editor: Lindsay Lefevere

Senior Copy Editor: Danielle Voirol

Technical Reviewers: David Herzog, Amy Nicklin

Editorial Program Coordinator: Joe Niesen

Editorial Manager: Michelle Hacker

Editorial Assistants: Jennette ElNaggar, David Lutton, Rachelle Amick

Cover Photo: © iStock / Alistair Forrester Shankie

Cartoons: Rich Tennant (www.the5thwave.com)

Composition Services

Project Coordinator: Sheree Montgomery

Layout and Graphics: Carrie A. Cesavice, Joyce Haughey, Ronald G. Terry

Proofreaders: Melanie Hoffman, Sossity R. Smith

Publishing and Editorial for Consumer Dummies

Diane Graves Steele, Vice President and Publisher, Consumer Dummies

Kristin Ferguson-Wagstaffe, Product Development Director, Consumer Dummies

Ensley Eikenburg, Associate Publisher, Travel

Kelly Regan, Editorial Director, Travel

Publishing for Technology Dummies

Andy Cummings, Vice President and Publisher, Dummies Technology/General User

Composition Services

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Introduction

Why do people often enter preschool excited about learning how to count and leave high school as young adults convinced that they can't do math? The answer to this question would probably take 20 books this size, but solving the problem of math aversion can begin right here.

Remember, just for a moment, an innocent time — a time before math inspired panic attacks or, at best, induced irresistible drowsiness. In this book, I take you from an understanding of the basics to the place where you're ready to enter any algebra class and succeed.

About This Book

Somewhere along the road from counting to algebra, most people experience the Great Math Breakdown. Please consider this book your personal roadside helper, and think of me as your friendly math mechanic (only much cheaper!). The tools for fixing the problem are in this book.

I've broken down the concepts into easy-to-understand sections. And because *Pre-Algebra Essentials For Dummies* is a reference book, you don't have to read the chapters or sections in order — you can look over only what you need. So feel free to jump around. Whenever I cover a topic that requires information from earlier in the book, I refer you to that section or chapter in case you want to refresh yourself on the essentials.

Note that this book covers only need-to-know info. For a broader look at pre-algebra, you can pick up a copy of *Basic Math & Pre-Algebra For Dummies* or the corresponding workbook.

Conventions Used in This Book

To help you navigate your way through this book, I use the following conventions:

- *Italicized* text highlights new words and defined terms.
- **Boldfaced** text indicates keywords in bulleted lists and the action part of numbered steps.
- `Monofont` text highlights Web addresses.
- Variables, such as x and y, are in italics.

Foolish Assumptions

If you're planning to read this book, you're likely

- A student who wants a solid understanding of the core concepts for a class or test you're taking
- A learner who struggled with algebra and wants a reference source to ensure success in the next level
- An adult who wants to improve skills in arithmetic, fractions, decimals, percentages, geometry, algebra, and so on for when you have to use math in the real world
- Someone who wants a refresher so you can help another person understand math

My only assumption about your skill level is that you can add, subtract, multiply, and divide. So to find out whether you're ready for this book, take this simple test:

$5 + 6 =$ ___

$10 - 7 =$ ___

$3 \times 5 =$ ___

$20 \div 4 =$ ___

If you can answer these four questions, you're ready to begin.

Icons Used in This Book

Throughout the book, I use three icons to highlight what's hot and what's not:

This icon points out key ideas that you need to know. Make sure you understand before reading on! Remember this info even after you close the book.

Tips are helpful hints that show you the quick and easy way to get things done. Try them out, especially if you're taking a math course.

Warnings flag common errors that you want to avoid. Get clear about where these little traps are hiding so you don't fall in.

Where to Go from Here

You can use this book in a few ways. If you're reading this book without immediate time pressure from a test or homework assignment, you can certainly start at the beginning and keep on going through to the end. The advantage to this method is that you realize how much math you *do* know — the first few chapters go very quickly. You gain a lot of confidence as well as some practical knowledge that can help you later on, because the early chapters also set you up to understand what follows.

Or how about this: When you're ready to work, read up on the topic you're studying. Leave the book on your nightstand and, just before bed, spend a few minutes reading the easy stuff from the early chapters. You'd be surprised how a little refresher on simple stuff can suddenly cause more-advanced concepts to click.

If your time is limited — especially if you're taking a math course and you're looking for help with your homework or an upcoming test — skip directly to the topic you're studying. Wherever you open the book, you can find a clear explanation of the topic at hand, as well as a variety of hints and tricks. Read through the examples and try to do them yourself, or use them as templates to help you with assigned problems.

The 5th Wave By Rich Tennant

"David's using algebra to calculate the tip. Barbara– would you mind being a fractional exponent?"

Chapter 1

Arming Yourself with Math Basics

In This Chapter

- Identifying four important sets of numbers
- Reviewing addition, subtraction, multiplication, and division
- Examining commutative, associative, and distributive operations
- Knowing exponents, roots, and absolute values
- Understanding how factors and multiples are related

You already know more about math than you think you know. In this chapter, you review and gain perspective on basic math ideas such as sets of numbers and concepts related to the Big Four operations (adding, subtracting, multiplying, and dividing). I introduce you (or reintroduce you) to properties and operations that will assist with solving problems. Finally, I explain the relationship between factors and multiples, taking you from what you may have missed to what you need to succeed as you move onward and upward in math.

Understanding Sets of Numbers

You can use the number line to deal with four important *sets* (or groups) of numbers. Each set builds on the one before it:

- **Counting numbers (also called natural numbers):** The set of numbers beginning 1, 2, 3, 4, ... and going on infinitely
- **Integers:** The set of counting numbers, zero, and negative counting numbers

- **Rational numbers:** The set of integers and fractions
- **Real numbers:** The set of rational and irrational numbers

Even if you filled in all the rational numbers, you'd still have points left unlabeled on the number line. These points are the irrational numbers.

An *irrational number* is a number that's neither a whole number nor a fraction. In fact, an irrational number can only be approximated as a *non-repeating decimal.* In other words, no matter how many decimal places you write down, you can always write down more; furthermore, the digits in this decimal never become repetitive or fall into any pattern. (For more on repeating decimals, see Chapter 5.)

The most famous irrational number is π (you find out more about π when I discuss the geometry of circles in Chapter 11):

$$\pi = 3.14159265358979323846264338327950288419716939937510...$$

Together, the rational and irrational numbers make up the *real numbers,* which comprise every point on the number line.

The Big Four Operations

When most folks think of math, the first thing that comes to mind is four little (or not-so-little) words: addition, subtraction, multiplication, and division. I call these operations the *Big Four* all through the book.

Adding things up

Addition is the first operation you find out about, and it's almost everybody's favorite. Addition is all about bringing things together, which is a positive thing. This operation uses only one sign — the plus sign (+).

When you add two numbers together, those two numbers are called *addends,* and the result is called the *sum.*