A PRACTICAL APPROACH TO SIGNALS AND SYSTEMS

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Contents

Preface

The increasing number of applications, requiring a knowledge of the theory of signals and systems, and the rapid developments in digital systems technology and fast numerical algorithms call for a change in the content and approach used in teaching the subject. I believe that a modern signals and systems course should emphasize the practical and computational aspects in presenting the basic theory. This approach to teaching the subject makes the student more effective in subsequent courses. In addition, students are exposed to practical and computational solutions that will be of use in their professional careers. This book is my attempt to adapt the theory of signals and systems to the use of computers as an efficient analysis tool.

A good knowledge of the fundamentals of the analysis of signals and systems is required to specialize in such areas as signal processing, communication, and control. As most of the practical signals are continuous functions of time, and since digital systems are mostly used to process them, the study of both continuous and discrete signals and systems is required. The primary objective of writing this book is to present the fundamentals of time-domain and frequency-domain methods of signal and linear time-invariant system analysis from a practical viewpoint. As discrete signals and systems are more often used in practice and their concepts are relatively easier to understand, for each topic, the discrete version is presented first, followed by the corresponding continuous version. Typical applications of the methods of analysis are also provided. Comprehensive coverage of the transform methods, and emphasis on practical methods of analysis and physical interpretation of the concepts are the key features of this book. The well-documented software, which is a supplement to this book and available on the website (www.wiley.com/go/sundararajan), greatly reduces much of the difficulty in understanding the concepts. Based on this software, a laboratory course can be tailored to suit individual course requirements.

This book is intended to be a textbook for a junior undergraduate level one-semester signals and systems course. This book will also be useful for self-study. Answers to selected exercises, marked $*$, are given at the end of the book. A Solutions manual and slides for instructors are also available on the website (www.wiley.com/go/sundararajan). I assume responsibility for any errors in this book and in the accompanying supplements, and would very much appreciate receiving readers' suggestions and pointing out any errors (email address: d_sundararajan@yahoo.com).

I am grateful to my editor and his team at Wiley for their help and encouragement in completing this project. I thank my family and my friend Dr A. Pedar for their support during this endeavor.

D. Sundararajan

Abbreviations

dc: Constant
DFT: Discrete Fourier transform
DTFT: Discrete-time Fourier transform
FT: Fourier transform
FS: Fourier series
IDFT: Inverse discrete Fourier transform
Im: Imaginary part of a complex number or expression
LTI: Linear time-invariant
Re: Real part of a complex number or expression
ROC: Region of convergence

1

Introduction

In typical applications of science and engineering, we have to process signals, using systems. While the applications vary from communication to control, the basic analysis and design tools are the same. In a signals and systems course, we study these tools: convolution, Fourier analysis, z-transform, and Laplace transform. The use of these tools in the analysis of linear time-invariant (LTI) systems with deterministic signals is presented in this book. While most practical systems are nonlinear to some extent, they can be analyzed, with acceptable accuracy, assuming linearity. In addition, the analysis is much easier with this assumption. A good grounding in LTI system analysis is also essential for further study of nonlinear systems and systems with random signals.

For most practical systems, input and output signals are continuous and these signals can be processed using continuous systems. However, due to advances in digital systems technology and numerical algorithms, it is advantageous to process continuous signals using digital systems (systems using digital devices) by converting the input signal into a digital signal. Therefore, the study of both continuous and digital systems is required. As most practical systems are digital and the concepts are relatively easier to understand, we describe discrete signals and systems first, immediately followed by the corresponding description of continuous signals and systems.

1.1 The Organization of this Book

Four topics are covered in this book. The time-domain analysis of signals and systems is presented in Chapters 2–5. The four versions of the Fourier analysis are described in Chapters 6–9. Generalized Fourier analysis, the z-transform and the Laplace transform, are presented in Chapters 10 and 11. State space analysis is introduced in Chapters 12 and 13.

The amplitude profile of practical signals is usually arbitrary. It is necessary to represent these signals in terms of well-defined basic signals in order to carry out

A Practical Approach to Signals and Systems D. Sundararajan

efficient signal and system analysis. The impulse and sinusoidal signals are fundamental in signal and system analysis. In Chapter 2, we present discrete signal classifications, basic signals, and signal operations. In Chapter 3, we present continuous signal classifications, basic signals, and signal operations.

The study of systems involves modeling, analysis, and design. In Chapter 4, we start with the modeling of a system with the difference equation. The classification of systems is presented next. Then, the convolution–summation model is introduced. The zero-input, zero-state, transient, and steady-state responses of a system are derived from this model. System stability is considered in terms of impulse response. The basic components of discrete systems are identified. In Chapter 5, we start with the classification of systems. The modeling of a system with the differential equation is presented next. Then, the convolution-integral model is introduced. The zero-input, zero-state, transient, and steady-state responses of a system are derived from this model. System stability is considered in terms of impulse response. The basic components of continuous systems are identified.

Basically, the analysis of signals and systems is carried out using impulse or sinusoidal signals. The impulse signal is used in time-domain analysis, which is presented in Chapters 4 and 5. Sinusoids (more generally complex exponentials) are used as the basic signals in frequency-domain analysis. As frequency-domain analysis is generally more efficient, it is most often used. Signals occur usually in the time-domain. In order to use frequency-domain analysis, signals and systems must be represented in the frequency-domain. Transforms are used to obtain the frequency-domain representation of a signal or a system from its time-domain representation. All the essential transforms required in signal and system analysis use the same family of basis signals, a set of complex exponential signals. However, each transform is more advantageous to analyze certain types of signal and to carry out certain types of system operations, since the basis signals consists of a finite or infinite set of complex exponential signals with different characteristics—continuous or discrete, and the exponent being complex or pure imaginary. The transforms that use the complex exponential with a pure imaginary exponent come under the heading of Fourier analysis. The other transforms use exponentials with complex exponents as their basis signals.

There are four versions of Fourier analysis, each primarily applicable to a different type of signals such as continuous or discrete, and periodic or aperiodic. The discrete Fourier transform (DFT) is the only one in which both the time- and frequency-domain representations are in finite and discrete form. Therefore, it can approximate other versions of Fourier analysis through efficient numerical procedures. In addition, the physical interpretation of the DFT is much easier. The basis signals of this transform is a finite set of harmonically related discrete exponentials with pure imaginary exponent. In Chapter 6, the DFT, its properties, and some of its applications are presented.

Fourier analysis of a continuous periodic signal, which is a generalization of the DFT, is called the Fourier series (FS). The FS uses an infinite set of harmonically related continuous exponentials with pure imaginary exponent as the basis signals.

This transform is useful in frequency-domain analysis and design of periodic signals and systems with continuous periodic signals. In Chapter 7, the FS, its properties, and some of its applications are presented.

Fourier analysis of a discrete aperiodic signal, which is also a generalization of the DFT, is called the discrete-time Fourier transform (DTFT). The DTFT uses a continuum of discrete exponentials, with pure imaginary exponent, over a finite frequency range as the basis signals. This transform is useful in frequency-domain analysis and design of discrete signals and systems. In Chapter 8, the DTFT, its properties, and some of its applications are presented.

Fourier analysis of a continuous aperiodic signal, which can be considered as a generalization of the FS or the DTFT, is called the Fourier transform (FT). The FT uses a continuum of continuous exponentials, with pure imaginary exponent, over an infinite frequency range as the basis signals. This transform is useful in frequency-domain analysis and design of continuous signals and systems. In addition, as the most general version of Fourier analysis, it can represent all types of signals and is very useful to analyze a system with different types of signals, such as continuous or discrete, and periodic or aperiodic. In Chapter 9, the FT, its properties, and some of its applications are presented.

Generalization of Fourier analysis for discrete signals results in the z-transform. This transform uses a continuum of discrete exponentials, with complex exponent, over a finite frequency range of oscillation as the basis signals. With a much larger set of basis signals, this transform is required for the design, and transient and stability analysis of discrete systems. In Chapter 10, the z-transform is derived from the DTFT and, its properties and some of its applications are presented. Procedures for obtaining the forward and inverse z-transforms are described.

Generalization of Fourier analysis for continuous signals results in the Laplace transform. This transform uses a continuum of continuous exponentials, with complex exponent, over an infinite frequency range of oscillation as the basis signals. With a much larger set of basis signals, this transform is required for the design, and transient and stability analysis of continuous systems. In Chapter 11, the Laplace transform is derived from the FT and, its properties and some of its applications are presented. Procedures for obtaining the forward and inverse Laplace transforms are described.

In Chapter 12, state-space analysis of discrete systems is presented. This type of analysis is more general in that it includes the internal description of a system in contrast to the input–output description of other types of analysis. In addition, this method is easier to extend to system analysis with multiple inputs and outputs, and nonlinear and time-varying system analysis. In Chapter 13, state-space analysis of continuous systems is presented.

In Appendix A, transform pairs and properties are listed. In Appendix B, useful mathematical formulas are given.

The basic problem in the study of systems is how to analyze systems with arbitrary input signals. The solution, in the case of linear time-invariant (LTI) systems, is to decompose the signal in terms of basic signals, such as the impulse or the sinusoid. Then, with knowledge of the response of a system to these basic signals, the response of the system to any arbitrary signal that we shall ever encounter in practice, can be obtained. Therefore, the study of the response of systems to the basic signals, along with the methods of decomposition of arbitrary signals in terms of the basic signals, constitute the study of the analysis of systems with arbitrary input signals.

2

Discrete Signals

A signal represents some information. Systems carry out tasks or produce output signals in response to input signals. A control system may set the speed of a motor in accordance with an input signal. In a room-temperature control system, the power to the heating system is regulated with respect to the room temperature. While signals may be electrical, mechanical, or of any other form, they are usually converted to electrical form for processing convenience. A speech signal is converted from a pressure signal to an electrical signal in a microphone. Signals, in almost all practical systems, have arbitrary amplitude profile. These signals must be represented in terms of simple and well-defined mathematical signals for ease of representation and processing. The response of a system is also represented in terms of these simple signals. In Section 2.1, signals are classified according to some properties. Commonly used basic discrete signals are described in Section 2.2. Discrete signal operations are presented in Section 2.3.

2.1 Classification of Signals

Signals are classified into different types and, the representation and processing of a signal depends on its type.

2.1.1 Continuous, Discrete and Digital Signals

A continuous signal is specified at every value of its independent variable. For example, the temperature of a room is a continuous signal. One cycle of the continuous complex exponential signal, $x(t) = e^{j(\frac{2\pi}{16}t+\frac{\pi}{3})}$, is shown in Figure 2.1(a). We denote a continuous signal, using the independent variable t, as $x(t)$. We call this representation the time-domain representation, although the independent variable is not time for some signals. Using Euler's identity, the signal can be expressed, in terms of cosine and

A Practical Approach to Signals and Systems D. Sundararajan

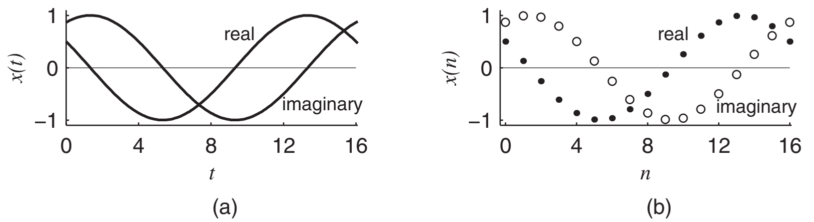

Figure 2.1 (a) The continuous complex exponential signal, $x(t) = e^{j(\frac{2\pi}{16}t+\frac{\pi}{3})}$; (b) the discrete complex exponential signal, $x(n) = e^{j(\frac{2\pi}{16}n+\frac{\pi}{3})}$

sine signals, as

$$x(t) = e^{j(\frac{2\pi}{16}t+\frac{\pi}{3})} = \cos\left(\frac{2\pi}{16}t + \frac{\pi}{3}\right) + j\sin\left(\frac{2\pi}{16}t + \frac{\pi}{3}\right)$$

The real part of $x(t)$ is the real sinusoid $\cos(\frac{2\pi}{16}t + \frac{\pi}{3})$ and the imaginary part is the real sinusoid $\sin(\frac{2\pi}{16}t + \frac{\pi}{3})$, as any complex signal is an ordered pair of real signals. While practical signals are real-valued with arbitrary amplitude profile, the mathematically well-defined complex exponential is predominantly used in signal and system analysis.

A discrete signal is specified only at discrete values of its independent variable. For example, a signal $x(t)$ is represented only at $t = nT_s$ as $x(nT_s)$, where T_s is the sampling interval and n is an integer. The discrete signal is usually denoted as $x(n)$, suppressing T_s in the argument of $x(nT_s)$. The important advantage of discrete signals is that they can be stored and processed efficiently using digital devices and fast numerical algorithms. As most practical signals are continuous signals, the discrete signal is often obtained by sampling the continuous signal. However, signals such as yearly population of a country and monthly sales of a company are inherently discrete signals. Whether a discrete signal arises inherently or by sampling, it is represented as a sequence of numbers $\{x(n), \ -\infty < n < \infty\}$, where the independent variable n is an integer. Although $x(n)$ represents a single sample, it is also used to denote the sequence instead of $\{x(n)\}$. One cycle of the discrete complex exponential signal, $x(n) = e^{j(\frac{2\pi}{16}n+\frac{\pi}{3})}$, is shown in Figure 2.1(b). This signal is obtained by sampling the signal (replacing t by nT_s) in Figure 2.1(a) with $T_s = 1$ s. In this book, we assume that the sampling interval, T_s, is a constant. In sampling a signal, the sampling interval, which depends on the frequency content of the signal, is an important parameter. The sampling interval is required again to convert the discrete signal back to its corresponding continuous form. However, when the signal is in discrete form, most of the processing is independent of the sampling interval. For example, summing of a set of samples of a signal is independent of the sampling interval.

When the sample values of a discrete signal are quantized, it becomes a digital signal. That is, both the dependent and independent variables of a digital signal are in discrete form. This form is actually used to process signals using digital devices, such as a digital computer.

2.1.2 Periodic and Aperiodic Signals

The smallest positive integer $N > 0$ satisfying the condition $x(n + N) = x(n)$, for all n, is the period of the periodic signal $x(n)$. Over the interval $-\infty < n < \infty$, a periodic signal repeats its values in any interval equal to its period, at intervals of its period. Cosine and sine waves, and the complex exponential, shown in Figure 2.1, are typical examples of a periodic signal. A signal with constant value (dc) is periodic with any period. In Fourier analysis, it is considered as $A\cos(\omega n)$ or $Ae^{j\omega n}$ with the frequency ω equal to zero (period equal to ∞).

When the period of a periodic signal approaches infinity, there is no repetition of a pattern and it degenerates into an aperiodic signal. Typical aperiodic signals are shown in Figure 2.3.

It is easier to decompose an arbitrary signal in terms of some periodic signals, such as complex exponentials, and the input–output relationship of LTI systems becomes a multiplication operation for this type of input signal. For these reasons, most of the analysis of practical signals, which are mostly aperiodic having arbitrary amplitude profile, is carried out using periodic basic signals.

2.1.3 Energy and Power Signals

The power or energy of a signal are also as important as its amplitude in its characterization. This measure involves the amplitude and the duration of the signal. Devices, such as amplifiers, transmitters, and motors, are specified by their output power. In signal processing systems, the desired signal is usually mixed up with a certain amount of noise. The quality of these systems is indicated by the signal-to-noise power ratio. Note that noise signals, which are typically of random type, are usually characterized by their average power. In the most common signal approximation method, Fourier analysis, the goodness of the approximation improves as more and more frequency components are used to represent a signal. The quality of the approximation is measured in terms of the square error, which is an indicator of the difference between the energy or power of a signal and that of its approximate version.

The instantaneous power dissipated in a resistor of 1Ω is $x^2(t)$, where $x(t)$ may be the voltage across it or the current through it. By integrating the power over the interval in which the power is applied, we get the energy dissipated. Similarly, the sum of the squared magnitude of the values of a discrete signal $x(n)$ is an indicator of its energy and is given as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

The use of the magnitude $|x(n)|$ makes the expression applicable to complex signals as well. Due to the squaring operation, the energy of a signal $2x(n)$, with double the amplitude, is four times that of $x(n)$. Aperiodic signals with finite energy are called energy signals. The energy of $x(n) = 4(0.5)^n, \ n \geq 0$ is

$$E = \sum_{n=0}^{\infty} |4(0.5)^n|^2 = \frac{16}{1 - 0.25} = \frac{64}{3}$$

If the energy of a signal is infinite, then it may be possible to characterize it in terms of its average power. The average power is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

For a periodic signal with period N, the average power can be determined as

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

Signals, periodic or aperiodic, with finite average power are called power signals. Cosine and sine waveforms are typical examples of power signals. The average power of the cosine wave $2\cos(\frac{2\pi}{4}n)$ is

$$P = \frac{1}{4} \sum_{n=0}^{3} |x(n)|^2 = \frac{1}{4}(2^2 + 0^2 + (-2)^2 + 0^2) = 2$$

A signal is an energy signal or a power signal, since the average power of an energy signal is zero while that of a power signal is finite. Signals with infinite average power and infinite energy, such as $x(n) = n, \ 0 \leq n < \infty$, are neither power signals nor energy signals. The measures of signal power and energy are indicators of the signal size, since the actual energy or power depends on the load.

2.1.4 Even- and Odd-symmetric Signals

The storage and processing requirements of a signal can be reduced by exploiting its symmetry. A signal $x(n)$ is even-symmetric, if $x(-n) = x(n)$ for all n. The signal is symmetrical about the vertical axis at the origin. The cosine waveform, shown in Figure 2.2(b), is an example of an even-symmetric signal. A signal $x(n)$ is odd-symmetric, if $x(-n) = -x(n)$ for all n. The signal is asymmetrical

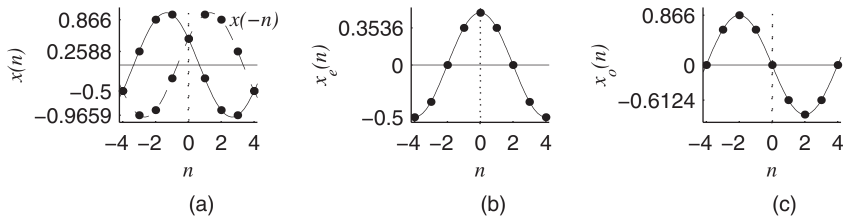

Figure 2.2 (a) The sinusoid $x(n) = \cos(\frac{2\pi}{8}n + \frac{\pi}{3})$ and its time-reversed version $x(-n)$; (b) its even component $x_\mathrm{e}(n) = \frac{1}{2}\cos(\frac{2\pi}{8}n)$; (c) its odd component $x_\mathrm{o}(n) = -\frac{\sqrt{3}}{2}\sin(\frac{2\pi}{8}n)$

about the vertical axis at the origin. For an odd-symmetric signal, $x(0) = 0$. The sine waveform, shown in Figure 2.2(c), is an example of an odd-symmetric signal.

The sum $(x(n) + y(n))$ of two odd-symmetric signals, $x(n)$ and $y(n)$, is an odd-symmetric signal, since $x(-n) + y(-n) = -x(n) - y(n) = -(x(n) + y(n))$. For example, the sum of two sine signals is an odd-symmetric signal. The sum $(x(n) + y(n))$ of two even-symmetric signals, $x(n)$ and $y(n)$, is an even-symmetric signal, since $x(-n) + y(-n) = (x(n) + y(n))$. For example, the sum of two cosine signals is an even-symmetric signal. The sum $(x(n) + y(n))$ of an odd-symmetric signal $x(n)$ and an even-symmetric signal $y(n)$ is neither even-symmetric nor odd-symmetric, since $x(-n) + y(-n) = -x(n) + y(n) = -(x(n) - y(n))$. For example, the sum of cosine and sine signals with nonzero amplitudes is neither even-symmetric nor odd-symmetric.

Since $x(n)y(n) = (-x(-n))(-y(-n)) = x(-n)y(-n)$, the product of two odd-symmetric or two even-symmetric signals is an even-symmetric signal. The product $z(n) = x(n)y(n)$ of an odd-symmetric signal $y(n)$ and an even-symmetric signal $x(n)$ is an odd-symmetric signal, since $z(-n) = x(-n)y(-n) = x(n)(-y(n)) = -z(n)$.

An arbitrary signal $x(n)$ can always be decomposed in terms of its even-symmetric and odd-symmetric components, $x_\mathrm{e}(n)$ and $x_\mathrm{o}(n)$, respectively. That is, $x(n) = x_\mathrm{e}(n) + x_\mathrm{o}(n)$. Replacing n by $-n$, we get $x(-n) = x_\mathrm{e}(-n) + x_\mathrm{o}(-n) = x_\mathrm{e}(n) - x_\mathrm{o}(n)$. Solving for $x_\mathrm{e}(n)$ and $x_\mathrm{o}(n)$, we get

$$x_\mathrm{e}(n) = \frac{x(n) + x(-n)}{2} \qquad \text{and} \qquad x_\mathrm{o}(n) = \frac{x(n) - x(-n)}{2}$$

As the sum of an odd-symmetric signal $x_\mathrm{o}(n)$, over symmetric limits, is zero,

$$\sum_{n=-N}^{N} x_\mathrm{o}(n) = 0 \qquad \sum_{n=-N}^{N} x(n) = \sum_{n=-N}^{N} x_\mathrm{e}(n) = x_\mathrm{e}(0) + 2\sum_{n=1}^{N} x_\mathrm{e}(n)$$

For example, the even-symmetric component of $x(n) = \cos(\frac{2\pi}{8}n + \frac{\pi}{3})$ is

$$x_{\mathrm{e}}(n) = \frac{x(n) + x(-n)}{2} = \frac{\cos\left(\frac{2\pi}{8}n + \frac{\pi}{3}\right) + \cos\left(\frac{2\pi}{8}(-n) + \frac{\pi}{3}\right)}{2}$$

$$= \frac{2\cos\left(\frac{2\pi}{8}n\right)\cos\left(\frac{\pi}{3}\right)}{2} = \frac{\cos\left(\frac{2\pi}{8}n\right)}{2}$$

The odd-symmetric component is

$$x_{\mathrm{o}}(n) = \frac{x(n) - x(-n)}{2} = \frac{\cos\left(\frac{2\pi}{8}n + \frac{\pi}{3}\right) - \cos\left(\frac{2\pi}{8}(-n) + \frac{\pi}{3}\right)}{2}$$

$$= \frac{-2\sin\left(\frac{2\pi}{8}n\right)\sin\left(\frac{\pi}{3}\right)}{2} = -\frac{\sqrt{3}}{2}\sin\left(\frac{2\pi}{8}n\right)$$

The sinusoid $x(n)$ and its time-reversed version $x(-n)$, its even component, and its odd component are shown, respectively, in Figures 2.2(a–c). As the even and odd components of a sinusoid are, respectively, cosine and sine functions of the same frequency as that of the sinusoid, these results can also be obtained by expanding the expression characterizing the sinusoid.

If a continuous signal is sampled with an adequate sampling rate, the samples uniquely correspond to that signal. Assuming that the sampling rate is adequate, in Figure 2.2 (and in other figures in this book), we have shown the corresponding continuous waveform only for clarity. It should be remembered that a discrete signal is represented only by its sample values.

2.1.5 Causal and Noncausal Signals

Most signals, in practice, occur at some finite time instant, usually chosen as $n = 0$, and are considered identically zero before this instant. These signals, with $x(n) = 0$ for $n < 0$, are called causal signals. Signals, with $x(n) \neq 0$ for $n < 0$, are called noncausal signals. Sine and cosine signals, shown in Figures 2.1 and 2.2, are noncausal signals. Typical causal signals are shown in Figure 2.3.

2.1.6 Deterministic and Random Signals

Signals such as $x(n) = \sin(\frac{2\pi}{8}n)$, whose values are known for any value of n, are called deterministic signals. Signals such as those generated by thermal noise in conductors or speech signals, whose future values are not exactly known, are called random signals. Despite the fact that rainfall record is available for several years in the past, the amount of future rainfall at a place cannot be exactly predicted. This type of signal is characterized by a probability model or a statistical model. The study of random

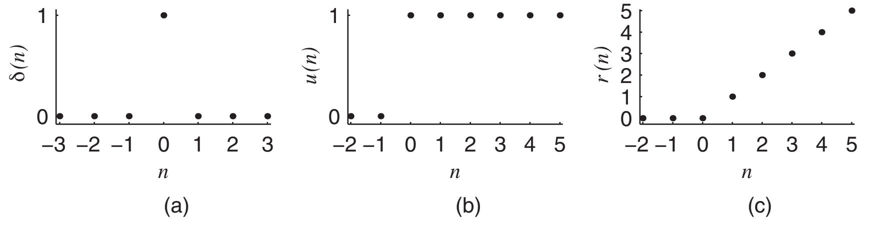

Figure 2.3 (a) The unit-impulse signal, $\delta(n)$; (b) the unit-step signal, $u(n)$; (c) the unit-ramp signal, $r(n)$

signals is important in practice, since all practical signals are random to some extent. However, the analysis of systems is much simpler, mathematically, with deterministic signals. The input–output relationship of a system remains the same whether the input signal is random or deterministic. The time-domain and frequency-domain methods of system analysis are common to both types of signals. The key difference is to find a suitable mathematical model for random signals. In this book, we confine ourselves to the study of deterministic signals.

2.2 Basic Signals

As we have already mentioned, most practical signals have arbitrary amplitude profile. These signals are, for processing convenience, decomposed in terms of mathematically well-defined and simple signals. These simple signals, such as the sinusoid with infinite duration, are not practical signals. However, they can be approximated to a desired accuracy.

2.2.1 Unit-impulse Signal

The unit-impulse signal, shown in Figure 2.3(a), is defined as

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

The unit-impulse signal is an all-zero sequence except that it has a value of one when its argument is equal to zero. A time-shifted unit-impulse signal $\delta(n-m)$, with argument $(n-m)$, has its only nonzero value at $n = m$. Therefore, $\sum_{n=-\infty}^{\infty} x(n)\delta(n-m) = x(m)$ is called the sampling or sifting property of the impulse. For example,

$$\sum_{n=-\infty}^{\infty} 2^n\delta(n) = 1 \quad \sum_{n=-2}^{0} 2^n\delta(n-1) = 0 \quad \sum_{n=-2}^{0} 2^n\delta(-n-1) = 0.5$$

$$\sum_{n=-2}^{0} 2^n\delta(n+1) = 0.5 \quad \sum_{n=-\infty}^{\infty} 2^n\delta(n+2) = 0.25 \quad \sum_{n=-\infty}^{\infty} 2^n\delta(n-3) = 8$$

In the second summation, the argument $n-1$ of the impulse never becomes zero within the limits of the summation.

The decomposition of an arbitrary signal in terms of scaled and shifted impulses is a major application of this signal. Consider the product of a signal with a shifted impulse $x(n)\delta(n-m) = x(m)\delta(n-m)$. Summing both sides with respect to m, we get

$$\sum_{m=-\infty}^{\infty} x(n)\delta(n-m) = x(n)\sum_{m=-\infty}^{\infty} \delta(n-m) = x(n) = \sum_{m=-\infty}^{\infty} x(m)\delta(n-m)$$

The general term $x(m)\delta(n-m)$ of the last sum, which is one of the constituent impulses of $x(n)$, is a shifted impulse $\delta(n-m)$ located at $n=m$ with value $x(m)$. The summation operation sums all these impulses to form $x(n)$. Therefore, the signal $x(n)$ is represented by the sum of scaled and shifted impulses with the value of the impulse at any n being $x(n)$. The unit-impulse is the basis function and $x(n)$ is its coefficient. As the value of the sum is nonzero only at $n=m$, the sum is effective only at that point. By varying the value of n, we can sift out all the values of $x(n)$. For example, consider the signal $x(-2)=2$, $x(0)=3$, $x(2)=-4$, $x(3)=1$, and $x(n)=0$ otherwise. This signal can be expressed, in terms of impulses, as

$$x(n) = 2\delta(n+2) + 3\delta(n) - 4\delta(n-2) + \delta(n-3)$$

With $n=2$, for instance,

$$x(2) = 2\delta(4) + 3\delta(2) - 4\delta(0) + \delta(-1) = -4$$

2.2.2 Unit-step Signal

The unit-step signal, shown in Figure 2.3(b), is defined as

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

The unit-step signal is an all-one sequence for positive values of its argument and is an all-zero sequence for negative values of its argument. The causal form of a signal $x(n)$, $x(n)$ is zero for $n<0$, is obtained by multiplying it by the unit-step signal as $x(n)u(n)$. For example, $\sin(\frac{2\pi}{6}n)$ has nonzero values in the range $-\infty < n < \infty$, whereas the values of $\sin(\frac{2\pi}{6}n)u(n)$ are zero for $n<0$ and $\sin(\frac{2\pi}{6}n)$ for $n \geq 0$. A shifted unit-step signal, for example $u(n-1)$, is $u(n)$ shifted by one sample interval to the right (the first nonzero value occurs at $n=1$). Using scaled and shifted unit-step signals, any signal, described differently over different intervals, can be specified, for easier mathematical analysis, by a single expression, valid for all n. For example, a