Dyslexia, Dyspraxia and Mathematics

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Foreword

Education is never still. There are constant changes and developments. Some of these enhance our knowledge of how children learn and thus how they can be taught most effectively. Some are merely cosmetic where the only increase in output is in administration. This book takes a proud place in the first category.

Our awareness of the diversity of special needs has increased greatly over the past thirty years. For example the term ‘specific learning difficulties’ was once promoted as the preferred term for ‘dyslexia’. Now we are aware of a cluster of specific learning difficulties, including dyspraxia.

Awareness of a learning difficulty is a good start, but it can result in stereotypical concepts and inadequate and inappropriate interventions. What Dorian Yeo has done in this book is to extend awareness to understanding and has then set the understanding of the individual within sound, clearly and thoroughly explained underlying principles.

Before the publication of this book there was a great need for material for the younger learner and for the dyspraxic learner. It is fascinating to see the comparisons between the problems experienced by dyslexic learners and dyspraxic learners. There seem to be more similarities than differences, which will not come as a surprise to those who work in special education. As important as this observation is, the realisation that good intervention for special needs is good intervention for all learners is even more important. Few learners (if any) are perfect, so this too should not be a surprise.

This book is about good practice. To paraphrase Professor Tim Miles, this good practice will help all learners, but it is an essential for dyspraxic and dyslexic learners.
Dorian Yeo's book is written from deep personal understanding and knowledge, but not from knowledge built on a sample of one, but of many similar yet diverse pupils. It combines a comprehensive explanation of the difficulties young children experience when learning numeracy with many practical, structured and developmental ideas for teaching. It has been a privilege to see Dorian's vision become reality.

Steve Chinn
August 2002
Preface

Emerson House is a small and intensive specialist teaching and learning centre which caters for the core learning needs of dyslexic and dyspraxic primary school children. When I established the maths department at the rapidly growing Emerson House some years ago, I already knew that many dyslexic children did not learn maths as easily as the majority of 'ordinary' children did. I had also discovered, through experience, that access to concrete materials could make a difference to the performance of dyslexic children. In the early days, however, I assumed that children with specific learning difficulties simply needed more practice – with concrete support – in order to make progress in learning the aspects of maths which they found hard. From working with our children and our teachers in a questioning way, I discovered that 'overlearning', however patiently orchestrated, was often not enough. Inspired by Steve Chinn and Richard Ashcroft's work with secondary school pupils, I realized that we needed to know more about how young children make sense of numbers and why some children – and dyslexic and dyspraxic children, in particular – can find the early stages of working with numbers so difficult. As we began to change aspects of how we taught maths at Emerson House, we realized that our most important task was to set out to make the foundations of number-work as simple, clear and easily understood as possible. In this book I have set out to describe the teaching ideas which have made a difference to the happiness, confidence, progress and attitude towards maths of the children whom we have taught.

I would like to thank Jane Emerson for the support and encouragement which she has always given me.
Dedication

For my husband, Dudley, and for my children, Lisa, Claire and Russell. Also for my sister, Kay, who, like me, survived; and for my brother, mother and father who did not.
PART I

Definitions and Premises
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CHAPTER 1

Background information

Introduction

This is a book which sets out to explore how primary school dyslexic and
dyspraxic children with varying degrees of maths learning difficulties
understand and learn maths. It discusses a number of important ideas
about some of the cognitive features which seem to underlie general maths
learning difficulties or which may underlie difficulties learning specific
aspects of maths, such as the times tables facts. It outlines the ways in which
children usually learn the foundation aspects of maths and considers the
special cognitive needs of dyslexic and dyspraxic children in this context.
It aims to offer practical support and detailed teaching suggestions to
teachers, tutors and parents who wish to help dyslexic or dyspraxic
children make real and sustained progress in learning maths.

The book has grown out of the experience of teaching maths to
primary school dyslexic and dyspraxic children, aged from about 6½
years upwards. From the outset the shape of this experience has been
profoundly influenced by the work of Steve Chinn and by Steve Chinn
and Richard Ashcroft's seminal and forward-looking book, Mathematics for
Dyslexics: A Teaching Handbook (1998). Above all, the experience has been
driven by the conviction, based on experience, and directly expressed in
Mathematics for Dyslexics, that to teach maths well to children with specific
learning difficulties, 'a different attitude and approach is needed' (Chinn

In the challenging and ongoing process of further developing and
refining 'a different attitude and approach' which aims to meet the needs
of younger primary school dyslexic and dyspraxic children, there have
been a number of other important sources and influences. It has, of
course, been possible to draw on the available research into 'dyslexia
specific' maths difficulties and, in particular, into their difficulties with
memorized *times tables* facts. There are also valuable teaching-based accounts of the ways in which many of the underlying cognitive difficulties associated with dyslexia seem to affect the maths learning abilities of dyslexic children. Such accounts include those of Chinn and Ashcroft (1998), Henderson (1989; 1998), Henderson and Miles (2001) and Miles and Miles (1992). However, as Steve Chinn frequently points out, there is a disappointingly limited body of research and a relatively limited range of literature exploring themes related to dyslexia and maths learning. In particular, there is very little consideration of the typical learning profiles and particular learning needs of young primary school dyslexic and dyspraxic children.

In recent years, on the other hand, there has been an explosion of research into how ‘ordinary’ young children make sense of numbers. Many studies set out to explore the ways in which children come to learn about, and make progress in, the crucial foundation stages of number-work. These studies have practical implications for how these ‘building block’ areas of maths are best taught. Furthermore, a number of contemporary researchers are interested in understanding the maths behaviours of the small numbers of children who can be found in any maths classroom who have difficulty making sense of numbers and who fail to make progress in number-work from the very earliest stages of mathematics. This contemporary research provides many illuminating insights into the maths learning profiles of primary school dyslexic and dyspraxic children. In particular, as we will see, the work of Karen Fuson and her colleagues in the US, Ian Thompson, Eddie Gray and Judith Anghileri, in the UK, and the work of the contemporary generation of Realistic Education proponents in The Netherlands have shaped many aspects of the ‘attitude and approach’ to teaching maths which are described in the book.

More general but illuminating books by Brian Butterworth (1999) and Stanislas Dehaene (1997) – both of which set out to explore the anthropological and biological bases of our knowledge of numbers – have also influenced some of the key discussions. In particular, I am indebted to Brian Butterworth’s very clear account of the structure of the number system and to Dehaene’s suggestive ideas about the *times tables*. I have also tried to make a layperson’s sense of their neuroscientific contribution towards understanding why and how we know about numbers in the first place and make some brief and tentative suggestions about what their studies and accounts of ‘maths in the brain’ seems to suggest about maths learning difficulties.
Which pupils does the book cater for?

Dyslexia, Dyspraxia and Mathematics explores ways of helping primary school dyslexic and dyspraxic children acquire a sound foundation in all of the key numeracy aspects of maths. Although many dyspraxic children also have difficulty with the non-number, spatial aspects of maths, these difficulties have not been addressed in this book.

In the last decade, or so, it has been increasingly widely accepted that a substantial proportion of dyslexic children have at least some difficulties learning the basic number aspects of maths. In recent years, as more children are diagnosed as belonging to the dyspraxic side of the specific learning difficulties spectrum, many teachers, educational psychologists and parents are finding that a very substantial proportion of young dyspraxic children have difficulties with number-work, too.

It is often noted that it is hard to generalize about the maths learning abilities and difficulties of dyslexic children. As Chinn and Ashcroft write,

many dyslexics have difficulty in at least some aspects of mathematics, but this is not necessarily in all aspects of mathematics. Indeed, some dyslexics are gifted problem solvers, despite persisting difficulties in, for example, rote learning of facts. (1998, p. 14)

Some researchers, for example Steeves (1983) and Miles and Miles (1992) have found that a number of dyslexics are broadly gifted in most aspects of maths and informal reports from teachers and parents seem to confirm Steeves's and Miles and Miles's finding.

Unfortunately there appears to be no detailed published research on the maths abilities of children who have been formally diagnosed as dyspraxic. Standard ability measures used by educational psychologists show that dyspraxic children are often weak at maths. Teachers report that a great many of the dyspraxic children in their classrooms have difficulties with the numeracy aspects of maths. Diagnostic assessments and teaching experience show that dyspraxic children with high verbal scores and with long-term and working-memory strengths frequently do well in the routine, procedural aspects of maths. However teachers report that a great many of the dyspraxic children whom they teach have severe problems in the very earliest stages of maths and that most dyspraxic children have serious word-problem-solving, 'number-puzzle'-solving, and pattern-solving weaknesses which persist throughout their primary school careers.

Although it is hard to generalize about the maths abilities of dyslexic children, as we have noted, it is well documented and widely acknowl-
edged that dyslexic children who show some ability in the numeracy aspects of maths nevertheless often have marked difficulty with two of the foundation aspects of number-work: dyslexic children typically have difficulty remembering exact maths facts, such as the times tables facts, and (like many dyspraxic children) dyslexic children also have difficulty with the place value conventions of the written number system. This book addresses these seemingly ‘dyslexia-typical’ and ‘dyspraxia-typical’ weaknesses and offers practical teaching suggestions which will help children in these areas.

While the available literature has offered a fairly clear picture of the number-related ‘gaps’ which can be described as ‘dyslexia-typical’ behaviours, it has been less widely acknowledged that some dyslexic children, and many dyspraxic children, have difficulties with number-work which are really very deep-seated and profound and which seem to go back to the very earliest stages of making sense of numbers. In fact, classroom teachers report that a sizeable proportion of dyslexic and dyspraxic children have quite severe all-round (global) maths learning difficulties. It is this group of children – the children who fail to make progress from the earliest stages of learning about numbers onwards – whom classroom teachers and parents are often most concerned about. It is also quite typically this group of children who do not seem to respond to ‘ordinary’ additional maths tuition and who seem to require ‘specialist’ understanding and help. This book sets out to examine the better understood ‘typical’ dyslexic and dyspraxic maths learning difficulties. It also sets out to begin charting the maths-learning profiles and apparent learning needs of the hitherto rather neglected group of dyslexic primary school maths learners who appear to have longstanding difficulties with all aspects of number-work and who can be described as children with very significant maths learning difficulties. As we will see later, these are also the children who are sometimes labelled dyscalculic.

Recent changes in maths teaching and a consideration of present-day maths learning situations

In broad maths educational terms this is an exciting but also potentially confusing time to be thinking about maths teaching and learning. In the wake of the explosion of research into how young children understand and learn about numbers, which was briefly mentioned above, far-reaching educational reforms have been introduced into primary school maths classrooms in many parts of the Western European world, including in the UK. The radical National Numeracy Strategy Framework was implemented in English state school classrooms in September 1999.
In general terms, many of the recent maths education reforms have been largely positive ones for dyslexic and dyspraxic maths learners. Influential contemporary maths educationalists, including Chinn and Ashcroft, have campaigned for a long time for some of the changes which have been enshrined in the Numeracy Strategy Report and in the Numeracy Strategy Framework. For instance, in reaction to the understood shortcomings of traditional maths teaching, the Numeracy Strategy, like most other newer maths teaching approaches, sets out to try and help children make genuine sense of mathematics. Instead of expecting that children simply learn facts and procedures solely by heart, or through rote 'drill', there is an emphasis on helping children understand logical principles, important concepts, and underlying patterns and structures. In keeping with this, there is a far greater emphasis on 'mental' mathematics, in general, and on logic-based and numeracy-friendly, informal ways of calculating.

The contemporary ideas about maths learning, together with the reforms they have inspired, have not, however, affected all primary school children in the UK in the same measure. The maths educational 'map' of what actually happens on the ground in maths classrooms is quite complex at present. While state-sector classrooms in England follow the Numeracy Strategy guidelines, the Numeracy Strategy has not been implemented in Scotland, Wales or Northern Ireland; and although Scotland and Wales have instituted their own maths teaching reforms, maths continues to be taught in quite traditional ways in Northern Ireland. Furthermore, private-sector schools in England are not bound to implement the National Numeracy Strategy.

In fact, the maths educational picture in private-sector schools in England is particularly complex and would seem to be in a state of flux at present. On one hand, it is evident that the Numeracy Strategy has had ripple effects on maths teaching in a number of private schools: for example, the teaching ideals, goals and recommendations of the Numeracy Strategy have shaped the ways that recent maths textbooks and schemes have been designed, and the Numeracy Strategy has also informed the content of the standardized maths National Curriculum Tests. Some head teachers and heads of maths departments in private schools have actively welcomed changes in maths teaching practices, and have looked favourably on the impetus towards reform – in particular, on the greater weight accorded in newer maths teaching approaches to mental maths. On the other hand, it is also evident that a considerable number of private schools have continued to teach maths in largely unchanged,
traditional ways. Many private schools continue to use older teaching methods, textbooks and materials, and many private schools also place greater weight on the results of hitherto more traditional 'Common Entrance' maths papers (or similar papers) than they do on the reform-based goals of the National Curriculum tests.

In the context of this book, it is of course the learning needs – and therefore the teaching requirements – of dyslexic and dyspraxic children which is the paramount consideration. Although the teaching issues are extremely complex (and some of the complexities will be explored, later on) it is also important to give an overview perspective on the ways in which different approaches to maths teaching and learning can affect the ability of dyslexic and dyspraxic children to make progress in learning maths.

On one hand, as we will see, and as Steve Chinn has frequently demonstrated, the memory requirements of traditional maths approaches create broadly unfavourable maths learning environments for dyslexic and dyspraxic maths learners and contribute to the severe difficulties that most dyslexic and dyspraxic children experience in the majority of traditional maths classrooms. The complex memory difficulties, which are commonly associated with dyslexia and dyspraxia, and which affect maths learning, will be explored in greater detail later on. In brief, however, traditional maths approaches require that maths facts are acquired through rote learning with little emphasis on the inter-relationships between facts. A good proportion of maths learning time is devoted to memorizing standard calculation procedures in columns. Learning the standard procedures depends on a good visual memory and a very good memory for sequential sets of instructions.

On the other hand, institutionalized, progressive approaches to teaching maths, such as that embodied in the Numeracy Strategy, create potentially favourable environments for dyslexic and dyspraxic children to learn maths. As suggested above, most of the principles and goals which lie behind recent maths reforms are principles and goals which apply, in essence, to dyslexic and dyspraxic maths learners, too. However, the cognitive weaknesses associated with dyslexia and dyspraxia, and the severity of the weaknesses affecting individual children, also influence the degree to which dyslexic and dyspraxic children are able to make progress in mainstream maths teaching approaches – approaches which are, in the main, designed to cater for the learning needs of 'ordinary' young children. It is perhaps not surprising that the progress of dyslexic and
dyspraxic children in, for example, Numeracy Strategy classrooms has been somewhat mixed to date. On the one hand teachers report that although maths fact acquisition continues to be an area of difficulty for most dyslexic children, more mathematically able dyslexic and dyspraxic are generally enjoying maths and are making good progress within the framework of the relatively flexible, pattern and logic-based approach to learning maths which characterizes the Numeracy Strategy. On the other hand, there have been more worrying reports that some children with specific learning difficulties – generally children who are found to have moderate to severe maths learning difficulties – are not faring particularly well in classrooms which are guided by the Numeracy Strategy approach. Indeed, since 1999 it has become increasingly clear that a significant number of dyslexic and dyspraxic children – together with other children who find maths learning difficult – are not making expected progress in otherwise successful state school classrooms. Many educationalists, teachers, and support teachers have recognized that if all dyslexic and dyspraxic children are to be helped to make the best possible progress in learning mathematics some teaching practices will have to be modified to take account of the number-related learning needs which dyslexic and dyspraxic children may have.

To sum up: Steve Chinn, and Steve Chinn and Richard Ashcroft, have convincingly shown that traditional maths teaching approaches do not suit the learning needs of the vast majority of dyslexic children. They have pioneered the argument, in the UK, that children with specific learning difficulties need to be taught in such a way that they are able to understand all aspects of the maths they are learning and that they need to be taught to reason effectively instead of being expected to rote learn facts and procedures. They have passionately fought for open-minded and flexible maths classroom environments. Contemporary progressive maths approaches, such as that of the Numeracy Strategy, share many of the maths teaching and learning ideals and principles for which Chinn and Ashcroft have campaigned. However overview assessments of children's progress at the primary school level shows that the particular maths learning needs of a great many dyslexic and dyspraxic children seem to require a degree of special consideration. This book is inspired by the aim of describing an understanding-based approach to teaching the numeracy aspects of primary school maths which also takes into account the special cognitive features of dyslexic and dyspraxic primary school children.
Basic definitions and important features associated with dyslexia and dyspraxia

While there can be marked cognitive and behavioural differences between \textit{classic dyslexic} and \textit{classic dyspraxic} learners, the distinctions are often harder to make in practice. As Madeline Portwood (2000) clearly explains, there is a very significant degree of overlap or comorbidity between different specific learning difficulties, such as dyslexia and dyspraxia. Madeline Portwood explains further that dyslexia and dyspraxia can also be comorbid with attention deficit disorders (ADD and ADHD) and Asperger's syndrome. From the point of view of this book this means that many children will have both dyslexic and dyspraxic cognitive and learning features. Dyslexia and dyspraxia also share a number of important learning-related characteristics, as we will see. This partly explains why so many classic dyspraxic learners have been diagnosed as dyslexic in the past.

Nevertheless it is important to acknowledge that \textit{classic dyslexia} and \textit{classic dyspraxia} are associated with some widely divergent weaknesses and strengths. The brief descriptions of dyslexia and dyspraxia, which follow, draw attention to some key differences and some of the key similarities in the cognitive profiles of dyslexic and dyspraxic children.

'Classic dyslexia'

Basic definitions of dyslexia usually centre on the difficulties which dyslexics experience in processing the symbolic aspects of language. Dyslexic children have difficulty learning to read and spell in large part because they have difficulty mapping segments of sound (phonemes) on to written symbols (graphemes). Some dyslexics have difficulties with phoneme awareness or the initial discrimination of sounds. Many dyslexic children have language acquisition, word finding or semantic (meaning-related) difficulties. Underlying cognitive weaknesses associated with dyslexia include: poor long-term verbal memory; poor working memory; poor sequencing skills and sequential memory; difficulties with auditory and/or visual perception and memory; and poor left/right discrimination. Because dyslexia is so strongly associated with difficulties to do with processing symbols, \textit{standardized assessment profiles} which are administered by educational psychologists – for example, the Wechsler Intelligence Scales or WISC – tend to show depressed \textit{verbal} scores in relation to overall \textit{intelligence} and \textit{performance} (non-verbal) stores. In terms of very broad brain function, dyslexia is often broadly characterized as a tendency towards
general left-hemispheric weakness. On the other hand, the *performance* scores of many dyslexic individuals show strengths in spatial or visuospatial areas. Such strengths, 'which may contribute to outstanding creative skills' (L. Peer in Smythe, 2000, p. 67), are sometimes said to be the 'compensatory gift' of dyslexia. Although some *classic dyslexics* do not have exceptionally strong *performance* skills, dyslexia is associated, generally speaking, with a tendency towards relative right hemispheric strengths in the brain (Portwood, 2000).

'Classic dyspraxia'

In essence, dyspraxia is associated with motor co-ordination difficulties – often with gross and fine motor co-ordination difficulties – and with perceptual and spatial-perceptual weaknesses. According to Portwood (2000, p. 26) all dyspraxics have 'co-ordination difficulties' and the vast majority 'show significant perceptual problems' (p. 26). Additional weaknesses associated with dyspraxia include: left/right confusion; poor tactile perceptual skills; poor hand–eye co-ordination; poor working memory; poor visual memory; poor sequencing skills; poor short-term visual or auditory memory; poor verbal memory; poor memory for verbal instructions; and finger agnosia (loss of 'finger sense' or an intuitive knowledge of the fingers). Standardized assessments of dyspraxics classically show depressed *performance* scores. They also show that 'on average verbal scores are higher than performance.' (Portwood, 2000, p. 47). In terms of hemispheric dominance this means that dyspraxia can be associated with a tendency towards right hemispheric weakness or 'immaturity' and with relative left hemispheric strength. In this regard it is worth noting that classic dyspraxia is not always associated with significant difficulties with learning to read although it is often associated with significant spelling difficulties.

Some cognitive weaknesses which dyslexic and dyspraxic children commonly share and which can affect maths learning

It is, of course, vital to acknowledge that each individual dyslexic or dyspraxic child will bring 'different combinations of strengths and weaknesses' to maths and that, as we have seen, there are 'enormous variations' in maths abilities among individual children with specific learning difficulties (Chinn and Ashcroft, 1998, p. 5). However, it is also vital to be aware of what one might call 'the big picture' in the relationship between
specific learning difficulties and maths learning difficulties. Before we go on to explore some of the significant ways in which dyslexic and dyspraxic learners can diverge in terms of how they process number tasks, it is important to start out by outlining some of the significant underlying learning constraints which a great many dyslexic and dyspraxic children have in common.

**Poor long-term memory in maths learning**

It is widely documented that dyslexic and dyspraxic children have difficulty automatizing maths facts and maths procedures – in other words, dyslexic and dyspraxic children have difficulty recalling number facts (such as subtraction facts, or multiplication facts) or the way to ‘do sums’. As we will see, working memory weaknesses and sequencing difficulties contribute to many of the long-term memory difficulties which dyslexic and dyspraxic children experience in maths. In addition to this, as Miles and Miles (1992) and Chinn and Ashcroft (1998) have shown, the majority of dyslexic children find it a ‘frustrating exercise’ to learn verbally encoded facts (Chinn and Ashcroft, 1998, p. 68) and find it almost impossible to recall many of them ‘in one’. Over and over again it is commented on that dyslexic children, and many dyspraxic children, fail to learn facts easily in the form of pure verbal associations. This has a disastrous impact on *times tables* learning, as we will see.

**Poor working memory**

Working memory weaknesses impact on maths learning in at least two key ways. First of all, most aspects of working with numbers, from basic counting onwards, are linear or step-by-step processes which involve holding several pieces of information in working memory at the same time. Children with learning difficulties often lose track of what they are doing, forget what the initial task was or forget the teacher’s instructions. ‘What was the sum again?’ is a classic ‘dyslexic’ or ‘dyspraxic’ question. As Chinn and Ashcroft (1998, p. 8) explain,

> The pupil may not be able to ‘hold’ the visual image of the sum he is trying to solve. He may not be able to hold the sum in visual or auditory memory while he searches for a necessary number fact.

As number-work becomes more demanding, a greater number of elements need to be held in working memory at once. At the most obvious
level, poor working memory affects the child's speed of thinking and calculating and, indeed, working in general in maths.

Secondly, memory weaknesses contribute to long-term memory difficulties and vice versa. The relationship between working memory and long-term memory is obviously complex, but, generally speaking, it would appear that in order for number information to enter long-term memory, working memory processes need to be relatively efficient. According to Ashcraft et al, the working memory has a limited capacity. In Eddie Gray's (1997) useful and succinct formulation, for information to enter long-term memory, in maths, it is important that the input, or question, and the output, or the answer to the question, are close together. In simple terms, poor working memory, or lengthy working memory processes, mean that number information is less likely to enter long-term memory. As we will see, sequencing difficulties contribute significantly to working memory difficulties in number-work, and ultimately – in a vicious cycle of cause and effect – to long-term memory difficulties. On the other hand, Ashcraft et al. (1996, p. 195) argue that long-term memory weaknesses (poor fact and procedure retrieval) contribute to working memory problems: long-term memory weaknesses drain processing resources or capacity from the 'executive' or managing component of the working memory in another complex cycle in which it is hard to disentangle cause and effect. In simple terms, limited working memory resources are 'drained' when children have to spend time trying to work out facts or trying to remember procedures and, once again, the ultimate outcome, or the steps of the procedure, are not remembered in the long-term.

Ashcraft et al. also maintain that maths anxiety affects the efficient functioning of working memory. We will touch on the theme of anxiety towards the end of this chapter, but one of Ashcraft et al.'s (1996, p. 193) significant suggestions is that anxiety causes 'intrusive thoughts and worry' to drain working memory resources. Hence Ashcraft et al. believe that maths anxiety affects cognitive functioning in maths tasks and therefore affects overall maths learning in a direct way. Chinn and Ashcroft (1998) and Henderson (1989, 1998) have noted that dyslexic children are often anxious about specific aspects of maths, such as the tables facts or division, and that some dyslexic children are anxious about maths in general. They show that many dyslexic and dyspraxic children are not confident enough to 'have a go' at answering 'challenging' maths questions: 'no attempt' errors contribute significantly to the poor scores which are commonly attained by dyslexic and dyspraxic pupils in maths assessments. Many parents of dyslexic and dyspraxic children volunteer
that their children are anxious about maths. In diagnostic assessments of primary school dyslexic and dyspraxic children more than 70 per cent of the children assessed stated that they ‘hated’ maths.

**Sequencing problems**

The broad label *sequencing problems* covers a number of complexly interrelated areas:

1. *Counting and the number system* Very early on, from the beginning stages of learning how to count, children have to make sense of, and have to learn to use, complex sequences of number words. As we will see, the ability to count also involves mapping words on to sequences of objects. Remembering sequences of words and seeing patterns within these sequences is a crucial aspect of learning to understand the complex structures of the number system. Many dyslexic and dyspraxic children learn to count later than their peers and fail to understand the structures of the number system. In consequence they have difficulty decoding large numbers and have problems solving mental and written large number calculations.

2. *Counting in number-work* As we will also see, much of early number-work is bound up with quite lengthy sequences of counting, for example, in counting on in addition, and in counting back in subtraction. Difficulty managing counting sequences, and especially ‘backwards’ sequences, impacts on working memory efficiency and on the automatization of facts.

3. *Sequences of instructions* It is widely noted that many dyslexic and dyspraxic children have difficulty remembering sequences of verbal instructions. All larger number calculations involve completing a sequence of steps. The familiar standard calculation methods are usually taught ‘procedurally’ or as a series of verbal instructions – ‘first, you do this, next you do this, next you do this, then you ...’ Standard methods are consequently particularly difficult for dyslexic and dyspraxic children to learn. The newer mental calculation methods are generally relatively easy to understand and do not require to be taught in ‘recipe-like’ ways. However, a number of teachers do, in fact, resort to teaching mental methods as rote-learned verbal routines: this happens, in particular, when teachers wish children to learn a number of different mental methods for specific operations.
Directional confusion

Although *directional confusion* can simply mean that young children muddle written digits, for example ‘2’ and ‘6’, directional difficulties usually become particularly significant when two-digit numbers need to be read, or decoded, and written or encoded. As we will see, the fact that the number system is structurally different in the crucial second decade between 10 and 20 means that many children become confused about which digit in a two-digit number to say or write first. These ‘normal’ difficulties are compounded if children confuse direction or ‘position.’ Directional confusion can mean that children who know that the larger value is usually read or written first may sometimes still be confused about what ‘first’ means in positional terms – in ‘left/right’ terminology a child may suddenly flounder as to whether ‘first’ means left or right.

Directional confusion can have particularly devastating consequences if children are taught column-based methods of multi-digit calculation from the outset. As we will see, column-based methods of addition and subtraction begin from the right whereas numbers, words and sentences are read from the left. As Chinn and Ashcroft point out, the starting point for the standard division algorithm, which is on the left, can upset children’s hard-won and overlearned ‘right-to-left in calculation’ response. In ‘borrowing’ in standard subtraction procedures, children have to move left and right and left in extremely taxing ways. Finally, the ‘crossover’ directional demands of formal long multiplication and the across-and-step-down demands of formal long division methods contribute to the difficulties which many dyslexic and dyspraxic children experience in trying to reproduce the standard ways of executing calculations.

Speed of working in oral and written work

Madeline Portwood (2000) suggests that the neurological immaturities (‘wiring immaturities’), which may contribute to dyslexia and dyspraxia, mean that children with learning difficulties tend to be slow to process incoming information. Even when maths information has been lodged in long-term memory, many dyslexic and dyspraxic children take longer to access this stored information. It is often noted that an over-emphasis on requiring dyslexic or dyspraxic children to give quick answers to maths facts questions, or to figure out mental calculations rapidly, has the effect of undermining the dyslexic or dyspraxic child’s ability to think. Of course
working memory difficulties, attention deficits and anxiety all compound delays in processing and retrieving information. As Chinn and Ashcroft argue, this often has the further result that children with learning difficulties complete written work more slowly and complete less work than their peers. This, in turn, can mean that children may not reach more challenging examples in written exercises or may fail to reach, and therefore have the opportunity to work through, certain exercises altogether.

**Poor ability to generalize in mathematics: a weak basic number-concept**

In an early analysis of the maths profiles of dyslexic learners, Joffe (1983b) observed that many dyslexic pupils do not easily generalize the knowledge they have acquired in number-work. In supporting this statement, Chinn and Ashcroft (1992, p. 98) say, 'in our experience of teaching dyslexics we have observed another handicapping factor: a poor ability to generalize and classify facts and rules in mathematics'. They go on to say that, in their experience, many of their secondary school students view maths 'as an amorphous, disjointed mixture of facts, rules and methods. Although they can understand these parts in isolation, they frequently have difficulty in mastering the interrelationships and cross generalizations.' These broad generalizations apply to many dyslexic and dyspraxic primary school children, too. It is often observed that primary school dyslexic and dyspraxic children can be particularly rigid and inflexible in their thinking and that they tend to see numbers and calculations in linear, action based, and rather tunnel-like ways.

As we will see, inflexibility in working with numbers at a primary school level usually springs from what is increasingly called poor number sense and a related weak basic number concept. In essence this means that children view numbers and calculations in primitive ones-based ways. Fuson, Wearne, Hiebert et al. (1997) call a ones-based number concept a unitary concept of numbers. An over-reliance on counting in ones and an inability to see patterns and connections means that it is hard for children to develop the broader, flexible understandings in number-work which underpin the ability to make links with other aspects of number-work. As Ashcraft et al. (1996) and Gray (1997) suggest, this is in large part because children who are weak at maths use calculation methods (mainly counting) which place very big demands on working memory. Instead of developing increasingly complex webs of understanding, many dyslexic and dyspraxic children tend to think along
isolated calculation tracks. An impoverished understanding of numbers and number relationships (which underpins a poor generalizing ability) has enormous repercussions, the most important of which have been well documented:

1. A large number of dyslexic and dyspraxic children can complete calculations presented in familiar, easily recognizable, standard ways, but cannot cope with unfamiliar presentations, or challenging tasks.
2. Dyslexic and dyspraxic children often have difficulty understanding which operation is involved in mixed word-problem work.
3. In mental calculation work, dyslexic and dyspraxic children often fail to select an appropriate ‘figuring out’ fact derived strategy or mental calculation method. In many instances, some dyslexic and dyspraxic children ‘may be so confused us to have no clues as to where to start’ (Chinn and Ashcroft, 1998, p. 8).

Diverging strengths and weaknesses

An interesting dyslexic maths learning personality

Although a large proportion of dyslexics have poor fact recall it has long been noted – by, for example, Tim Miles, Chinn and Ashcroft, and Anne Henderson – that a significant number of dyslexic children are good at the ‘thinking’, conceptual, or problem-solving aspects of mathematics. Teaching experience confirms that some dyslexic children solve certain maths tasks spectacularly quickly and without appearing to do much calculation. While fact-retrieval difficulties can slow down their calculation some dyslexic children quickly grasp the principles of logico-mathematical (or ‘thinking’) calculation strategies. With the right support, some dyslexic children are able to invent ways of figuring out difficult calculations for themselves. On the other hand, it is also commonly noted that the innate mathematical ‘flair’ of these dyslexic children can be difficult for them to harness, particularly in more traditional maths classrooms. Typically, dyslexic children who are able ‘thinkers’ are not able to explain the methods that they used to solve calculations or word problems. Since they appear to be working intuitively, they often seem genuinely unable to record their methods in written ‘workings’ or recording. In part because their exact fact knowledge is limited, their answers are often inaccurate, too.
A consideration of the numeracy profiles of dyspraxic children

Unfortunately there is very little available detail about the ‘typical’ numeracy profiles of dyspraxic children and general comments and teachers’ responses tend to be somewhat gloomy. As we have already seen, Portwood notes that the majority of dyspraxic children have significant difficulties learning maths. Studies of children with ‘spatial weaknesses’ indicate that poor spatial ability correlates with weaknesses in the very earliest stages of number-work. One interpretation of this is that very early number-work depends on physical counting activities, and objects are often hard for children with spatial difficulties to manipulate; concrete work is also hard for children with poor spatial ability to process and visualize (Carter, Crawley and Lewis, 1999a). As we have already noted, too, teachers consistently report that the majority of dyspraxic children struggle to understand concepts, logico-mathematical ways of reasoning (such as fact-derived strategies, or mental calculation methods), word problems and number puzzles. It is often commented on that dyspraxic children seem to be particularly rigid maths thinkers. While there are no references in the available literature on dyspraxia or in ‘mainstream’ maths learning literature to surprising areas of ability among dyspraxic maths learners, some dyspraxic children are able to do reasonably well in certain areas of maths. As we have suggested, this is largely because some dyspraxic children have strong verbal memory abilities. While it is not the case that all dyspraxic children master times tables as verbal associations, it would seem that a small but significant proportion of dyspraxic children are able to do so. We have already commented on the fact that some dyspraxic children are able to learn the routine, procedural aspects of maths, and are able to perform well in familiar calculation situations.

Ways of interpreting the very different maths learning personality possibilities among dyslexic and dyspraxic maths learners

Very crudely speaking, able dyslexic maths learners and (relatively) able dyspraxic maths learners seem to have abilities at opposite ends of the broad spectrum of numeracy requirements. In simple terms, a small number of dyslexic children have ‘thinking’ abilities in maths whereas most dyspraxic children find the ‘thinking’ aspects of maths especially hard to manage. On the other hand, a small number of dyspraxic children have