Introduction to Biological Physics for the Health and Life Sciences

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Staff at the University of Otago,
New Zealand

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Contents

PREFACE

Physics is central to an understanding of biomedical science. We are aware that many students studying for a career in biomedicine are not primarily motivated by physics; they are interested in other areas of science. We are also aware that no currently available first-year textbook takes the physics needs of health-science students seriously.

In this textbook we have several goals. Firstly, we are trying to present the necessary base concepts of physics as clearly as possible. Secondly, the textbook is designed to remove any unnecessary conceptual load from students by removing all physics that is not absolutely necessary for health-science students. The decision as to which parts of physics are necessary has been determined in close collaboration with the physicists and teachers of the Department of Physics and the professional clinicians and academics in the Faculty of Health Science at the University of Otago. Thirdly, we are keenly aware that student motivation is always an issue in the study of physics for the health sciences. We have tried to add as many applications to the biomedical sciences as possible to the text in an attempt to aid this motivation. The companion website for this book is available at www.wiley.com/go/biological_physics.

The production of a textbook is an enormous task and this textbook is no exception. In writing this book, we have relied on the expertise and goodwill of a large group of academic colleagues. We would like to express our gratitude to Mr Gordon Sanderson of the Ophthalmology Department and Professor Terence Doyle of the Radiology Department in the University of Otago Medical School. We would like to thank Dai Redshaw for the many hours he has spent reading through the text and working through the problem sets and Dr Phil Sheard from the Department of Physiology for his inspiring review lectures on bioelectricity. We would particularly like to thank Dr Don Warrington for his diligent and careful reading of the entire manuscript, and for his many corrections and suggestions. Finally we would like to thank the staff of the Department of Physics at Otago for the time and support that they have rendered over the past years. While the staff of the Department of Physics are listed as authors of this textbook we would particularly like to thank Gerry Carrington, Pat Langhorne, Craig Rodger, Rob Ballagh, Neil Thomson and Bob Lloyd.

Finally, the goal of this textbook is to provide for the needs of our students. In order to achieve this goal, we have depended on the feedback provided by our students. There will of course still be errors which have escaped our editing process, and for these we apologise in advance, and we welcome feedback from our readers.

I

Mechanics

Mechanics is the study of motion. It may be divided into two related areas: kinematics and dynamics. Kinematics is the study of the fundamental properties of motion: displacement, acceleration, velocity, distance, and speed. These concepts allow us to quantify motion and this allows for its scientific study. Dynamics is the study of force as described by Newton's three laws. Forces produce accelerations and thus cause changes in the motion of objects.

Mechanics is the most fundamental subject in physics: it shows how the forces of nature produce the changes which are observed in nature. The concepts introduced in mechanics will be used throughout the rest of this book.

Mechanics is of central importance in the health sciences. The applications of mechanics in biological systems appear whenever the concepts of force, energy or momentum appear. There are, however, many more direct applications of the ideas of mechanics. The working of the musculoskeletal system in humans and other vertebrates cannot be understood without an understanding of mechanical concepts such as torque, force, levers and tension. The energy and forces required for everyday activities in nature – jumping, flying, accelerating to elude capture – can only be evaluated using the techniques introduced in this section.

1

KINEMATICS

1.1 Introduction

Kinematics is that part of mechanics which is concerned with the description of motion. This is a vital first step in coming to an understanding of motion, since we will not be able to describe its causes, or how it changes, without a clear understanding of the properties of motion. Kinematics is about the definition and clarification of those concepts necessary for the complete description of motion. Only six concepts are needed: time, distance, displacement, speed, velocity and acceleration.

We will begin by focussing on linear motion in one dimension. Later we will expand this to include motion in two and three dimensions, and we will then look at three particularly important special cases of motion in one and two dimensions: circular motion, simple harmonic motion, and wave motion.

Key Objectives

- To develop an understanding of the concepts used to describe motion: time, distance, displacement, speed, velocity and acceleration.
- To understand the relationships between time, displacement, velocity and acceleration.
- To understand the distinction between average and instantaneous velocity and acceleration.
- To understand that the horizontal and vertical components of vector quantities, such as acceleration and velocity, may be treated independently.

1.2 Distance and Displacement

Motion is characterised by the direction of movement, as well as the amount of movement involved. It is not surprising that we must use vector quantities in kinematics. The **distance** an object travels is defined as the length of the path that the object took in travelling from one place to another. Distance is a scalar quantity. **Displacement**, on the other hand, is the distance travelled, but with a direction associated. Thus a road trip of 100 km to the north covers the same distance as a road trip of 100 km to the south, but these two trips have quite different displacements. The use of displacement rather than distance to give directions is commonplace.

1.3 Speed and Velocity

We are accustomed to talking about the speed at which an object is moving. We also talk about the velocity with which an object is moving. In normal usage these two words mean the same thing. I can talk about the speed with which a car is travelling, or I can talk about its velocity. In physics, we redefine these two words, **speed** and **velocity**, so that they have similar, but distinct meanings.

Key concept:
The **velocity** of an object is the change in its position, divided by the time it took for this change to occur. Velocity is a vector and has both a magnitude and a direction.

Mathematically, the velocity of an object is

$$\boldsymbol{v} = \frac{\Delta \boldsymbol{x}}{\Delta t} \tag{1.1}$$

where $\boldsymbol{v}$ is the velocity vector, $\Delta\boldsymbol{x}$ is the displacement vector and Δt is the time interval over which the displacement occurs. Note that we will use bold symbols, such as $\boldsymbol{v}$, for vectors and normal-weight symbols, such as v, for scalar quantities. Note also that the Greek letter Δ (capital delta) represents the change in a quantity. In the above expression, Eq. (1.1), for example, the change in the position of an object is its final position minus its initial position:

$$\Delta x = x_{\mathrm{f}} - x_{\mathrm{i}} \tag{1.2}$$

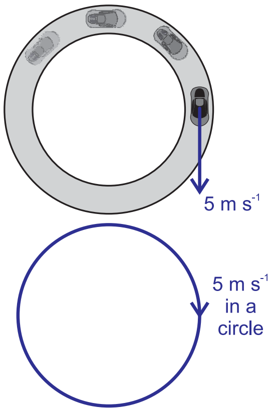

Figure 1.1 A toy car on a race track. How do we characterise its motion?

Vector equations vs. scalar equations
When demonstrating numerical calculations the vector character that many quantities possess will not be explicitly addressed in the equation itself. Most numerical examples will be treated as scalar problems without any attempt to represent the various quantities used as vectors. This is to keep problems simple and readable. Please note that this does NOT mean that vector properties are ignored, but rather that they are addressed in the process of constructing the problem.

Key concept:
Speed is the magnitude of the velocity. Speed is a scalar, and it does not have a direction.

The speed of an object is the distance travelled, divided by the time it took to travel that distance:

$$v = \frac{\Delta x}{\Delta t} \tag{1.3}$$

Note the differences between Eq. (1.1) and Eq. (1.3). In Eq. (1.1), we use bold symbols for both the $\boldsymbol{v}$ and the $\boldsymbol{x}$, indicating that we are referring to the velocity and the displacement in this equation. In Eq. (1.3) we use normal weight symbols, v and x, indicating that we are referring to the speed and distance in this equation.

Many textbooks use d to represent distances and $\boldsymbol{d}$ to represent displacements rather than Δx and $\Delta\boldsymbol{x}$. We will often follow this practice when specific reference to the initial and final positions is not called for.

Consider Figure 1.1. A toy car is travelling in a circle around a toy race track and we wish to characterise its motion. If we are interested only in how fast the car is going, we could say it is travelling at 5 m s^{-1} (= 18 km h^{-1}). Two cars travelling on the same circle will be perfectly well distinguished by noting the different lengths of the circle they traverse in the same time.

Now consider the situation illustrated in Figure 1.2. In this case, two cars approach the same intersection from different directions. In this situation, we might point out that one of the cars is travelling at 18 km h^{-1}, while the other is travelling at 12 km h^{-1}. However, this will not cover all of the differences between the two cars. Another important fact about them is that they are travelling in different directions. If we wanted to predict where these two cars would be in an hour (for example) it would not be enough to just use the magnitude of their velocity; we would also need to take into account their directions.

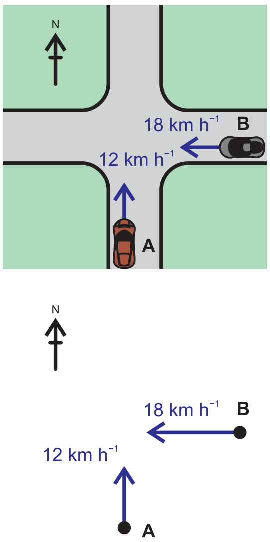

Figure 1.2 Linear motion in two directions. The cars are travelling at different speeds and in different directions.

1.4 Acceleration

In kinematics, the **acceleration**, $\boldsymbol{a}$, is a vector which quantifies changes in velocity. In everyday conversation we use the word acceleration to mean that the speed of an object is increasing. If an object was slowing down we would say that the object was decelerating. The concept of acceleration in physics is more general and applies to a larger set of situations. In physics, acceleration is defined to be the rate of change (in time) of the velocity:

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t} \tag{1.4}$$

This definition implies several characteristics of the acceleration:

1. Acceleration is a vector: it has a direction as well as a magnitude. The acceleration is the rate of change of the velocity, and velocity is a vector, therefore acceleration must also be a vector.

2. The acceleration vector of an object may point in the opposite direction to that object's velocity vector. When this happens, the object's velocity will decrease and may even reverse direction. This means that deceleration (slowing down) is just another acceleration, but in a particular direction.

3. An object may have an acceleration without its speed changing at all. Should the acceleration vector point in a direction perpendicular to the velocity vector, the direction of the velocity vector will change, but its length will not. A good example of this is when an object moves in a circle. In this case, the acceleration is always perpendicular to the velocity, so the speed of the object is constant, but its velocity is constantly changing.

To illustrate these ideas, consider a car which starts from rest ($\boldsymbol{v}_{\mathrm{i}} = 0$) and accelerates along a straight road so that its velocity increases by 2 m s^{-1} every second. The velocity of this car is illustrated at a series of later times in Figure 1.3.

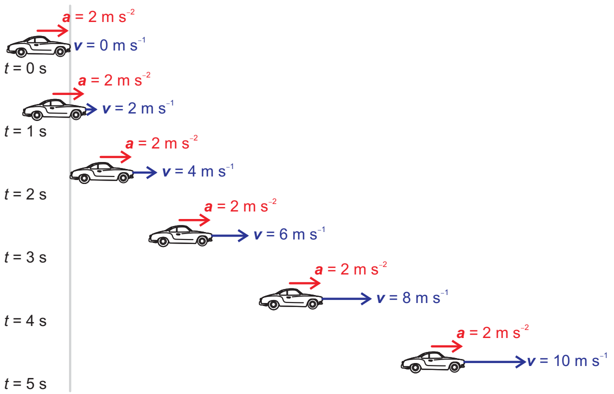

Figure 1.3 A car accelerating at 2 m s^{-2} for 5 s.

Since the velocity changes by the same amount every second (2 m s^{-1}), the acceleration of the car is constant. The velocity is changing at a rate of 2 m s^{-1} per second, or 2 metres per second per second. This acceleration would normally be written as $\boldsymbol{a} = 2$ m s^{-2} (or 2 m/s^2) to the right.

We can calculate the velocity at any time. Since we know how much the velocity increases every second and we also know that the car was initially stationary, we just multiply this rate by the time elapsed since the acceleration began, i.e., we use the equation

$$\boldsymbol{v} = \boldsymbol{a}t \tag{1.5}$$

Note that this is a vector equation, so that the velocity is in the same direction as the acceleration. For the car in this example, which is accelerating in a straight line at a constant rate of 2 m s^{-2} from rest, after 4 s the speed is $v = at = 2$ m s^{-2} $\times$ 4 s = 8 m s^{-1}, and so on.

What if the car had not been at rest initially? Suppose that the car in the previous example had been travelling at a constant velocity of 5 m s^{-1} for some unspecified length of time, and then began to accelerate at 2 m s^{-2}. Figure 1.4 shows this car at a sequence of later times. Compare this figure with Figure 1.3.

In one most important respect, the situation has not changed. The velocity of the car still increases at the same rate, so that the *change* in velocity after the acceleration begins is given by the equation

$$\Delta\boldsymbol{v} = \boldsymbol{a}t \tag{1.6}$$

The difference between Eq. (1.5) and Eq. (1.6) is that we now explicitly recognise that it is the *change* in velocity that we are calculating. In the previous example we calculated the change in velocity, but since the car started at rest, the velocity of the car was the same as how much the velocity had increased. Since we now have a nonzero initial velocity, we must recognise that the change in velocity is the final velocity minus the initial velocity, so

$$\boldsymbol{v}_{\mathrm{f}} - \boldsymbol{v}_{\mathrm{i}} = \boldsymbol{a}t$$

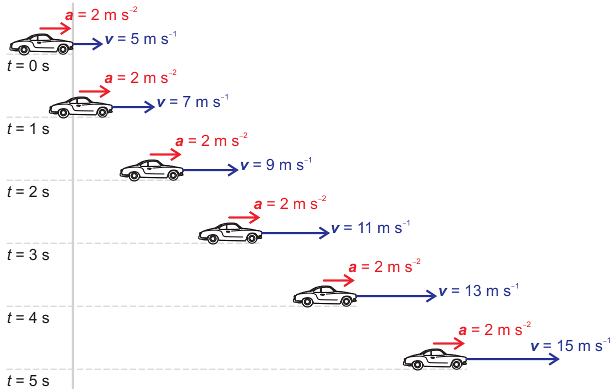

Figure 1.4 A car accelerates from an initial velocity of 5 m s^{-1} with a constant acceleration of 2 m s^{-2}.

Thus after 5 s we would find that 15 m s^{-1} – 5 m s^{-1} = 2 m s^{-2} × 5 s. We are normally interested in calculating the final velocity, so we write the above equation in the form

$$\boldsymbol{v}_\mathrm{f} = \boldsymbol{v}_\mathrm{i} + \boldsymbol{a}t \tag{1.7}$$

We can use this equation to find the final velocity at any later time, so long as the acceleration has not changed.

Note that the velocity calculated using this formula is the *instantaneous* velocity of the car at that time. This is the velocity that you would read off the car's speedometer. We will discuss instantaneous and average velocities in more detail next.

1.5 Average Velocity or Speed

The discussion above allows us to calculate the instantaneous velocity of an object moving with constant acceleration. This is the velocity of the object at a particular instant of time. It is also often useful, when we are dealing with motion in a straight line, to use the average velocity of an object to solve problems. The average velocity is

$$\boldsymbol{v}_\mathrm{av} = \frac{\text{total displacement}}{\text{total time}} = \frac{\boldsymbol{d}}{t} \tag{1.8}$$

If you drive a car from Dunedin to Christchurch (370 km away) in 5 h, your average speed is given by

$$v_\mathrm{av} = \frac{d}{t} = \frac{370\,\mathrm{km}}{5\,\mathrm{h}} = 74\,\mathrm{km\,h^{-1}}$$

It is important to realise that this calculation does not require any knowledge of the details of your trip. You may have traveled at a constant 74 km h^{-1} the whole way, or (more likely) you may have varied your speed significantly. You may even have stopped to look at the view and eat lunch for half an hour. These details are not needed for the calculation of the average velocity.

We will now derive a general relationship between the distance travelled by an object, its initial velocity, and its constant acceleration. If an object, for example a car, a plane or a soccer ball, has constant acceleration, then the displacement, $\boldsymbol{d}$, occurring in some given time, t, is

$$\begin{aligned}\boldsymbol{d} = \boldsymbol{v}_\mathrm{av}t &= \frac{1}{2}(\boldsymbol{v}_\mathrm{i} + \boldsymbol{v}_\mathrm{f})\,t \\ &= \frac{1}{2}(\boldsymbol{v}_\mathrm{i} + \boldsymbol{v}_\mathrm{i} + \boldsymbol{a}t)\,t \\ &= \left(\boldsymbol{v}_\mathrm{i} + \frac{1}{2}\boldsymbol{a}t\right)t\end{aligned}$$

$$\boldsymbol{d} = \boldsymbol{v}_\mathrm{i}t + \frac{1}{2}\boldsymbol{a}t^2 \tag{1.9}$$

Often $\boldsymbol{v}_{\mathrm{i}} = 0$, i.e., the object is starting from rest, so then

$$\boldsymbol{d} = \frac{1}{2}\boldsymbol{a}t^2 \tag{1.10}$$

We will now investigate an acceleration which is particularly important for the motion of objects near the surface of the earth: the *acceleration due to gravity*.

Example 1.1 ***Falling ball (1D kinematics)***

Problem: If you drop a cricket ball from a 125 m high tower, how far will it fall in 5 s?

Solution: We can solve this problem in two different ways. We can find the average velocity of the ball over the first 5 s and use this average velocity to calculate a displacement, or we can calculate a displacement directly.

(a) The acceleration due to gravity is $10\ \mathrm{m\ s^{-2}}$ downwards and so the velocity increases by $10\ \mathrm{m\ s^{-1}}$ in the downwards direction every second. The initial velocity is $0\ \mathrm{m\ s^{-1}}$ so the final velocity must be $5\ \mathrm{s} \times 10\ \mathrm{m\ s^{-2}} = 50\ \mathrm{m\ s^{-1}}$ in the downwards direction. The average velocity of the cricket ball is therefore

$$\boldsymbol{v}_{\mathrm{av}} = \frac{0\ \mathrm{m\ s^{-1}} + 50\ \mathrm{m\ s^{-1}}}{2} = 25\ \mathrm{m\ s^{-1}}\ \text{downwards}$$

Using this average velocity, the distance that the cricket ball will fall in 5 s is $25\ \mathrm{m\ s^{-1}} \times 5\ \mathrm{s} = 125\ \mathrm{m}$.

(b) The second technique uses Eq. (1.10) (since the initial velocity is zero) using g for the acceleration. The change in displacement of the ball is

$$d = \frac{1}{2}gt^2 = \frac{1}{2} \times 10\ \mathrm{m\ s^{-2}} \times (5\ \mathrm{s})^2 = 125\ \mathrm{m}$$

which is the same answer as we found with the previous method.

1.6 The Acceleration Due to Gravity

Galileo found (and countless experiments since have also shown) that all objects falling freely towards the Earth have the same acceleration. (In order to see this effect, we must take into account the effect of air resistance when this is significant.) Thus every object in free fall has its downward speed increased by $10\ \mathrm{m\ s^{-1}}$ in every second regardless of its mass. Galileo claimed that this was an experimental fact and is reported to have shown it by dropping two balls of unequal mass from the top of the Leaning Tower of Pisa. Later we will discuss the theoretical explanation for this experimental fact when we investigate the relationship between acceleration and the concept of *force*. The value of this constant acceleration is given by

$$g = 9.81\ \mathrm{m\ s^{-2}} \approx 10\ \mathrm{m\ s^{-2}} \tag{1.11}$$

This quantity, g, is called the **acceleration due to gravity**. (The value of $9.8\ \mathrm{m\ s^{-1}}$ is the value at sea level on the surface of the Earth; the value will change with altitude.)

Consider an object released from rest and accelerating in free fall. Assuming that the air resistance is negligible, we are able to calculate its velocity after 5 s:

$$v = gt = 10\ \mathrm{m\ s^{-2}} \times 5\ \mathrm{s} = 50\ \mathrm{m\ s^{-1}}$$

Note that the mass of this object is not even mentioned in the original question. Also note that we have not considered the vectorial character of either the acceleration due to gravity or of the velocity achieved by this object after 5 seconds. These quantities are of course vectors, but their directions may be assumed to be towards the centre of the Earth and need not be considered in this problem. This is not always the case.

Air resistance
Actually, the acceleration is only the same in a vacuum. Objects falling in air are affected by air resistance which reduces the acceleration. Often this is small enough to be ignored.

Precision and *g*
In mathematical examples where we wish to find an accurate result, we will often use a value for g of 9.8 or $9.81\ \mathrm{m\ s^{-2}}$. However, in cases where we are more interested in the concepts and methods than the exact answer we will frequently approximate the value as $10\ \mathrm{m\ s^{-2}}$ to keep things simpler.

Example 1.2 ***A ball thrown straight up (I) (1D kinematics)***

Problem: If you throw a cricket ball straight up at 12 m s^{-1}, how high will it go?

Solution: This problem is very similar to the previous one, except now the cricket ball has an initial velocity. This initial velocity is in the opposite direction to the acceleration due to gravity. This means that, at first, gravity will *reduce* the upward velocity of the cricket ball by 10 m s^{-1} every second. At some point the upward velocity of the cricket ball will have been reduced to zero – the cricket ball has stopped travelling up so it has reached its maximum height.

To calculate the maximum height that the ball reaches we need to find the time taken for the upward velocity to decrease to zero as this is also the time for the ball to reach its maximum height. We then calculate the average velocity and use this combined with the elapsed time to calculate the change in displacement of the ball.

For this problem we will define the upwards direction as positive, and define $d = 0$ m to be the height at which the ball was released.

The change in velocity of the ball is $v_\mathrm{f} - v_\mathrm{i} = -12\,\mathrm{m\,s^{-1}}$ (i.e., the velocity goes from $+12\,\mathrm{m\,s^{-1}}$ initially to $0\,\mathrm{m\,s^{-1}}$ at its highest point). The time it takes the acceleration due to gravity ($-10\,\mathrm{m\,s^{-2}}$, as it points in the downwards direction) to cause this change in velocity is,

$$\begin{aligned}\Delta v &= gt\\ t &= \frac{\Delta v}{g} = \frac{-12\,\mathrm{m\,s^{-1}}}{-10\,\mathrm{m\,s^{-1}}} = 1.2\,\mathrm{s}\end{aligned}$$

The average velocity is $6\,\mathrm{m\,s^{-1}}$ ($v_\mathrm{av} = \frac{1}{2}\left(v_f + v_i\right) = \frac{1}{2}\left(12\,\mathrm{m\,s^{-1}} + 0\,\mathrm{m\,s^{-1}}\right) = 6\,\mathrm{m\,s^{-1}}$) and using this we calculate the change in displacement of the cricket ball during the 1.2 seconds it takes to reach the maximum height,

$$d = v_\mathrm{av}t = 6\,\mathrm{m\,s^{-1}} \times 1.2\,\mathrm{s} = 7.2\,\mathrm{m}$$

The highest point the ball reaches is 7.2 m above the point at which it was released.

Example 1.3 ***A ball thrown straight up (II) (1D kinematics)***

Problem: How long does it take the ball in Example 1.2 to fall to its original position from its maximum height?

Solution: The ball reaches a maximum height of 7.2 m above the point at which it is thrown. At its maximum height it has a velocity of $0\,\mathrm{m\,s^{-1}}$. The time taken to fall a distance of 7.2 m back to its original position can be found using Eq. (1.9):

$$d = v_\mathrm{i}t + \frac{1}{2}at^2$$

Since the initial velocity, v_i, is the velocity of the ball at its maximum height, i.e., $v_\mathrm{i} = 0\,\mathrm{m\,s^{-1}}$ this can be reduced to

$$d = \frac{1}{2}at^2$$

The time in this equation, t, is the time the ball takes to fall back to its original position. We rearrange this equation to solve for t where (using the same sign convention as in Example 1.2) $d = -7.2$ m and $a = g = -10\,\mathrm{m\,s^{-2}}$

$$\begin{aligned}t &= \sqrt{\frac{2d}{g}} = \sqrt{\frac{2\times -7.2\,\mathrm{m}}{-10\,\mathrm{m\,s^{-2}}}}\\ &= 1.2\,\mathrm{s}\end{aligned}$$

Note that this is the same time it took the ball to reach its maximum height from the point at which it was initially thrown in Example 1.2. This is a useful general result. Projectile motion is symmetrical about the point of maximum height. It takes the same amount of time to reach the maximum height from a starting height as it does to get back to that height from the maximum height.

Example 1.4 ***A ball thrown straight up (III)***

Problem: What is the velocity of the ball in Examples 1.2 and 1.3 when it falls back to the height at which it was released?

Solution: As in the previous problem we can think of the ball starting at its maximum height and falling 7.2 m under the influence of gravity to its original height, which we found would take 1.2 s, the same as the time taken to reach its maximum height from the point at which it was released. The change in velocity over the three seconds in which it is falling from its maximum height is

$$\begin{aligned}\Delta v &= at = -10\,\mathrm{m\,s^{-2}} \times 1.2\,\mathrm{s} \\ &= -12\,\mathrm{m\,s^{-1}}\end{aligned}$$

The ball is travelling at the same speed at it was when released, but in the opposite direction! Again this is a useful general result we can apply to many kinematics problems without going through an extensive derivation.

Example 1.5 ***A ball thrown straight up (IV) (1D kinematics)***

Problem: If the ball in the Example 1.2 was released (travelling upwards) at a height of 1.2 m above the ground, what is the velocity of the ball just before it hits the ground?

Solution: We will first need to find the time it takes for the ball to hit the ground and then use $\Delta v = at$ to find the change in velocity, and hence the final velocity. Initially it is tempting to try to use Eq. 1.9 to solve this problem directly.

$$d = v_i t + \frac{1}{2} a t^2$$

We know the change in position of the ball (as, in this case, the ball ends up 1.2 m below it's starting point), the initial velocity of the ball ($v_i = 12\,\mathrm{m\,s^{-1}}$), and acceleration ($g = -9.8\,\mathrm{m\,s^{-2}}$). In order to use this equation, however, we would need to solve for t, which would require solving a quadratic equation. Even using the shortcuts highlighted in the previous two examples does not allow us to avoid this quadratic as we still end up with two terms featuring t. If you're confident doing this, that is great. If you are not very confident at solving quadratic equations however, all is not lost.

We can simplify the problem by using the fact that we know a bit more about the situation than is apparent from the question. From Example 1.2, we know that the ball reaches a maximum height of 7.2 m above the point at which it was released. This means that the ball will reach a height of $7.2 + 1.2 = 8.4$ m above the ground. At this maximum height, the velocity of the ball is $0\,\mathrm{m\,s^{-1}}$, and by ignoring the first part of the ball's motion, we can simplify Eq. (1.9) to $d = \frac{1}{2}at^2$, where d is the change in displacement of the ball as it moves from its maximum height to the ground, $d = -8.4$ m, and $a = g = -9.8\,\mathrm{m\,s^{-2}}$:

$$\begin{aligned} d &= \tfrac{1}{2} a t^2 \\ t &= \sqrt{\tfrac{2d}{g}} = \sqrt{\tfrac{2 \times -8.4\,\mathrm{m}}{-9.8\,\mathrm{m\,s^{-2}}}} \\ &= 1.3\,\mathrm{s}\end{aligned}$$

This gives a change in velocity of $\Delta v = gt = -9.8\,\mathrm{m\,s^{-2}} \times 1.3\,\mathrm{s} = -13\,\mathrm{m\,s^{-1}}$, so the ball will be travelling at $13\,\mathrm{m\,s^{-1}}$ in the downwards direction as it hits the ground.

1.7 Independence of Motion in 2D

In the previous section we considered objects which move vertically up and down. This means that in these cases the velocity vector is always parallel to the acceleration vector. These objects would go straight upward and fall straight downward. They will not move horizontally since there is no initial velocity in the horizontal direction, and no acceleration in the horizontal direction to cause a non-zero horizontal velocity to develop. What would happen if the initial velocity was not straight upward? What would have happened if the initial velocity was at some angle to the vertical? This is the situation shown in Figure 1.5, using the example of a cricket ball launched upward at an angle. In Figure 1.5, the cricket ball has a vertical velocity and a horizontal velocity and

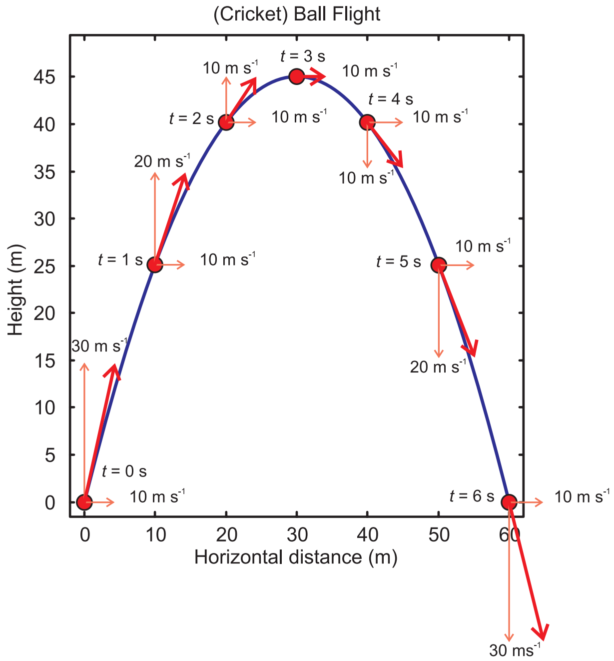

Figure 1.5 The trajectory of a cricket ball initially launched upward at an angle to the vertical. The vertical and horizontal components of the ball's velocity are shown at a number of times in its trajectory.

this results in a net velocity at an angle to the vertical. The acceleration due to gravity, however, acts only in the vertical direction, and changes only the vertical component of the velocity. The horizontal component of the velocity is initially 10 m s^{-1} and is still 10 m s^{-1} when the ball reaches its maximum height at 45 m after 3 s, and is 10 m s^{-1} when the ball reaches the ground again after a total of 6 s. There is no acceleration in the horizontal direction, so the velocity component in this direction *cannot change*.

The vertical component of the velocity is changed by the acceleration due to gravity. This can be seen in Figure 1.5 as well. In point of fact, the vertical component of the velocity behaves in exactly the same way as it did in the examples above. The vertical velocity is initially 30 m s^{-1} upward. After 3 s this has dropped to 0 m s^{-1} when the ball reaches its maximum height. The vertical velocity is again 30 m s^{-1} just before the ball hits the ground after 6 s, but now the velocity is in the downward direction.

This is what we mean when we say that the horizontal and vertical components of the velocity vector are independent. These components are acted on separately by the accelerations in those directions. An acceleration in the horizontal direction would not change the velocity component in the vertical direction. This means that when we are attempting to solve a kinematics problem in three dimensions, we may look at the components of the velocity and acceleration in a given direction independently of their components in the other two directions.

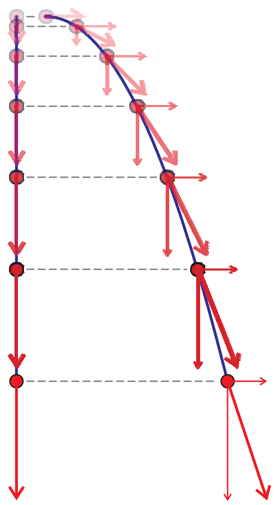

Figure 1.6 Independence of the vertical and horizontal components of velocity.

This effect may be seen in the following experiment. Suppose that we have two identical balls which are held on a platform in a darkened room. (The balls do not need to have the same mass for this experiment to work, but we will simplify the discussion by assuming that they do.) Now suppose that we drop one ball directly downward from a platform and at the same instant fire the other ball horizontally out from the same platform. As the balls fall, a strobe light flashes at regular intervals and the trajectory of the two balls is recorded on a camera with a very long exposure time. Figure 1.6 is an example of the sort of image that would be obtained from this experiment.

In this figure, we observe that the balls are at the same height at each interval, i.e., at each flash of the strobe light. This means that their vertical velocity components are the same at each time. Their horizontal velocity components are quite different, however. The ball which is simply dropped has no horizontal velocity component, whereas the horizontal velocity component of the other ball is constant.

Example 1.6 ***Projectile motion (2D kinematics)***

Problem: A modern artist throws a bottle of paint towards the wall of a nearby building. The bottle leaves the artist's hand at a height of 2.00 m, a speed of 16.0 m s^{-1}, and at an angle of 30.0° above the horizontal. If the building is 15.0 m away

(a) At what height H does the bottle hit the wall?

(b) At what velocity v is the bottle travelling as it hits the wall?

Solution:

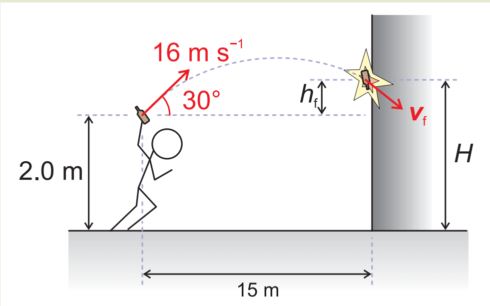

Figure 1.7 An artist throws a bottle against a wall.

A good first step for this kind of problem is to draw a diagram like Figure 1.7. Remember that when dealing with 2D kinematics you can always separate out the horizontal and vertical motions. Note also that since we are given numerical quantities to two significant figures, we will use this level of numerical precision throughout the problem, i.e., we will use $g = -9.8\,\text{m s}^{-2}$.

In order to answer both of the questions, we will need to know how long after the bottle leaves the artist's hand it hits the wall. We can calculate this time by looking at the horizontal motion of the bottle. As the acceleration due to gravity is in the vertical direction only, we know that the horizontal velocity is constant and has a magnitude of $v_x = 16.0\,\text{m s}^{-1} \times \cos 30° = 13.9\,\text{m s}^{-1}$. So the time it takes the bottle to travel the 15 m horizontally to the wall is

$$d_x = v_x t$$
$$t = \frac{d_x}{v_x} = \frac{15\,\text{m}}{13.9\,\text{m s}^{-1}} = 1.08\,\text{s}$$

(a) The difference in height between the bottle's initial height of 2.0 m and its final height of H is h_f. We can calculate this height by using the initial vertical velocity of the ball ($v_{yi} = 16\,\text{m s}^{-1} \times \sin 30° = 8.0\,\text{m s}^{-1}$) and the fact that the ball is accelerating in the vertical direction at a rate of $a = g = -9.8\,\text{m s}^{-2}$.

$$\begin{aligned} h_f &= v_i t + \frac{1}{2} g t^2 \\ &= 8.0\,\text{m s}^{-1} \times 1.08\,\text{s} + \frac{1}{2} \times \left(-9.8\,\text{m s}^{-2}\right) \times (1.08)^2 \\ &= 2.93\,\text{m} \end{aligned}$$

So the bottle must hit the wall a total of 4.9 m above the ground (giving solution to two significant figures).

(b) In order to find the final velocity of the bottle we will have to add together the vertical and horizontal components of the bottle's velocity as it hits the wall. We already know that the horizontal velocity of the bottle is constant as there is no acceleration in the horizontal direction, therefore the horizontal component of the final velocity is $v_x = 14\,\text{m s}^{-1}$. To find the final vertical velocity we can use $v_f = v_i + at$ where $v_i = v_{yi} = 8.0\,\text{m s}^{-1}$ and $a = g = -9.8\,\text{m s}^{-2}$.

$$\begin{aligned} v_{yf} &= v_{yi} + g t \\ &= 8.0\,\text{m s}^{-1} + \left(-9.8\,\text{m s}^{-2}\right) \times 1.08\,\text{s} = -2.58\,\text{m s}^{-1} \end{aligned}$$

By the time the bottle has hit the wall, it has reached its maximum height (at which $v_y = \text{m s}^{-1}$) and has started moving back down, hence the negative vertical velocity.

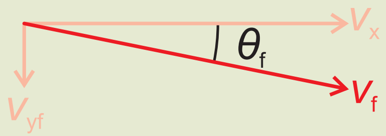

Figure 1.8 The velocity vector components of the bottle as it hits the wall.

We can get the magnitude of the final velocity by vector addition of the two components v_x, and v_{yf}:

$$|v_f| = \sqrt{v_x^2 + v_{yf}^2}$$
$$= \sqrt{(13.9\,\mathrm{m\,s^{-1}})^2 + (-2.58\,\mathrm{m\,s^{-1}})^2} = 14.14\,\mathrm{m\,s^{-1}}$$

The direction in which the bottle is traveling can be found by using trigonometry.

$$\tan\theta_f = \frac{v_{yf}}{v_x}$$
$$\theta_f = \tan^{-1}\left(\frac{v_{yf}}{v_x}\right)$$
$$= \tan^{-1}\left(\frac{2.58\,\mathrm{m\,s^{-1}}}{13.9\,\mathrm{m\,s^{-1}}}\right) = 10.52°$$

So as the bottle hits the wall it is travelling at a speed of $14\,\mathrm{m\,s^{-1}}$, 11° below the horizontal (to 2 s.f.).

1.8 Summary

Key Concepts

elapsed time (Δt) The time interval between two events

distance (d **or** Δx) The length of a path between two spatial positions.

displacement ($\boldsymbol{d}$ **or** $\Delta \boldsymbol{x}$) The vector equivalent of distance, which specifies the distance and direction of one point in space relative to another. It depends only on the initial and final spatial positions, and is independent of the path taken from one position to the other.

speed (v) A scalar measure of the rate of motion. The SI unit of speed is metres per second (m/s or $\mathrm{m\,s^{-1}}$).

velocity ($\boldsymbol{v}$) A vector measure of the rate of motion, which specifies both the magnitude and direction of the rate of motion.

acceleration ($\boldsymbol{a}$) A measure of the rate of change of the velocity. Acceleration is a vector quantity. The SI units of acceleration are $\mathrm{m/s^2}$ or $\mathrm{m\,s^{-2}}$.

Equations

$$\boldsymbol{d} = \boldsymbol{v}_{av}\,t$$
$$\Delta\boldsymbol{v} = \boldsymbol{a}\,t$$
$$\boldsymbol{v}_{av} = \frac{1}{2}(\boldsymbol{v}_i + \boldsymbol{v}_f)$$
$$\boldsymbol{d} = \boldsymbol{v}_i\,t + \frac{1}{2}\boldsymbol{a}\,t^2$$

1.9 Problems

1.1 A dog chasing a ball starts at rest and accelerates uniformly over a distance of 5 meters. It takes the dog 1 s to cover that first 5 m. What is the dog's acceleration, and what speed is the dog travelling when it reaches the 5 m point?

1.2 During a particular car crash, it takes just 0.18 s for the car to come to a complete stop from 50 km h^{-1}.

(a) At what rate is the car accelerating during the crash?

(b) How many times larger than the acceleration due to gravity is this?

1.3 A jogger starting their morning run accelerates from a standstill to their steady jogging pace of 8.0 km h^{-1}. They reach a speed of 8.0 km h^{-1}, 5 s after starting. How long does it take the jogger to reach the end of their 20 m driveway?

1.4 A driver in a blue car travelling at 50 km h^{-1} sees a red car approaching in his rear-view mirror. The red car is travelling at 60 km h^{-1} and is 30 m behind the blue car when first spotted.

(a) How many seconds from the time the driver of the blue car first noticed it until the red car passes the blue car?

(b) How much farther down the road will the blue car travel in this time?

1.5 You are abducted by aliens who transport you to their home world in a galaxy far far away. Oddly, the only thing you can think of doing is measuring the acceleration due to gravity on this strange new world. You drop an alien paperweight from a height of 12 m and use an alien stopwatch to measure the interval of 1.36 s it takes the paperweight to hit the ground below. What is the acceleration due to gravity on the alien home world?

1.6 In a bid to escape from your alien captors you hurl your paperweight straight up towards the door switch on a space ship above you. If the switch is 25 m above you how fast does the paperweight need to leave your hand?

1.7 An initially stationary hovercraft sits on a large lake. When a whistle blows the hovercraft accelerates due north at a rate of 1.2 m s^{-2} for 10 s, does not accelerate at all for the next 10 s, and then accelerates at a rate of 0.6 m s^{-2} due east for another 10 s. The hovercraft then coasts for another 10 s without any acceleration.

(a) What is the velocity of the hovercraft 40 s after the whistle blows?

(b) What is the displacement of the hovercraft 40 s after the whistle blows?

1.8 A jogger takes the following route to the entrance of their local park: north 120 m, west 100 m, south 35 m, and finally west 50 m. It takes them 2 minutes 18 seconds to reach the park entrance.

(a) What distance did the jogger travel?

(b) What is the displacement of the jogger as she enters the park?

(c) What is the average speed of the jogger?

(d) What is the average velocity of the jogger?

1.9 A tennis ball is hit down at an angle of 30° below the horizontal from a height of 2 m. It is initially travelling at 5.0 m s^{-1}. What is the velocity of the ball when it hits the ground if we can neglect air resistance?

1.10 A stunt rider is propelled upward from his motorbike by a spring loaded ejector seat. The rider was travelling horizontally at 60 km h^{-1} when the ejector seat was triggered, and as they leave the seat they are travelling with a vertical velocity of 15 m s^{-1}.The seat is 1.0 m off the ground.

(a) What is the initial velocity of the stunt rider (in km h^{-1})?

(b) How high does the stunt rider reach?

(c) How far along the track does the stunt rider land on the ground?

(d) What is the velocity of the stunt rider when they hit the ground (in km h^{-1})?

1.11 A bullet is fired horizontally from a gun that is 1.5 m from the ground. The bullet travels at 1000 m s^{-1} and strikes a tree 150 m away. How far up the tree from the ground does the bullet hit? [Neglect air resistance.]

Force and Newton's Laws of Motion

2

2.1 Introduction

We now have a clear and complete description of motion. This description relates the primary properties of motion – displacement, velocity, acceleration and elapsed time – to each other. The question now arises: what causes a change in the motion of an object? This amounts to asking: 'Where does acceleration come from?' The answer to this question is deceptively simple: 'Accelerations are caused by the application of forces.' The purpose of this chapter is to explain clearly what a 'force' is in physics and how it can be used to solve problems relating to the motion of objects. When forces are included in the discussion of motion, it is called dynamics.

Key Objectives

- To understand the concept of force.
- To understand the relationship between force and motion.
- To be able to identify action–reaction pairs of forces.
- To understand normal, friction and tension forces.
- To be able to solve straightforward problems in dynamics.

2.2 The Concept of Force

In everyday conversation we use the word 'force' quite liberally. The Oxford English Dictionary gives a number of definitions of the noun 'force':

1. physical strength or energy accompanying action or movement.
2. (Physics) a measurable influence that causes something to move.
3. pressure to do something backed by the use of threat of violence.
4. influence or power.
5. a person or thing having influence: *a force for peace*.
6. an organised group of soldiers, police or workers.

The word is also used as a verb, as in sentences like: 'She forced the committee to consider her application seriously.' In physics, the word 'force' has a very precise meaning, and this is given by Newton's laws. These laws define force by listing the essential properties of a force. If some phenomenon does not have all of these properties, then it is not a force. In this section we will go through Newton's Laws and explain each of them in turn.

Force vectors in figures

In this text *force vectors* are represented as acting on the *centre of mass* of an object. It should be noted that other texts use differing conventions for contact force vectors in which a contact force is shown acting at the surface between two objects. The distinction between the two ways of representing contact force vectors is partially style, although when dealing with contact forces that may act to rotate an object the exact position of action of a force is important. Such cases will not be dealt with in this text and so the simpler standard in which all force vectors are shown to act at the centre of mass of an object has been used in the interests of clarity.

Newton's First Law

In the list of Oxford English Dictionary definitions of force given above, the second item is closest to the definition used by physicists. A force is essentially anything that is measurable and causes a *change* in the motion of an object. For example, if an object is at rest, then we must apply a force to it to cause it to accelerate and to develop a non-zero velocity.

Newton's first law states that any object continues at rest, or at constant velocity (constant speed in a straight line), unless an external force acts on it. Figure 2.1 illustrates the effect of a force on the motion of an object. This object is travelling in a straight line until an external force acts on it in a direction perpendicular to its motion. This causes the object to be deflected from its straight-line motion.

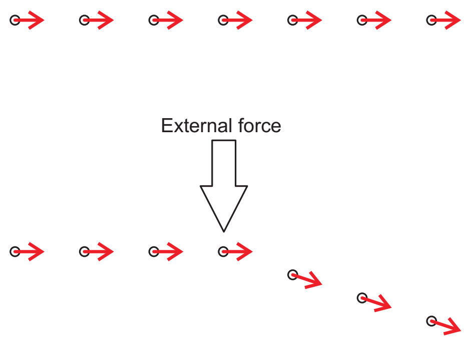

Figure 2.1 A pair of objects initially travel in the same direction. The lower object is subject to a force and its motion then deviates from a straight line while the force acts on it, but continues on in a new straight line after the force ceases to act.

Newton's Second Law

Key concept:
An external force gives an object an acceleration. The acceleration produced is proportional to the force applied, and the constant of proportionality is the **mass**.

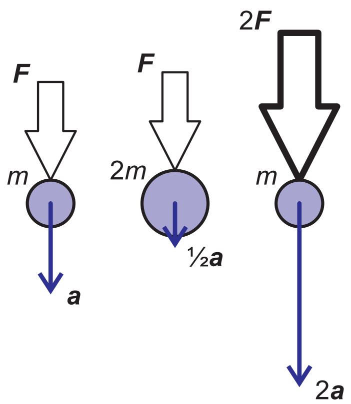

Figure 2.2 If the force applied to an object does not change, but the mass of the object is doubled, then the acceleration is halved. If the mass does not change and the force is doubled, then the acceleration is doubled also.

Newton's second law can be summarised with the following equation:

$$\boldsymbol{F} = m\boldsymbol{a} \tag{2.1}$$

In this equation, $\boldsymbol{a}$ is the acceleration (in m s^{-2}) as usual, m is the mass (in kg), and $\boldsymbol{F}$ is the force (in N) (N = newton). The SI unit of force is the newton; one newton (1 N) is the force which would accelerate a 1 kg mass at 1 m s^{-2} (i.e., would cause its velocity to increase by 1 m s^{-1} in every second).

As is illustrated in Figure 2.2, if the mass of an object on which the force is applied does not change, but the force is doubled, then the acceleration of this object will also be doubled (see diagram to the right in Figure 2.2). If the applied force is not changed, but the mass of the object is doubled, then the acceleration will be halved (see centre diagram of Figure 2.2). For numerical examples of the relationship between mass, force and acceleration, see Table 2.1.

Weight and Mass

Since any object of mass m near the surface of the Earth falls with acceleration g downwards, it must be acted on by a force:

$$\begin{aligned} \boldsymbol{F} &= m\boldsymbol{a} \\ &= mg \text{ downwards} \end{aligned}$$

This downward force is due to the gravitational attraction between the Earth and the object and is often called the object's **weight**, i.e., the magnitude of the force, W, is

$$W = mg \tag{2.2}$$

Key concept:
The weight of an object is a force, not a mass.

m (kg)	a (m s^{-2})	F (N)
1	1	1
1	2	2
2	1	2
2	2	4
4	5	20

Table 2.1 Newton's second law and the relationship between force, mass and acceleration.

When we step on our bathroom scales and are told our mass in kilograms, we are effectively being given our weight in newtons with a zero removed. The scales do not directly measure our mass or weight – they measure the magnitude of the contact force (also referred to as the normal force or support force) between the scales and our feet. If we were to step onto our scales while resident on the International Space Station (ISS), we would read a weight of nearly zero kilograms. Our mass has not changed; what has happened is that both the scales and ourselves are in free fall, and there is no significant contact force between the scales and our feet.

Interestingly, because the ISS orbits at an altitude of only 350 km (6.34×10^6 m + 0.35×10^6 m = 6.69×10^6 m from the centre of the Earth), the difference in the gravitational force, and hence acceleration due to gravity, between an object on the surface of the Earth and on the ISS is quite small ($F_{\text{grav, ISS}} \approx 0.90 \times F_{\text{grav, sea level}}$). (The strength of the gravitational force is discussed in Section 2.4.

Forces are Vectors

When I push against an object, say, an apple, I push hard or not so hard, and I push in a particular direction. For example, I could be pushing the apple across the table towards you, or I could be pulling the apple toward myself. It seems that forces have both magnitude and direction; this means that we must represent forces as *vectors*. This fits with what we have said so far about forces. Thus to find the total or net force on an object, we must find the *vector sum* of all of the individual forces on that object. It is the net force acting on an object that produces an acceleration. Since acceleration is a vector, and the mass is not, the force that acts on the object must also be a vector (see Eq. (2.1).) This means that if several forces are acting on the same object, we find the total force using vector addition to add up all of the applied forces.

As in the case of the velocity and acceleration vectors, we may treat the components of a force vector as separate independent vectors. Thus, in calculations we are able to look at the behaviour of the horizontal components of the force, acceleration, velocity and displacement, and then the vertical components of these vectors. Once we have completed these separate calculations we can combine the components of the relevant vector quantities to find the total force, velocity or acceleration.

Newton's Third Law

Key concept:
For every action there is an equal and opposite reaction.

Newton's third law states that forces come in pairs – for every force that is applied to a body, there is a force applied *by* that body. The 'action' referred to in the box above is the force applied by one object on the other. Suppose that the 'action' is the force exerted by object 1 on object 2. Newton's third law then states that object 2 will exert a force of equal size, but in the opposite direction, on object 1. This second force is the 'reaction' to the force exerted by object 1.

Forces act in pairs and each force acts between a pair of objects. These force pairs are called **action–reaction pairs** or **third-law force pairs**. It is important to correctly identify the action–reaction pairs in a problem. For example, the weight force of an object is due to the gravitational interaction between that object and the Earth. Thus the Earth exerts a force on the object and the object exerts an equal, but opposite, force on the Earth. The object will accelerate toward the Earth under the influence of the

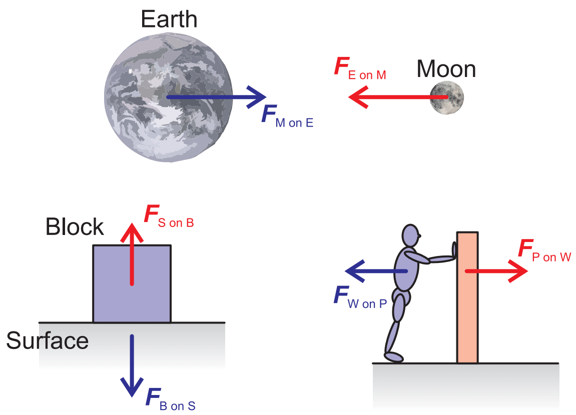

Figure 2.3 Three examples of action–reaction pairs. The gravitational force between the Earth and the Moon, the downward contact force a block exerts on a surface and the upwards support force the surface exerts on the block, and a person pushing against a wall.

gravitational force exerted by the Earth – this is source of the acceleration due to gravity. These equal and opposite forces exist whether the object under discussion is in free fall toward the Earth or is sitting on a set of bathroom scales being weighed; in both of these cases the action–reaction pair is the force of the Earth on the object and the force of the object on the Earth. Later we will discuss the electric force, the force described by Coulomb's Law. This is a force which again acts between two objects (which are charged), with the force on one being equal and opposite to the force on the other.

In Figure 2.3 we illustrate Newton's third law with three examples. In the first example, the Earth (E) exerts an attractive gravitational force on the Moon (M), $F_{\text{E on M}}$. In turn, the Moon exerts and equal, but opposite (and hence also attractive), force on the Earth, $F_{\text{M on E}}$. In the second example, the surface (S) on which a block (B) is sitting exerts an upward (normal or support) force $F_{\text{S on B}}$, on the block, which exerts an equal and opposite force, $F_{\text{B on S}}$, on the surface. Note that the downward force exerted by the block is not necessarily the weight force of the block. If I push down on the block, the downward force is the sum of the forces acting downward, and the support force provided by the surface will increase to equal it. Finally, a person (P) pushing against a wall (W) applies a force $F_{\text{P on W}}$ to the wall, and the wall applies an equal and opposite force $F_{\text{W on P}}$ to the person. It is important to note that each force in an action–reaction pair acts on a different object. It is *always* the case that action–reaction pairs act on different objects.

Example 2.1 *A box on a box*

Problem:

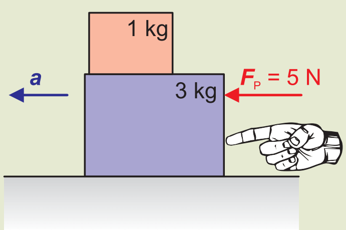

Figure 2.4 Two boxes being pushed along a frictionless surface

As shown in Figure 2.4, a large 3 kg box is being pushed with a horizontal force of F_P = 5 N and as a result is accelerating along the horizontal frictionless surface upon which it rests. The large box has a smaller 1 kg box resting on top of it. This box does NOT slide from the top of the big box as it accelerates.

(a) At what rate are the boxes accelerating?

(b) What are the magnitudes and directions of the friction forces acting on each box?

(c) What are the magnitudes and directions of the normal forces acting on each box?

Solution: Because the small box does not slide around on top of the large one, the small box accelerates at the same rate as the large one $\boldsymbol{a}_S = \boldsymbol{a}_L$. Drawing a diagram of all the forces acting on each box will be helpful at this point. Because each box is accelerating horizontally, there must be a net force on each box which is horizontal. The sum of all vertical forces on each box must be zero.