DESIGN OF ULTRA WIDEBAND POWER TRANSFER NETWORKS

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This book is dedicated to my wife, Prof. Dr Md. Sema Yarman, and my son, Dr Can Evren Yarman, for their continuous support, endless patience and love.
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About the Author

Professor Dr. B. S. Yarman received his BSc, MEE and PhD degrees from Istanbul Technical University; Stevens Institute of Technology, Hoboken, NJ; and Cornell University, Ithaca, NY, respectively. He was a Member of Technical Staff at RCA David Sarnoff Research Center, Princeton, NJ, where he was responsible for designing various broadband microwave and satellite communication systems for commercial and military use.

Professor Yarman has taught and done research at Anadolu University, Middle East Technical University, Istanbul University and Istanbul Technical University in Turkey; Stevens Institute of Technology and Cornell University in the USA; Ruhr University in Germany; and Tokyo Institute of Technology in Japan.

He is currently the Chairman of the Department of Electrical–Electronics Engineering and the Coordinator of Scientific Research Projects of Istanbul University, Turkey.

He was the founding president of Isik University and was one of the founders of International Education Research and Engineering Consulting Inc. in Maryland USA; Savronik Defence Electronics Corp. of Turkey; and ARES Electronic Security Inc., Istanbul, Turkey.

He has served as a consultant on the design of various broadband matching networks and microwave amplifiers for many commercial and military agencies in the USA, Europe and Asia Pacific, as well as in Turkey.

He has published more than 300 journal and conference papers as well as technical reports in the area of broadband matching networks, microwave amplifiers, digital phase shifters, speech and biomedical signal processing (ECG, EEC, EMG, etc.) and decision making. He is the author of the books ‘Design of Multistage Microwave Amplifiers via Simplified Real Frequency Technique’ published by Scientific Research and Technology Council of Turkey, 1986; ‘Design of Ultra Wideband Antenna Matching Networks’ by Springer-Verlag’ 2008; and ‘Design of Ultra Wideband Power Transfer Networks’ by John Wiley & Sons, Ltd, 2009. He also holds four US patents as assigned to the US Air Force.

Professor Yarman is the recipient of the Young Turkish Scientist Award, Technology Award of National Science-Technology and Research Counsel of Turkey. He is a Fellow of the Alexander Von Humboldt Foundation, Germany; a Member of the New York Academy of Science; ‘Man of the Year in Science and Technology’ in 1998 of Cambridge Biography Centre, UK; and IEEE Fellow for his contribution to ‘Computer Aided Design of Broadband Amplifiers’.

He is married to Prof. Dr. Md. Sema Yarman of Istanbul University and is the father of Dr. Can Evren Yarman of Schlumberger Houston, Texas.
Power transfer networks (PTNs) are essential units of communications systems. For example, if the system is a transmitter, a PTN must be placed between the output of the power amplifier and the antenna. If the system is a receiver, the PTN is placed between the antenna and the low-noise amplifier. Any interface or interstage connection must be made over a PTN.

In general, PTNs are lossless two-ports. They transfer the frequency-dependent power between ports over a prescribed frequency band. Depending on the application, they are referred to as filters, matching networks or equalizers.

From the circuit theory point of view, a port may be modeled as a simple resistor or as a complex impedance. In this regard, the power transfer problem is defined as the ‘construction of a lossless two-port between the given terminations over a specified frequency band’.

In the course of the PTN design process, power transfer is maximized from the source to the receiving end over the band of interest.

From the physical nature of the problem, we can only transfer a fraction of the available power of the generator. In this case, our concern is with the power transfer ratio, which is defined as the power delivered to the load in relation to the available power of the generator. This ratio is called transducer power gain (TPG).

In practice, our desire is to make the power transfer as flat and as high as possible over the passband. It is well established that flat TPG level is restricted by the complex terminations. This is called the gain–bandwidth limit of the power transfer problem under consideration. In the classical literature, gain–bandwidth problems are known as broadband matching problems. They may be classified as follows:

- Filter or insertion loss problem: In this problem, a lossless two-port is constructed between the resistive terminations over the specified passband. In other words, the goal of the filter problem is to restrict the power transfer to a selected frequency band. In this case, ideally, the flat TPG level can be unity if an infinite number of reactive elements are used in the lossless two-port.
- Single matching problem: In this problem, a lossless two-port is constructed between a resistive generator and a complex load. It has been shown that the ideal flat TPG level is dictated by the complex load and is less than unity.
- Double matching problem: This is the generalized version of the single matching problem where both the generator and load networks are complex impedances. Therefore, the flat TPG level is even further reduced than those of the single matching problems defined by either the generator or load impedance.
- Active matching problem: In this problem an active device is matched to a complex generator at the frontend and also simultaneously matched to a complex load at the backend.
- Equalization problem: In this problem, a lossless two-port is constructed between resistive terminations which approximate predefined arbitrary TPG shape over a prescribed frequency band.
This book covers all the power transfer problems comprehensively. Solutions to many practical problems are provided with design software (S/W) packages developed on MATLAB®. In this regard, the book is unique.

In order to tackle power transfer problems thoroughly, an understanding of circuit theory with lumped and distributed elements is essential. Furthermore, the practical implementation of PTNs requires a straightforward application of electromagnetic field theory. Hence, the book is organized as follows.

Chapter 1 covers the basic ingredients of circuit theory from the power transfer point of view. In this chapter, it is emphasized that lumped elements are dimensionless. Furthermore, they ‘do not care’ about the velocity of power transfer. Therefore, they are ideal and excellent tools for designing PTNs. However, in practice we need more.

Chapter 2 is devoted to electromagnetic fields and waves, where we define all the passive lumped circuit components from the field theory perspective by introducing material properties and geometric layouts. Moreover, major properties of ideal transmission lines are derived by employing electromagnetic field theory, which makes power transfer issues physically understandable.

In Chapter 3, transmission lines are introduced as viable practical circuit components having geometric dimensions. From a practical implementation point of view, it is shown that a short transmission line may act like an inductor in series configuration, or like a capacitor in shunt configuration. It may even behave like a transformer or resonance circuit if its length or operating frequency is adjusted properly. In this chapter, we also introduce a complex variable denoted by $\lambda = \Sigma + j\Omega$ which is called the Richard variable. It is shown that lossless networks constructed with equal length or commensurate transmission lines can be described by means of classical network functions such as impedance and admittance functions by $\lambda = \tanh(p\tau)$. In this representation, $p = \sigma + j\omega$ is the classical complex domain variable which is used to describe the functions derived from the networks constructed with lumped elements and $\tau$ is the constant delay of the commensurate transmission lines.

In Chapter 4, the concept of unit element (UE) is introduced and the properties of circuits constructed with commensurate transmission lines or UEs are presented. Natural definitions of incident and reflected waves are given and then a scattering description of lossless two-ports is introduced in the Richard domain. The power transfer issue is studied by means of the scattering parameters. Features of filters designed with commensurate transmission lines are provided from experiments run on MATLAB employing the design tools developed for this chapter. Many practical examples are presented to demonstrate the utilization of the design packages.

In Chapter 5, the general equalization problem is solved by employing UEs via the scattering approach, which is known as the simplified real frequency technique (SRFT). Examples and design S/W are provided.

In Chapter 6, the properties of lossless two-ports constructed with lumped circuit elements are investigated by means of scattering parameters. A formal definition of power transfer is introduced and a definition of transfer scattering parameters is given. The power transfer properties of cascade connections of two-ports are then derived. Examples are presented to help the reader understand the properties of lossless two-ports from a design point of view.

As far as computer-aided design of PTNs is concerned, descriptions of lossless two-ports in terms of ‘easy to use’ parameters are crucial. For example, a lossless two-port can be described by means of the component values of a chosen circuit topology. In this regard, TPG is expressed as a function of the unknown element values. Then it is optimized to satisfy the design specifications, which in turn yield the element values of the chosen topology. Unfortunately, this task is very difficult since TPG is highly nonlinear in terms of the unknown element values. On the other hand, it is always desirable to deal with quadratic objective functions in optimization problems.

In 1977, Professor H. J. Carlin of Cornell University proposed a design method for the solution of single matching problems, which deals with quadratic objective functions. The new method of broadband matching is called the ‘real frequency line segment technique (RFLT)’ and it is based on the famous theorem of Darlington, who proved that any positive real (PR) immittance function can be realized as a
lossless two-port in resistive termination. In other words, TPG of the power transfer problem can be expressed in terms of the driving point input immittance of the lossless two-port. Then, the PR immittance function is determined in such a way that TPG is optimized. In Carlin’s approach, over the real frequencies, the real part of a PR driving point function is described by the mean of line segments; it is shown that TPG is quadratic in terms of the selected points of line segments. Therefore, in the optimization process, we are able to deal with a quadratic objective function which makes the optimization almost always convergent.

Based on the above explanations, in designing PTNs, generation of PR immittance is quite important. Therefore, Chapters 7 and 8 are devoted to generating and modeling the realizable PR driving point functions using a parametric approach and the Gewertz procedure respectively. In this regard, MATLAB S/W tools are developed to solve many practical problems. Examples are presented to show the utilization of S/W tools.

Chapter 9 deals with Darlington synthesis of a PR immittance function, which is essential for the construction of lossless two-ports for real frequency techniques.

In order to understand the nature of the power transfer problem, the analytic theory of broadband matching is indispensable. Therefore, Chapter 10 is devoted to the analytic theory of broadband matching. It is shown that, beyond simple problems, the theory is inaccessible. Nevertheless, it is shown that filter theory can be expanded to solve simple single and double matching problems analytically. Hence, in this chapter, programs are developed to solve practical matching problems by utilizing the modified filter theory. Several examples are presented to show the utilization of the S/W.

The early 1980s witnessed the derivation of the analytic theory of double matching and also the expanded RFLT concept to design complicated single and double matching networks as well as microwave amplifiers. The new techniques are called the ‘real frequency direct computational technique or RFDT’, the ‘RF parametric approach’ and the ‘simplified real frequency technique or SRFT’. Thus, Chapter 11 deals with all the versions of real frequency techniques. Many complicated real-life problems are solved using the RF design tools developed with MATLAB. Examples include the design of matching networks for a complicated monopole antenna, for a helix GPS antenna and for an ultrasonic piezoelectric transducer. Furthermore, the design of ultra wideband microwave amplifiers using lumped and distributed elements is also presented. Obviously, the reader of this book can utilize the S/W tools provided to solve many crucial matching problems.

In many engineering applications, modeling of the measured immittance data is inevitable. For example, in RFLT, the driving point immittance of the lossless equalizer is generated point by point to optimize TPG. At the end of the design process, computer-generated immittance data must be modeled as a realizable positive real function so that it can be synthesized as a lossless two-port in resistive termination, yielding the desired matching network. Similarly, the analytic theory of matching requires models for both measured generator and load impedances. Therefore, in Chapter 12, we introduce our modeling tools via linear interpolation techniques.

The practical design of broadband matching networks must include both lumped and distributed elements, where all the geometric sizes and related parasitic elements are imbedded into the design. This is a very difficult task from a circuit theory point of view. However, our continuing efforts in the field recently matured in the design of broadband matching networks with mixed lumped and distributed elements. Chapter 13 covers this design. The MATLAB codes provided with this book can be found at http://www.wiley.com/go/yarman_wideband.

Acknowledgments

I should mention that all the design S/W provided with this book has been developed in the scientific spirit of sharing our knowledge, accumulated over the last 30 years. Including myself, the S/W reflects the blessed labor of many outstanding researchers, namely S. Darlington, H. W. Bode, C. Gewertz, R. M.
Fano, V. Belevitch, D. C. Youla, R. Levy, W. K. Chen, H. J. Carlin and A. Fettweis. The S/W is by no means professional and may include some bugs. Nevertheless, it provides solutions to all the worked examples in this book.

On the other hand, our design of broadband matching networks has been developed with the programs provided in this book. As the input, we feed in measured data; as the output, we automatically receive the optimum circuit topology with element values which optimize TPG as desired. This is nice, despite the bugs. At this point, I should mention that lumped element Darlington synthesis of positive real functions is essential to obtain lossless equalizers. The synthesis program in this book was developed by Dr Ali Kilinc of Okan University, Istanbul, Turkey. In this regard, his continuous support is gratefully acknowledged.

It is my hope that, having this book as a base, the readers, namely researchers and professional engineers in the field, will develop outstanding user-friendly design tools to construct optimum matching networks for various kinds of commercial and military applications. Therefore, they should feel free to contact me at yarman@istanbul.edu.tr in case help is needed.

Finally, I would like to take this opportunity to thank to my dear friends Mrs Asli Divris and Dr Birep Aygun for their careful reading of and corrections to the manuscript. I should also acknowledge the constructive guidance of Miss Skinner of Wiley in the course of completing the book.

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August 2009
In Memory of H. J. Carlin

It would not have been possible to complete this book without the spiritual guidance of Professor H. J. Carlin, who passed away on February 9, 2009. He was the initiator of the real frequency techniques which facilitated the design and implementation process of all kinds of power transfer networks.
1 Circuit Theory for Power Transfer Networks

1.1 Introduction

In circuit theory, a power transfer network is known as a lossless two-port which matches a given voltage generator with internal impedance $Z_G$ to a load $Z_L$. The lossless two-port consists of lossless circuit elements such as capacitors, inductors, coupled coils, transmission lines and transformers.

In practice, the complex impedances $Z_G$ and $Z_L$ are measured and modeled using idealized lossy and reactive circuit elements. In circuit theory, losses are associated with resistors. Reactive elements can be considered as capacitors, inductors, transmission lines or a combination of these.

It is well known that passive or lossy impedances consume energy. This is also known as power dissipation (i.e. energy consumption per unit time).

For given design specifications, such as the frequency band of operations and a desirable minimum flat gain level, the design problem of a power transfer network involves fundamental concepts of circuit theory. On the other hand, the fundamentals of circuit theory stem from electromagnetic fields. Especially at high frequencies, where the size of the circuit components is comparable to the wavelength of operational signals, use of electromagnetic field theory becomes inevitable for assessing the performance of the circuits. Therefore, at high frequencies, circuit design procedures must include electromagnetic field-dependent behavior of circuit components to produce actual reliable electrical performance.

In designing power transfer networks, we usually deal with mathematical functions employed in classical circuit theory. These functions are determined directly from the given design specifications by means of optimization. Eventually, they are synthesized at the component level, yielding the desired power transfer network. Therefore, a formal understanding of circuit functions and their electromagnetic field assessments is essential for dealing with design problems.

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\[1\] Circuit functions may be described as positive real driving point impedance or admittance functions or corresponding bounded real input reflection coefficients. The mathematical properties of these functions will be given in the following chapters.
As mentioned above, power transfer networks are designed as lossless two-ports which may include only reactive lumped elements,\textsuperscript{2} or only distributed elements, or a combination of both; that is, lumped and distributed elements. Usually, distributed elements are considered as ideal transmission lines.\textsuperscript{3}

In Figure 1.1, a conceptual power transfer network is shown. The input port may be driven by an amplifier which is modeled as a Thévenin voltage source with complex internal impedance $Z_G$. The output port may be terminated by an antenna which is considered as a complex passive impedance $Z_L$.

At this point, it may be appropriate to give the formal definitions of ideal circuit components so that we can build some concrete properties of network functions.

![Figure 1.1 Conceptual power transfer network](image)

**1.2 Ideal Circuit Elements**

In classical circuit theory, circuit elements may be described in terms of their terminal or port-related quantities such as voltage and current or incident and reflected wave relations.

In essence, descriptive port quantities are related to power delivered to that port. Referring to Figure 1.2, multiplication of port voltage $v(t)$ by port current $i(t)$ yields the power delivered to that port at any time $t$.

![Figure 1.2 Ideal one-port circuit components](image)

For a dissipative or lossy one-port the delivered power

$$P(t) = v(t) \cdot i(t)$$  \hspace{1cm} (1.1)

\textsuperscript{2} Reactive elements are also known as lossless circuit components such as capacitors and inductors.

\textsuperscript{3} An ideal transmission line is lossless and propagates uniform transverse electromagnetic waves. These waves are called uniform plane waves.
must be positive. Consequently, the total energy consumed by that ‘one-port’ is given as the integral of the delivered power such that

\[
W = \int_{-\infty}^{+\infty} P(t)dt = \int_{-\infty}^{+\infty} v(t)i(t)dt < 0 \tag{1.2}
\]

Specifically, for a lossless one-port, \(W = 0\) since there is no power consumption on it.

Now let us elaborate the concept of power by means of the following examples.

**Example 1.1**: Let the applied voltage to a port be \(v(t) = 3\) volts (or V) (DC) and the corresponding current response be \(i(t) = 1\) ampere (or A) (DC) over the entire time domain. Find the power dissipation of the one-port under consideration.

**Solution**: Power delivered to the port is given by Equation (1.1). Thus,

\[
P(t) = v(t)i(t) = 3V \times 1A = 3\text{ watts (or W)}. \tag{1.1}
\]

**Example 1.2**: Let the applied voltage to a port be \(v(t) = 3\sin(2\pi \times 50t)\) volts (50 Hz AC) and the corresponding current response be \(i(t) = 1\sin(2\pi \times 50t)\) amps (50 Hz AC) over the time domain \(t \geq 0\). Find the power dissipation of the one-port at time \(t = 5\) milliseconds.

**Solution**: Instantaneous power dissipation at any time \(t \geq 0\) is given by

\[
P(t) = v(t)\times i(t) = 3\sin(2\pi \times 50t)\text{volts} \times 1\sin(2\pi \times 50t)\text{amps}
\]

\[
= 3\sin^2(2\pi \times 50t)\text{watts}
\]

Hence, \(P(t = 50\text{ ms}) = 3\text{ W} \).

Note that, in this problem, the ‘voltage and current’ pair is sinusoidal with a frequency of \(f = 50\) Hz; or equivalently with the time period of \(T = \frac{1}{50} = 20\) ms. In practice, however, we are interested in average power dissipation over a period. Now let us define the average power dissipation as follows.

### 1.3 Average Power Dissipation and Effective Voltage and Current

For a one-port, let the port voltage and current pair be specified as

\[
\begin{align*}
v(t) &= V_m\sin(\omega_0 t - \varphi_v) \\
i(t) &= I_m\sin(\omega_0 t - \varphi_i)
\end{align*} \tag{1.3}
\]

where

\[
\omega_0 = 2\pi f_0 = \frac{2\pi}{T} \tag{1.4}
\]

is the angular frequency with frequency \(f_0\) and the period

\[
T = \frac{1}{f_0} \tag{1.5}
\]
Then, for a periodic voltage and current pair, the average power dissipation over a period $T$ is defined as

\[
P_{av} = \frac{1}{T} \int_0^T v(t)i(t)dt
= \frac{V_m I_m}{2T} \int_0^T \sin \left( \frac{2\pi}{T} t - \varphi_v \right) \sin \left( \frac{2\pi}{T} t - \varphi_i \right) dt
\]  

(1.6)

Note that

\[
\sin(\alpha) \cdot \sin(\beta) = \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \cos(\alpha + \beta)
\]

Furthermore,

\[
\cos(A) = \cos(-A)
\]

In the above trigonometric equalities, by replacing $\alpha$ by $(2\pi/T)t - \phi_v$ and $\beta$ by $(2\pi/T)t - \phi_i$, one obtains

\[
P_{av} = \frac{V_m I_m}{2T} \cos(\varphi_i - \varphi_v) \int_0^T dt - \frac{V_m I_m}{2T} \int_0^T \cos \left( \frac{4\pi}{T} t - \varphi_v - \varphi_i \right) dt
\]  

(1.7)

Note that the second integral is zero since the area under the cosine function is zero over a full period $T$. Hence, we have

\[
P_{av} = \frac{1}{2} V_m I_m \cos(\varphi_i - \varphi_v)
\]  

or

\[
P_{av} = \left[ \frac{V_m}{\sqrt{2}} \right] \left[ \frac{I_m}{\sqrt{2}} \right] \cos(\varphi_i - \varphi_v)
\]  

(1.8)

In the above form, the quantities

\[
V_{eff} = \frac{V_m}{\sqrt{2}}
\]
and
\[
I_{eff} = \frac{I_m}{\sqrt{2}}
\]

are called the effective values of the peak voltage $V_m$ and the peak current $I_m$ respectively.
1.4 Definitions of Voltage and Current Phasors

In the classical circuit theory literature, complex quantities can be expressed in terms of the Euler formula. For example,

\[ e^{j\varphi} = \cos(\varphi) + j\sin(\varphi) \] (1.10)

Furthermore, sinusoidal time domain signals can be expressed using the Euler formula such that

\[ v(t) = V_m\cos(\omega t - \varphi_v) = \text{real}\{e^{j\omega t}[V_m e^{-j\varphi_v}]\} \] (1.11)

In Equation (1.11) the quantity

\[ V \triangleq [V_m e^{-j\varphi_v}] \] (1.12)

is called the voltage phasor. Similarly, the current phasor is defined as

\[ I = [I_m e^{-j\varphi_i}] \]

In terms of the current phasor, the actual current is given by

\[ i(t) = \text{real}\{Ie^{j\omega t}\} = I_m\cos(\omega t - \varphi_i) \] (1.13)

By means of voltage and current phasors, average power can be expressed as

\[ P_{av} = \text{real}\{VI^*\} = \text{real}\{V^*I\} = \frac{1}{2}V_mI_m\cos(\varphi_v - \varphi_i) = V_{eff}I_{eff}\cos(\varphi_v - \varphi_i) \]

Example 1.3: Let \( v(t) = 10\cos(\omega t - 10^\circ) \) and \( i(t) = 20\cos(\omega t - 40^\circ) \).

(a) Find the voltage and current phasors.
(b) Find the average power dissipated over a period \( T \).

Solution:
(a) By definition, voltage phasor is \( V = 10.0 e^{j10^\circ} \). Similarly, the current phasor is given by \( I = 20.0 e^{j40^\circ} \).
(b) The average power is \( P_{av} = \frac{1}{2} \times 10.0 \times 20.0\cos(10^\circ - 40^\circ) = 100.0\cos(30^\circ) = 86.6 \text{ W} \).

Example 1.4: Let the voltage phasor be \( V = 1.0 e^{j60^\circ} \). Find the steady state time domain form of the voltage at \( \omega = 10 \text{ rad/s} \).

Solution: By formal definition of phasor within this book, we can write \( v(t) = \text{real}\{Ve^{j10t}\} = \cos(10t - 60^\circ) \). For the sake of completeness, it should be noted that the steady state voltage \( v(t) \) may also be defined as the imaginary part of \( \{Ve^{j10t}\} \) if the input drive is \( v_{in}(t) = \sin(\omega t) \).

In general, usage of phasor notation facilitates the sinusoidal steady state analysis of a circuit in the time domain. In principle, network equations (more specifically, equations originating from Kirchhoff’s voltage and current laws) are written using voltage and current phasors. Eventually, time domain expressions can easily be obtained by Equation (1.11), like mappings.\(^4\)

---

\(^4\) Here, what we mean is that any steady state time domain expression of a phasor \( A = A_m e^{j\phi_A} \) \( A = A_m e^{j\phi_A} \) can be obtained as \( A(t) = \text{real}\{A e^{j\omega t}\} \). In this representation \( A(t) \) may designate any node or mesh voltage and current in a network.
1.5 Definitions of Active, Passive and Lossless One-ports

Referring to Figure 1.2, let \( v(t) \) and \( i(t) \) be the voltage and current pair with designated polarity and direction of an ideal circuit component. We assume that these quantities are given as a function of time \( t \). Based on the given polarity and directions:

- A one-port is called passive if \( W = \int_{-\infty}^{\infty} P(t) \, dt = v(t) \cdot i(t) < 0 \).
- A one port is called lossless if \( W = \int_{-\infty}^{\infty} P(t) \, dt = v(t) \cdot i(t) = 0 \).
- On the other hand, if \( W = \int_{-\infty}^{\infty} P(t) \, dt = v(t) \cdot i(t) < 0 \), then the one-port is called active. Obviously, a conventional voltage or current source is an active one-port.

In the following section, we will present elementary definitions of passive and lossless circuit components based on their port voltages and currents.

An ideal circuit component may be a resistor \( R \), a capacitor \( C \) or an inductor \( L \). Formal circuit theory definitions of these components are given next.

1.6 Definition of Resistor

A resistor \( R \) is a lumped one-port circuit element which is defined by means of Ohm’s law:\(^5\)

\[
\begin{align*}
\text{or} & \\
v_R(t) & \triangleq R i_R(t) \\
i_R & = \frac{v_R}{R} \triangleq G v_R \quad \text{G} = \frac{1}{R}
\end{align*}
\]

where \( R \) is called the resistance and it is measured by means of the ratio of port voltage to port current. The symbol ‘ \( \triangleq \) ‘ refers to equality by definition.

The units of voltage \( v(t) \) and current \( i(t) \) are volt (V) and ampere (A) respectively. The unit of resistance \( R \) is given by V/A, which is called the ohm and designated by \( \Omega \). \( G \) is called conductance and it is measured in siemens or \( \Omega^{-1} \).\(^6\) The power dissipated on a resistor is given by multiplication of its port voltage and current such that

\[
P_R(t) = v_R(t) i_R = R i_R^2(t) = R i_R(t)^2 \geq 0
\]

Dissipated power is always non-negative.\(^7\) Therefore, the value of resistance must always be non-negative (i.e.\( R \geq 0 \)).\(^8\)

\(^5\) A one-port circuit element is placed between two nodes and described in terms of its port quantities such as voltage and current pairs. These nodes are referred to as terminals of the one-port.

\(^6\) \( \Omega \) is a Greek letter read as omega.

\(^7\) That is, \( P_R(t) \geq 0, \forall t \).

\(^8\) Here, it should be noted that for a real physical system, time is measured as a real number; voltage and current in the time domain are measured as real numbers with respect to selected references. Therefore, energy and power quantities are also measured as real numbers which in turn yield a non-negative real resistance value for the port under consideration.
The unit of power is volt × ampere which is called the watt and designated by W; 1 watt describes 1 joule of energy (1 J) dissipated per second (s).  

### 1.7 Definition of Capacitor

In electromagnetic field theory, we talk about energy stored both in electric and magnetic fields which produce actual work when applied to a moving electric charge. With this understanding, electric energy is stored on a circuit element called a capacitor and is usually designated by the letter C. As an ideal lumped circuit element, a capacitor C is described in terms of its port voltage \( v_C \) and port current \( i_C \) as

\[
i_C(t) \triangleq C \frac{dv_C(t)}{dt}
\]

where \( C \) is the capacitance and its unit is the farad (F).

Total electric energy stored in a capacitor C is given in terms of the time integral of the power flow \( P_C(t) = v_C(t) \cdot i_C(t) \) by

\[
W_C = \int_{-\infty}^{t} v_C(\tau) i_C(\tau) d\tau = C \int_{-\infty}^{t} v_C(\tau) dv_C = \frac{1}{2} C v_C^2
\]

provided that initially the capacitor is empty, i.e. \( v(-\infty) = 0 \). 

Since the stored electric energy \( W_C \) must be non-negative (or positive), then capacitance \( C \) must always be non-negative (or positive) (i.e. \( C \geq 0 \)). At this point we should mention that this is potential electric energy. It is not dissipation. In other words, it is not consumed by the capacitor; rather it is stored. However, it may generate work or, equivalently, it can be transformed into kinetic energy when it is applied to a moving charge.

In practice, a capacitor is charged with a constant voltage source \( E_G \), say a simple battery which has a series internal resistance \( R_G \). When the charging process is completed within \( T_C \) seconds, the capacitor is said to be full and passes no current (i.e. \( i_C(T_C) = 0 \)). The voltage \( v_C(T_C) \) across its plates becomes constant and is equal to \( E_G \). In this case, the total stored electric energy is given by \( W_C = \frac{1}{2} C E_G^2 \). However, consumed energy will be zero since \( i_C(T_C) = 0 \). In this explanation, any transient process is ignored and the charging time period \( T_C = 0^+ \) seconds is assumed. This means that the capacitor is immediately charged having \( v_C(t < 0^+) = E_G \) and \( i_C(t < 0^+) = 0 \), yielding no power dissipation (i.e. \( P(t) = 0 \)) or equivalently total energy consumption \( W = 0 \) (Figure 1.3).

---

9. That is, 1 W = 1 J/s.
10. In this book all the units are given in the International Standard Unit (ISU) system. Basic units of ISU are the meter, kilogram and second (MKS). Therefore, ISU is also known as the MKS unit system. In MKS, voltage and current units are given as volt (V) and ampere (A).
1.8 Definition of Inductor

An inductor L is an ideal lumped circuit element. It stores magnetic energy. Its formal definition is given in terms of its port voltage $v_L(t)$ and port current $i_L(t)$ as

$$v_L(t) = L \frac{di_L(t)}{dt}$$

where $L$ called inductance and its unit is given by the henry (H).

Total magnetic energy $W_L$ stored in an inductor L over an interval of time $(-\infty, t]$ is given by

$$W_L = \int_{-\infty}^{t} v_L(\tau) i_L(\tau) d\tau = \int_{-\infty}^{t} L i_L(\tau) di = \frac{1}{2} L i_L^2$$

Since the stored magnetic field energy must be non-negative (or positive), then inductance $L$ must be non-negative (or positive) (i.e. $L \geq 0$).

In a similar manner to that of a capacitor, as an ideal lumped circuit element, an inductor L is lossless. This means that it does not dissipate power but rather holds magnetic energy over a specified period of time unless it is emptied. When an inductor is connected to an excitation, say to a constant current source $I_G$ with an internal shunt resistance $R_G$, at time $t = 0$ seconds, a constant current $I_L = I_G = i_L(t < 0^+)$ immediately builds up over a very short period of time ending at $t = 0^+$ seconds. Then, this current circulates indefinitely within the circuit. Let the voltage drop on $R_G$ be equal to $R_G I_G$ at time $t = 0^+$ seconds. Roughly speaking, when the inductor L is connected to the current source $I_G$, this voltage immediately appears on the inductor $v_L(t = 0) = R_G I_G$ and rapidly reduces to zero within $T_L = 0^+$ seconds while the inductor current $i_L$ rises to the level of $I_G$, yielding zero power transfer. During this process, as $i_L$ increases, current through the shunt resistance $R_G$ goes to zero resulting in zero voltage across inductor L (see Figure 1.4).

We should emphasize that this is a macroscopic explanation. Details are skipped here.

Just to summarize the above discussions based on the definitions, as ideal circuit elements a capacitor or an inductor is a lossless one-port, and it can only store energy. On the other hand, a resistor is a lossy circuit element which dissipates or consumes energy by heating itself. In practice, however, there is no ideal circuit element; one can always associate a real dissipation perhaps in series with an ideal inductance $L$, say $r_L$, or in parallel with an ideal capacitance $C$ which may be designated as conductance $G_C$ as shown in Figure 1.5.

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11 It should be noted that in Equation (1.16) and (1.19), initially capacitor C and inductor L were assumed to be empty. Therefore, in these equations the integration constant is set to zero.