Modelling Single-name and Multi-name Credit Derivatives

Dominic O'Kane



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To Penny, Rory and Fergal.

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> Dominic O'Kane April 2008.

About the Author _____

Dominic O'Kane is an affiliated Professor of Finance at the French business school EDHEC which is based in Nice, France. Until May 2006, Dominic O'Kane was a managing director and ran the European Fixed Income Quantitative Research group at Lehman Brothers, the US investment bank. Dominic spent seven of his nine years at Lehman Brothers working as a quant for the credit derivatives trading desk. Dominic has a doctorate in theoretical physics from Oxford University.

Introduction

The aim of this book is to present an up-to-date, comprehensive, accessible and practical guide to the models used to price and risk-manage credit derivatives. It is both a detailed introduction to credit derivative modelling and a reference for those who are already practitioners.

This book is up-to-date as it covers many of the important developments which have occurred in the credit derivatives market in the past 4–5 years. These include the arrival of the CDS portfolio indices and all of the products based on these indices. In terms of models, this book covers the challenge of modelling single-tranche CDOs in the presence of the correlation skew, as well as the pricing and risk of more recent products such as constant maturity CDS, portfolio swaptions, CDO squareds, credit CPPI and credit CPDOs.

For each model, the reader is taken through the underlying assumptions and then the mathematical derivation. The application of the model to pricing and risk-management is explained with the goal of trying to build intuition. There is also a focus on the efficient implementation of each model.

Product coverage is extensive and is split into two parts. Part I covers single-name credit derivatives and Part II covers multi-name credit derivatives. We begin the first part with the traditional credit products including the credit risky bond, the floating rate note and the asset swap. Although they are not credit derivatives, these are included for the sake of completeness and because they are a pricing reference for the credit default swap. We then move on to the credit default swap (CDS) which, reflecting its importance, is covered in considerable detail. We also include a discussion of digital CDS, forward CDS and loan CDS. We then cover other single-name products including the constant maturity default swap and the default swaption.

Part II covers products whose risk is linked to the credit performance of more than one credit. These are known as 'multi-name' products. We begin with the biggest growth product of the credit derivatives market, the CDS index. This then leads us to the many product innovations which have resulted from the arrival of these CDS indices. These include the tranched CDS indices and CDS index swaptions. We also cover advanced correlation products such as the CDO-squared and the leveraged super senior. Dynamically managed structures such as the credit CPPI and CPDO are also examined. Towards the end of the book we consider a number of the newer products which are beginning to be traded. These include forward starting tranches and tranche options.

In this book we set out in detail the models which have been developed to address the challenges posed by these products. Of these challenges, the most important has been the modelling of default correlation. We therefore cover in detail the Gaussian copula and the modelling of default dependency in general, especially within a copula framework. After establishing the arbitrage-free conditions for a correlation model, we devote an entire chapter to *base correlation*, which has become a widely used pricing and risk-management approach. We discuss its implementation and in doing so highlight the advantages and disadvantages of base correlation as a pricing and risk-management framework. We then discuss a range of specific copula models, highlighting the pros and cons of each. This takes us to the subject of much current research – the development of usable dynamic correlation models. In the final two chapters of this book we discuss the two main categories of dynamic models known as bottom-up and top-down. We also set out in detail some specific models which fall into these two categories.

The credit derivatives market has changed significantly in the past four to five years and most of these developments are contained within this book. However, the market continues to evolve. As a result, I would suggest that readers keen to keep abreast of the latest modelling and market developments periodically visit www.defaultrisk.com and the technical forums of www.wilmott.com.

Notation _____

Symbol	Description
t	today (valuation date).
t = 0	Contract initiation date.
t_s	Contract settlement date.
t_E	Option expiry date.
Т	Contract maturity date.
t_n	<i>n</i> th cash flow date on the premium leg. Usually $t_0 = t$ and $t_N = T$.
$\Delta(t_{n-1},t_n,b)$	The year fraction from date t_{n-1} to t_n in a basis <i>b</i> . We typically drop the <i>b</i> .
Z(t,T)	Libor zero coupon bond price from time t to T . We sometimes use
	Z(t) = Z(0, t).
$\hat{Z}(t,T)$	Zero recovery credit risky zero coupon bond price from today time t to
	time T.
$L(T_{1}, T_{2})$	The observed value of the Libor rate which sets at time T_1 for a period $[T_1, T_2]$.
$L(t, T_1, T_2)$	The value at time <i>t</i> of a forward Libor rate which sets at time T_1 for a period
	$[T_1, T_2].$
D(t,T)	Present value of 1 paid at the time of default as seen at time t .
$Q_i(t,T)$	Survival probability from today time t to time T for issuer i . We sometimes
	use $Q_i(t) = Q_i(0, t)$.
S(t,T)	CDS contractual spread at time <i>t</i> for a contract which matures at time <i>T</i> .
$S(t,t_F,T)$	Forward CDS contractual spread at time t for a contract with forward start t_F which matures at time T. Note that $S(t, t, T) = S(t, T)$
PV01(t T)	The time t present value of a \$1 Libor quality annuity which matures at time T
RPV01(t,T)	The time t present value of a credit risky $\$1$ annuity which matures at time T. The time t present value of a credit risky $\$1$ annuity which matures at time T.
r(t)	The time <i>t</i> present value of a createrinsky ϕ t annuly which matures at time <i>t</i> . The risk-free short interest rate at time <i>t</i>
$\beta(t)$	Value of the rolled-up money-market account $\beta(t) = \exp(\int^t r(s) ds)$
$\beta(t)$ $\beta(t T)$	Value of $\beta(T)\beta(t)^{-1}$
$\lambda(t)$	The bazard rate or intensity process at time t
Pr(r)	The probability that r is true
$C(\mu, \mu)$	The <i>n</i> -dimensional default conula
$\hat{C}(u_1,\ldots,u_n)$	The <i>n</i> dimensional default copula.
$C(u_1,\ldots,u_n)$	rne <i>n</i> -unnensional survival copula.

XX

Symbol	Description
$\phi(a)$	The Gaussian density.
$\Phi(a)$	The uni-variate Gaussian cumulative distribution function.
$\Phi_2(a, b, \rho)$	The bi-variate Gaussian cumulative distribution function.
$\Phi_n(\mathbf{a}, \Sigma)$	The <i>n</i> -variate Gaussian cumulative distribution function with correlation matrix Σ .
$t_{\nu}(x)$	The uni-variate Student- <i>t</i> cumulative distribution function with v degrees of freedom.
L(T)	The fractional portfolio loss at horizon time T.
$L(T, K_1, K_2)$	The fractional tranche loss at horizon time <i>T</i> .
F(x)	The cumulative portfolio loss distribution, i.e. $F(x) = Pr(L \le x)$.
f(x)	The density of the portfolio loss distribution $f(x) = \partial F(x) / \partial x$.
O(x)	'Order of' x. If $f(x)$ is $O(x^n)$ then n is the exponent of the dominant polynomial term in $f(x)$.
int(x)	Function which returns the integer part of a number without rounding, e.g. int $(5.7322) = 5$.
ceil(x)	Function which returns the smallest integer value greater than or equal to x , e.g. ceil (5.7322) = 6.

The Credit Derivatives Market

1.1 INTRODUCTION

Without a doubt, credit derivatives have revolutionised the trading and management of credit risk. They have made it easier for banks, who have historically been the warehouses of credit risk, to hedge and diversify their credit risk. Credit derivatives have also enabled the creation of products which can be customised to the risk–return profile of specific investors. As a result, credit derivatives have provided something new to both hedgers and investors and this has been a major factor in the growth of the credit derivatives market.

From its beginning in the mid-1990s, the size of the credit derivatives market has grown at an astonishing rate and it now exceeds the size of the credit bond market. According to a recent ISDA survey,¹ the notional amount outstanding of credit derivatives as of mid-year 2007 was estimated to be \$45.46 trillion. This significantly exceeds the size of the US corporate bond market which is currently \$5.7 trillion and the US Treasury market which is currently \$4.3 trillion.² It also exceeds the size of the equity derivatives market which ISDA also estimated in mid-2007 to have a total notional amount outstanding of \$10.01 trillion.

In addition to its size, what is also astonishing about the credit derivatives market is the breadth and liquidity it has attained. This has been due largely to the efforts of the dealer community which has sought to structure products in a way that maximises tradabililty and standardisation and hence liquidity. The CDS indices, introduced in 2002 and discussed extensively in this book, are a prime example of this. They cover over 600 of the most important corporate and sovereign credits. They typically trade with a bid–offer spread of less than 1 basis point and frequently as low as a quarter of a basis point.³

To understand the success of the credit derivatives market, we need to understand what it can do. In its early days, the credit derivatives market was dominated by banks who found credit derivatives to be a very useful way to hedge the credit risk of a bond or loan that was held on their balance sheet. Credit derivatives could also be used by banks to manage their regulatory capital more efficiently. More recently, the credit derivatives market has become much more of an investor driven market, with a focus on developing products which present an attractive risk–return profile. However, to really understand the appeal of the credit derivatives market, it is worth listing the many uses which credit derivatives present:

• Credit derivatives make it easier to go short credit risk either as a way to hedge an existing credit exposure or as a way to express a negative view on the credit market.

¹ These numbers are based on the ISDA Mid-Year 2007 Market Survey report.

² Securities Industry and Financial Markets Association Research Quarterly, August 2007.

³ This has certainly been the case for the CDX Investment Grade North America index. In price terms, this is roughly equivalent to a bid–offer of 1 cent on a five-year bond with a price of \$100.

2 Modelling Single-name and Multi-name Credit Derivatives

- Most credit derivatives are unfunded. This means that unlike a bond, a credit derivative contract requires no initial payment. As a consequence, the investor in a credit derivative does not have to fund any initial payment. This means that credit derivatives may present a cheaper alternative to buying cash bonds for investors who fund above Libor. It also makes it easier to leverage a credit exposure.
- Credit derivatives increase liquidity by taking illiquid assets and repackaging them into a form which better matches the risk-reward profiles of investors.
- Credit derivatives enable better diversification of credit risk as the breadth and liquidity of the credit derivatives market is greater than that of the corporate bond market.
- Credit derivatives add transparency to the pricing of credit risk by broadening the range of traded credits and their liquidity. We estimate that there are over 600 corporate and sovereign names which have good liquidity across the credit derivatives market.⁴ The scope of the credits is global as it includes European, North American and Asian corporate credits plus Emerging Market sovereigns.
- Credit derivatives shift the credit risk which has historically resided on bank loan books into the capital markets and in doing so it has reduced the concentrations of credit risk in the banking sector. However, this does raise the concern of whether this credit risk is better managed in less regulated entities which sit outside the banking sector.
- Credit derivatives allow for the creation of new asset classes which are exposed to new risks such as credit volatility and credit correlation. These can be used to diversify investment portfolios.

The relatively short history of the credit derivatives market has not been uneventful. Even before the current credit crisis of 2007–2008, the credit derivative market has weathered the 1997 Asian Crisis, the 1998 Russian default, the events of 11 September 2001, the defaults of Conseco, Railtrack, Enron, WorldCom and others, and the downgrades of Ford and General Motors. What has been striking about all of these events is the ability of the credit derivatives market to work through these events and to emerge stronger. This has been largely due to the willingness of the market participants to resolve any problems which these events may have exposed in either the mechanics of the products or their legal documentation. Each of these events has also strengthened the market by demonstrating that it is often the only practical way to go short and therefore hedge these credit risks.

In this chapter, we discuss the growth in the credit derivatives market size. We present an overview of the different credit derivatives and discuss a market survey which shows how the importance of these products has evolved over time. We then discuss the structure of the credit derivatives market in terms of its participants.

1.2 MARKET GROWTH

The growth of the credit derivatives market has been phenomenal. Although there are different ways to measure this growth, each with its own particular approach, when plotted as a function

⁴ We calculated this by summing the number of distinct credits across the main CDS indices listed in Table 10.1. These credits have been selected for their liquidity.

of time, they all show the same exponential growth shown in Figure 1.1. Let us consider the three sources of market size data:

- The British Bankers' Association (BBA) surveys the credit derivatives market via a questionnaire every two or so years. Their questionnaire is sent to about 30 of the largest investment banks who act as dealers in the credit derivatives market. Their latest report was published in 2006 and estimated the total market notional at the end of 2006 to be \$20.207 trillion.
- 2. The International Swaps and Derivatives Association (ISDA) conducts a twice-yearly survey of the market. In the most recent, they surveyed 88 of their member firms including the main credit derivatives dealers about the size of their credit derivatives positions. The collected numbers were adjusted to correct for double-counting.⁵ The mid-2007 survey estimated the size of the credit derivatives market to be \$45.25 trillion, an increase of 32% in the first six months of 2007.
- 3. The US Office for the Comptroller of the Currency conducts a quarterly survey of the credit derivatives market size. The survey covers just the US commercial bank sector. The June 2007 survey found that the total notional amount of credit derivatives held by US commercial banks was \$10.2 trillion, an increase of 86% on the first quarter of 2006. This number is lower than the others partly because it excludes trades done by many non-US commercial banks and investment banks.

Although these numbers all differ because of the differing methodologies and timings, what is beyond doubt is the rapid growth that has been experienced by the credit derivatives market.



Figure 1.1 Evolution of the credit derivatives market size using estimates calculated in the BBA Credit Derivatives Report 2006. Source: British Banker's Association

⁵ A lot of credit derivatives trades occur between dealers. If two dealers have taken the opposite side of a contract, then that contract is only counted once.

Although the size of the credit derivatives market is significant, it is important to realise that credit derivatives do not increase the overall amount of credit risk in the financial system. This is because every credit derivative contract has a buyer and a seller of the credit risk and so the net increase of credit risk is zero. However, credit derivatives can in certain cases be used to increase the amount of credit risk in the *capital markets*. For example, suppose a bank has made a \$10 million loan to a large corporate and this loan is sitting on their loan book portfolio. As we will see later, to hedge this risk, the bank can use a credit default swap (CDS) contract. The credit risk of the corporate is therefore transferred from the bank to the CDS counterparty who may then transfer this risk on to another counterparty using another CDS contract. At the end of this chain of transactions will be someone, typically an investor, who is happy to hold on to the risk of the corporate and views that the premium from the CDS is more than sufficient to compensate them for assuming this risk. The loan that was sitting on the bank's loan book is still there. However, the risk has been transferred via the CDS contracts to this investor in the capital markets. Because the credit risk has been transferred without any actual sale of the loan, the credit risk produced by a CDS is 'synthetic'. If a default of the corporate does occur, the loss compensation is paid from the investor to the counterparty and on down the chain of contracts to the bank which was the initial buyer of protection. The bank has successfully hedged its credit exposure to this loan.

The only way in which credit risk has increased is through the counterparty credit risk associated with each contract. This is the risk that the protection seller does not make good a payment of the default loss compensation to the protection buyer. In practice, this risk is usually mitigated through the use of collateral posting as explained in Section 8.8.

1.3 PRODUCTS

The simplest and most important credit derivative is the credit default swap (CDS). This is a bilateral contract which transfers the credit risk of a specific company or sovereign from one party to another for a specified period of time. It is designed to protect an investor against the loss from par on a bond or loan following the default of the issuing company or sovereign. In return for this protection the buyer of the CDS pays a premium.

We note that someone who is assuming the credit risk in a credit derivatives contract like a CDS is called a *protection seller*. The person taking the other side of this trade is insuring themselves against this credit risk and is called a *protection buyer*.

An important extension of the CDS is the CDS index. This is a product which allows the investor to enter into a portfolio consisting of 100 or more different CDS in one transaction. For example, one of the most liquid indices is the CDX NA IG index which consists of 125 investment grade corporate credits which are domiciled in North America. We call this a *multiname* product because it exposes the issuer to the default risk of more than one credit or 'name'. The considerable liquidity and diversified nature of the CDS index have meant that it has also become a building block for a range of other credit derivatives products.

There are also a number of option-based credit derivatives. These include single-name default swaptions in which the option buyer has the option to enter into a CDS contract on a future date. More recently we have seen growth in the market for portfolio swaptions. As the name suggests, these grant the option holder the option to enter into a CDS index.

Then there are the multi-name contracts such as default baskets and synthetic CDOs which are built on top of a portfolio of CDS. These contracts work by 'tranching' up the credit risk of

the underlying portfolio. Tranching is a mechanism by which different securities or 'tranches' are structured so that any default losses in the portfolio are incurred in a specific order. The first default losses are incurred by the riskiest equity tranche. If the size of these losses exceeds the face value of the equity tranche then the remaining losses are incurred by the mezzanine tranche. If there are still remaining losses after this, then these are incurred by the senior tranches. The risk of this credit derivatives contract is sensitive to the tendency of the credits in the portfolio to default together. This is known as default correlation and, for this reason, these derivatives are known as correlation products.

Finally, we have the credit CPPI structure and the more recent CPDO structure. These structures exploit a rule-based dynamic trading strategy typically involving a CDS index. This trading strategy is designed to produce an attractive risk–return profile for the investor. In the case of CPPI, it is designed to provided a leveraged credit exposure while protecting the investor's principal. In the case of CPDO, the strategy is designed to produce a high coupon with low default risk.

Product type	Market share(%)			
	2002	2004	2006	
Single-name credit default swaps	45.0	51.0	32.9	
Credit-linked notes	8.0	6.0	3.1	
Credit spread options	5.0	2.0	1.3	
Full index trades	_	9.0	30.1	
Synthetic CDOs	_	16.0	16.3	
Tranched index trades	_	2.0	7.6	
Basket products	6.0	4.0	1.8	
Swaptions	_	1.0	0.8	
Others	36.0	8.0	5.7	
Market size (\$bn)	1 952	5 021	20 207	

Table 1.1Market share of different credit derivatives products measured by marketoutstanding notional. We compare the results of the BBA survey for 2002, 2004 and2006. Source: BBA Credit Derivatives Report 2006

Table 1.1 shows a breakdown of the various credit derivatives by their market outstanding notional. The data is sourced from the BBA survey of 2006 mentioned earlier. Note that this survey does not consider the CPPI and CPDO products since these have only become important in the time since this survey was carried out. This table already enables us to make the following observations about the current state and also the trends of the credit derivatives market:

- Many of the products which appeared in the 2004 and 2006 surveys did not exist in 2002. The most notable examples of this are the full index trades, which refer to trades on the CDX and iTraxx indices which were launched after 2002. We also see the establishment of a number of synthetic CDO categories and a tranched index trade category.
- We see a relative decline in the importance of more traditional credit derivatives products such as credit-linked notes and spread options. However, this decline in market percentage share is actually an increase in absolute size given the fourfold growth of the credit derivatives market over the 2004–2006 period.

• The market share of CDS fell from 51% to 32.9% over the 2004–2006 period. Over the same period, the portfolio indices rose from 9.1% to 30.1% of a much larger market. This suggests a trend away from single-name credit towards portfolio products. In absolute terms, the CDS market size actually grew significantly over this period.

There is a clear trend towards portfolio index products, i.e. multi-name products.

1.4 MARKET PARTICIPANTS

There are several different types of participants in the credit derivatives market. Each has its own specific rationale for using credit derivatives. Table 1.2 presents a breakdown of the market

Year of survey	2004		2006	
Type of market participant	Protection buyer (%)	Protection seller (%)	Protection buyer (%)	Protection seller (%)
Banks (including securities firms)	67	54	59	44
Trading activities	_	_	39	35
Loan portfolio	_	_	20	9
Insurers	7	20	6	17
Hedge funds	16	15	28	32
Pension funds	3	4	2	4
Mutual funds	3	3	2	2
Corporates	3	2	2	1
Other	1	1	1	1

Table 1.2 Market share of different market participants. Source: BBA Credit Derivatives Report 2006

share of the credit derivatives market by participant. This data is taken from the 2006 BBA Credit Derivatives Report. From this, we make the following observations:

- Banks are the largest participant in the credit derivatives market, both as buyers and sellers of protection. Table 1.2 splits the category of banks into trading activities and loan portfolio which we now consider separately.
- Many banks, in particular the securities houses, have significant trading activities as they act as dealers in the credit derivatives market. Dealers provide liquidity to the credit derivatives market by being willing to take risk onto their trading books which they then attempt to hedge. As a result, they buy roughly as much protection as they sell. They also act as issuers of structured products such as synthetic CDOs which they also hedge dynamically.
- Commercial banks possess loan portfolios. They use credit derivatives to buy protection in
 order to synthetically remove credit risk concentrations from their loan portfolio. They sell
 protection on other credits in order to earn income which can be used to fund these hedges,
 and to diversify their credit risk. One of the main drivers of bank behaviour is their regulatory
 framework. Until recently, this was based on the 1988 Basel Accord in which the capital a
 bank had to reserve against a loan or credit exposure was linked to whether the issuer of the

loan was an OECD⁶ member government, bank or other. However, the regulatory regime has recently changed to the Basel II capital accord in which the regulatory capital is linked to the credit rating of the asset.

- Insurance companies mainly use credit derivatives as a form of investment which sits on the asset side of their business. They are principally sellers of credit protection and tend to prefer highly rated credits such as the senior tranches of CDOs.
- Hedge funds have grown their credit derivatives activity and have become significant players in the credit derivatives market. They are attracted by the unfunded⁷ nature of most credit derivatives products which makes leverage possible. The fact that the credit derivatives market makes it easy to go short credit is another big attraction. Furthermore, credit derivatives also facilitate a number of additional trading strategies including cash versus CDS basis trading, correlation trading and credit volatility trading which hedge funds are free to exploit.
- Mutual and pension funds are not particularly large participants in the credit derivatives market. As investors, they would be primarily sellers of protection. They buy protection to hedge existing exposures. Often, they have restrictions on what sort of assets they can hold which preclude credit derivatives. However, the exact permissions depend on both their investment mandate and the investment regulations governing the jurisdiction in which they operate. Typically, one of their main restrictions is that the credits owned should be investment grade quality.
- Although credit derivatives could be used to try to hedge the credit risk of receivables, corporates have not become significant players in the credit derivatives market. There are a number of reasons why. First, standard credit default swaps do not trigger on the non-payment of receivables since receivables are not classified as *borrowed money* the term used to encompass the range of obligations which are covered. Typically a contract will only trigger if there is a default of bonds and loans. Second, the payout from a standard credit default swap may not be consistent with the actual loss since it is linked to the delivery of a senior unsecured bond or loan. Finally, the range of traded credits may not overlap with the companies to which the corporate has a credit exposure.

Understanding the motivations of these different participants assists us in understanding why different products are favoured by different participants.

1.5 SUMMARY

In this chapter we have explained the features of credit derivatives which have led to their success and then discussed the market in terms of its size, the types of credit derivatives which are traded and who uses them.

⁶ Organisation for Economic Co-operation and Development.

⁷ Unfunded means that the credit derivatives transaction can be entered into at zero initial cost. Unlike a bond, which is a funded transaction, there is no need to pay an initial bond price of around \$100 in order to have a credit exposure of \$100.