“An extremely useful book for anyone interested in actually applying MATLAB based computational techniques to fixed-income problems. Theoretically sound and practically useful: a rare combination. I decided to buy MATLAB after reading it.”

Dr Riccardo Rebonato,
Global Head of Market Risk and Quant Analytics, RBS.

“This book is a wonderfully practical ‘how to’ guide for bond market empirics implemented in MATLAB, with particular strength in dynamic yield curve models. It will interest students and practitioners alike.”

Francis X. Diebold, Joseph M. Cohen
Professor of Economics, Finance and Statistics, and
Co-Director of the Wharton Financial Institutions Center,
University of Pennsylvania.

“This book offers a unique opportunity for mathematically savvy readers with little prior exposure to finance to ‘hit the ground running’ in fixed income modeling. It provides a broad but thorough introduction to fixed-income finance, from basic elements like price-yield conversions to more complex topics such as term structure modeling and strategic asset allocation. No prior financial knowledge is assumed; an effort is made to develop all models presented from first principles, and references are provided for those who wish to probe a given topic in greater depth. At the same time, the book also serves as an introduction to Matlab, which is used throughout the book to provide working code examples for all of the models discussed in the text. These code fragments, which may be downloaded from the internet, can help readers jump-start their own efforts at writing Matlab code for financial applications. This book can thus be useful in different ways to different people. It offers an excellent first exposure to finance for scientists and engineers interested in joining the field. For experienced practitioners, the book and accompanying code fragments can greatly minimize the time required to start implementing models in Matlab, a very powerful programming tool.”

Lev Dynkin, Managing Director,
Quantitative Portfolio Strategies, Lehman Brothers.

“Strategic Asset Allocation in Fixed Income Markets is a carefully thought through introduction to fixed income asset allocation with an applied focus on Matlab implementation. A great resource for anyone who wants to get started solving realistic asset allocation problems.”

Michael W. Brandt, Professor of finance,
The Fuqua School of Business, Duke University.
Strategic Asset Allocation in
Fixed-Income Markets
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Strategic Asset Allocation in Fixed-Income Markets

A MATLAB-Based User’s Guide

Ken Nyholm

John Wiley & Sons, Ltd
To Laura, Clara and Emilia
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The work contained in this book was initially intended as background reading for students at the Finance Department of the Frankfurt University for a course on ‘Fixed Income Strategic Asset Allocation in Practice’, which I started to teach some years ago. A literature search for an appropriate textbook that combines fixed-income analysis, portfolio theory and in-depth computer-based applications concluded unsuccessfully. Consequently, I set out to write lecture notes, aiming to explain and to illustrate, by the use of examples, some of the topics from the intersection between fixed-income portfolio management and applied econometrics. As a computer tool I decided to rely on MATLAB.¹

Eventually, the lecture notes ended up forming this book. Its chief purpose is to assist readers who are interested in the implementation of financial and econometric models in MATLAB. While the provided annotated examples by no means claim to be ‘the best way’ to program and implement such models and techniques, it is hoped that the text still can serve as a useful introduction relevant for students and practitioners alike. In particular, many practitioners (and students, it seems) cannot find the time (although probably for different reasons) to do the necessary foot work to get started implementing models in MATLAB; with those in mind, the aim of this book is to lower the ‘barrier of entry’ to the successful implementation of financial models.

A disclaimer is warranted at this point. Given my work-related background in strategic asset allocation for debt instruments, the book has a significant skew towards fixed-income markets. This means that the book does not cover equity and options-related material at all. Also, the book does not claim or aim to give a unified treatment of strategic asset allocation methods, nor how to construct an asset allocation framework. Rather, it presents a smorgasbord of different ideas and techniques, and it shows how these can be implemented using MATLAB.

¹ More information about MATLAB and how to use it can be found on the Internet, see, for example, www.mathworks.com and www.econphd.net/notes. MATLAB is a registered trademark of The MathWorks Inc.
I believe that a thorough understanding of financial concepts, their strengths, weaknesses and practical applicability is best facilitated by the use of concrete examples. This idea is the underpinning pedagogical philosophy of the book: all central concepts and theories are illustrated by MATLAB implementations, which are accompanied by detailed descriptions of the needed programming steps. In this way, the book can equally well be used as an introduction to finance and MATLAB.

The main prerequisites for reading the book are: a keen interest in exploring financial models in practice and an educational background roughly equivalent to a bachelor’s degree in a moderately technical field. All concepts and techniques are introduced from the basic level, so the latter prerequisite is easily compensated by the former. Mathematically adept individuals will probably find the text and examples relatively easy, while individuals with somewhat rusty mathematical skills (probably) will find the text refreshing.

It should be noted that the notation used in the book is chapter specific. This means that one symbol can be used for different objects in two different chapters. For example, $r$ is used to denote the short rate in Chapter 5, and to denote the vector of returns in Chapter 6. When a model is introduced in a given chapter, it is always accompanied with an explanation of the notation used. This should limit the scope for notational confusion. I personally find the notation used in some books very difficult to read, and have therefore adopted the approach of using as few variable names as possible, and I have tried to make the notation as intuitive as possible. The advantage of this choice is that each chapter is easy to read; the disadvantage is obviously that the notation changes from one chapter to the next. Also, I use the asterisk $\ast$ in the text with the aim of bridging the MATLAB code and the formulae presented in the text (so that multiplication looks identical in the text and in the MATLAB code).

While having a different focus and coverage compared with the current exposition, it should be mentioned that the above-referred literature search did uncover some very interesting books in the juncture between finance and MATLAB programming: see for example McLeish (2005) for a thorough description of efficient simulation methods mainly applied to the area of derivatives pricing and variance reduction techniques; Meucci (2005) gives an in-depth account of risk and asset allocation drawing on elements from the theory of multivariate statistics; Down (2005) contains a good introduction to the measurement of financial risks; and Brandimarte (2002) focuses on optimisation methods useful in financial applications.

The book is accompanied by an online resource: www.kennyholm.com where MATLAB programs referred to in the text can be downloaded. The site also contains other materials that may be of interest.

It should also be mentioned that the techniques, methodologies and views expressed in the book are mine alone, and they are by no means necessarily supported by my employer, the European Central Bank.

Many books include quotes and historical references. This practice is probably meant to set a given book’s material in a wider context or to serve as a means to
put the material into perspective. I have always found this practice a bit corny. However, it turns out that I also cannot resist the temptation of inserting a quote, so here goes:

*Why do we fall, Sir? So that we might better learn to pick ourselves up.*

Alfred Pennyworth to Batman

(*Batman Begins*, Warner Bros., 2005)
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