Microstrip Filters for RF/Microwave Applications

Second Edition

JIA-SHENG HONG

WILEY

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Microstrip Filters for RF/Microwave Applications

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Preface to the Second Edition

The first edition of "Microstrip Filters for RF/Microwave Applications" was published in 2001. Over the years, this book has been well received and is used extensively in both academy and industry by microwave researchers and engineers. From its inception to publication, the book is almost 10 years old. While the fundamentals of filter circuit have not changed, further innovations in filter realizations and other applications have occurred, including changes in technology and use of new fabrication processes. There have been recent advances in RF MEMS and ferroelectric films for tunable filters; the use of liquid crystal polymer (LCP) substrates for multilayer circuits, as well as the new filters for multiband and ultra-wideband (UWB) applications.

Although the microstrip remains as a main transmission line medium for these new developments, there has been a new trend of the combined use of other planar transmission line structures, such as co-planar waveguide (CPW), slotline and defected or slotted ground structures, for novel physical implementations beyond single layer in order to achieve filter miniaturization and better performance. Over the years, practitioners have also suggested topics that should be added for completeness, or deleted in some cases, as they were not very useful in practice.

It is in response to these concerns that the 2nd edition of *Microstrip Filters for RF/Microwave Applications* has been written. The extensively revised book will offer a thoroughly up-to-date professional reference focusing on microstrip and planar filters, which find wide applications in today's wireless, microwave, communications, and radar systems. It offers a unique comprehensive treatment of filters based on the microstrip and planar structures and includes full design methodologies that are applicable to waveguide and other transmission-line filters. The new edition includes a wealth of new materials including

- CPW and slotlines
- General coupling matrix, including source and load
- Multiband filters
- Nondegenerate dual-mode filters

- Filters with defected ground structures
- Substrate-integrated waveguide filters
- Liquid crystal polymer (LCP) and LTCC multilayer filters
- HTS filters for mobile/satellite communications and radio astronomy
- Ultra-wideband (UWB) filters
- Tunable and reconfigurable filters

Microstrip Filters for RF/Microwave Applications utilize numerous design examples to demonstrate and emphasize computer-aided design with commercially available software. This intensively revised book, with cutting-edge information, remains not only a valuable design resource for partitions, but also a handy reference for students and researchers in RF and microwave engineering.

I wish to acknowledge the financial supports of the UK EPSRC, Scottish Enterprise, BAE Systems (UK), and SELEX Galileo (UK). I would like to thanks all of my former and current research associates, PhD students, and visiting scholars, including Eamon McErlean, Dr. Young-Hoon Chun, Dr. Zhang-Cheng Hao, Dr. Neil Thomson, Dr. Hussein Shaman, Dr. Sultan Alotaibi, Shuzhou Li, Wenxing Tang, and Alexander Miller, for their works, some of which are presented in the book. In addition, I would like to express my gratitude to several national and international collaborators, including Prof. Michael Lancaster and Dr. Tim Jackson (both at University of Birmingham, UK), Dr. Paul Kirby (University of Cranfield, UK), Dr. Zheng Cui (Rutherford Appleton Laboratory, UK), Prof. Yusheng He (CAS, China), Alan Burdis and Colin Bird (both at SELEX Galileo, UK), and Dr. Keren Li (NiCT, Japan). The support provided by Dr. James Rautio and other members of staff at Sonnet Software Inc., USA, is acknowledged. I also wish to thank the colleagues who I have worked with at Heriot-Watt University, including Prof. Marc Desmulliez, Prof. Alan Sangster, Dr. George Goussetis, Prof. Duncan Hand, Dr. Changhai Wang, and Dr. Paul Record.

Needless to say, I am indebted to many researchers for their published work, which have been rich sources of reference. My sincere gratitude extends to the Editor of Wiley series in microwave and optical engineering, Prof. Kai Chang; and the Executive Editor of Wiley-Interscience, George Telecki, for their encouragement in writing this new edition book. I am also indebted to my wife, Kai, and my son, Haide, without their support, writing this book would not have been possible.

JIA-SHENG HONG

2010

Preface to the First Edition

Filters play important roles in many RF/microwave applications. Emerging applications such as wireless communications continue to challenge RF/microwave filters with ever more stringent requirements — higher performance, smaller size, lighter weight, and lower cost. The recent advance of novel materials and fabrication technologies, including high-temperature superconductors (HTS), low-temperature cofired ceramics (LTCC), monolithic microwave-integrated circuit (MMIC), microelectromechanic system (MEMS), and micro-machining technology, have stimulated the rapid development of new microstrip and other filters for RF/microwave applications. In the meantime, advances in computer-aided design (CAD) tools, such as full-wave electromagnetic (EM) simulators, have revolutionized the filter design. Many novel microstrip filters with advanced filtering characteristics have been demonstrated. However, up until now there has not been a single book dedicated to this subject.

Microstrip Filters for RF/Microwave Applications offers a unique and comprehensive treatment of RF/microwave filters based on the microstrip structure, providing a link to applications of computer-aided design tools and advanced materials and technologies. Many novel and sophisticated filters using computer-aided design are discussed, from basic concepts to practical realizations. The book is self-contained — it is not only a valuable design resource, but also a handy reference for students, researchers, and engineers in microwave engineering. It can also be used for RF/microwave education.

The outstanding features of this book include discussion of many novel microstrip filter configurations with advanced filtering characteristics, new design techniques, and methods for filter miniaturization. The book emphasizes computer analysis and synthesis and full-wave electromagnetic (EM) simulation through a large number of design examples. Applications of commercially available software are demonstrated. Commercial applications are included as are design theories and methodologies, which are not only for microstrip filters, but also directly applicable to other types of filters, such as waveguide and other transmission-line filters. Therefore, this book is more than just a text on microstrip filters.

Much of work described herein has resulted from the authors' research. The authors wish to acknowledge the financial supports of the UK EPSRC and the European Commission through the Advanced Communications Technologies and Services (ACTS) program. They would also like to acknowledge their national and international collaborators, including Professor Heinz Chaloupka at Wuppertal University (Germany), Robert Greed at Marconi Research Center (U.K.), Dr. Jean-Claude Mage at Thomson-CSF/CNRS (France), and Dieter Jedamzik formerly with GEC-Marconi Materials Technology (U.K.).

The authors are indebted to many researchers for their published work, which were rich sources of reference. Their sincere gratitude extends to the Editor of Wiley series in microwave and optical engineering, Professor Kai Chang; the Executive Editor of Wiley-Interscience, George Telecki; and the reviewers for their support in writing the book. The help provided by Cassie Craig and other members of the staff at Wiley is most appreciated. The authors also wish to thank their colleagues at the University of Birmingham, including Professor Peter Hall, Dr. Fred Huang, Dr. Adrian Porch, and Dr. Peter Gardener.

In addition, Jia-Sheng Hong would like to thank Professor John Allen at the University of Oxford (U.K.), Professor Werner Wiesbeck at Kalsruhe Universiy (Germany), and Dr. Nicholas Edwards at the British Telecom (U.K.) for their many years' support and friendship. Professor Joseph Helszajn at Heriot-Watt University (U.K.), who sent his own book on filters to Jia-Sheng Hong, is also acknowledged.

Finally, Jia-Sheng Hong would like to express his deep appreciation to his wife, Kai, and his son, Haide, for their tolerance and support, which allowed him to write the book at home over many evenings, weekends, and holidays. In particular, without the help of Kai, completing this book on time would not have been possible.

JIA-SHENG HONG
M. J. LANCASTER

2001

CHAPTER ONE

Introduction

The term *microwaves* may be used to describe electromagnetic (EM) waves with frequencies ranging from 300 MHz to 300 GHz, which correspond to wavelengths (in free space) from 1 m to 1 mm. The EM waves with frequencies above 30 and up to 300 GHz are also called *millimeter waves,* because their wavelengths are in the millimeter range (1−10 mm). Above the millimeter wave spectrum is the infrared, which comprises electromagnetic waves with wavelengths between 1 μm (10^{-6} m) and 1 mm. Beyond the infrared spectrum is the visible optical spectrum, the ultraviolet spectrum, and x rays. Below the microwave frequency spectrum is the radio-frequency (RF) spectrum. The frequency boundary between RF and microwaves is somewhat arbitrary, depending on the particular technologies developed for the exploitation of that specific frequency range. Therefore, by extension, the RF/microwave applications can be referred to as communications, radar, navigation, radio astronomy, sensing, medical instrumentation, and others that explore the usage of frequency spectrums in the range, for example, 300 kHz up to 300 GHz (Fig. 1.1). For convenience, some of these frequency spectrums are further divided into many frequency bands, as indicated in Fig. 1.1.

Filters play important roles in many RF/microwave applications. They are used to separate or combine different frequencies. The electromagnetic spectrum is limited and has to be shared; filters are used to select or confine the RF/microwave signals within assigned spectral limits. Emerging applications, such as wireless communications, continue to challenge RF/microwave filters with ever more stringent requirements — higher performance, more functionalities such as tunable or reconfigurable, smaller size, lighter weight, and lower cost. Depending on the requirements and specifications, RF/microwave filters may be designed as lumped element or distributed element circuits, they may be realized in various transmission-line structures, such as waveguide, coaxial line, coplanar waveguide (CPW), slotline, and microstrip.

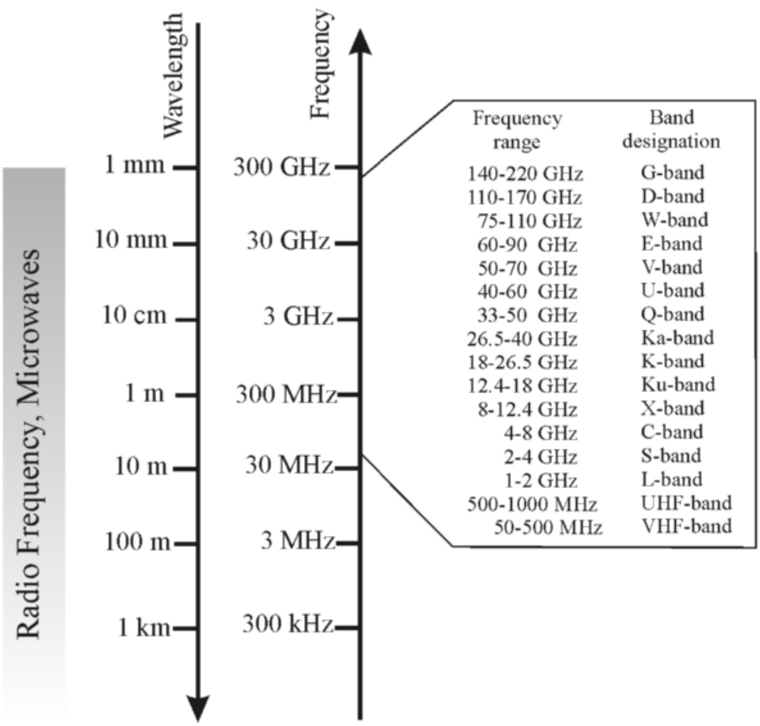

FIGURE 1.1 RF/microwave spectrums.

The recent advance of novel materials and fabrication technologies, including monolithic microwave integrated circuit (MMIC), microelectromechanic system (MEMS) or micromachining, ferroelectrics, high-temperature superconductor (HTS), low-temperature co-fired ceramics (LTCC), and liquid crystal polymers (LCP), has stimulated the rapid development of new microstrip and other filters. In the meantime, advances in computer-aided design (CAD) tools, such as full-wave electromagnetic (EM) simulators, have revolutionized the filter design. Many novel microstrip filters with advanced filtering characteristics have been demonstrated.

The main objective of this book is to offer a unique and comprehensive treatment of RF/microwave filters, based on the microstrip structure, providing a link to applications of CAD tools, advanced materials, and technologies (see Fig. 1.2). However, it is not the intention of this book to include everything that has been published on microstrip filters; such a work would be out of scale in terms of space and knowledge involved. Moreover, design theories and methods described in the book are not only for microstrip filters, but directly applicable to other types of filters, such as waveguide filters.

Although the physical realization of filters at RF/microwave frequencies may vary, the circuit network topology is common to all. Therefore, the technique content of the book begins with Chapter 2, which describes various network concepts

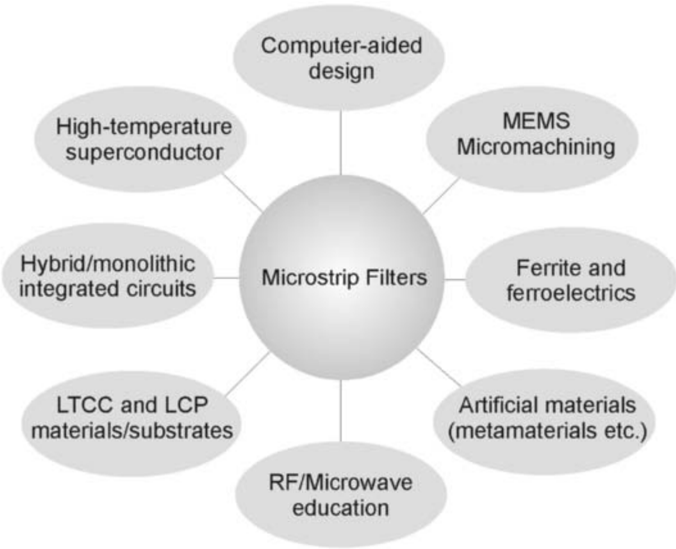

FIGURE 1.2 Microstrip filter linkage.

and equations; these are useful for the analysis of filter networks. Chapter 3 then introduces basic concepts and theories for designing general RF/microwave filters (including microstrip filters). The topics cover filter transfer functions (such as Butterworth, Chebyshev, elliptic function, all-pass, and Gaussian response), lowpass prototype filters and elements, frequency and element transformations, immittance (impedance/admittance) inverters, Richards' transformation, and Kuroda identities for distributed elements. Effects of dissipation and unloaded quality factor of filter elements on the filter performance are also discussed.

Chapter 4 summarizes basic concepts and design equations for microstrip lines, coupled microstrip lines, discontinuities, lumped and distributed components, as well as coplanar waveguide (CPW), and slotlines, which are useful for design of filters. In Chapter 5, conventional microstrip lowpass and bandpass filters, such as stepped-impedance filters, open-stub filters, semilumped element filters, end- and parallel-coupled half-wavelength resonator filters, hairpin-line filters, interdigital and combline filters, pseudocombline filters and stubline filters, are discussed with instructive design examples.

Chapter 6 discusses some typical microstrip highpass and bandstop filters. These include quasilumped element and optimum distributed highpass filters, narrow-band and wide-band bandstop filters, as well as filters for RF chokes. Design equations, tables, and examples are presented for easy reference.

The remaining chapters of the book deal with more advanced topics. Chapter 7, presents a comprehensive treatment of subjects regarding coupled resonator circuits. These are of importance for design of RF/microwave filters, in particular, the narrow-band bandpass filters that play a significant role in many applications. There is a general technique for designing coupled resonator filters, which can be applied to any type of resonator despite its physical structure. For examples, it can be applied for the design of waveguide filters, dielectric resonator filters, ceramic combline filters, microstrip filters, superconducting filters, and micromachined filters. This design method is based on coupling coefficients of intercoupled resonators and the external quality factors of the input and output resonators. Since this design technique is so useful and flexible, it would be desirable to have a deep understanding of not only its approach, but also its theory. For this purpose, the subjects cover (1) the formulation of general coupling matrix, which is of importance for representing a wide range of coupled-resonator filter topologies, and (2) the general theory of couplings for establishing the relationship between the coupling coefficient, and the physical structure of coupled resonators. This leads to a very useful formulation for extracting coupling coefficients from EM simulations or measurements. Formulations for extracting the external quality factors from frequency responses of the externally loaded input/output resonators are derived next. Numerical examples are followed to demonstrate how to use these formulations to extract coupling coefficients and external quality factors of microwave coupling structures for filter designs. In addition, a more advanced topic on general coupling matrix involving source and load is addressed.

Chapter 8 is concerned with computer-aided design (CAD). Generally speaking, any design involves using computers may be called CAD. There have been extraordinary recent advances in CAD of RF/microwave circuits, particularly in full-wave electromagnetic (EM) simulations. They have been implemented both in commercial and specific in-house software and are being applied to microwave filters simulation, modeling, design, and validation. The developments in this area are certainly stimulated by increasing computer power. Another driving force for the developments is the requirement of CAD for low-cost and high-volume production. In general, the investment for tooling, materials, and labor mainly affect the cost of filter production. Labor costs include those for design, fabrication, testing, and tuning. Here the costs for the design and tuning can be reduced greatly by using CAD, which can provide more accurate design with less design iterations, leading to first-pass or tuneless filters. This chapter discusses computer simulation and/or computer optimization. It summarizes some basic concepts and methods regarding filter design by CAD. Typical examples of the applications, including filter synthesis by optimization, are described. Many more CAD examples, particularly those based on full-wave EM simulation, can be found throughout this book.

In Chapter 9, we discuss the designs of some advanced filters, including selective filters with a single pair of transmission zeros, cascaded quadruplet (CQ) filters, trisection and cascaded trisection (CT) filters, cross-coupled filters using transmission-line inserted inverters, linear phase filters for group-delay equalization, extracted-pole filters, canonical filters, and multiband filters. These types of filters, which are different from conventional Chebyshev filters, must meet stringent requirements from RF/microwave systems, particularly from wireless communications systems.

Chapter 10 is intended to describe novel concepts, methodologies, and designs for compact filters and filter miniaturization. The new types of filters discussed include compact open-loop and hairpin resonator filters, slow-wave resonator filters, miniaturized dual-mode filters using degenerate or nondegenerate modes, lumped-element filters, filters using high dielectric constant substrates, and multilayer filters. The last topic covers aperture-coupled resonator filters, filters with defected or slotted-ground structures, substrate integrated waveguide filters, as well as low-temperature cofired ceramics (LTCC) and liquid crystal polymer (LCP) filters.

Chapter 11 introduces high-temperature superconductors (HTS) for RF/microwave filter applications. It covers some important properties of superconductors and substrates for growing HTS films, which are essential for the design of HTS microstrip filters. Typical superconducting filters with super performance for mobile and satellite communications, as well as radio astronomy and radar applications, are described in this chapter.

Chapter 12 focuses on ultra-wideband (UWB) filters, which are a key component for many promising modern applications of UWB technology. In this chapter, typical types of UWB filters are described. This includes UWB filters comprised of short-circuit stubs, UWB filters using coupled single-mode or multimode resonators, quasilumped element UWB filters, UWB filters based on cascaded highpass and lowpass filters, and UWB filters with single- or multiple-notched bands.

The final chapter of the book (Chapter 13) is concerned with electronically tunable and reconfigurable filters. In general, to develop an electronically reconfigurable filter, active switching or tuning elements, such as semiconductor p-i-n and varactor diodes, RF MEMS or other functional material-based components, including ferroelectric varactors and piezoelectric transducers need to be integrated within a passive filtering structure. Typical filters of these types are described in this chapter, which include tunable combline filters, tunable open-loop filters without using any via-hole connections, reconfigurable dual-mode filters, wideband filters with reconfigurable bandwidth, reconfigurable UWB filters, RF MEMS reconfigurable filters, piezoelectric transducer tunable filters, and ferroelectric tunable filters.

CHAPTER TWO

Network Analysis

Filter networks are essential building elements in many areas of RF/microwave engineering. Such networks are used to select/reject or separate/combine signals at different frequencies in a host of RF/microwave systems and equipments. Although the physical realization of filters at RF/microwave frequencies may vary, the circuit network topology is common to all.

At microwave frequencies, the use of voltmeters and ammeters for the direct measurement of voltages and currents do not exist. For this reason, voltage and current, as a measure of the level of electrical excitation of a network, do not play a primary role at microwave frequencies. On the other hand, it is useful to be able to describe the operation of a microwave network, such as a filter, in terms of voltages, currents, and impedances in order to make optimum use of low-frequency network concepts.

It is the purpose of this chapter to describe various network concepts and provide equations [1–10] that are useful for the analysis of filter networks.

2.1 NETWORK VARIABLES

Most RF/microwave filters and filter components can be represented by a two-port network, as shown in Figure 2.1, where V_1, V_2 and I_1, I_2 are the voltage and current variables at ports 1 and 2, respectively, Z_{01} and Z_{02} are the terminal impedances, and E_s is the source or generator voltage. Note that the voltage and current variables are complex amplitudes when we consider sinusoidal quantities. For example, a sinusoidal voltage at port 1 is given by

$$v_1(t) = |V_1|\cos(\omega t + \phi) \tag{2.1}$$

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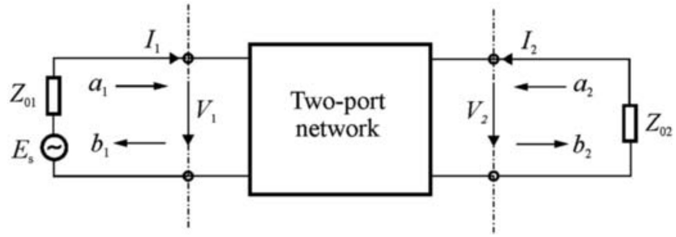


FIGURE 2.1 Two-port network showing network variables.

We can then make the following transformations

$$v_1(t) = |V_1| \cos(\omega t + \phi) = \mathrm{Re}\left(|V_1|\, e^{j(\omega t+\phi)}\right) = \mathrm{Re}\left(V_1 e^{j\omega t}\right) \tag{2.2}$$

where Re denotes "the real part of" the expression that follows it. Therefore, one can identify the complex amplitude V_1 defined by

$$V_1 = |V_1|\, e^{j\phi} \tag{2.3}$$

Because it is difficult to measure the voltage and current at microwave frequencies, the wave variables a_1, b_1 and a_2, b_2 are introduced, with a indicating the incident waves and b the reflected waves. The relationships between the wave variables and the voltage and current variables are defined as

$$\begin{aligned} V_n &= \sqrt{Z_{0n}}\,(a_n + b_n) \\ I_n &= \frac{1}{\sqrt{Z_{0n}}}\,(a_n - b_n) \end{aligned} \qquad n = 1 \text{ and } 2 \tag{2.4a}$$

or

$$\begin{aligned} a_n &= \frac{1}{2}\left(\frac{V_n}{\sqrt{Z_{0n}}} + \sqrt{Z_{0n}}\,I_n\right) \\ b_n &= \frac{1}{2}\left(\frac{V_n}{\sqrt{Z_{0n}}} - \sqrt{Z_{0n}}\,I_n\right) \end{aligned} \qquad n = 1 \text{ and } 2 \tag{2.4b}$$

The above definitions guarantees that the power at port n is

$$P_n = \frac{1}{2}\mathrm{Re}\left(V_n \cdot I_n^*\right) = \frac{1}{2}\left(a_n a_n^* - b_n b_n^*\right) \tag{2.5}$$

where the asterisk denotes a conjugate quantity. It can be recognized that $a_n a_n^*/2$ is the incident wave power and $b_n b_n^*/2$ is the reflected wave power at port n.

2.2 SCATTERING PARAMETERS

The scattering or S parameters of a two-port network are defined in terms of the wave variables as

$$\begin{aligned} S_{11} &= \left.\frac{b_1}{a_1}\right|_{a_2=0} & S_{12} &= \left.\frac{b_1}{a_2}\right|_{a_1=0} \\ S_{21} &= \left.\frac{b_2}{a_1}\right|_{a_2=0} & S_{22} &= \left.\frac{b_2}{a_2}\right|_{a_1=0} \end{aligned} \tag{2.6}$$

where $a_n = 0$ implies a perfect impedance match (no reflection from terminal impedance) at port n. These definitions may be written as

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{2.7}$$

where the matrix containing the S parameters is referred to as the scattering matrix or S matrix, which may simply be denoted by $[S]$.

The parameters S_{11} and S_{22} are also called the reflection coefficients, whereas S_{12} and S_{21} are the transmission coefficients. These are the parameters directly measurable at microwave frequencies. The S parameters are, in general, complex, and it is convenient to express them in terms of amplitudes and phases, that is, $S_{mn} = |S_{mn}|\, e^{j\phi_{mn}}$ for $m, n = 1, 2$. Often their amplitudes are given in decibels (dB), which are defined as

$$20 \log |S_{mn}| \quad \text{dB} \quad m, n = 1, 2 \tag{2.8}$$

where the logarithm operation is base 10. This will be assumed through this book unless otherwise stated. For filter characterization, we may define two parameters

$$\begin{aligned} L_A &= -20 \log |S_{mn}| \quad \text{dB} \quad m, n = 1, 2 (m \neq n) \\ L_R &= 20 \log |S_{nn}| \quad \text{dB} \quad n = 1, 2 \end{aligned} \tag{2.9}$$

where L_A denotes the insertion loss between ports n and m and L_R represents the return loss at port n. Instead of using the return loss, the voltage-standing wave ratio $VSWR$ may be used. The definition of $VSWR$ is

$$VSWR = \frac{1 + |S_{nn}|}{1 - |S_{nn}|} \tag{2.10}$$

Whenever a signal is transmitted through a frequency-selective network, such as a filter, some delay is introduced into the output signal in relation to the input signal. There are two other parameters that play a role in characterizing filter performance related to this delay. The first one is the phase delay, defined by

$$\tau_p = \frac{\phi_{21}}{\omega}\ \mathrm{s} \tag{2.11}$$

where ϕ_{21} is in radians and ω is in rad/s. Port 1 is the input port and port 2 is the output port. The phase delay is actually the time delay for a steady sinusoidal signal and is not necessarily the true signal delay, because a steady sinusoidal signal does not carry information; sometimes, it is also referred to as the carrier delay [1]. The more important parameter is the group delay, defined by

$$\tau_d = -\frac{d\phi_{21}}{d\omega}\ \mathrm{s} \tag{2.12}$$

This represents the true signal (baseband signal) delay and is also referred to as the envelope delay.

In network analysis or synthesis, it may be desirable to express the reflection parameter S_{11} in terms of the terminal impedance Z_{01} and the so-called input impedance $Z_{in1} = V_1/I_1$, which is the impedance looking into port 1 of the network. Such an expression can be deduced by evaluating S_{11} in Eq. (2.6) in terms of the voltage and current variables using the relationships defined in Eq. (2.4b). This gives

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{V_1/\sqrt{Z_{01}} - \sqrt{Z_{01}}I_1}{V_1/\sqrt{Z_{01}} + \sqrt{Z_{01}}I_1} \tag{2.13}$$

Replacing V_1 by $Z_{in1}I_1$ results in the desired expression

$$S_{11} = \frac{Z_{in1} - Z_{01}}{Z_{in1} + Z_{01}} \tag{2.14}$$

Similarly, we can have

$$S_{22} = \frac{Z_{in2} - Z_{02}}{Z_{in2} + Z_{02}} \tag{2.15}$$

where $Z_{in2} = V_2/I_2$ is the input impedance looking into port 2 of the network. Equations (2.14) and (2.15) indicate the impedance matching of the network with respect to its terminal impedances.

The S parameters have several properties that are useful for network analysis. For a reciprocal network we have $S_{12} = S_{21}$. If the network is symmetrical, an addition property, $S_{11} = S_{22}$, holds. Hence, the symmetrical network is also reciprocal. For a lossless passive network, the transmitting power and the reflected power must equal to the total incident power. The mathematical statements of this power conservation condition are

$$\begin{aligned} S_{21}S_{21}^* + S_{11}S_{11}^* &= 1 \quad \text{or} \quad |S_{21}|^2 + |S_{11}|^2 = 1 \\ S_{12}S_{12}^* + S_{22}S_{22}^* &= 1 \quad \text{or} \quad |S_{12}|^2 + |S_{22}|^2 = 1 \end{aligned} \tag{2.16}$$

2.3 SHORT-CIRCUIT ADMITTANCE PARAMETERS

The short-circuit admittance or Y parameters of a two-port network are defined as

$$\begin{aligned} Y_{11} &= \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \\ Y_{21} &= \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \end{aligned} \tag{2.17}$$

in which $V_n = 0$ implies a perfect short-circuit at port n. The definitions of the Y parameters may also be written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{2.18}$$

where the matrix containing the Y parameters is called the short-circuit admittance or simply Y matrix and may be denoted by $[Y]$. For reciprocal networks $Y_{12} = Y_{21}$. In addition to this, if networks are symmetrical, then $Y_{11} = Y_{22}$. For a lossless network, the Y parameters are all purely imaginary.

2.4 OPEN-CIRCUIT IMPEDANCE PARAMETERS

The open-circuit impedance or Z parameters of a two-port network are defined as

$$\begin{aligned} Z_{11} &= \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \\ Z_{21} &= \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \end{aligned} \tag{2.19}$$

where $I_n = 0$ implies a perfect open-circuit at port n. These definitions can be written as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{2.20}$$

The matrix, which contains the Z parameters, is known as the open-circuit impedance or Z matrix denoted by $[Z]$. For reciprocal networks, we have $Z_{12} = Z_{21}$. If networks are symmetrical, then $Z_{12} = Z_{21}$ and $Z_{11} = Z_{22}$. For a lossless network, the Z parameters are all purely imaginary.

Inspecting Eqs. (2.18) and (2.20), we immediately obtain an important relation

$$[Z] = [Y]^{-1} \tag{2.21}$$

2.5 *ABCD* PARAMETERS

The *ABCD* parameters of a two-port network are given by

$$\begin{aligned} A &= \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \\ C &= \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \end{aligned} \tag{2.22}$$

These parameters are actually defined in a set of linear equations in matrix notation

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{2.23}$$

where the matrix comprised of the *ABCD* parameters is called the *ABCD* matrix. Sometimes, it may also be referred to as the transfer or chain matrix. The *ABCD* parameters have following properties:

$$AD - BC = 1 \qquad \text{for a reciprocal network} \tag{2.24}$$

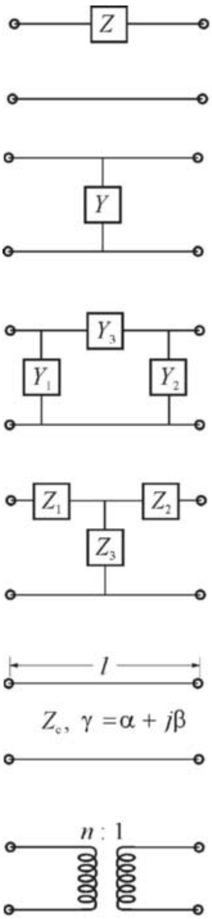
$$A = D \qquad \text{for a symmetrical network} \tag{2.25}$$

If the network is lossless, then A and D will be purely real and B and C will be purely imaginary.

If the network in Figure 2.1 is turned around, then the transfer matrix defined in Eq. (2.23) becomes

$$\begin{bmatrix} A_t & B_t \\ C_t & D_t \end{bmatrix} = \begin{bmatrix} D & B \\ C & A \end{bmatrix} \tag{2.26}$$

where the parameters with t subscripts are for the network after being turned around, and the parameters without subscripts are for the network before being turned around (with its original orientation). In both cases, V_1 and I_1 are at the left terminal and V_2 and I_2 are at the right terminal.



$$A = 1 \quad B = Z$$
$$C = 0 \quad D = 1$$

$$A = 1 \quad B = 0$$
$$C = Y \quad D = 1$$

$$A = 1 + \frac{Y_2}{Y_3} \qquad B = \frac{1}{Y_3}$$
$$C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3} \quad D = 1 + \frac{Y_1}{Y_3}$$

$$A = 1 + \frac{Z_1}{Z_3} \quad B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$
$$C = \frac{1}{Z_3} \qquad D = 1 + \frac{Z_2}{Z_3}$$

$$A = \cosh\gamma l \quad B = Z_c \sinh\gamma l$$
$$C = \frac{\sinh\gamma l}{Z_c} \quad D = \cosh\gamma l$$

$$A = n \quad B = 0$$
$$C = 0 \quad D = \frac{1}{n}$$

FIGURE 2.2 Some useful two-port networks and their *ABCD* parameters.

The *ABCD* parameters are very useful for analysis of a complex two-port network that may be divided into two or more cascaded subnetworks. Figure 2.2 gives the *ABCD* parameters of some useful two-port networks.

2.6 TRANSMISSION-LINE NETWORKS

Since $V_2 = -I_2 Z_{02}$, the input impedance of the two-port network in Figure 2.1 is given by

$$Z_{in1} = \frac{V_1}{I_1} = \frac{Z_{02}A + B}{Z_{02}C + D} \tag{2.27}$$