

**Market Risk Analysis**  
**Volume III**

**Pricing, Hedging and Trading**  
**Financial Instruments**

**Carol Alexander**



John Wiley & Sons, Ltd



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To Jacques Pézier





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## Foreword

How many children dream of one day becoming risk managers? I very much doubt little Carol Jenkins, as she was called then, did. She dreamt about being a wild white horse, or a mermaid swimming with dolphins, as any normal little girl does. As I start crunching into two kilos of Toblerone that Carol Alexander-Pézier gave me for Valentine's day (perhaps to coax me into writing this foreword), I see the distinctive silhouette of the Matterhorn on the yellow package and I am reminded of my own dreams of climbing mountains and travelling to distant planets. Yes, adventure and danger! That is the stuff of happiness, especially when you daydream as a child with a warm cup of cocoa in your hands.

As we grow up, dreams lose their naivety but not necessarily their power. Knowledge makes us discover new possibilities and raises new questions. We grow to understand better the consequences of our actions, yet the world remains full of surprises. We taste the sweetness of success and the bitterness of failure. We grow to be responsible members of society and to care for the welfare of others. We discover purpose, confidence and a role to fulfil; but we also find that we continuously have to deal with risks.

Leafing through the hundreds of pages of this four-volume series you will discover one of the goals that Carol gave herself in life: to set the standards for a new profession, that of market risk manager, and to provide the means of achieving those standards. Why is market risk management so important? Because in our modern economies, market prices balance the supply and demand of most goods and services that fulfil our needs and desires. We can hardly take a decision, such as buying a house or saving for a later day, without taking some market risks. Financial firms, be they in banking, insurance or asset management, manage these risks on a grand scale. Capital markets and derivative products offer endless ways to transfer these risks among economic agents.

But should market risk management be regarded as a professional activity? Sampling the material in these four volumes will convince you, if need be, of the vast amount of knowledge and skills required. A good market risk manager should master the basics of calculus, linear algebra, probability – including stochastic calculus – statistics and econometrics. He should be an astute student of the markets, familiar with the vast array of modern financial instruments and market mechanisms, and of the econometric properties of prices and returns in these markets. If he works in the financial industry, he should also be well versed in regulations and understand how they affect his firm. That sets the academic syllabus for the profession.

Carol takes the reader step by step through all these topics, from basic definitions and principles to advanced problems and solution methods. She uses a clear language, realistic illustrations with recent market data, consistent notation throughout all chapters, and provides a huge range of worked-out exercises on Excel spreadsheets, some of which demonstrate analytical tools only available in the best commercial software packages. Many chapters on

advanced subjects such as GARCH models, copulas, quantile regressions, portfolio theory, options and volatility surfaces are as informative as and easier to understand than entire books devoted to these subjects. Indeed, this is the first series of books entirely dedicated to the discipline of market risk analysis written by one person, and a very good teacher at that.

A profession, however, is more than an academic discipline; it is an activity that fulfils some societal needs, that provides solutions in the face of evolving challenges, that calls for a special code of conduct; it is something one can aspire to. Does market risk management face such challenges? Can it achieve significant economic benefits?

As market economies grow, more ordinary people of all ages with different needs and risk appetites have financial assets to manage and borrowings to control. What kind of mortgages should they take? What provisions should they make for their pensions? The range of investment products offered to them has widened far beyond the traditional cash, bond and equity classes to include actively managed funds (traditional or hedge funds), private equity, real estate investment trusts, structured products and derivative products facilitating the trading of more exotic risks – commodities, credit risks, volatilities and correlations, weather, carbon emissions, etc. – and offering markedly different return characteristics from those of traditional asset classes. Managing personal finances is largely about managing market risks. How well educated are we to do that?

Corporates have also become more exposed to market risks. Beyond the traditional exposure to interest rate fluctuations, most corporates are now exposed to foreign exchange risks and commodity risks because of globalization. A company may produce and sell exclusively in its domestic market and yet be exposed to currency fluctuations because of foreign competition. Risks that can be hedged effectively by shareholders, if they wish, do not have to be hedged in-house. But hedging some risks in-house may bring benefits (e.g. reduction of tax burden, smoothing of returns, easier planning) that are not directly attainable by the shareholder.

Financial firms, of course, should be the experts at managing market risks; it is their *métier*. Indeed, over the last generation, there has been a marked increase in the size of market risks handled by banks in comparison to a reduction in the size of their credit risks. Since the 1980s, banks have provided products (e.g. interest rate swaps, currency protection, index linked loans, capital guaranteed investments) to facilitate the risk management of their customers. They have also built up arbitrage and proprietary trading books to profit from perceived market anomalies and take advantage of their market views. More recently, banks have started to manage credit risks actively by transferring them to the capital markets instead of warehousing them. Bonds are replacing loans, mortgages and other loans are securitized, and many of the remaining credit risks can now be covered with credit default swaps. Thus credit risks are being converted into market risks.

The rapid development of capital markets and, in particular, of derivative products bears witness to these changes. At the time of writing this foreword, the total notional size of all derivative products exceeds \$500 trillion whereas, in rough figures, the bond and money markets stand at about \$80 trillion, the equity markets half that and loans half that again. Credit derivatives by themselves are climbing through the \$30 trillion mark. These derivative markets are zero-sum games; they are all about market risk management – hedging, arbitrage and speculation.

This does not mean, however, that all market risk management problems have been resolved. We may have developed the means and the techniques, but we do not necessarily



understand how to address the problems. Regulators and other experts setting standards and policies are particularly concerned with several fundamental issues. To name a few:

1. How do we decide what market risks should be assessed and over what time horizons? For example, should the loan books of banks or long-term liabilities of pension funds be marked to market, or should we not be concerned with pricing things that will not be traded in the near future? We think there is no general answer to this question about the most appropriate description of risks. The descriptions must be adapted to specific management problems.
2. In what contexts should market risks be assessed? Thus, what is more risky, fixed or floating rate financing? Answers to such questions are often dictated by accounting standards or other conventions that must be followed and therefore take on economic significance. But the adequacy of standards must be regularly reassessed. To wit, the development of International Accounting Standards favouring mark-to-market and hedge accounting where possible (whereby offsetting risks can be reported together).
3. To what extent should risk assessments be 'objective'? Modern regulations of financial firms (Basel II Amendment, 1996) have been a major driver in the development of risk assessment methods. Regulators naturally want a 'level playing field' and objective rules. This reinforces a natural tendency to assess risks purely on the basis of statistical evidence and to neglect personal, forward-looking views. Thus one speaks too often about risk 'measurements' as if risks were physical objects instead of risk 'assessments' indicating that risks are potentialities that can only be guessed by making a number of assumptions (i.e. by using models). Regulators try to compensate for this tendency by asking risk managers to draw scenarios and to stress-test their models.

There are many other fundamental issues to be debated, such as the natural tendency to focus on micro risk management – because it is easy – rather than to integrate all significant risks and to consider their global effect – because that is more difficult. In particular, the assessment and control of systemic risks by supervisory authorities is still in its infancy. But I would like to conclude by calling attention to a particular danger faced by a nascent market risk management profession, that of separating risks from returns and focusing on downside-risk limits.

It is central to the ethics of risk managers to be independent and to act with integrity. Thus risk managers should not be under the direct control of line managers of profit centres and they should be well remunerated independently of company results. But in some firms this is also understood as denying risk managers access to profit information. I remember a risk commission that had to approve or reject projects but, for internal political reasons, could not have any information about their expected profitability. For decades, credit officers in most banks operated under such constraints: they were supposed to accept or reject deals *a priori*, without knowledge of their pricing. Times have changed. We understand now, at least in principle, that the essence of risk management is not simply to reduce or control risks but to achieve an optimal balance between risks and returns.

Yet, whether for organizational reasons or out of ignorance, risk management is often confined to setting and enforcing risk limits. Most firms, especially financial firms, claim to have well-thought-out risk management policies, but few actually state trade-offs between risks and returns. Attention to risk limits may be unwittingly reinforced by regulators. Of course it is not the role of the supervisory authorities to suggest risk–return trade-offs; so supervisors impose risk limits, such as value at risk relative to capital, to ensure safety and

fair competition in the financial industry. But a regulatory limit implies severe penalties if breached, and thus a probabilistic constraint acquires an economic value. Banks must therefore pay attention to the uncertainty in their value-at-risk estimates. The effect would be rather perverse if banks ended up paying more attention to the probability of a probability than to their entire return distribution.

With *Market Risk Analysis* readers will learn to understand these long-term problems in a realistic context. Carol is an academic with a strong applied interest. She has helped to design the curriculum for the Professional Risk Managers' International Association (PRMIA) qualifications, to set the standards for their professional qualifications, and she maintains numerous contacts with the financial industry through consulting and seminars. In *Market Risk Analysis* theoretical developments may be more rigorous and reach a more advanced level than in many other books, but they always lead to practical applications with numerous examples in interactive Excel spreadsheets. For example, unlike 90% of the finance literature on hedging that is of no use to practitioners, if not misleading at times, her concise expositions on this subject give solutions to real problems.

In summary, if there is any good reason for not treating market risk management as a separate discipline, it is that market risk management should be the business of *all* decision makers involved in finance, with primary responsibilities on the shoulders of the most senior managers and board members. However, there is so much to be learnt and so much to be further researched on this subject that it is proper for professional people to specialize in it. These four volumes will fulfil most of their needs. They only have to remember that, to be effective, they have to be good communicators and ensure that their assessments are properly integrated in their firm's decision-making process.

*Jacques Pézier*

## Preface to Volume III

A *financial instrument* is a legal contract between two or more parties that defines conditions under which the various parties incur costs and receive benefits. A cost or benefit need not be a monetary amount; it could be a commodity, for instance. The simplest type of financial instrument is a *financial asset*, which is a legal claim on a real asset such as a company, a commodity, cash, gold or a building. A *financial security* is a standardized form of financial asset that is traded in an organized market. For instance, equity securities (shares on a company's stock) are traded on exchanges and debt securities such as bonds and money market instruments (including bills, notes and repurchase agreements) are traded in brokers' markets.

A *derivative contract*, usually called a 'derivative' for short, is another type of financial instrument which is a contract on one or more *underlying* financial instruments. The underlying of a derivative does not have to be a traded asset or an interest rate. For instance, futures on carbon emissions or temperature have started trading on exchanges during the last few years. Derivatives are the fastest-growing class of financial instruments and the notional amount outstanding now far exceeds the size of ordinary securities markets. For instance, in 2007 the Bank for International Settlements estimated the total size of the debt securities market (including all corporate, government and municipal bonds and money market instruments) to be approximately US\$70 trillion. However, the amount outstanding on all interest rate derivatives was nearly \$300 trillion.

The most common types of financial derivatives are futures and forwards, swaps and options, and within each broad category there are numerous subcategories, so there is a huge diversity of financial derivatives. For instance, the vast majority of the trading in swaps is on interest rate swaps, but credit default swaps and cross-currency basis swaps are also heavily traded. Other swaps include variance swaps, covariance swaps, equity swaps and contracts for differences. But the greatest diversity amongst all derivative instruments can be found in the category of options. Options can be defined on virtually any underlying contract, including options on derivatives such as futures, swaps and other options. Many options, mostly standard calls and puts, are traded on exchanges, but there is a very active over-the-counter (OTC) market in non-standard options. Since the two parties in an OTC contract are free to define whatever terms they please, the pay-off to the holder of an OTC option can be freely defined. This means that ever more exotic options are continually being introduced, with pay-off profiles that can take virtually any shape.

A *portfolio* is a collection of financial instruments. An investor holds a portfolio with the aim of obtaining a particular return on his investment and to spread his risk. The more

differences between the financial instruments available to the investor, the better he can diversify his risk. Risk can be substantially reduced in large, well-diversified portfolios, but there can never be zero risk associated with any return above the risk free rate, and some investors are more averse to taking risks than others. The main reason for the terrific number of different financial instruments is that the risk–return profiles of different investors are not the same. Each new type of instrument is introduced specifically because it purports to provide its own unique profile of risk and return.

## AIMS AND SCOPE

This book is designed as a text for advanced university and professional courses in finance. It provides a pedagogical and complete treatment of the characteristics of the main categories of financial instruments, i.e. bonds, swaps, futures and forwards, options and volatility. Given the tremendous diversity of financial instruments, it is not surprising that there are many books that deal with just one type of financial instrument. Often the textbooks that cover fixed income securities alone, or just futures and forwards, or swaps or options, are large books that go into considerable details about specific market conventions. Some present each subcategory of instrument in its own unique theoretical framework, or include all mathematical details. By contrast, this book adopts a general framework whenever possible and provides a concise but rigorous treatment of only the essential mathematics.

To cover all major financial instruments (excluding credit derivatives) in one volume, one has to be very selective in the material presented. The reason why I have decided to exclude credit derivatives is that this book series is on market risk and not credit risk. Also I have not set up the background in Volume I, *Quantitative Methods in Finance*, to be able to cover credit derivatives in the same detail as I can analyse swaps, futures, options and volatility. Also we do not have a chapter specifically devoted to cash equity in this volume. This material naturally belongs in the Econometrics volume of *Market Risk Analysis*. A large part of Volume II, *Practical Financial Econometrics*, concerns cash equity portfolios, including the regression factor models that are used to analyse their risk and return and more advanced equity trading strategies (e.g. pairs trading based on cointegration).

Readers will appreciate the need to be concise, and whilst a mathematically rigorous approach is adopted some detailed proofs are omitted. Instead we refer readers to tractable sources where proofs may be perused, if required. My purpose is really to focus on the important concepts and to illustrate their application with practical examples. Even though this book omits some of the detailed arguments that are found in other textbooks on financial instruments, I have made considerable effort not to be obscure in any way. Each term is carefully defined, or a cross-reference is provided where readers may seek further enlightenment in other volumes of *Market Risk Analysis*. We assume no prior knowledge of finance, but readers should be comfortable with the scope of the mathematical material in Volume I and will preferably have that volume to hand. In order to make the exposition accessible to a wide audience, illustrative examples are provided immediately after the introduction of each new concept and virtually all of these examples are also worked through in interactive Excel spreadsheets.

This book is much shorter than other general books on financial instruments such as Wilmott (2006), Hull (2008) and Fabozzi (2002), one reason being that we omit credit derivatives. Many other textbooks in this area focus on just one particular category

of financial instrument. Thus there is overlap with several existing books. For instance, Chapter 3 on *Options* covers the same topics as much of the material in James (2003). A similar remark applies to Gatheral (2006), which has content similar to the first 75 pages of Chapter 4, on *Volatility* but in Gatheral's book this is covered in greater mathematical depth.

The readership of this volume is likely to be equally divided between finance professionals and academics. The main professional audience will be amongst traders, quants and risk managers, particularly those whose work concerns the pricing and hedging of bonds, swaps, futures and forwards, options and volatility. The main academic audience is for faculty involved with teaching and research and for students at the advanced master's or PhD level in finance, mathematical finance or quantitative market risk management. There are only five (extremely long) chapters and each aims to provide sufficient material for a one-semester postgraduate course, or for a week's professional training course.

### OUTLINE OF VOLUME III

Chapter 1, *Bonds and Swaps*, begins by introducing fundamental concepts such as the compounding of interest and the relationship between spot and forward rates, by providing a catalogue of fixed and floating coupon bonds by issuer and maturity and by performing a basic analysis of fixed coupon bonds, including the price–yield relationship, the characteristics of the zero coupon spot yield curve and the term structure of forward interest rates. We cover duration and convexity for single bonds and then for bond portfolios, the Taylor expansion to approximate the change in portfolio price for a parallel shift in the yield curve, and the traditional approach to bond portfolio immunization. Then we look at floating rate notes, forward rate agreements and interest rate swaps and explain their relationship; we analyse the market risk of an interest rate swap and introduce the PV01 and the dollar duration of cash flow. Bootstrapping, splines and parametric yield curve fitting methods and convertible bonds are also covered in this chapter.

Chapter 2, *Futures and Forwards*, gives details of the futures and forward markets in interest rates, bonds, currencies, commodities, stocks, stock indices, exchange traded funds, volatility indices, credit spreads, weather, real estate and pollution. Then we introduce the no arbitrage pricing argument, examine the components of basis risk for different types of underlying contract, and explain how to hedge with futures and forwards. Mean–variance, minimum variance and proxy hedging are all covered. We illustrate how futures hedges are implemented in practice: to hedge international portfolios with forex forwards, stock portfolios with index futures, and bond portfolios with portfolios of notional bond futures. The residual risk of a hedged portfolio is disaggregated into different components, showing which uncertainties cannot be fully hedged, and we include an Excel case study that analyses the book of an energy futures trader, identifying the key risk factors facing the trader and providing simple ways for the trader to reduce his risks.

Chapter 3, *Options*, introduces the basic principles of option pricing, and the options trading strategies that are commonly used by investors and speculators; describes the characteristics of different types of options; explains how providers of options hedge their risks; derives and interprets the Black–Scholes–Merton pricing model, and a standard trader's adjustment to this model for stochastic volatility; explains how to price interest rate options and how to calibrate the LIBOR model; and provides pricing models for European exotic options. It

begins with a relatively non-technical overview of the foundations of option pricing theory, including some elementary stochastic calculus, deriving the principle of risk neutral valuation and explaining the binomial option pricing model. The scope of the chapter is very broad, covering the pricing of European and American options with and without path-dependent pay-offs, but only under the assumption of constant volatility. ‘Greeks’ are introduced and analysed thoroughly and numerical examples how to hedge the risks of trading options. For interest rate options we derive the prices of European caps, floors and swaptions and survey the family of mean-reverting interest rate models, including a case study on the LIBOR model. Formulae for numerous exotics are given and these, along with more than 20 other numerical examples for this chapter, are all implemented in Excel.

Chapter 4, *Volatility*, begins by explaining how to model the market implied and market local volatility surfaces and discusses the properties of model implied and model local volatility surfaces. A long case study, spread over three Excel workbooks, develops a dynamic model of the market implied volatility surface based on principal component analysis and uses this to estimate price hedge ratios that are adjusted for implied volatility dynamics. Another main focus of the chapter is on option pricing models with stochastic volatility and jumps. The model implied and local volatility surfaces corresponding to any stochastic volatility model are defined intuitively and several stochastic volatility models, including their applications to options pricing and hedging, are discussed. We cover a few specific models with jumps, such as the Heston jump model (but not Lévy processes) and introduce a new type of volatility jump model as the continuous version of Markov switching GARCH. We explain why the models for tradable assets (but not necessarily interest rates) must be scale invariant and why it does not matter which scale invariant model we use for dynamic delta–gamma hedging of virtually any claim (!). Then we describe the market and the characteristics of variance swaps, volatility futures and volatility options and explain how to construct a term structure of volatility indices, using for illustration the Vftse, a volatility index that is not currently quoted on any exchange. At 94 pages, it is one of the longest and most comprehensive chapters in the book.

Chapter 5, *Portfolio Mapping*, is essential for hedging market risks and also lays the foundations for Volume IV, *Value-at-Risk Models*. It begins by summarizing a portfolio’s risk factors and its sensitivities to these factors for various categories of financial instruments, including cash and futures or forward portfolios on equities, bonds, currencies and commodities and portfolios of options. Then it covers present value, duration, volatility and PV01 invariant cash flow mapping, illustrating these with simple interactive Excel spreadsheets. Risk factor mapping of futures and forward portfolios, and that of commodity futures portfolios in particular, and mappings for options portfolios are covered, with all technical details supported with Excel spreadsheets. Mapping a volatility surface is not easy and most vega bucketing techniques are too crude, so this is illustrated with a case study based on the Vftse index. Statistical techniques such as regression and principal component analysis are used to reduce the dimension of the risk factor space and the chapter also requires some knowledge of matrix algebra for multivariate delta–gamma mapping.

## ABOUT THE CD-ROM

Virtually all the concepts in this book are illustrated using numerical and empirical examples which are stored in Excel workbooks for each chapter. These may be found on the accompanying CD-ROM in the folder labelled by the chapter number. Within these spreadsheets