Suspension Geometry and Computation

John C. Dixon, PhD, F.I.Mech.E., F.R.Ae.S.

Senior Lecturer in Engineering Mechanics
The Open University, Great Britain.
Suspension Geometry and Computation
By the same author:

Tires, Suspension and Handling, 2nd edn (SAE, Arnold).
The High-Performance Two-Stroke Engine (Haynes)
Suspension Geometry and Computation

John C. Dixon, PhD, F.I.Mech.E., F.R.Ae.S.

Senior Lecturer in Engineering Mechanics
The Open University, Great Britain.
This work is dedicated to

Aythe

the beautiful goddess of truth, hence also of science and mathematics, and of good computer programs.

Her holy book is the book of nature.
# Contents

## Preface

### 1 Introduction and History

1.1 Introduction 1
1.2 Early Steering History 1
1.3 Leaf-Spring Axles 3
1.4 Transverse Leaf Springs 8
1.5 Early Independent Fronts 10
1.6 Independent Front Suspension 13
1.7 Driven Rigid Axles 20
1.8 De Dion Rigid Axles 24
1.9 Undriven Rigid Axles 24
1.10 Independent Rear Driven 26
1.11 Independent Rear Undriven 32
1.12 Trailing-Twist Axles 34
1.13 Some Unusual Suspensions 35
   References 42

### 2 Road Geometry

2.1 Introduction 43
2.2 The Road 45
2.3 Road Curvatures 48
2.4 Pitch Gradient and Curvature 49
2.5 Road Bank Angle 51
2.6 Combined Gradient and Banking 53
2.7 Path Analysis 53
2.8 Particle-Vehicle Analysis 55
2.9 Two-Axle-Vehicle Analysis 57
2.10 Road Cross-Sectional Shape 59
2.11 Road Torsion 61
2.12 Logger Data Analysis 61
   References 63

### 3 Road Profiles

3.1 Introduction 65
3.2 Isolated Ramps 65
3.3 Isolated Bumps 67
3.4 Sinusoidal Single Paths 69
3.5 Sinusoidal Roads 71
3.6 Fixed Waveform 74
3.7 Fourier Analysis 75
3.8 Road Wavelengths 77
3.9 Stochastic Roads 77
References 82

4 Ride Geometry 83
4.1 Introduction 83
4.2 Wheel and Tyre Geometry 83
4.3 Suspension Bump 88
4.4 Ride Positions 88
4.5 Pitch 90
4.6 Roll 90
4.7 Ride Height 92
4.8 Time-Domain Ride Analysis 95
4.9 Frequency-Domain Ride Analysis 96
4.10 Workspace 97

5 Vehicle Steering 99
5.1 Introduction 99
5.2 Turning Geometry – Single Track 100
5.3 Ackermann Factor 103
5.4 Turning Geometry – Large Vehicles 108
5.5 Steering Ratio 111
5.6 Steering Systems 112
5.7 Wheel Spin Axis 113
5.8 Wheel Bottom Point 116
5.9 Wheel Steering Axis 118
5.10 Caster Angle 118
5.11 Camber Angle 119
5.12 Kingpin Angle Analysis 120
5.13 Kingpin Axis Steered 123
5.14 Steer Jacking 124
References 125

6 Bump and Roll Steer 127
6.1 Introduction 127
6.2 Wheel Bump Steer 127
6.3 Axle Steer Angles 131
6.4 Roll Steer and Understeer 132
6.5 Axle Linear Bump Steer and Roll Steer 133
6.6 Axle Non-Linear Bump Steer and Roll Steer 134
6.7 Axle Double-Bump Steer 136
6.8 Vehicle Roll Steer 136
6.9 Vehicle Heave Steer 137
15.12 Spheres 307
15.13 Circles 308
15.14 Routine PointFPL2P 309
15.15 Routine PointFPLPDC 309
15.16 Routine PointITinit 310
15.17 Routine PointIT 312
15.18 Routine PointFPT 313
15.19 Routine Plane3P 313
15.20 Routine PointFP 314
15.21 Routine PointFPP13P 314
15.22 Routine PointATinit 315
15.23 Routine PointAT 316
15.24 Routine Points3S 316
15.25 Routine Points2SHP 318
15.26 Routine Point3Pl 319
15.27 Routine ‘PointLP’ 320
15.28 Routine Point3SV 321
15.29 Routine PointITV 321
15.30 Routine PointATV 322
15.31 Rotations 323

16 Programming Considerations 325
16.1 Introduction 325
16.2 The RASER Value 325
16.3 Failure Modes Analysis 326
16.4 Reliability 327
16.5 Bad Conditioning 328
16.6 Data Sensitivity 329
16.7 Accuracy 330
16.8 Speed 331
16.9 Ease of Use 332
16.10 The Assembly Problem 332
16.11 Checksums 334

17 Iteration 335
17.1 Introduction 335
17.2 Three Phases of Iteration 336
17.3 Convergence 337
17.4 Binary Search 338
17.5 Linear Iterations 339
17.6 Iterative Exits 340
17.7 Fixed-Point Iteration 343
17.8 Accelerated Convergence 344
17.9 Higher Orders without Derivatives 346
17.10 Newton’s Iterations 348
17.11 Other Derivative Methods 350
17.12 Polynomial Roots 351
17.13 Testing 354
  References 357
Appendix A: Nomenclature 359
Appendix B: Units 377
Appendix C: Greek Alphabet 379
Appendix D: Quaternions for Engineers 381
Appendix E: Frenet, Serret and Darboux 393
Appendix F: The Fourier Transform 395
References and Bibliography 403
Index 407
The motor car is over one hundred years old. The suspension is an important part of its design, and there have been many research papers and several books on the topic. However, suspension analysis and design are so complex and bring together so many fields of study that they seem inexhaustible. Certainly, the number of different suspension designs that have been used over the years is considerable, and each one was right for a particular vehicle in that designer’s opinion. This breadth of the field, at least, is a justification for another book. It is not that the existing ones are not good, but that new perspectives are possible, and often valuable, and there are new things to say. Here, the focus is on the most fundamental aspect of all, the geometry of the road, the vehicle and the suspension, the basic measurement which is the foundation of all subsequent dynamic analysis.

The process of modern engineering has been deeply affected by the computer, not always for the better because analytical solutions may be neglected with loss of insight to the problem. In this case the solution of complex three-dimensional geometrical problems is greatly facilitated by true coordinate geometry solutions or by iteration, methods which are discussed here in some detail.

In principle, geometry is not really conceptually difficult. Wrestling with actual problems shows otherwise. Analytical geometry, particularly on the computer with its many digits of precision, mercilessly shows up any approximations and errors, and, surprisingly, often reveals incomplete understanding of deep principles.

New material presented in detail here includes relationships between bump, heave and roll coefficients (Table 8.10.1), detailed analysis of linear and non-linear bump steer, design methods for determining wishbone arm lengths and angles, methods of two-dimensional and three-dimensional solutions of suspension-related geometry, and details of numerical iterative methods applied to three-dimensional suspensions, with examples.

As in my previous work, I have tried to present the basic core of theory and practice, so that the book will be of lasting value. I would be delighted to hear from readers who wish to suggest any improvements to presentation or coverage.

John C. Dixon.
1

Introduction and History

1.1 Introduction

To understand vehicle performance and cornering, it is essential to have an in-depth understanding of the basic geometric properties of roads and suspensions, including characteristics such as bump steer, roll steer, the various kinds of roll centre, and the relationships between them.

Of course, the vehicle is mainly a device for moving passengers or other payload from A to B, although in some cases, such as a passenger car tour, a motor race or rally, it is used for the interest of the movement itself. The route depends on the terrain, and is the basic challenge to be overcome. Therefore road characteristics are examined in detail in Chapter 2. This includes the road undulations giving ride quality problems, and road lateral curvature giving handling requirements. These give rise to the need for suspension, and lead to definite requirements for suspension geometry optimisation.

Chapter 3 analyses the geometry of road profiles, essential to the analysis of ride quality and handling on rough roads. Chapter 4 covers suspension geometry as required for ride analysis. Chapter 6 deals with steering geometry. Chapters 6–9 study the geometry of suspensions as required for handling analysis, including bump steer, roll steer, camber, roll centres, compliance steer, etc., in general terms.

Subsequent chapters deal with the properties of the main particular types of suspension, using the methods introduced in the earlier chapters. Then the computational methods required for solution of suspension geometry problems are studied, including two- and three-dimensional coordinate geometry, and numerical iteration.

This chapter gives an overview of suspensions in qualitative terms, with illustrations to show the main types. It is possible to show only a sample of the innumerable designs that have been used.

1.2 Early Steering History

The first common wheeled vehicles were probably single-axle hand carts with the wheels rotating independently on the axle, this being the simplest possible method, allowing variations of direction without any steering mechanism. This is also the basis of the lightweight horse-drawn chariot, already important many thousands of years ago for its military applications. Sporting use also goes back to antiquity, as illustrated in films such as Ben Hur with the famous chariot race. Suspension, such as it was, must have been important for use on rough ground, for some degree of comfort, and also to minimise the stress of the structure, and was based on general compliance rather than the inclusion of special spring members. The axle can be made long and allowed to bend vertically and longitudinally to ride the bumps. Another important factor in riding over rough roads is to use large wheels.
For more mundane transport of goods, a heavier low-speed two-axle cart was desirable, and this requires some form of steering mechanism. Initially this was achieved by the simple means of allowing the entire front axle to rotate, as shown in Figure 1.2.1(a).

Figure 1.2.1  Steering: (a) basic cart steering by rotating the whole axle; (b) Langensperger’s independent steering of 1816.

For more mundane transport of goods, a heavier low-speed two-axle cart was desirable, and this requires some form of steering mechanism. Initially this was achieved by the simple means of allowing the entire front axle to rotate, as shown in Figure 1.2.1(a).

Figure 1.2.2  Ackermann steering effect achieved by two cams on L’Obeissante, designed by Amedée Bollée in 1873.
Steering by the movement of the whole axle gives good geometric positioning, with easy low-speed manoeuvring, but the movement of the axle takes up useful space. To overcome this, the next stage was to steer the wheels independently, each turning about an axis close to the wheel. The first steps in this direction were taken by Erasmus Darwin (1731–1802), who had built a carriage for his doctor’s practice, allowing larger-diameter wheels of great help on the rough roads. However, if the two wheels are steered through the same angle then they must slip sideways somewhat during cornering, which greatly increases the resistance to motion in tight turns. This is very obvious when a parallel-steered cart is being moved by hand. To solve this, the two wheels must be steered through different angles, as in Figure 1.2.1(b). The origin of this notion may be due to Erasmus Darwin himself in 1758, or to Richard Edgeworth, who produced the earliest known drawing of such a system. Later, in 1816 Langensperger obtained a German patent for such a concept, and in 1817 Rudolf Ackermann, acting as Langensperger’s agent, obtained a British patent. The name Ackermann has since then been firmly attached to this steering design. The first application of this steering to a motor vehicle, rather than hand or horse-drawn carts, was by Edward Butler. The simplest way to achieve the desired geometry is to angle the steering arms inwards in the straight-ahead position, and to link them by a tie rod (also known as a track rod), as was done by Langensperger. However, there are certainly other methods, as demonstrated by French engineer Amedée Bollée in 1873, Figure 1.2.2, possibly allowing a greater range of action, that is, a smaller minimum turning circle.

The ‘La Mancelle’ vehicle of 1878 (the name refers to a person or thing from Le Mans) achieved the required results with parallel steering arms and a central triangular member, Figure 1.2.3. In 1893 Benz obtained a German patent for the same system, Figure 1.2.4. This shows tiller control of the steering, the common method of the time. In 1897 Benz introduced the steering wheel, a much superior system to the tiller, for cars. This was rapidly adopted by all manufacturers. For comparison, it is interesting to note that dinghies use tillers, where it is suitable, being convenient and economic, but ships use a large wheel, and aircraft use a joystick for pitch and roll, although sometimes they have a partial wheel on top of a joystick with only fore–aft stick movement.

1.3 Leaf-Spring Axles

Early stage coaches required suspension of some kind. With the limited technology of the period, simple wrought-iron beam springs were the practical method, and these were made in several layers to obtain the required combination of compliance with strength. These multiple-leaf springs became known simply as leaf springs. To increase the compliance, a pair of leaf springs were mounted back-to-back. They were curved, and so then known, imprecisely, as elliptical springs, or elliptics for short. Single ones were called
semi-elliptics. In the very earliest days of motoring, these were carried over from the stage coaches as the one practical form of suspension, as may be seen in Figure 1.3.1.

The leaf spring was developed in numerous variations over the next 50 years, for example as in Figure 1.3.2. With improving quality of steels in the early twentieth century, despite the increasing average

Figure 1.2.4  German patent of 1893 by Benz for a mechanism to achieve the Ackermann steering effect, the same mechanism as La Mancelle.

Figure 1.3.1  Selden’s 1895 patent showing the use of fully-elliptic leaf springs at the front A and rear B. The steering wheel is C and the foot brake D.
weight of motor cars, the simpler semi-elliptic leaf springs became sufficient, and became widely standardised in principle, although with many detailed variations, not least in the mounting systems, position of the shackle, which is necessary to permit length variation, and so on. The complete vehicle of Figure 1.3.3 shows representative applications at the front and rear, the front having a single compression shackle, the rear two tension shackles. A very real advantage of the leaf spring in the early days was that the spring provides lateral and longitudinal location of the axle in addition to the springing compliance action. However, as engine power and speeds increased, the poor location geometry of the leaf spring became an increasing problem, particularly at the front, where the steering system caused many problems in bump and roll. To minimise these difficulties, the suspension was made stiff, which caused poor ride quality.

Figures 1.3.4 and 1.3.5 show representative examples of the application of the leaf spring at the rear of normal configuration motor cars of the 1950s and 1960s, using a single compression shackle. Greatly improved production machinery by the 1930s made possible the mass production of good quality coil springs, which progressively replaced the leaf spring for passenger cars. However, leaf-spring use on passenger cars continued through into the 1970s, and even then it functioned competitively, at the rear at least, Figure 1.3.6. The leaf spring is still widely used for heavily loaded axles on trucks and military vehicles, and has some advantages for use in remote areas where only basic maintenance is possible, so leaf-spring geometry problems are still of real practical interest.

Figure 1.3.2  Some examples of the variation of leaf springs in the early days. As is apparent here, the adjective ‘elliptical’ is used only loosely.

Figure 1.3.3  Grand Prix car of 1908, with application of semi-elliptic leaf springs at the front and rear (Mercedes-Benz).
At the front, the leaf spring was much less satisfactory, because of the steering geometry difficulties (bump steer, roll steer, brake wind-up steering effects, and shimmy vibration problems). Figure 1.3.7 shows a representative layout of the typical passenger car rigid-front-axle system up to about 1933. In bump, the axle arc of movement is centred at the front of the spring, but the steering arm arc is centred at

Figure 1.3.4 A representative rear leaf-spring assembly (Vauxhall).

Figure 1.3.5 A 1964 live rigid rear axle with leaf springs, anti-roll bar and telescopic dampers. The axle clamps on top of the springs (Maserati).
the steering box. These conflicting arcs give a large and problematic bump steer effect. The large bump steer angle change also contributed to the shimmy problems by causing gyroscopic precession moments on the wheels. Figure 1.3.8 shows an improved system with a transverse connection.

Truck and van steering with a leaf spring generally has the steering box ahead of the axle, to give the maximum payload space, as seen in Figure 1.3.9. In bump, the arc of motion of the steering arm and the axle on the spring are in much better agreement than with the rear box arrangement of Figure 1.3.7, so bump steer is reduced. Also, the springs are likely to be much stiffer, with reduced range of suspension movement, generally reducing the geometric problems.

Figure 1.3.6  Amongst the last of the passenger car leaf-spring rear axles used by a major manufacturer was that of the Ford Capri. Road testers at the time found this system in no way inferior to more modern designs.
1.4 Transverse Leaf Springs

Leaf springs were not used only in longitudinal alignment. There have been many applications with transverse leaf springs. In some cases, these were axles or wheel uprights located by separate links, to overcome the geometry problems, with the leaf spring providing only limited location service, or only the springing action. Some transverse leaf examples are given in Figures 1.4.1–1.4.4

Figure 1.3.8 Alternative application of the rigid axle at the front of a passenger car, with a transverse steering link between the steering box on the sprung mass and the axle, reducing bump steer problems.

Figure 1.3.9 Van or truck steering typically has a much steeper steering column with a steering box forward of the axle, as here. The steering geometry problems are different in detail, but may be less overall because a stiffer suspension is more acceptable.
Figure 1.4.1  A transverse leaf spring at the top also provides upper lateral and longitudinal location on this front axle, with a lower wishbone (early BMW).

Figure 1.4.2  This more modern small car front suspension has a transverse leaf spring at the bottom with an upper wishbone (Fiat).

Figure 1.4.3  Two transverse leaf springs providing complete hub location acting as equal-length wishbones without any additional links (1931, Mercedes Benz).
1.5 Early Independent Fronts

Through the 1920s, the rigid axle at the front was increasingly a problem. Despite considerable thought and experimentation by suspension design engineers, no way had been found to make a steering system that worked accurately. In other words, there were major problems with bump steer, roll steer and spring wind-up, particularly during braking. Any one of these problems might be solved, but not all at once. With increasing engine power and vehicle speeds, this was becoming increasingly dangerous, and hard front springs were required to ameliorate the problem, limiting the axle movement, but this caused very poor ride comfort. The answer was to use independent front suspension, for which a consistently accurate steering system could be made, allowing much softer springs and greater comfort. Early independent suspension designs were produced by Andrée Dubonnet in France in the late 1920s, and a little later for Rolls-Royce by Donald Bastow and Maurice Olley in England. These successful applications of independent suspension became known in the USA, and General Motors president Alfred P. Sloan took action, as he describes in his autobiography (Sloan, 1963).

Around 1930, Sloan considered the problem of ride quality as one of the most pressing and most complex in automotive engineering, and the problem was getting worse as car speeds increased. The early solid rubber tyres had been replaced by vented thick rubber, and then by inflated tyres. In the 1920s, tyres became even softer, which introduced increased problems of handling stability and axle vibrations. On a trip to Europe, Sloan met French engineer Andrée Dubonnet who had patented a successful independent suspension, and had him visit the US to make contact with GM engineers. Also, by 1933 Rolls-Royce already had an independent front suspension, which was on cars imported to the USA. Maurice Olley, who had previously worked for Rolls-Royce, was employed by GM, and worked on the introduction of independent suspensions there. In Sloan’s autobiography, a letter from Olley describes an early ride meter, which was simply an open-topped container of water, which was weighed after a measured mile at various speeds. Rolls-Royce had been looking carefully at ride dynamics, including measuring body inertia, trying to get a sound scientific understanding of the problem, and Olley introduced this approach at GM. In 1932 they built the K-squared rig (i.e. radius of gyration squared), a test car with various heavy added masses right at the front and rear to alter the pitch inertia in a controlled way. This brought home the realisation that a much superior ride could be achieved by the use of softer front springs, but soft springs...
caused shimmy problems and bad handling. Two experimental Cadillac cars were built, one using Dubonnet’s type of suspension, the other with a double-wishbone (double A-arm) suspension of GM’s design. The engineers were pleased with the ride and handling, but shimmy steering vibration was a persistent problem requiring intensive development work. In March 1933 these two experimental cars were demonstrated to GM’s top management, along with an automatic transmission. Within a couple of miles, the ‘flat ride’ suspension was evidently well received.

March 1933 was during the Great Depression, and financial constraints on car manufacturing and retail prices were pressing, but the independent front suspension designs were enthusiastically accepted, and shown to the public in 1934. In 1935 Chevrolet and Pontiac had cars available with Dubonnet suspension, whilst Cadillac, Buick and Oldsmobile offered double-wishbone front suspension, and the rigid front axle was effectively history, for passenger cars at least. A serious concern for production was the ability of the machine tool industry to produce enough suitable centreless grinders to make all the coil springs that would be required. With some practical experience, it became apparent that with development the wishbone suspension was easier and cheaper to manufacture, and also more reliable, and was universally adopted.

Figure 1.5.1 shows the 1934 Cadillac independent suspension system, with double wishbones on each side, in which it may be seen that the basic steering concept is recognisably related to the ones described earlier. As covered in detail in Chapter 6, the track-rod length and angle can be adjusted to give good steering characteristics, controlling bump steer and roll steer. The dampers were the lever-operated double-piston type, incorporated into the upper wishbone arms. Such a system would still be usable today.

Figure 1.5.2 shows the Dubonnet type suspension, used by several other manufacturers, which was unusually compact. The wheels are on leading or trailing arms, with the spring contained in a tube on the

![Figure 1.5.1](image1)

**Figure 1.5.1** The new Cadillac steering and independent suspension of 1934.
Figure 1.5.2  The Dubonnet type suspension in plan view, front at the top: (a) with trailing links; (b) with leading links (1938 Opel).

Figure 1.5.3  Broulhet ball-spline sliding pillar independent suspension.