Praise for Fourth Edition of Cost of Capital WORKBOOK AND TECHNICAL SUPPLEMENT

“Pratt and Grabowski went the extra mile to supplement their magnum opus by providing this Workbook and Technical Supplement. As a finance professor for many years, I know from experience that students and teachers really value supplements to textbooks. It allows the teacher to help the student to review and apply what was presented in the text, and the PowerPoints are a great service to teachers in course preparation. The website provides various worksheets that show the inner workings of the models. I enthusiastically recommend the Workbook and Technical Supplement to finance professors and teachers and their students.”
—Daniel L. McConaughy, PhD, ASA, Professor of Finance, California State University, Northridge, Valuation Services, Crowe Horwath LLP

“The Workbook and Technical Supplement provides a detailed tutorial on understanding and executing the theoretical concepts explained in the Fourth Edition. This supplement is three books in one. Part One is a step-by-step tutorial on estimating certain key components of the cost of equity capital. Part Two provides a bridge between the theory and some practical applications, such as estimating the cost of capital for real property. Parts Three and Four allow the readers to test their comprehension of the concepts and identify areas for a review. It is almost as good as having Professor Pratt and Grabowski looking over your shoulder to ensure that one is both comprehending and correctly implementing the complex concepts.”
—Ashok Abbott, PhD, Associate Professor of Finance, College of Business & Economics, West Virginia University

“This text provides the most comprehensive coverage of cost of capital issues that I have seen to date. Messrs. Pratt and Grabowski have created a very accessible and lucid treatment of what most would consider an opaque subject. The Fourth Edition is especially important for its new topics as well as expanded coverage of concepts from earlier editions. Of particular interest is the review of the extreme market conditions during the 2008–2009 crisis and the effect that the unprecedented volatility had on traditional cost of capital models. For years, Pratt and Grabowski’s research has informed the business valuation curriculum of the American Society of Appraisers. This book will be added to our reading list, and thousands of students worldwide will benefit from the state-of-the-art content of the Fourth Edition and the companion Workbook and Technical Supplement. Furthermore, Cost of Capital, Fourth Edition should be a mandatory part of every valuation practitioners library. If you buy this book, you can expect it to become well worn and remain on your desk within arm’s length until the publication of the Fifth Edition.”
—John Barton, ASA, CPA, Chairman, Business Valuation Committee, ASA

“Cost of capital is so much more complex than it used to be. With so many additional considerations regarding each variable of the cost of capital formula, this book is a must for anyone that needs to understand or develop a discount rate. Even the most experienced practitioner will benefit from the outstanding work of Pratt and Grabowski. This book has to become part of your library.”
—Gary R. Trupman, CPA/ABV, MCBA, ASA, MVS, President, Trupman Valuation Associates, Inc.

SHANNON P. PRATT, CFA, ARM, ABA, FASA, CM&AA, referred to as the father of business valuations, is the author of several bestselling Wiley business valuation books and a sought-after speaker at business valuation industry conferences. He is the managing owner of Shannon Pratt Valuations Portland, Oregon, and has served as supervisory analyst for over 3,000 business valuation engagements in forty years and as an expert witness in numerous state and federal courts on contested business valuations.

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Cost of Capital Fourth Edition Workbook and Technical Supplement
Cost of Capital Fourth Edition Workbook and Technical Supplement

SHANNON P. PRATT
ROGER J. GRABOWSKI

John Wiley & Sons, Inc.
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Preface

Why did we add the *Workbook and Technical Supplement* to the *Cost of Capital: Applications and Examples, 4th ed.?* We wanted to further assist practitioners in better understanding how to estimate the cost of capital. This text adds more detailed examples to the *Cost of Capital: Applications and Examples, 4th ed.* It also contains questions and problems covering the material contained in the *Cost of Capital: Applications and Examples, 4th ed.* designed to help the reader better grasp that material.

This book uses the identical notation and abbreviations as those used in the text. Those can be referenced either in *Cost of Capital: Applications and Examples, 4th ed.* or on the companion web site (see later).

Part One contains technical supplements to several chapters. These will help the reader to be better able to implement the methods of analyses discussed in the main book.

Part Two contains an example of specific applications of applying the theory to the cost of capital for private investment companies, including one approach to incorporating the discount for lack of control and lack of marketability into the cost of capital. It also contains chapters extending the general concepts of developing cost of capital to real estate properties and real estate entities. These investments have their own set of terminologies unique to the industry, and we cover the terminology and methods of analysis commonly used in the industry in detail.

Part Three contains learning objectives, questions, and problems to help the practitioner better understand the content of the first 34 chapters of the *Cost of Capital: Applications and Examples, 4th ed.*

Part Four contains the answers to the questions and solutions to the problems presented in Part Three.

Finally, this book includes a companion web site, which can be found at www.wiley.com/go/coc4e. The web site includes the following:

1. The notation system and abbreviations used in this book.
2. The worksheets that are presented as exhibits in Chapters 5, Iterative Process Using CAPM to Calculate the Cost of Equity Component of the Weighted Average Cost of Capital When Capital Structure Is Constant; in Chapter 6, Iterative Process Using CAPM to Calculate the Cost of Equity Component of the Weighted Average Cost of Capital When Capital Structure Is Changing; and in Chapter 8, Cost of Capital of Private Investment Company Interests. These exhibits are provided for your reference so that you can track the methodologies discussed in the book and see how they are implemented through the Microsoft Excel worksheets. This will assist you in building models of your own using the worksheets as templates. Note that each file, however, typically contains additional information in different worksheets within that file.
3. Three appendices:
   - Appendix I: Sample Report Submitted to U.S. Tax Court (Supplement to Chapters 7 and 17) is an example of a report submitted to the U.S. Tax Court to help readers communicate the cost of capital methods in a straightforward way to the nontechnical reader.
   - Appendix II discusses the ValuSource Valuation Software, which is a helpful tool for the practitioner.
   - Appendix III contains a comprehensive review of the statistics discussed in the Cost of Capital: Applications and Examples, 4th ed. and used in developing the cost of capital. We included this appendix so practitioners who may need a refresher in basic statistics do not need to try to locate their statistics books from college. It covers many topics including probability theory (important for understanding and measuring risk), the statistics (e.g., mean, mode, standard deviation, beta, etc.) that are used to summarize return and risk data, and basic concepts of risk neutral payoffs and probabilities. It also includes formulas, terminology, and the statistical tools of the Microsoft Excel Analysis Toolpak.

4. PowerPoints that accompany the chapters of Cost of Capital: Applications and Examples, 4th ed. to assist those that want to use the book in seminars.
About the Authors

Dr. Shannon P. Pratt, CFA, FASA, ARM, MCBA, ABAR, CM&AA, is the chairman and CEO of Shannon Pratt Valuations, Inc., a nationally recognized business valuation firm headquartered in Portland, Oregon. He is also the founder and editor emeritus of Business Valuation Resources, LLC, and one of the founders of Willamette Management Associates, for which he was a managing director for almost 35 years.

He has performed valuation assignments for these purposes: transaction (acquisition, divestiture, reorganization, public offerings, public companies going private), taxation (federal income, gift, and estate and local ad valorem), financing (securitization, recapitalization, restructuring), litigation support and dispute resolution (including dissenting stockholder suits, damage cases, and corporate and marital dissolution cases), and management information and planning. He has also managed a variety of fairness opinion and solvency opinion engagements. He regularly reviews business valuation reports for attorneys in litigation matters.

Dr. Pratt has testified on hundreds of occasions in such litigated matters as dissenting stockholder suits, various types of damage cases (including breach of contract, antitrust, and breach of fiduciary duty), divorces, and estate and gift tax cases. Among the cases in which he has testified are Estate of Mark S. Gallo v. Commissioner, Charles S. Foltz, et al. v. U.S. News & World Report et al., Estate of Martha Watts v. Commissioner, and Okerlund v. United States. He has also served as appointed arbitrator in numerous cases.

Previous Experience

Before founding Willamette Management Associates in 1969, Dr. Pratt was a professor of business administration at Portland State University. During this time, he directed a research center known as the Investment Analysis Center, which worked closely with the University of Chicago’s Center for Research in Security Prices.

Education

Doctor of Business Administration, Finance, Indiana University.
Bachelor of Arts, Business Administration, University of Washington.

Professional Affiliations

Dr. Pratt is an Accredited Senior Appraiser and Fellow (FASA), Certified in Business Valuation, of the American Society of Appraisers (their highest designation) and is also accredited in Appraisal Review and Management (ARM). He is a Chartered Financial Analyst (CFA), a Master Certified Business Appraiser.
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Dr. Pratt is a life member of the American Society of Appraisers, a life member of the Business Valuation Committee of that organization, and a teacher of courses for the organization. He is also a lifetime member emeritus of the Advisory Committee on Valuations of the ESOP Association. He is a recipient of the magna cum laude award of the National Association of Certified Valuation Analysts for service to the business valuation profession. He is also the first life member of the Institute of Business Appraisers. He is a member and a past president of the Portland Society of Financial Analysts, the recipient of the 2002 Distinguished Achievement Award, and a member of the Association for Corporate Growth. Dr. Pratt is a past trustee of the Appraisal Foundation and is currently an outside director and chair of the audit committee of Paulson Capital Corp., a NASDAQ-listed investment banking firm specializing in small initial public offerings (usually under $50 million).

Publications


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Mr. Grabowski has directed valuations of businesses, partial interests in businesses, intellectual property, intangible assets, real property, and machinery and equipment for various purposes, including tax (income and ad valorem) and financial reporting; mergers, acquisitions, formation of joint ventures, divestitures, and financing. He developed methodologies and statistical programs for analyzing useful lives of tangible and intangible assets, such as customers and subscribers. His experience includes work in a wide range of industries, including sports, movies, recording, broadcast and other entertainment businesses; newspapers, magazines, music, and other publishing businesses; retail; banking, insurance, consumer credit, and other financial services businesses; railroads and
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Mr. Grabowski has testified in court as an expert witness on the value of closely held businesses and business interests; matters of solvency, valuation, and amortization of intangible assets; and other valuation issues. His testimony in U.S. District Court was referenced in the U.S. Supreme Court opinion decided in his client’s favor in the landmark Newark Morning Ledger income tax case. Among other cases in which he has testified are Herbert V. Kohler Jr., et al. v. Comm. (value of stock of The Kohler Company); The Northern Trust Company, et al. v. Comm. (the first U.S. Tax Court case that recognized the use of the discounted cash flow method for valuing a closely held business); Oakland Raiders v. Oakland–Alameda County Coliseum Inc. et al. (valuation of the Oakland Raiders); In re: Louisiana Riverboat Gaming Partnership, et al. Debtors (valuation of business enterprise owning two riverboat casinos and feasibility of plan of reorganization); ABC-NACO, Inc. et al., Debtors, and The Official Committee of Unsecured Creditors of ABC-NACO v. Bank of America, N.A. (valuation of collateral); Wisniewski and Walsh v. Walsh (oppressed shareholder action); and TMR Energy Limited v. The State Property Fund of Ukraine (arbitration on behalf of world’s largest private company in Stockholm, Sweden, on cost of capital for oil refinery in Ukraine in a contract dispute).

Previous Experience

Mr. Grabowski was formerly managing director of the Standard & Poor’s Corporate Value Consulting practice and a partner of PricewaterhouseCoopers, LLP, and one of its predecessor firms, Price Waterhouse (where he founded its U.S. Valuation Services practice and managed the real estate appraisal practice). Prior to Price Waterhouse, he was a finance instructor at Loyola University of Chicago, a cofounder of Valtec Associates, and a vice president of American Valuation Consultants.

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Publications

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Dr. Grissom was formerly Professor of Real Estate and Urban Land Economics at Georgia State University, Atlanta, in the Robinson College of Business. Prior to his tenure at GSU, he was Vice-President of Investment Research for Equitable Real Estate Investment Management, an institutional investment advisory for pension funds, insurance companies, and other financial institutions. From 1992 through October 1994, he was the National Research Director for Price Waterhouse’s Financial Services Industry Practice.

Dr. Grissom has published more than 100 academic and professional articles, monographs, and working papers in his career to this point. He has also authored, co-authored, and edited four books concerning real estate appraisal and investment analysis, market analysis, and real estate development and land economics. He has also authored chapters in books on real estate development, investment analysis, business and property valuation techniques, and education theory and practice for both academics and practitioners and for both domestic and international audiences.


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About the Authors

Mr. Ptashne contributed Chapters 2 and 4 of the Cost of Capital: Applications and Examples, 4th ed. Workbook and Technical Supplement.


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A charter member of the LA Society of CPA’s Litigation Services Committee, Mr. Shirley has remained active since the committee’s formation. He has been an adjunct faculty member at the National Judicial College, University of Nevada, Reno, since 1998. Mr. Shirley also serves on the Advisory Panel for Mdex Online; The Daubert Tracker, an online Daubert research database; and the Ethics Oversight Board for the NACVA.

Since 1985, Mr. Shirley has provided expert witness testimony before the U.S. Tax Court, Federal District Court, Louisiana district courts, Tunica-Biloxi Indian Tribal Court, and local specialty courts. Court appointments have been received in various matters adjudicated before the Louisiana Nineteenth Judicial District Court.

The NACVA has recognized Mr. Shirley’s contributions to professional education by awarding him the Circle of Light in 2002, Instructor of the Year in 2000–2001, and multiple recognitions as Outstanding Member and Award of Excellence.

Mr. Shirley contributed Chapter 3 of the Cost of Capital: Applications and Examples, 4th ed. Workbook and Technical Supplement and Appendix III of the Workbook and Technical Supplement which appears on the John Wiley & Sons web site.
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We thank all of the people singled out for their assistance. Of course, any errors are our responsibility.¹

Shannon Pratt
Roger Grabowski

¹ Any opinions presented in this book are those of the authors. The opinions of Mr. Grabowski do not represent the official position of Duff & Phelps, LLC. This material is offered for educational purposes with the understanding that neither the authors nor Duff & Phelps, LLC, are engaged in rendering legal, accounting, or any other professional service through presentation of this material. The information presented in this book has been obtained with the greatest of care from sources believed to be reliable, but is not guaranteed to be complete, accurate, or timely. The authors and Duff & Phelps LLC expressly disclaim any liability, including incidental or consequential damages, arising from the use of this material or any errors or omissions that may be contained in it.
Notation System and Abbreviations Used in This Book

A source of confusion for those trying to understand financial theory and methods is that financial writers have not adopted a standard system of notation. The notation system used in this volume is adapted from the fifth edition of *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, by Shannon P. Pratt (New York: McGraw-Hill, 2008).

**VALUE AT A POINT IN TIME**

\[
\begin{align*}
P_n & = \text{Stock price in period } n \\
P_0 & = \text{Stock price at valuation period} \\
P_i & = \text{Price per share for company } i \text{ (seen elsewhere as PV)} \\
PV & = \text{Present value} \\
PV_{b} & = \text{Present value of net cash flows due to business operations before cost of financing} \\
PV_{keu} & = \text{Present value of net cash flows using unlevered cost of equity capital, } k_{eu}, \text{ as the discount rate} \\
PV_{ts} & = \text{Present value of tax shield due to interest expense on debt capital} \\
PV_{dc} & = \text{Present value of net distress-related costs} \\
PV_{TSn} & = \text{Present value of the tax shield as of time } = n \\
PV_{i} & = \text{Present value of invested capital} \\
TV_{n} & = \text{Terminal value at time } n \\
M_{e} & = \text{Market value of equity capital (stock)} \\
M_{d} & = \text{Market value of debt capital} \\
M_{p} & = \text{Market value of preferred equity} \\
MVIC & = \text{Market value of invested capital} \\
& = \text{Enterprise value} \\
& = M_{e} + M_{d} + M_{p} \\
BV & = \text{Book value of net assets} \\
BV_{n} & = \text{Book value of equity at time } = n \\
BV_{i} & = \text{Measure of book value (typically book value to market value) of stock of company } i \\
F_{d} & = \text{Fair value of debt} \\
FV_{RU} & = \text{Fair value of reporting unit} \\
FV_{NWCRU} & = \text{Fair value of net working capital of the reporting unit} \\
FV_{ICRU} & = \text{Fair value of invested capital of the reporting unit} \\
FV_{FARU} & = \text{Fair value of fixed assets of the reporting unit}
\end{align*}
\]
FV_{IARU} = \text{Fair value on intangible assets, identified and individually valued, of the reporting unit}

FV_{UIVRU} = \text{Fair value of unidentified intangibles value (i.e., goodwill) of the reporting unit}

FV_{dRU} = \text{Fair value of debt capital of the reporting unit}

FV_{eRU} = \text{Fair value of equity capital of the reporting unit}

FMV_{BE} = \text{Fair market value of the business enterprise}

FMV_{NWC} = \text{Fair market value of net working capital}

FMV_{FA} = \text{Fair market value of fixed assets}

FMV_{IA} = \text{Fair market value on intangible assets}

FMV_{UIV} = \text{Fair market value of unidentified intangibles value (i.e., goodwill)}

FMV_{e} = \text{Fair market value of equity capital}

FMV_{e,n,up} = \text{Fair market value of equity at time =} n \text{ assuming “up” scenario (value of BE increases)}

FMV_{BE,n,down} = \text{Fair market value of business enterprise at time =} n \text{ assuming “down” scenario (value of BE decreases)}

FMV_{e,n,down} = \text{Fair market value of equity at time =} n \text{ assuming “down” scenario (value of BE decreases)}

\text{COST OF CAPITAL AND RATE OF RETURN VARIABLES}

k = \text{Discount rate (generalized)}

k_{C} = \text{Country cost of equity}

k_{e} = \text{Discount rate for common equity capital (cost of common equity capital). Unless otherwise stated, it generally is assumed that this discount rate is applicable to net cash flow available to common equity.}

k_{e,local} = \text{Discount rate for equity capital in local country for discounting expected cash flows in local currency}

k_{e,u.s.} = \text{Discount rate for equity capital in the United States}

k_{BV} = \text{Rate of return on book value, retained portion of net income, usually estimated as} = \frac{NI_{i,in}}{BV_{n}}

k_{eu} = \text{Cost of equity capital, unlevered (cost of equity capital assuming firm financed with all equity)}

k_{local} = \text{Cost of equity capital in local country}

k_{i} = \text{Discount rate for company i}

k_{ni} = \text{Discount rate for equity capital when net income rather than net cash flow is the measure of economic income being discounted}

k_{(pt)} = \text{Discount rate applicable to pretax cash flows}

k_{p} = \text{Discount rate for preferred equity capital}

k_{d} = \text{Discount rate for debt (net of tax effect, if any) (Note: For complex capital structures, there could be more than one class of capital in any of the preceding categories, requiring expanded subscripts.)} = k_{d,(pt)} \times (1 - \text{tax rate})

k_{d,(pt)} = \text{Cost of debt prior to tax effect}

k_{A} = \text{Discount rate for the firm’s assets}

k_{TS} = \text{Rate of return used to present value tax savings due to deducting interest expense on debt capital financing}

k_{eRU} = \text{After tax rate of return on equity capital of reporting unit}
$k_{NWCRU}$ = Rate of return for net working capital of the reporting unit financed with debt capital (return measured net of the tax effect on debt financing, if any) and equity capital

$k_{FARU}$ = Rate of return for fixed assets financed with debt capital (return measured net of the tax effect on debt financing, if any) and equity capital

$k_{IARU}$ = Rate of return for identified and individually valued intangible assets financed with debt capital (return measured net of the tax effect on debt financing, if any) and equity capital

$k_{UIVRU}$ = Rate of return for unidentified intangibles value of the reporting unit financed with debt capital (return measured net of the tax effect on debt financing, if any) and equity capital

$k_{IA+UIV(pt)}$ = Pretax rate of return on all intangible assets, identified and individually valued, plus the unidentified intangible value financed with debt capital (measured before interest tax shield) and equity capital

$c$ = Capitalization rate

$c_{(pt)}$ = Capitalization rate on pretax cash flows (Note: For complex capital structures, there could be more than one class of capital in any of the preceding categories, requiring expanded subscripts.)

$D/P_0$ = Dividend yield on stock

$DR_j$ = Downside risk in the local market (U.S. dollars)

$DR_{global}$ = Downside risk in global ("world") market (U.S. dollars)

$R_i$ = Rate of return

$R_j$ = Return on stock $i$

$R_{d}$ = Rate of return on subject debt (e.g., bond) capital

$R_{m,n}$ = Return on market portfolio in current month $n$

$R_f$ = Rate of return on a risk-free security

$R_{f,n}$ = Risk-free rate in current month $n$

$R_{f,local}$ = Return on the local country government’s (default-risk-free) paper

$R_{f,us}$ = U.S. risk-free rate

$R_{local\ euro\ issuance}$ = Current market interest rate on debt issued by the local country government denominated in U.S. dollars ("euro-dollar" debt), same maturity as debt issued by the local country government denominated in U.S. dollars

$-R_{f,us}$ = Yield spread between government bonds issued by the local country versus U.S. government bonds

$R_{i}$ = Return on individual security subject stock in current month

$R_{m}$ = Historical rate of return on the "market"

$RP$ = Risk premium

$RP_{m}$ = Risk premium for the "market" (usually used in the context of a market for equity securities, such as the NYSE or S&P 500)

$RP_{s}$ = Risk premium for "small" stocks (usually average size of lowest quintile or decile of NYSE as measured by market value of common equity) over and above $RP_{m}$

$RP_{m+s}$ = Risk premium for the market plus risk premium for size (Duff & Phelps Risk Premium Report data for use in build-up method)
\[ RP_{s+u} = \text{Risk premium for small size plus risk premium attributable to the specific distressed company} \]
\[ RP_{m+s+u} = \text{Risk premium for the “market” plus risk premium for size plus risk attributable to the specific company} \]
\[ RP_u = \text{Risk premium for company-specific or unsystematic risk attributable to the specific company} \]
\[ RP_i = \text{The equity risk premium on a “world” diversified portfolio} \]
\[ RP_{i,s} = \frac{B_{i,s} \times S_i}{C^2} = \text{Risk premium for size of company } i \]
\[ RP_{i,BV} = \frac{B_{i,BV} \times BV_i}{C^2} = \text{Risk premium for book value of company } i \]
\[ RP_{i,u} = \frac{B_{i,u} \times U_i}{C^2} = \text{Risk premium for unique or unsystematic risk of company } i \]
\[ RP_{\text{local}} = \text{Equity risk premium in local country’s stock market} \]
\[ RI_{IL} = \text{Full-information levered beta estimate of the subject company} \]
\[ E(R) = \text{Expected rate of return} \]
\[ E(R_{m}) = \text{Expected rate of return on the “market” (usually used in the context of a market for equity securities, such as the New York Stock Exchange [NYSE] or Standard & Poor’s [S&P] 500)} \]
\[ E(R_i) = \text{Expected rate of return on security } i \]
\[ E(R_{div}) = \text{Expected rate of return on dividend} \]
\[ E(R_{cap-gains}) = \text{Expected rate of return on capital gains} \]
\[ E(R_{i,j}) = \text{Expected rate of return on security } i \text{ for undiversified investor } j \]
\[ B = \text{Beta (a coefficient, usually used to modify a rate of return variable)} \]
\[ B_i = \text{Expected beta of the stock of company } i \]
\[ B_L = \text{Levered beta for (equity) capital} \]
\[ B_U = \text{Unlevered beta for (equity) capital} \]
\[ B_{LS} = \text{Levered segment beta} \]
\[ B_d = \text{Beta for debt capital} \]
\[ B_p = \text{Beta of preferred capital} \]
\[ B_e = \text{Beta (equity) expanded} \]
\[ B_{op} = \text{Operating beta (beta with effects of fixed operating expense removed)} \]
\[ B_i = \text{Beta of company } i \text{ (F-F beta)} \]
\[ B_{i,m} = \text{Sensitivity of return of stock of company } i \text{ to the market risk premium or ERP} \]
\[ B_{i,s} = \text{Sensitivity of return of stock of company } i \text{ to a measure of size, } S_i \text{ of company } i \]
\[ B_{i,BV} = \text{Sensitivity of return of stock of company } i \text{ to a measure of book value (typically measure of book-value-to-market-value) of stock of company } i \]
\[ B_{i,u} = \text{Sensitivity of return of stock of company } i \text{ to a measure of unique or unsystematic risk of company } i \]
\[ B_n = \text{Estimated market coefficient based on sensitivity to excess returns on market portfolio in current month} \]
\[ B_{local} = \text{Market risk of the subject company measured with respect to the local securities market} \]
\[ B_w = \text{Market or systematic risk measured with respect to a “world” portfolio of stocks} \]
\[ B_{i1} \ldots B_{im} = \text{Sensitivity of security } i \text{ to each risk factor relative to the market average sensitivity to that factor} \]
\[ B^E_i = \text{True beta estimate for stock of company } i \text{ based on relationship to excess returns on market portfolio of equity plus debt, } M_E + M_D \]
\[ B_{u.s.} \times RP_{u.s.} = \text{Risk premium appropriate for a U.S. company in similar industry as the subject company in local country, expressed in U.S. dollar-denominated returns} \]
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**Notation System and Abbreviations Used in This Book**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FI\text{-Beta}$</td>
<td>Full-information beta for industry</td>
</tr>
<tr>
<td>$TB_i$</td>
<td>Total beta for security $i$</td>
</tr>
<tr>
<td>$\beta_{rc}$</td>
<td>Country covariance with region</td>
</tr>
<tr>
<td>$\beta_{cw}$</td>
<td>Country covariance with world</td>
</tr>
<tr>
<td>$S_i$</td>
<td>Measure of size of company $i$</td>
</tr>
<tr>
<td>$U_i$</td>
<td>Measure of unique or unsystematic risk of company $i$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>A measure of individual stock’s liquidity</td>
</tr>
<tr>
<td>$RP_1 \ldots RP_n$</td>
<td>Risk premium associated with risk factor 1 through $n$ for the average asset in the market (used in conjunction with arbitrage pricing theory)</td>
</tr>
<tr>
<td>$s_i$</td>
<td>Small-minus-big coefficient in the Fama-French regression</td>
</tr>
<tr>
<td>$SMBP$</td>
<td>Expected small-minus-big risk premium, estimated as the difference between the historical average annual returns on the small-cap and large-cap portfolios (also shown as SMB)</td>
</tr>
<tr>
<td>$h_i$</td>
<td>High-minus-low coefficient in the Fama-French regression</td>
</tr>
<tr>
<td>$HMLP$</td>
<td>Expected high-minus-low risk premium, estimated as the difference between the historical average annual returns on the high book-to-market and low book-to-market portfolios (also shown as HML)</td>
</tr>
<tr>
<td>$F_d$</td>
<td>Face value of outstanding debt</td>
</tr>
<tr>
<td>$b$</td>
<td>$1 - Payout ratio = retention ratio</td>
</tr>
<tr>
<td>$WACC_{(pc)}$</td>
<td>Weighted average cost of capital (before interest tax shield)</td>
</tr>
<tr>
<td>$WACC_{RU}$</td>
<td>Overall rate of return for the reporting unit</td>
</tr>
<tr>
<td>$WACC_{(pc)RU}$</td>
<td>Before interest tax shield WACC of the reporting unit</td>
</tr>
<tr>
<td>$\sigma_i^2$</td>
<td>Variance of returns for security $i$</td>
</tr>
<tr>
<td>$\sigma_{bp}$</td>
<td>Variance of the returns on the market portfolio (e.g., S&amp;P 500)</td>
</tr>
<tr>
<td>$\sigma_{e}$</td>
<td>Variance of error terms</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Standard deviation of returns on firm’s common equity</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Standard deviation of returns on firm’s assets</td>
</tr>
<tr>
<td>$\sigma_B$</td>
<td>Standard deviation of operating cash flows of business before cost of financing</td>
</tr>
<tr>
<td>$\sigma_{rev}$</td>
<td>Standard deviation of revenues</td>
</tr>
<tr>
<td>$\sigma_{BE}$</td>
<td>Standard deviation of value of business enterprise</td>
</tr>
<tr>
<td>$\sigma_{local}$</td>
<td>Volatility of subject (local) stock market</td>
</tr>
<tr>
<td>$\sigma_{U.S.}$</td>
<td>Volatility of U.S. stock market</td>
</tr>
<tr>
<td>$\sigma_{stock}$</td>
<td>Volatility of local country’s stock market</td>
</tr>
<tr>
<td>$\sigma_{bond}$</td>
<td>Volatility of local country’s bond market</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Standard deviation of returns for security $i$</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Standard deviation of returns for the market portfolio (e.g., S&amp;P 500)</td>
</tr>
<tr>
<td>$\sigma_{im}$</td>
<td>Variance of returns on the security, $i$, and the market, $m$</td>
</tr>
<tr>
<td>$\sigma_p^2$</td>
<td>Variance in excess returns on market of debt</td>
</tr>
<tr>
<td>$\sigma_{ME+MD}^2$</td>
<td>Variance in excess returns on market portfolio of equity plus debt, $ME + MD$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Correlation coefficient between the returns on the security, $i$, and the market, $m$</td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>Regional risk not included in $RP_w$</td>
</tr>
<tr>
<td>$CCR_{local}$</td>
<td>Country credit rating of local country</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Company’s exposure to the local country risk</td>
</tr>
<tr>
<td>$t$</td>
<td>Tax rate (expressed as a percentage of pretax income)</td>
</tr>
<tr>
<td>$h$</td>
<td>Holding period</td>
</tr>
<tr>
<td>$Inflation_{local}$</td>
<td>Expected rate of inflation in local country</td>
</tr>
<tr>
<td>$Inflation_{U.S.}$</td>
<td>Expected rate of inflation in U.S.</td>
</tr>
</tbody>
</table>
**INCOME VARIABLES**

- $E$ = Expected economic income (in a generalized sense; i.e., could be dividends, any of several possible definitions of cash flows, net income, etc.)
- $F$ = Fixed operating assets (without regard to costs of financing)
- $F_c$ = Fixed operating costs of the business
- $NI$ = Net income (after entity-level taxes)
- $NCI_{e,n}$ = Net comprehensive income to common equity in period $n$, which includes income terms reported directly in the equity account rather than in the income statement
- $NCI_{f,n}$ = Net comprehensive income to the firm in period $n$, which includes income terms reported directly in the equity account rather than in the income statement
- $CF$ = Cash flow for a specific period
- $NCF_e$ = Net cash flow (free cash flow) to equity
- $NCF_f$ = Net cash flow (free cash flow) to the firm (to overall invested capital, or entire capital structure, including all equity and long-term debt)
- $NCF_{ue}$ = Net cash flow to unlevered equity
- $D$ = Dividends
- $D_{e,n}$ = Distributions to common equity, net of new issues of common equity in period $n$
- $D_{f,n}$ = Distributions to total capital, net of new issues of debt or equity capital in period $n$
- $RI_{e,n}$ = Residual income for common equity capital
- $TS$ = Present value of tax savings due to deducting interest expense on debt capital financing
- $EBT$ = Earnings before taxes
- $EBIT$ = Earnings before interest and taxes
- $EBITDA$ = Earnings before interest, taxes, depreciation, and amortization
- $V$ = Variable operating costs
- $AEG$ = Abnormal earnings growth

**PERIODS OR VARIABLES IN A SERIES**

- $i$ = $i^{th}$ period or $i^{th}$ variable in a series (may be extended to the $j^{th}$ variable, the $k^{th}$ variable, etc.)
- $n$ = Number of periods or variables in a series, or the last number in a series
- $0$ = Period 0, the base period, usually the latest year immediately preceding the valuation date
- $p_y$ = Partial year of first year following the valuation date

**WEIGHTINGS**

- $W$ = Weight
- $W_e$ = Weight of common equity in capital structure
  
  $= M_e/(M_e + M_d + M_p)$

- $W_p$ = Weight of preferred equity in capital structure
Notation System and Abbreviations Used in This Book

\[ W_d = M_d / (M_e + M_d + M_p) \]
\[ W_{dRU} = \text{Weight of debt capital in capital structure of reporting unit} \]
\[ W_s = \text{Weight of segment data to total business (e.g., sales, operating income)} \]
\[ W_{NWC} = \text{Weight of net working capital in } FMV_{BE} \]
\[ W_{NWCRU} = \text{Weight of net working capital in } FV_{RU} \]
\[ W_{FA} = \text{Weight of fixed assets in } FMV_{BE} \]
\[ W_{FARU} = \text{Weight of fixed assets in } FV_{RU} \]
\[ W_{IARU} = \text{Weight of intangible assets in } FV_{RU} \]
\[ W_{UIVRU} = \text{Weight of unidentified intangibles value } FV_{RU} \]
\[ W_{TS} = \text{Weight of TS in } FMV_{BE} \]

GROWTH

\[ g = \text{Rate of growth in a variable (e.g., net cash flow)} \]
\[ g_i = \text{Dividend growth rate for company } i \]
\[ g_{ni} = \text{Rate of growth in net income} \]

MATHEMATICAL FUNCTIONS

\[ \Sigma = \text{Sum of (add all the variables that follow)} \]
\[ \cap = \text{Product of (multiply together all the variables that follow)} \]
\[ X = \text{Mean average (the sum of the values of the variables divided by the number of variables)} \]
\[ G = \text{Geometric average (the sum of the values of the variables taken to the root of the number of variables)} \]
\[ \alpha = \text{Regression constant} \]
\[ \varepsilon = \text{Regression error term} \]
\[ \varepsilon_i = \text{Error term, difference between predicted return and realized return, } R_i \]
\[ \infty = \text{Infinity} \]
\[ N(\ast) = \text{Cumulative normal density function (the area under the normal probability distribution)} \]
\[ \Delta = \text{Change in . . . (whatever follows)} \]
NOTATION FOR REAL PROPERTY VALUATION (CHAPTER 9 OF WORKBOOK AND TECHNICAL SUPPLEMENT)

\[
\begin{align*}
DSCR & = \text{Debt service coverage ratio} \\
EGIM & = \text{Effective gross income multiplier} \\
NOI, I_p & = \text{Net operating income} \\
OER & = \text{Operating expense rates} \\
PV_p & = \text{Overall value or present value of the property} \\
k_e & = \text{Equity discount or yield rate (dividend plus appreciation)} \\
k_m & = \text{Mortgage interest rate} \\
k_p & = \text{Property yield discount rate} \\
c_p & = \text{Overall property capitalization rate} \\
c_e & = \text{Dividend to equity capitalization rate} \\
c_m & = \text{Mortgage capitalization rate or constant} \\
c_a & = \text{Terminal or residual or going-out capitalization rate} \\
c_B & = \text{Building capitalization rate} \\
c_L & = \text{Land capitalization rate} \\
c_{LF} & = \text{Leased fee capitalization rate} \\
c_{LH} & = \text{Leasehold capitalization rate} \\
A & = \text{Change in income and value (adjustment factor)} \\
P & = \text{Principal paid off over the holding period} \\
1/S_n & = \text{Sinking fund factor at the equity discount or yield rate (}k_e\text{)} \\
\Delta_P & = \text{Change in value over the holding period} \\
SC\% & = \text{Cost of sale} \\
PGI & = \text{Potential gross income} \\
PGIM & = \text{Potential gross income multiplier} \\
EGI & = \text{Effective gross income} \\
NIM & = \text{Net income multiplier} \\
F_p/PV_p & = \text{Face value of debt (loan amount outstanding) to value ratio} \\
\left[1 - \left(F_p/PV_p\right)\right] & = \text{Equity to value ratio} \\
M_B & = \text{Building value} \\
M_m & = \text{Mortgage value} \\
M_L & = \text{Land value} \\
M_{LF} & = \text{Leased fee value} \\
M_{LH} & = \text{Leasehold value} \\
I_p & = \text{Overall income to the property} \\
I_L & = \text{Residual income to the land} \\
I_B & = \text{Residual income to the building} \\
I_e & = \text{Equity income} \\
I_m & = \text{Mortgage income} \\
I_{LF} & = \text{Income to the leased fee} \\
I_{LH} & = \text{Income to the leasehold}
\end{align*}
\]

ABBREVIATIONS

\[
\begin{align*}
ERP & = \text{Equity risk premium (usually the general equity risk premium for which the} \\
& \text{benchmark for equities is either the S&P 500 stocks or the NYSE stocks)} \\
WACC & = \text{Weighted average cost of capital} \\
WARA & = \text{Weighted average return on assets}
\end{align*}
\]