Frequently Asked Questions In Quantitative Finance

... including key models, important formulae, popular contracts, essays and opinions, a history of quantitative finance, sundry lists, the commonest mistakes in quant finance, brainteasers, plenty of straight-talking, the Modellers’ Manifesto and lots more

www.wilmott.com

Paul Wilmott
Frequently Asked Questions In Quantitative Finance

... including key models, important formulæ, popular contracts, essays and opinions, a history of quantitative finance, sundry lists, the commonest mistakes in quant finance, brainteasers, plenty of straight-talking, the Modellers’ Manifesto and lots more

www.wilmott.com

Paul Wilmott
To my parents
Paul Wilmott has been called ‘the smartest of the quants, he may be the only smart quant’ (Portfolio magazine/Nassim Nicholas Taleb), ‘cult derivatives lecturer’ (Financial Times), ‘expert on quantitative easing,’ (Guardian)*, ‘the finance industry’s Mozart’ (Sunday Business) and ‘financial mathematics guru’ (BBC).

* This was, of course, a typical Grauniad mistake.
<table>
<thead>
<tr>
<th>Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface to the Second Edition</td>
<td>x</td>
</tr>
<tr>
<td>Preface to the First Edition</td>
<td>xiii</td>
</tr>
<tr>
<td>1 The Quantitative Finance Timeline</td>
<td>1</td>
</tr>
<tr>
<td>2 FAQs</td>
<td>21</td>
</tr>
<tr>
<td>3 The Financial Modellers’ Manifesto</td>
<td>253</td>
</tr>
<tr>
<td>4 Essays</td>
<td>259</td>
</tr>
<tr>
<td>5 The Commonest Mistakes in Quantitative Finance: A Dozen Basic Lessons in Commonsense for Quants and Risk Managers and the Traders Who Rely on Them</td>
<td>313</td>
</tr>
<tr>
<td>6 The Most Popular Probability Distributions and Their Uses in Finance</td>
<td>383</td>
</tr>
<tr>
<td>7 Twelve Different Ways to Derive Black–Scholes</td>
<td>401</td>
</tr>
<tr>
<td>8 Models and Equations</td>
<td>429</td>
</tr>
<tr>
<td>9 The Black–Scholes Formulae and the Greeks</td>
<td>453</td>
</tr>
<tr>
<td>10 Common Contracts</td>
<td>459</td>
</tr>
<tr>
<td>11 Popular Quant Books</td>
<td>483</td>
</tr>
<tr>
<td>12 The Most Popular Search Words and Phrases on Wilmott.com</td>
<td>497</td>
</tr>
<tr>
<td>13 Brainteasers</td>
<td>507</td>
</tr>
<tr>
<td>14 Paul &amp; Dominic’s Guide to Getting a Quant Job</td>
<td>557</td>
</tr>
<tr>
<td>Index</td>
<td>579</td>
</tr>
</tbody>
</table>
## Frequently Asked Questions

1. What are the different types of mathematics found in quantitative finance? 22

2. What is arbitrage? 27

3. What is put–call parity? 30

4. What is the Central Limit Theorem and what are its Implications for finance? 33

5. How is risk defined in mathematical terms? 38

6. What is value at risk and how is it used? 42

7. What is Extreme Value Theory? 46

8. What is CrashMetrics? 48

9. What is a coherent risk measure and what are its properties? 52

10. What is Modern Portfolio Theory? 55

11. What is the Capital Asset Pricing Model? 58

12. What is Arbitrage Pricing Theory? 62

13. What is Maximum Likelihood Estimation? 65

14. What is cointegration? 71

15. What is the Kelly criterion? 74

16. Why hedge? 77

17. What is marking to market and how does it affect risk management in derivatives trading? 83
18. What is the Efficient Markets Hypothesis? 87
19. What are the most useful performance measures? 90
20. What is a utility function and how is it used? 93
21. What is the difference between a quant and an actuary? 97
22. What is a Wiener process/Brownian motion and what are its uses in finance? 100
23. What is Jensen’s Inequality and what is its role in finance? 103
24. What is Itô’s lemma? 106
26. What is Girsanov’s theorem, and why is it important in finance? 113
27. What are the greeks? 116
29. What are the forward and backward equations? 125
30. What is the Black–Scholes equation? 129
31. Which numerical method should I use and when? 132
32. What is Monte Carlo simulation? 141
33. What is the finite-difference method? 145
34. What is a Poisson process and what are its uses in finance? 150
35. What is a jump-diffusion model and how does it affect option values? 152
36. What is meant by ‘complete’ and ‘incomplete’ markets? 155
37. Can I use real probabilities to price derivatives? 160
38. What is volatility? 162
39. What is the volatility smile? 167
40. What is GARCH? 174
41. How do I dynamically hedge? 179
42. What is serial autocorrelation and does it have a role in derivatives? 185
43. What is dispersion trading? 188
44. What is bootstrapping using discount factors? 191
45. What is the LIBOR market model and its principal applications in finance? 196
46. What is meant by the ‘value’ of a contract? 200
47. What is calibration? 203
48. What is Option Adjusted Spread? 206
49. What is the market price of risk? 208
50. Can I reverse engineer a partial differential equation to get at the model and contract? 212
51. What is the difference between the equilibrium approach and the no-arbitrage approach to modelling? 216
52. How good is the assumption of normal distributions for financial returns? 219
53. How robust is the Black–Scholes model? 223
54. Why is the lognormal distribution important? 226
55. What are copulas and how are they used in quantitative finance? 229
56. What is asymptotic analysis and how is it used in financial modelling? 233
57. What is a free-boundary problem and what is the optimal-stopping time for an American option? 236
58. What are low-discrepancy numbers? 240
59. What are the bastard greeks? 245
60. What are the stupidest things people have said about risk neutrality? 248
61. What is the best-kept secret in quantitative finance? 250
The previous edition of this book was aimed to some extent at those people wanting to get their first job in quantitative finance and perhaps needed something to refresh their memories just as they were going into an interview. In part it was meant to be orthogonal to those interview-prep books that focus almost exclusively on the math. Math is important, but I think it’s the easy bit of this business. It’s where the math breaks down that I think is crucial, and so there’s a lot of that in both the previous edition and this one.

In one respect I couldn’t have been more wrong when I said that the last book was for newbies. Recent events have shown that some of those most in need of a refresher in the fundamentals are not necessarily the newbies, but sometimes also the ‘experienced’ quants, the ‘respected’ academics and the ‘genius’ Nobel laureates. I think this book has an additional purpose now if you are going for a job. Yes, take this book with you, but use it to interrogate the interviewer. If he can’t answer your questions then don’t accept a job from him, his bank might not be around for much longer!

This second edition is not an ‘updating.’ Nothing in the previous edition is out of date … And that’s not a statement that many quant finance authors can make! One of the themes in my research and writing is that much of quantitative finance is too theoretical, it’s far too mathematical, and somewhere along the way common sense has got left behind. In 2000 I wrote that there needed to be a change in modelling methods if there was not to be a “mathematician-led market meltdown.” There wasn’t, so there was. In 2006 I narrowed this down to credit instruments and credit models. Sadly, money making got in the way of good modelling, and you no doubt know what ensued. But now more and more people are starting to appreciate the importance of getting the level of mathematics right, and this has to be a good thing for the industry. We ran a survey on wilmott.com, our famous ‘Name and Shame in Our New Blame Game!’ in which we asked members
to tell us which are the worst finance models. The results are published towards the end of this book so you'll see that common sense and robustness of modelling are on their way back.

To the thankees in the first edition I would like to add Emanuel Derman for allowing me to republish the “Financial Modelers’ Manifesto” that we wrote together at the start of 2009.

**Some more about the author**

Paul Wilmott is still learning the guitar, after 36 years trying. He now knows six chords. His only hobby at which he has been successful is reading, always fiction. But even so, he has been stuck at half way through James Joyce’s *Ulysses* for a decade, and has never got beyond the first 10 pages of anything by Salman Rushdie. Paul divides his time between his home in London and airport lounges around the world, where he can often be found nursing a dry martini.
Preface to the First Edition

This book grew out of a suggestion by wilmott.com Member ‘bayes’ for a Forum (as in ‘internet discussion group’) dedicated to gathering together answers to the most common quanty questions. We responded positively, as is our wont, and the Wilmott Quantitative Finance FAQs Project was born. This Forum may be found at www.wilmott.com/faq. (There anyone may read the FAQ answers, but to post a message you must be a member. Fortunately, this is entirely free!) The FAQs project is one of the many collaborations between Members of wilmott.com.

As well as being an ongoing online project, the FAQs have inspired the book you are holding. It includes FAQs and their answers and also sections on common models and formulæ, many different ways to derive the Black–Scholes model, the history of quantitative finance, a selection of brainteasers and a couple of sections for those who like lists (there are lists of the most popular quant books and search items on wilmott.com). Right at the end is an excerpt from Paul and Dominic’s Guide to Getting a Quant Job, this will be of interest to those of you seeking their first quant role.

FAQs in QF is not a shortcut to an in-depth knowledge of quantitative finance. There is no such shortcut. However, it will give you tips and tricks of the trade, and insight, to help
you to do your job or to get you through initial job interviews. It will serve as an aide memoire to fundamental concepts (including why theory and practice diverge) and some of the basic Black–Scholes formulæ and greeks. The subject is forever evolving, and although the foundations are fairly robust and static there are always going to be new products and models. So, if there are questions you would like to see answered in future editions please drop me an email at paul@wilmott.com.

I would like to thank all Members of the forum for their participation and in particular the following, more prolific, Members for their contributions to the online FAQs and Brain-teasers: Aaron, adas, Alan, bayes, Cuchulainn, exotiq, HA, kr, mj, mrbadguy, N, Omar, reza, WaaghBakri and zerdna. Thanks also to DCFC for his advice concerning the book.

I am grateful to Caitlin Cornish, Emily Pears, Graham Russel, Jenny McCall, Sarah Stevens, Steve Smith, Tom Clark and Viv Wickham at John Wiley & Sons Ltd for their continued support, and to Dave Thompson for his entertaining cartoons.

I am also especially indebted to James Fahy for making the Forum happen and run smoothly. Mahalo and aloha to my ever-encouraging wife, Andrea.

About the author

Paul Wilmott is an author, researcher, consultant and trainer in quantitative finance. He owns wilmott.com, is the Editor in Chief of the bimonthly quant magazine Wilmott and is the Course Director for the Certificate in Quantitative Finance (cqf.com). He is the author of the student text Paul Wilmott Introduces Quantitative Finance, which covers classical quant finance from the ground up, and Paul Wilmott on Quantitative Finance, the three-volume research-level epic. Both are also published by John Wiley & Sons, Ltd.
There follows a speedy, roller-coaster of a ride through the official history of quantitative finance, passing through both the highs and lows. Where possible I give dates, name names and refer to the original sources.¹

1827 Brown  The Scottish botanist, Robert Brown, gave his name to the random motion of small particles in a liquid. This idea of the random walk has permeated many scientific fields and is commonly used as the model mechanism behind a variety of unpredictable continuous-time processes. The lognormal random walk based on Brownian motion is the classical paradigm for the stock market. See Brown (1827).

1900 Bachelier  Louis Bachelier was the first to quantify the concept of Brownian motion. He developed a mathematical theory for random walks, a theory rediscovered later by Einstein. He proposed a model for equity prices, a simple normal distribution, and built on it a model for pricing the almost unheard of options. His model contained many of the seeds for later work, but lay ‘dormant’ for many, many years. It is told that his thesis was not a great success and, naturally, Bachelier’s work was not appreciated in his lifetime. See Bachelier (1995).

1905 Einstein  Albert Einstein proposed a scientific foundation for Brownian motion in 1905. He did some other clever stuff as well. See Stachel (1990).

1911 Richardson  Most option models result in diffusion-type equations. And often these have to be solved numerically. The two main ways of doing this are Monte Carlo and finite differences (a sophisticated version of the binomial model).

¹A version of this chapter was first published in *New Directions in Mathematical Finance*, edited by Paul Wilmott and Henrik Rasmussen, John Wiley & Sons Ltd, 2002.
The very first use of the finite-difference method, in which a differential equation is discretized into a difference equation, was by Lewis Fry Richardson in 1911, and used to solve the diffusion equation associated with weather forecasting. See Richardson (1922). Richardson later worked on the mathematics for the causes of war. During his work on the relationship between the probability of war and the length of common borders between countries he stumbled upon the concept of fractals, observing that the length of borders depended on the length of the ‘ruler.’ The fractal nature of turbulence was summed up in his poem “Big whorls have little whorls that feed on their velocity, and little whorls have smaller whorls and so on to viscosity.”

1923 Wiener Norbert Wiener developed a rigorous theory for Brownian motion, the mathematics of which was to become a necessary modelling device for quantitative finance decades later. The starting point for almost all financial models, the first equation written down in most technical papers, includes the Wiener process as the representation for randomness in asset prices. See Wiener (1923).

1950s Samuelson The 1970 Nobel Laureate in Economics, Paul Samuelson, was responsible for setting the tone for subsequent generations of economists. Samuelson ‘mathematized’ both macro and micro economics. He rediscovered Bachelier’s thesis and laid the foundations for later option pricing theories. His approach to derivative pricing was via expectations, real as opposed to the much later risk-neutral ones. See Samuelson (1955).

1951 Itô Where would we be without stochastic or Itô calculus? (Some people even think finance is only about Itô calculus.) Kiyosi Itô showed the relationship between a stochastic differential equation for some independent variable and the stochastic differential equation for a function of that variable. One of the starting points for classical derivatives theory is
the lognormal stochastic differential equation for the evolution of an asset. Itô’s lemma tells us the stochastic differential equation for the value of an option on that asset.

In mathematical terms, if we have a Wiener process $X$ with increments $dX$ that are normally distributed with mean zero and variance $dt$, then the increment of a function $F(X)$ is given by

$$dF = \frac{dF}{dX} dX + \frac{1}{2} \frac{d^2 F}{dX^2} dt$$

This is a very loose definition of Itô’s lemma but will suffice. See Itô (1951).

1952 Markowitz  
Harry Markowitz was the first to propose a modern quantitative methodology for portfolio selection. This required knowledge of assets’ volatilities and the correlation between assets. The idea was extremely elegant, resulting in novel ideas such as ‘efficiency’ and ‘market portfolios.’ In this Modern Portfolio Theory, Markowitz showed that combinations of assets could have better properties than any individual assets. What did ‘better’ mean? Markowitz quantified a portfolio’s possible future performance in terms of its expected return and its standard deviation. The latter was to be interpreted as its risk. He showed how to optimize a portfolio to give the maximum expected return for a given level of risk. Such a portfolio was said to be ‘efficient.’ The work later won Markowitz a Nobel Prize for Economics but is problematic to use in practice because of the difficulty in measuring the parameters ‘volatility,’ and, especially, ‘correlation,’ and their instability.

1963 Sharpe, Lintner and Mossin  
William Sharpe of Stanford, John Lintner of Harvard and Norwegian economist Jan Mossin independently developed a simple model for pricing risky assets. This Capital Asset Pricing Model (CAPM) also reduced the number of parameters needed for portfolio selection from those needed by Markowitz’s Modern Portfolio Theory,

**1966 Fama** Eugene Fama concluded that stock prices were unpredictable and coined the phrase ‘market efficiency.’ Although there are various forms of market efficiency, in a nutshell the idea is that stock market prices reflect all publicly available information, and that no person can gain an edge over another by fair means. See Fama (1966).

**1960s Sobol’, Faure, Hammersley, Haselgrove and Halton**... Many people were associated with the definition and development of quasi random number theory or low-discrepancy sequence theory. The subject concerns the distribution of points in an arbitrary number of dimensions in order to cover the space as efficiently as possible, with as few points as possible (see Figure 1.1). The methodology is used in the evaluation of multiple integrals among other things. These ideas would find a use in finance almost three decades later. See Sobol’ (1967), Faure (1969), Hammersley & Handscomb (1964), Haselgrove (1961) and Halton (1960).

**1968 Thorp** Ed Thorp’s first claim to fame was that he figured out how to win at casino Blackjack, ideas that were put into practice by Thorp himself and written about in his best-selling *Beat the Dealer*, the “book that made Las Vegas change its rules.” His second claim to fame is that he invented and built, with Claude Shannon, the information theorist, the world’s first wearable computer. His third claim to fame is that he used the ‘correct’ formulæ for pricing options, formulæ that were rediscovered and originally published several years later by the next three people on our list. Thorp used these formulæ to make a fortune for himself and his clients in the first ever quantitative finance-based hedge fund. He proposed dynamic hedging as a way of
removing more risk than static hedging. See Thorp (2002) for the story behind the discovery of the Black–Scholes formulæ.


---

**Figure 1.1:** They may not look like it, but these dots are distributed deterministically so as to have very useful properties.
The Black–Scholes model is based on geometric Brownian motion for the asset price $S$

$$dS = \mu S \, dt + \sigma S \, dX.$$ 

The Black–Scholes partial differential equation for the value $V$ of an option is then

$$\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0$$

1974 Merton, again In 1974 Robert Merton (Merton, 1974) introduced the idea of modelling the value of a company as a call option on its assets, with the company’s debt being related to the strike price and the maturity of the debt being the option’s expiration. Thus was born the structural approach to modelling risk of default, for if the option expired out of the money (i.e. assets had less value than the debt at maturity) then the firm would have to go bankrupt.

Credit risk became big, huge, in the 1990s. Theory and practice progressed at rapid speed during this period, urged on by some significant credit-led events, such as the Long Term Capital Management mess. One of the principals of LTCM was Merton who had worked on credit risk two decades earlier. Now the subject really took off, not just along the lines proposed by Merton but also using the Poisson process as the model for the random arrival of an event, such as bankruptcy or default. For a list of key research in this area see Schönbucher (2003).

1977 Boyle Phelim Boyle related the pricing of options to the simulation of random asset paths (Figure 1.2). He showed how to find the fair value of an option by generating lots of possible future paths for an asset and then looking at the average that the option had paid off. The future important
role of Monte Carlo simulations in finance was assured. See Boyle (1977).

1977 Vasicek So far quantitative finance hadn’t had much to say about pricing interest rate products. Some people were using equity option formulæ for pricing interest rate options, but a consistent framework for interest rates had not been developed. This was addressed by Vasicek. He started by modelling a short-term interest rate as a random walk and concluded that interest rate derivatives could be valued using equations similar to the Black–Scholes partial differential equation.
Oldrich Vasicek represented the short-term interest rate by a stochastic differential equation of the form

\[ dr = \mu(r, t) \, dt + \sigma(r, t) \, dX. \]

The bond pricing equation is a parabolic partial differential equation, similar to the Black–Scholes equation. See Vasicek (1977).

1979 Cox, Ross and Rubinstein Boyle had shown how to price options via simulations, an important and intuitively reasonable idea, but it was these three, John Cox, Stephen Ross and Mark Rubinstein, who gave option-pricing capability to the masses.

The Black–Scholes equation was derived using stochastic calculus and resulted in a partial differential equation. This was not likely to endear it to the thousands of students interested in a career in finance. At that time these were typically MBA students, not the mathematicians and physicists that are nowadays found on Wall Street. How could MBAs cope? An MBA was a necessary requirement for a prestigious career in finance, but an ability to count beans is not the same as an ability to understand mathematics. Fortunately Cox, Ross and Rubinstein were able to distil the fundamental concepts of option pricing into a simple algorithm requiring only addition, subtraction, multiplication and (twice) division. Even MBAs could now join in the fun. See Cox, Ross & Rubinstein (1979) and Figure 1.3.

1979–81 Harrison, Kreps and Pliska Until these three came onto the scene quantitative finance was the domain of either economists or applied mathematicians. Mike Harrison and David Kreps, in 1979, showed the relationship between option prices and advanced probability theory, originally in discrete time. Harrison and Stan Pliska in 1981 used the same ideas but in continuous time. From that moment until the mid 1990s applied mathematicians hardly got a look in. Theorem,

1986 Ho and Lee One of the problems with the Vasicek framework for interest-rate derivative products was that it didn’t give very good prices for bonds, the simplest of fixed-income products. If the model couldn’t even get bond prices right, how could it hope to correctly value bond options? Thomas Ho and Sang-Bin Lee found a way around this, introducing the idea of yield-curve fitting or calibration. See Ho & Lee (1986).

1992 Heath, Jarrow and Morton Although Ho and Lee showed how to match theoretical and market prices for simple bonds, the methodology was rather cumbersome and not easily generalized. David Heath, Robert Jarrow and Andrew Morton (HJM) took a different approach. Instead of modelling just a short rate and deducing the whole yield curve, they modelled the random evolution of the whole yield curve. The initial yield curve, and hence the value of simple interest
rate instruments, was an input to the model. The model cannot easily be expressed in differential equation terms and so relies on either Monte Carlo simulation or tree building. The work was well known via a working paper, but was finally published, and therefore made respectable in Heath, Jarrow & Morton (1992).

1990s Cheyette, Barrett, Moore and Wilmott When there are many underlyings, all following lognormal random walks, you can write down the value of any European non-path-dependent option as a multiple integral, one dimension for each asset. Valuing such options then becomes equivalent to calculating an integral. The usual methods for quadrature are very inefficient in high dimensions, but simulations can prove quite effective. Monte Carlo evaluation of integrals is based on the idea that an integral is just an average multiplied by a ‘volume.’ And since one way of estimating an average is by picking numbers at random we can value a multiple integral by picking integrand values at random and summing. With \( N \) function evaluations, taking a time of \( O(N) \) you can expect an accuracy of \( O(1/N^{1/2}) \), independent of the number of dimensions. As mentioned above, breakthroughs in the 1960s on low-discrepancy sequences showed how clever, non-random, distributions could be used for an accuracy of \( O(1/N) \), to leading order. (There is a weak dependence on the dimension.) In the early 1990s several groups of people were simultaneously working on valuation of multi-asset options. Their work was less of a breakthrough than a transfer of technology.

They used ideas from the field of number theory and applied them to finance. Nowadays, these low-discrepancy sequences are commonly used for option valuation whenever random numbers are needed. A few years after these researchers made their work public, a completely unrelated group at Columbia University successfully patented the work. See Oren Cheyette (1990) and John Barrett, Gerald Moore & Paul Wilmott (1992).
Another discovery was made independently and simultaneously by three groups of researchers in the subject of option pricing with deterministic volatility. One of the perceived problems with classical option pricing is that the assumption of constant volatility is inconsistent with market prices of exchange-traded instruments. A model is needed that can correctly price vanilla contracts, and then price exotic contracts consistently. The new methodology, which quickly became standard market practice, was to find the volatility as a function of underlying and time that when put into the Black–Scholes equation and solved, usually numerically, gave resulting option prices which matched market prices. This is what is known as an inverse problem: use the ‘answer’ to find the coefficients in the governing equation. On the plus side, this is not too difficult to do in theory. On the minus side, the practice is much harder, the sought volatility function depending very sensitively on the initial data. From a scientific point of view there is much to be said against the methodology. The resulting volatility structure never matches actual volatility, and even if exotics are priced consistently it is not clear how to best hedge exotics with vanillas in order to minimize any model error. Such concerns seem to carry little weight, since the method is so ubiquitous. As so often happens in finance, once a technique becomes popular it is hard to go against the majority. There is job safety in numbers. See Emanuel Derman & Iraj Kani (1994), Bruno Dupire (1994) and Mark Rubinstein (1994).

Marco Avellaneda and Antonio Parás were, together with Arnon Levy and Terry Lyons, the creators of the uncertain volatility model for option pricing. It was a great breakthrough for the rigorous, scientific side of finance theory, but the best was yet to come. This model, and many that succeeded it, was nonlinear. Nonlinearity in an option pricing model means that the value of a portfolio of contracts is not necessarily the same as the sum of the values of its constituent parts. An option will have a different
value depending on what else is in the portfolio with it, and an exotic will have a different value depending on what it is statically hedged with. Avellaneda and Parás defined an exotic option’s value as the highest possible marginal value for that contract when hedged with any or all available exchange-traded contracts. The result was that the method of option pricing also came with its own technique for static hedging with other options. Prior to their work the only result of an option pricing model was its value and its delta, only dynamic hedging was theoretically necessary. With this new concept, theory became a major step closer to practice. Another result of this technique was that the theoretical price of an exchange-traded option exactly matched its market price. The convoluted calibration of volatility surface models was redundant. See Avellaneda & Parás (1996).

1997 Brace, Gatarek and Musiela Although the HJM interest rate model had addressed the main problem with stochastic spot rate models, and others of that ilk, it still had two major drawbacks. It required the existence of a spot rate and it assumed a continuous distribution of forward rates. Alan Brace, Dariusz Gatarek & Marek Musiela (1997) got around both of those difficulties by introducing a model which only relied on a discrete set of rates – ones that actually are traded. As with the HJM model the initial data are the forward rates so that bond prices are calibrated automatically. One specifies a number of random factors, their volatilities and correlations between them, and the requirement of no arbitrage then determines the risk-neutral drifts. Although B, G and M have their names associated with this idea many others worked on it simultaneously.

2000 Li As already mentioned, the 1990s saw an explosion in the number of credit instruments available, and also in the growth of derivatives with multiple underlyings. It’s not a great step to imagine contracts depending on the default of many underlyings. Examples of these are the once ubiquitous
Collateralized Debt Obligations (CDOs). But to price such complicated instruments requires a model for the interaction of many companies during the process of default. A probabilistic approach based on copulas was proposed by David Li (2000). The copula approach allows one to join together (hence the word ‘copula’) default models for individual companies in isolation to make a model for the probabilities of their joint default. The idea was adopted universally as a practical solution to a complicated problem. However with the recent financial crisis the concept has come in for a lot of criticism.

2002 Hagan, Kumar, Lesniewski and Woodward There has always been a need for models that are both fast and match traded prices well. The interest-rate model of Pat Hagan, Deep Kumar, Andrew Lesniewski and Diana Woodward (2002), which has come to be called the SABR (stochastic, $\alpha$, $\beta$, $\rho$) model, is a model for a forward rate and its volatility, both of which are stochastic. This model is made tractable by exploiting an asymptotic approximation to the governing equation that is highly accurate in practice. The asymptotic analysis simplifies a problem that would otherwise have to be solved numerically. Although asymptotic analysis has been used in financial problems before, for example in modelling transaction costs, this was the first time it really entered mainstream quantitative finance.

August 2007 quantitative finance in disrepute In early August 2007 several hedge funds using quantitative strategies experienced losses on such a scale as to bring the field of quantitative finance into disrepute. From then, and through 2008, trading of complex derivative products in obscene amounts using simplistic mathematical models almost brought the global financial market to its knees: Lend to the less-than-totally-creditworthy for home purchase, repackage these mortgages for selling on from one bank to another, at each stage adding complexity, combine with overoptimistic