Practical Portfolio Performance Measurement and Attribution

Carl R. Bacon
Practical Portfolio Performance Measurement and Attribution
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This book is dedicated to Alex and Matt

Thanks for the support, black coffee and suffering in silence the temporary suspension of normal family life
Contents

About the Author  xii
Acknowledgements  xiii

1  Introduction  1
   Why measure portfolio performance?  1
   The purpose of this book  2
   Reference  3

2  The Mathematics of Portfolio Return  5
   Simple return  5
   Money-weighted returns  7
      Internal rate of return (IRR)  7
      Simple internal rate of return  7
      Modified internal rate of return  8
      Simple Dietz  9
      ICAA method  11
      Modified Dietz  12
   Time-weighted returns  13
      True time-weighted  13
      Unit price method  15
   Time-weighted versus money-weighted rates of return  16
   Approximations to the time-weighted return  18
      Index substitution  18
      Regression method (or β method)  19
      Analyst’s test  20
   Hybrid methodologies  21
      Linked modified Dietz  21
      BAI method  22
   Which method to use?  22
   Self-selection  23
   Annualized returns  25
   Continuously compounded returns  28
Gross- and net-of-fee calculations 29
Estimating gross- and net-of-fee returns 30
Performance fees 30
Portfolio component returns 32
Component weight 33
Carve-outs 34
Multi-period component returns 34
Base currency and local returns 35
References 36

3 **Benchmarks**

Benchmarks 39
Benchmark attributes 39
Commercial indexes 40
Calculation methodologies 40
Index turnover 40
Hedged indexes 41
Customized (or composite) indexes 41
Fixed weight and dynamized benchmarks 42
Capped indexes 44
Blended (or spliced) indexes 44
Peer groups and universes 45
Percentile rank 45
Notional funds 46
Normal portfolio 47
Growth and value 47
Excess return 47
Arithmetic excess return 48
Geometric excess return 48

4 **Risk**

Definition of risk 53
Risk management versus risk control 54
Risk aversion 54
Risk measures 54
*Ex post* and *ex ante* risk 54
Variability 54
Mean absolute deviation 54
Variance 55
Standard deviation 55
Sharpe ratio (reward to variability) 56
Risk-adjusted return: $M^2$ 58
$M^2$ excess return 59
Differential return 60
Regression analysis 61
Regression equation 62
Regression alpha ($\alpha_R$) 62
Regression beta ($\beta_R$) 62
Regression epsilon ($\varepsilon_R$) 62
Capital Asset Pricing Model (CAPM) 62
Beta ($\beta$) (systematic risk or volatility) 62
Jensen’s alpha (or Jensen’s measure or Jensen’s differential return) 63
Bull beta ($\beta^+$) 63
Bear beta ($\beta^-$) 63
Beta timing ratio 63
Covariance 64
Correlation ($\rho$) 64
$R^2$ (or coefficient of determination) 66
Systematic risk 66
Specific or residual risk 66
Treynor ratio (reward to volatility) 66
Modified Treynor ratio 68
$M^2$ for beta 68
Appraisal ratio (Sharpe ratio adjusted for systematic risk) 68
Modified Jensen 69
Fama decomposition 69
Selectivity 69
Diversification 69
Net selectivity 70
Relative risk 70
Tracking error 71
Information ratio (or modified Sharpe ratio) 71
Return distributions 74
Normal distribution 74
Skewness 74
Kurtosis 74
d ratio 75
Downside risk 75
Sortino ratio 76
$M^2$ for Sortino 76
Upside potential ratio 77
Omega excess return 77
Volatility skewness 77
Value at Risk (VaR) 78
VaR ratio 78
Hurst index 80
Fixed income risk 80
Duration 80
Macaulay duration 81
Modified duration 81
Effective duration 81
Convexity 82
Modified convexity 82
Effective convexity 82
## 5 Performance Attribution

### Arithmetic attribution
- Brinson, Hood and Beebower
- Asset allocation
- Security (or stock) selection
- Interaction

### Geometric excess return attribution
- Asset allocation
- Stock selection

### Sector weights
- Buy-and-hold (or holding-based) attribution
- Security-level attribution

### Multi-period attribution

### Smoothing algorithms
- Carino
- Menchero
- GRAP method
- Frongello
- Davies and Laker
- Multi-period geometric attribution

### Risk-adjusted attribution
- Selectivity

### Multi-currency attribution
- Ankrim and Hensel
- Karnosky and Singer

### Geometric multi-currency attribution
- Naive currency attribution
- Compounding effects
- Interest-rate differentials
- Currency allocation
- Cost of hedging
- Currency timing (or currency selection)
- Summarizing
- Other currency issues

### Fixed income attribution
- Weighted duration attribution

### Attribution standards
- Evolution of performance attribution methodologies

### References
6 Performance Presentation Standards

Why do we need performance presentation standards? 163
Advantages for asset managers 164
The standards 165
Verification 167
Investment Performance Council 167
Country Standards Subcommittee (CSSC) 168
Verification Subcommittee 169
Interpretation Subcommittee 169
Guidance statements 170
Definition of firm 170
Carve-outs 170
Portability 171
Supplemental information 172
Achieving compliance 172
Maintaining compliance 173
Reference 174

Appendix A Simple Attribution 175

Appendix B Multi-currency Attribution Methodology 178

Appendix C EIPC Guidance for Users of Attribution Analysis 186

Appendix D European Investment Performance Committee – Guidance on Performance Attribution Presentation 191

Appendix E The Global Investment Performance Standards 204

Bibliography 215

Index 219
Carl Bacon joined StatPro Group plc as Chairman in April 2000. StatPro develops and markets specialist middle-office reporting software to the asset management industry. Carl also runs his own consultancy business providing advice to asset managers on various risk and performance measurement issues.

Prior to joining StatPro Carl was Director of Risk Control and Performance at Foreign & Colonial Management Ltd, Vice President Head of Performance (Europe) for J P Morgan Investment Management Inc., and Head of Performance for Royal Insurance Asset Management.

Carl holds a B.Sc. Hons. in Mathematics from Manchester University and is a member of the UK Investment Performance Committee (UKIPC), the European Investment Performance Committee (EIPC) and the Investment Performance Council (IPC). An original GIPS committee member, Carl also chairs the IPC Interpretations Sub-Committee, is ex-chair of the IPC Verification Sub-committee and is a member of the Advisory Board of the Journal of Performance Measurement.
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This book developed from the series of performance measurement trainings courses I have had the pleasure of running around the world since the mid-1990s. I have learned so much and continue to learn from the questions and observations of the participants over the years, all of whom must be thanked.

I should also like to thank the many individuals at work, at conferences and in various IPC committee meetings who have influenced my views over the years and are not mentioned specifically.

Naturally from the practitioner’s perspective, I’ve favoured certain methodologies over others – apologies to those who may feel their methods have been unfairly treated.

I am particularly grateful to Stefan Illmer for his useful corrections and suggestions for additional sections.

Of course, all errors and omissions are my own.

Carl R. Bacon
Deeping St James
September 2004
WHY MEASURE PORTFOLIO PERFORMANCE?

Whether we manage our own investment assets or choose to hire others to manage the assets on our behalf we are keen to know “how well” our collection, or portfolio of assets are performing.

The process of adding value via benchmarking, asset allocation, security analysis, portfolio construction and executing transactions is collectively described as the investment decision process. The measurement of portfolio performance should be part of the investment decision process, not external to it.

Clearly there are many stakeholders in the investment decision process; this book focuses on the investors or owners of capital and the firms managing their assets (asset managers or individual portfolio managers). Other stakeholders in the investment decision process include independent consultants tasked with providing advice to clients, custodians, independent performance measurers and audit firms.

Portfolio performance measurement answers the three basic questions central to the relationship between asset managers and the owners of capital:

(1) *What* is the return on assets?
(2) *Why* has the portfolio performed that way?
(3) *How* can we improve performance?

Portfolio performance measurement is the quality control of the investment decision process and provides the necessary information to enable asset managers and clients to assess exactly how the money has been invested and the results of the process. The US Bank Administration Institute (BAI) laid down the foundations of the performance measurement process as early as 1968. The main conclusions of their study hold today:

(1) Performance measurement returns should be based on asset values measured at market value not at cost.
Returns should be “total” returns (i.e., they should include both income and changes in market value – realized and unrealized capital appreciation).

Returns should be time-weighted.

Measurement should include risk as well as return.

THE PURPOSE OF THIS BOOK

The vocabulary of performance measurement and the multiple methodologies open to performance analysts worldwide are extremely varied and complex.

My purpose in writing this book is an attempt to provide a reference of the available methodologies and to hopefully provide some consistency in their definition.

Despite the development and global success of performance measurement standards there are considerable differences in terminology, methodology and attitude to performance measurement throughout the world.

Few books are dedicated to portfolio performance measurement; the aim of this one is to promote the role of performance measurers and to provide some insights into the tools at their disposal.

With its practical examples this book should meet the needs of performance analysts, portfolio managers, senior management within asset management firms, custodians, verifiers and ultimately the clients.

Performance measurement is a key function in an asset management firm, it deserves better than being grouped with the back office. Performance measurers provide real added value, with feedback into the investment decision process and analysis of structural issues. Since their role is to understand in full and communicate the sources of return within portfolios they are often the only independent source equipped to understand the performance of all the portfolios and strategies operating within the asset management firm.

Performance measurers are in effect alternative risk controllers able to protect the firm from rogue managers and the unfortunate impact of failing to meet client expectations.

The chapters of this book are structured in the same order as the performance measurement process itself, namely:

(1) Calculation of portfolio returns.
(2) Comparison against a benchmark.
(3) Proper assessment of the reward received for the risk taken.
(4) Attribution of the sources of return.
(5) Presentation and communicating the results.

First, we must establish what has been the return on assets and to make some assessment of that return compared with a benchmark or the available competition.

In Chapter 2 the “what” of performance measurement is introduced describing the many forms of return calculation, including the relative merits of each method together with calculation examples.

Performance returns in isolation add little value; we must compare these returns
against a suitable benchmark. Chapter 3 discusses the merits of good and bad benchmarks and examines the detailed calculation of commercial and customized indexes.

Clients should be aware of the increased risk taken in order to achieve higher rates of return; Chapter 4 discusses the multiple risk measures available to enhance understanding about the quality of return and to facilitate the assessment of the reward achieved for risk taken.

Chapter 5 examines the sources of excess return with the help of a number of performance attribution techniques.

Finally, in Chapter 6 we turn to the presentation of performance and consider the global development of performance presentation standards.

**REFERENCE**

Mathematics has given economics rigour, alas also mortis.

Robert Helibroner

SIMPLE RETURN

In measuring the performance of a “portfolio” or collection of investment assets we are concerned with the increase or decrease in the value of those assets over a specific time period – in other words, the change in “wealth”.

This change in wealth can be expressed either as a “wealth ratio” or a “rate of return”.

The wealth ratio describes the ratio of the end value of the portfolio relative to the start value, mathematically:

\[
\frac{V_E}{V_S}
\]

where:

- \( V_E \) = the end value of the portfolio
- \( V_S \) = the start value of the portfolio.

A wealth ratio greater than one indicates an increase in value, a ratio less than one a decrease in value.

Starting with a simple example, take a portfolio valued at £100m initially and valued at £112m at the end of the period. The wealth ratio is calculated as follows:

<table>
<thead>
<tr>
<th>Exhibit 2.1</th>
<th>Wealth ratio</th>
</tr>
</thead>
</table>
| \[
\frac{112}{100} = 1.12
\] |

The value of a portfolio of assets is not always easy to obtain, but should represent a reasonable estimate of the current economic value of the assets. Firms should ensure internal valuation policies are in place and consistently applied over time. A change in valuation policy may generate spurious performance over a specific time period.

Economic value implies that the traded market value, rather than the settlement value of the portfolio should be used. For example, if an individual security has been
bought but the trade has not been settled (i.e., paid for) then the portfolio is economically exposed to any change in price of that security. Similarly, any dividend declared and not yet paid or interest accrued on a fixed income asset is an entitlement of the portfolio and should be included in the valuation.

The rate of return, denoted \( r \), describes the gain (or loss) in value of the portfolio relative to the starting value, mathematically:

\[
r = \frac{V_E - V_S}{V_S}
\]  

(2.2)

Rewriting Equation (2.2):

\[
r = \frac{V_E}{V_S} - \frac{V_S}{V_S} = \frac{V_E}{V_S} - 1
\]

(2.3)

Using the previous example the rate of return is:

<table>
<thead>
<tr>
<th>Exhibit 2.2</th>
<th>Rate of return</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{112}{100} - 1 = 12% ]</td>
<td></td>
</tr>
</tbody>
</table>

Equation (2.3) can be conveniently rewritten as:

\[
1 + r = \frac{V_E}{V_S}
\]

(2.4)

Hence, the wealth ratio is actually the rate of return plus one.

Where there are no “external cash flows” it is easy to show that the rate of return for the entire period is the “compounded return” over multiple sub-periods.

Let \( V_t \) equal the value of the portfolio after the end of period \( t \) then:

\[
\frac{V_1}{V_S} \times \frac{V_2}{V_1} \times \frac{V_3}{V_2} \times \cdots \times \frac{V_{n-1}}{V_{n-2}} \times \frac{V_E}{V_{n-1}} = \frac{V_E}{V_S} = 1 + r
\]

(2.5)

External cash flow is defined as any new money added to or taken from the portfolio, whether in the form of cash or other assets. Dividend and coupon payments, purchases and sales, and corporate transactions funded from within the portfolio are not considered external cash flows.

Substituting Equation (2.4) into Equation (2.5) we establish Equation (2.6):

\[
(1 + r_1) \times (1 + r_2) \times (1 + r_3) \times \cdots \times (1 + r_{n-1}) \times (1 + r_n) = (1 + r)
\]

(2.6)

This process (demonstrated in Exhibit 2.3) of compounding a series of sub-period returns to calculate the entire period return is called “geometric” or “chain” linking.
Exhibit 2.3  Chain linking

<table>
<thead>
<tr>
<th></th>
<th>Market value (£m)</th>
<th>Return (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start value</td>
<td>$V_S$</td>
<td>100</td>
</tr>
<tr>
<td>End of period 1</td>
<td>$V_1$</td>
<td>112</td>
</tr>
<tr>
<td>End of period 2</td>
<td>$V_2$</td>
<td>95</td>
</tr>
<tr>
<td>End of period 3</td>
<td>$V_3$</td>
<td>99</td>
</tr>
<tr>
<td>End of period 4</td>
<td>$V_4$</td>
<td>107</td>
</tr>
<tr>
<td>End value</td>
<td>$V_E$</td>
<td>115</td>
</tr>
</tbody>
</table>

\[
\frac{112}{100} \times \frac{95}{112} \times \frac{99}{95} \times \frac{107}{99} \times \frac{115}{107} = \frac{115}{100} = 1.15 \quad \text{or} \quad 15.0\% \]

\[
1.12 \times 0.8482 \times 1.0421 \times 1.0808 \times 1.0748 = 1.15 \quad \text{or} \quad 15.0\%
\]

**MONEY-WEIGHTED RETURNS**

Unfortunately, in the event of external cash flows we cannot continue to use the ratio of market values to calculate wealth ratios and hence rates of return. The cash flow itself will make a contribution to the valuation. Therefore, we must develop alternative methodologies that adjust for external cash flow.

**Internal rate of return (IRR)**

To make allowance for external cash flow we can borrow a methodology from economics and accountancy, the “internal rate of return” or IRR.

The internal rate of return has been used for many decades to assess the value of capital investment or other business ventures over the future lifetime of a project. Normally, the initial outlay, estimated costs and expected returns are well known and the internal rate of return of the project can be calculated to determine if the investment is worth undertaking. The IRR is often used to calculate the future rate of return on a bond and called the yield to redemption.

**Simple internal rate of return**

In the context of the measurement of investment assets for a single period the IRR method in its most simple form requires that a return $r$ be found that satisfies the following equation:

\[
V_E = V_S \times (1 + r) + C \times (1 + r)^{0.5} \quad (2.7)
\]

where:  \( C = \) external cash flow.
In this form we are making an assumption that all cash flows are received at the mid-point of the period under analysis. To calculate the simple IRR we need only the start and end market values, and the total external cash flow as shown in Exhibit 2.4:

Exhibit 2.4  Simple IRR

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Market start value</td>
<td>$74.2m</td>
</tr>
<tr>
<td>Market end value</td>
<td>$104.4m</td>
</tr>
<tr>
<td>External cash flow</td>
<td>$37.1m</td>
</tr>
</tbody>
</table>

\[
104.4 = 74.2 \times (1 + r) + 37.1 \times (1 + r)^{0.5}
\]

We can see \( r = -7.41\% \) satisfies the above equation:

\[
74.2 \times (0.9259) + 37.1 \times (0.9259)^{0.5} = 104.4
\]

Modified internal rate of return

Making the assumption that all cash flows are received midway through the period of analysis is a fairly crude estimate. The midpoint assumption can be modified for all cash flows to adjust for the fraction of the period of measurement that the cash flow is available for investment as follows:

\[
V_E = V_S \times (1 + r) + \sum_{t=1}^{t=T} C_t \times (1 + r)^{W_t}
\]  

(2.8)

where: \( C_t \) = the external cash flow on day \( t \)

\( W_t \) = weighting ratio to be applied on day \( t \).

Obviously, there will be no external cash flow for most days:

\[
W_t = \frac{TD - D_t}{TD}
\]  

(2.9)

where: \( TD \) = total number of days within the period of measurement

\( D_t \) = number of days since the beginning of the period including weekends and public holidays.

In addition to the information in Exhibit 2.4 to calculate the modified internal rate of return shown in Exhibit 2.5 we need to know the date of the cash flow and the length of the period of analysis:

Exhibit 2.5  Modified IRR

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Market start value</td>
<td>31 December</td>
</tr>
<tr>
<td>Market end value</td>
<td>31 January</td>
</tr>
<tr>
<td>External cash flow</td>
<td>14 January</td>
</tr>
</tbody>
</table>
Assuming the cash flow at the end of day 14 is:

\[ 104.4 = 74.2 \times (1 + r) + 37.1 \times (1 + r)^{17/31} \]

We can see \( r = -7.27\% \) satisfies the above equation:

\[ 74.2 \times (0.9273) + 37.1 \times (0.9273)^{17/31} = 104.4 \]

The standard internal rate of return method in Equation (2.8) is often described by performance measurers as the modified internal rate of return method to differentiate it from the simple internal rate of return method described in Equation (2.7) which assumes midpoint cash flows. Students of finance would find the addition of the word “modified” puzzling and unnecessary.

This method assumes a single, constant force of return throughout the period of measurement, an assumption we know not to be true since the returns of investment assets are rarely constant. This assumption also means we cannot disaggregate the IRR into different asset categories since we cannot continue to use the single constant rate.

For project appraisal or calculating the redemption yield of a bond this assumption is not a problem since we are calculating a future return for which we must make some assumptions.

IRR is an example of a money-weighted return methodology: each amount or dollar invested is assumed to achieve the same effective rate of return irrespective of when it was invested. In the US the term “dollar-weighted” rather than “money-weighted” is used.

The weight of money invested at any point of time will ultimately impact the final return calculation. Therefore, if using this methodology it is important to perform well when the amount of money invested is largest.

To calculate the “annual” internal rate of return rather than the “cumulative” rate of return for the entire period we need to solve for \( r \), using the following formula:

\[ V_E = V_S \times (1 + r)^Y + \sum_{t=1}^{T} C_t \times (1 + r)^{W_t^Y} \]

where: \( Y = \) length of time period to be measured in years

\( W_t^Y = \) factor to be applied to external cash flow on day \( t \).

This factor is the time available for investment after the cash flow given by:

\[ W_t^Y = Y - Y_t \]

where: \( Y_t = \) number of years since the beginning of the period of measurement.

For example, assume cash flow occurs on the 236th day of the 3rd year for a total measurement period of 5 years. Then:

\[ W_t^Y = 5 - 2 \times \frac{236}{365} = 2 \times \frac{129}{365} \]

Simple Dietz

Even in its simple form the internal rate of return is not a particularly practical calculation, especially over longer periods with multiple cash flows. Peter Dietz