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# Foreign exchange option pricing

## *A Practitioner's Guide*

IAIN J. CLARK

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**Iain J. Clark**

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For Isabel

# Contents

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I would also like to thank my amazing wife, to whom I owe more than I can possibly say. I am as always grateful to my parents John and Joan for their love, support and tolerance of my difficult questions and interest in science and mathematics – I'm glad to say some things haven't changed so much! Also to my extended family, whom I don't get to see as often as I would like, thanks for keeping us in your thoughts and all your messages of encouragement. They mean a lot to an author.

Finally, to my young nieces and nephews in Canada – Andrew, Bradley, Isabel, Mackenzie and William – who asked me if my mathematics book for grown-ups was going to have 'very hard sums' like 1 000 010 −1 000 000 012, I have a *very* hard sum just for you:

$$\begin{array}{r} 101\,598\,490 \\ +\ \underline{21\,858\,299} \end{array}$$

This book is for you and for all students, young and old, of the mathematical arts. I wish you all the very best with your studies and your work.

**Web page for this book**

`www.fxoptionpricing.com`

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# 1
# Introduction

This book covers foreign exchange (FX) options from the point of view of a practitioner in the area. With content developed with input from industry professionals and with examples using real-world data, this book introduces many of the more commonly requested products from FX options trading desks, together with the models that capture the risk characteristics necessary to price these products accurately, an area often neglected in the literature, which is nevertheless of paramount importance in the real financial marketplace. Essentially this is a mathematical practitioner's cookbook that contains all the information necessary to price both vanilla and exotic FX options in a professional context.

Connecting mathematically rigorous theory with practice, and inspired by the questions asked daily by junior quantitative analysts (quants) and other colleagues (both from FX and other asset classes) this book is aimed at quants, quant developers, traders, structurers and anyone who works with them. Basically, this is the book I wish I'd had when I started in the industry. This book will also be of real benefit to academics, students of mathematical finance across all asset classes and anyone wishing to enter this area of finance.

The level of knowledge assumed is about at the level of Hull (1997) and Baxter and Rennie (1996) – both excellent introductory works. This work extends that knowledge base specifically into FX and I hope will be useful to those joining (or hoping to join) the finance industry, to industry practitioners who wish to learn more about FX as an asset class or the numerical techniques used in FX, and last but not least to academics – both in regard to their own work and as a reference for their students.

## 1.1 A GENTLE INTRODUCTION TO FX MARKETS

The simplest foreign exchange transaction one can imagine is going to a *bureau de change*, such as one might find in an airport, and exchanging a certain number of banknotes or coins of one currency for a certain amount of notes and coins of another realm. For example, on 24 September 2009, the currency converter at www.oanda.com was quoting a GBPUSD rate of 1.63935 US dollars per pound sterling (or conversely, 0.6100 pounds sterling per US dollar). Thus, neglecting two-sided bid–offer pricing and commissions, a holidaymaker at Heathrow seeking to buy \$100.00 for his/her holiday in Miami should expect to pay £61.00.

This transaction is, to within a minute or two, immediate. Now let us suppose instead that the transaction is larger in notional by a factor of 1000 – perhaps motivated by investment purposes. Suppose that our traveller is seeking to transfer pounds sterling into a US account as a deposit on the purchase of a condo in Miami. The traveller is clearly not going to pull out £61 000.00 in the Heathrow departures hall and expect to collect \$100 000.00 in crisp unmarked US dollar bills. A trade of this size will be executed in the FX spot market, and instead of the US dollar funds being available in a minute or two (and the UK pound funds being transferred away from the client), the exchange of funds happens at the spot date, which is generally in a couple of days (this is vague; see Section 1.4 for exact details). This lag is largely for historical reasons. On that day, the US dollar funds appear in the client's US account

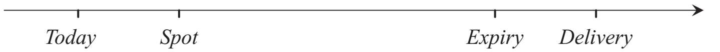


**Figure 1.1** Dates of importance for FX trading

and the UK pound sterling funds are transferred out of the client's UK account. This process is referred to as *settlement*. The risk that one of these payments goes through but the other does not is referred to as foreign exchange settlement risk or Herstatt risk (after the famous example of Herstatt bank defaulting on dollar payments on 26 June 1974).

Another possibility is that perhaps the traveller is flying to Miami to see a new build apartment building being built, and he/she knows that the \$100 000.00 will be needed in six months' time. To lock in the currency rate today and protect against currency risk, he/she could enter into an FX forward, which fixes the rate today and requires the funds to be transferred in six months. The date in the future when the settlement must take place is called the *delivery date*.

A third possibility is that the traveller, being structurally long in pounds sterling, could buy an option to protect against depreciation of the sterling amount over the six month interval – in other words, an option to buy USD (and, equivalently, sell GBP) – i.e. a put on the GBPUSD exchange rate, at a prearranged strike price. This removes any downside risk, at the cost of the option premium. Since the transaction is deferred into the future, we have the delivery date (just as for the forward) but also an *expiry date* when the option holder must decide whether or not to exercise the option.

There are, therefore, as many as four dates of importance: today (sometimes called the *horizon date*), spot, expiry and delivery (see Figure 1.1).

Being a practitioner's guide, this chapter seeks to describe exactly how all these important dates are determined, and the next chapter describes how they impact the price of foreign currency options. A good introductory discussion can be found in Section 3.3 of Wystup (2006), but the devil is very much in the detail.

## 1.2 QUOTATION STYLES

Unlike other asset classes, in FX there is no natural numeraire currency. While no sensible investor would denominate his or her wealth in the actual number of IBM or Lehman Brothers stocks he or she owns, it is perfectly natural for investors to measure their wealth in US dollars, euros, Aussie dollars, etc. As a result, there is no special reason to quote spot or forward rates for foreign currency in any particular order. The choice of which way around they are quoted is purely market convention.

For British pounds against the US dollar, with ISO codes[1] of GBP and USD respectively, the market standard quote could be GBPUSD (the price of 1 GBP in USD) or USDGBP (the price of 1 USD in GBP). For this particular currency pair, it is the former – GBPUSD.

Note that the mainstream financial press, such as the *Financial Times* and the *Economist* (in the tables inside the back cover), report the values of all currencies in the same quote terms – for example, the value of one US dollar in each of AUD, CAD, EUR, MXN, . . . , ZAR. While easier to understand, this is *not* the way spot rates are quoted in FX markets.

[1] ISO 4217 code, from the International Organization for Standardization; www.iso.org.

**Table 1.1** Currency pair quotation conventions and market terminology

| Currency pair | Common trading floor jargon |
|---|---|
| EURUSD | Euro-dollar |
| USDJPY | Dollar-yen |
| EURJPY | Euro-yen |
| GBPUSD | Cable (from the late 1800s transatlantic telegraph cables) |
| EURGBP | Euro-sterling |
| USDCHF | Dollar-swiss |
| AUDUSD | Aussie-dollar |
| NZDUSD | Kiwi-dollar |
| USDCAD | Dollar-cad (or dollar-canada or, less commonly, dollar-loonie) |
| EURNOK | Euro-nokkie |
| EURSEK | Euro-stockie (from Stockholm) |
| EURDKK | Euro-danish |
| EURHUF | Euro-huff ('huff', not H-U-F) |
| EURPLN | Euro-polish |
| USDTRY | Dollar-turkey (or dollar-try, pron. 'try') |
| USDZAR | Dollar-rand (or dollar-zar, pron. 'zar') |
| USDMXN | Dollar-mex ('mex', not M-E-X) |
| USDBRL | Dollar-brazil |
| USDSGD | Dollar-sing |

For a currency pair quoted[2] as ccy1ccy2, the spot rate $S_t$ at time $t$ is the number of units of ccy2 (also known as the domestic currency, the terms currency or the quote currency) required to buy one unit of ccy1 (the foreign currency or sometimes the base currency) – the spot rate being fixed today and with settlement occurring on the spot date. The spot rate is therefore dimensionally equal to units of ccy2 per ccy1 – this is why pedantic quants such as myself tend to discourage currency pairs from being written with a slash ('/') between the currency ISO codes. It's all too easy to read 'GBP/USD' as 'GBP per USD', which is *not* what the market quotes. The GBPUSD quote is for US dollars per pound sterling, so if GBPUSD is 1.63935, then one British pound can be bought for $1.63935 in the spot market. It's a USD per GBP price. It's the cost of one pound, in dollars.

Note that on FX trading floors, the spot rate $S_t$ is invariably read aloud with the word 'spot' used to indicate the decimal place, e.g. 'one **spot** six three nine three five' in the GBPUSD example above. The exception is where there are no digits trailing the decimal place; e.g. if the spot rate for USDJPY was 92.00, we would read this out as '92 **the figure**'.

When I was first learning this, I found it convenient to remember that precious metals are quoted in the same way as currencies (gold has the ISO code XAU, silver has the ISO code XAG and similarly we have XPT and XPD) and are quoted, for example, as 'currency' pair XAUUSD, with spot rate close to 1000.00 (in September 2009). That number means $1000 per ounce – 1000 USD per XAU. Having a physical ounce of gold to think about may help you keep your bearings where this whole currency 1/currency 2 thing is concerned.

In fact, to sound the part on a trading floor, the names in Table 1.1 above are recommended (FX practitioners rarely refer to a currency pair by spelling out the three-letter currency

[2] The use of monikers such as ccy1 and ccy2 is standard among FX market practitioners. It is also very confusing when one gets into the world of quantos and multicurrency options and therefore is generally avoided in that context.

ISO code if they can possibly avoid it – though you may hear it for some emerging market currencies).

So how do we know which way round a particular currency pair should be quoted? Regrettably, it's purely market convention and seems to be quite arbitrary, as the examples shown in the first column of Table 1.1 demonstrate. There are, however, a few guidelines that hint at which ordering is more likely.

### *Currency quote styles – heuristic rules*

1. Precious metals are always ccy1 against *any* currency, e.g. XAGUSD.
2. Euro is always ccy1, unless rule (1) takes precedence, e.g. EURCHF.
3. Emerging market (EM) currencies strongly tend to be ccy2, e.g. USDBRL, USDMXN, USDZAR, EURTRY, etc., as EM currencies are very often weaker than the majors and this quote style gives a spot rate greater than unity, which is easier for bookkeeping. The same heuristic holds even if the currency is revalued, as in the revaluation of the Turkish currency from TRL to TRY in 2005.
4. Currencies that historically[3] were subdivided into nondecimal units (e.g. pounds, shillings and pence[4]), such as GBP, AUD and NZD, tend to be ccy1 for ease of accounting. To give an example, it was much easier back in the late 1940s to quote GBPUSD as \$4.03 per pound sterling than to quote USDGBP as 4 shillings and $11\frac{1}{2}$ pence per US dollar.[5] For INR (and presumably PKR), heuristic (3) takes precedence.
5. For currency pairs where the spot rate would be either markedly greater than or markedly less than unity, the quote style tends to end up being the quote style that gives a spot rate significantly greater than unity, once again for ease of bookkeeping – e.g. USDJPY, with spot levels around 100.0 rather than JPYUSD, which would have spot levels around 0.01. Similarly USDNOK would be around 5.8.

Counterexamples to the above heuristic rules exist with probability one, but the general pattern holds surprisingly often. What I hope to convey above is that while the market conventions are arbitrary, there is at least *some* underlying logic.

A useful hierarchy of which of the major currencies dominate in their propensity to be ccy1 can be written:

$$\text{EUR} > \text{GBP} > \text{AUD} > \text{NZD} > \text{USD} > \text{CAD} > \text{CHF} > \text{JPY}.$$

As spot rates are quoted to finite precision, the least significant digit in the spot rate is called a pip. It represents the smallest usual price increment possible in the FX spot market (though half-pips are becoming more common with tighter spreads). A big figure is invariably 100 pips and is often tacitly assumed when quoting a rate. A few examples are: if the spot rate for EURUSD is 1.4591, the big figure is 1.45 and there are 91 additional pips in the price.

[3] In the 20th century, anyway.

[4] An interesting historical note is that the florin, a very early precursor to decimal coinage worth 2 shillings, was introduced in Britain in 1849 bearing the inscription 'one tenth of a pound', to test whether the public would be comfortable with the idea of decimal coinage. They weren't – the coin stayed, but the inscription was dropped.

[5] There are only two currencies in the world that still have nondecimal currency subunits – those of Mauritania and Madagascar, with ISO codes of MRO and MGA respectively.

If the spot rate for EURJPY is 131.25, the big figure is 131 and there are 25 additional pips. Finally, to return to our earlier example, if the spot rate for GBPUSD is 1.63935, the big figure is 1.63 and there are $93\frac{1}{2}$ additional pips.

Most currency pairs are quoted to five significant figures, except for those currencies that fall through a particular level and lose a digit – e.g. USDJPY used to trade at levels well above 100.00 and had a pip value of 0.01, but in late 2009, with the currency pair trading closer to 90.00, only four significant figures remain. Many currency pairs have a pip value of 0.0001, with some exceptions. A couple of examples are: majors against the Japanese yen, which have a pip value of 0.01, and majors against the Korean won, with a pip value of 0.1. Other exceptions are easy to find.

## 1.3 RISK CONSIDERATIONS

In equities, it should be pretty clear whether one is considering upside or downside risk. Downside risk is if the value of the stock goes down and upside risk is if the stock appreciates dramatically. Further, if one is long the stock, then upside risk is positive to the stockholder and downside risk is negative.

In FX, the complexity of having two currencies to consider makes this more subtle. Perhaps one is a euro based investor who is long USD dollars. In that case the investor, regarding this particular exposure, is long US dollars and commensurately short euros. More complicated situations arise when options are introduced: if the euro based investor above attempts to hedge his or her long USD/short EUR position by buying a USD put/EUR call option. Since the premium for such an option is generally quoted in USD (see Section 3.3.1), purchasing the FX option hedge will make the option holder's position somewhat shorter US dollars (as intended) and longer euros from holding the option – and additionally, short the number of USD required to purchase the option.

## 1.4 SPOT SETTLEMENT RULES

A foreign currency spot transaction, if entered into today with spot reference $S_0$, will involve the exchange of $N_\mathrm{d}$ units of domestic currency for $N_f$ units of foreign currency, where the two notionals are related by the spot rate $S_0$, i.e. $N_d = S_0 \cdot N_f$. However, these two payments are in general *not* made on the day the transaction is agreed. The day on which the two payments, in domestic and foreign currency, are made is known as the *value date*. For conventional spot trades, which are almost always the case, it is known as the *spot date*. So we have two dates of special interest: today and spot.

It is often believed that FX trades settle two business days (2bd) after the trade date, known as T+2 settlement, so that the spot date is obtained by counting forward to the second good business day after today. This is mostly correct, but with some notable exceptions and with the specific handling of holidays requiring some further explanation.

The first point to make is that not all currency pairs trade with T+2 settlement. The most commonly cited exception is USDCAD which trades with T+1 settlement; i.e. the currencies are exchanged one good business day after today. Eventually we shall probably see some currency pairs trading with T+0 settlement. In the meantime, at the time of compilation of this work, the exceptions to T+2 settlement I could find are listed in Table 1.2. For notational

**Table 1.2** Currency pair exceptions to T+2 settlement

| | ccy2 | | | | |
|---|---|---|---|---|---|
| ccy1 | USD | EUR | CAD | TRY | RUB |
| USD | | | T+1 | T+1 | T+1 |
| EUR | T+2 | | T+2 | T+1 | T+1 |
| CAD | | | | T+1 | T+1 |
| TRY | | | | | T+1 |
| RUB | | | | | |

ease, we shall refer to T+$x$ settlement. Now we need to define the concept of a good business day for currency pairs.

**Definition:** A day is a good business day for a currency ccy if it is not a weekend (Saturday and Sunday for most currencies, but not always for Islamic countries). For currency pairs ccy1ccy2, a day is only a good business day if it is a good business day for both ccy1 and ccy2.

The situation with respect to currencies in the Islamic world is complicated, and appears to vary by institution and country. The reason for this is that Friday is a particularly holy day for Jumu'ah prayers in the Muslim faith, influencing trading calendars similarly to Sunday in the Christian faith.

Weekends in Islamic countries are generally constructed with this in mind. For some countries such as Saudi Arabia, the weekend is taken on Thursday and Friday. It is, however, becoming more common for Islamic countries to change to observing weekends on Friday and Saturday – such as is the case in Algeria, Bahrain, Egypt, Iraq, Jordan, Kuwait, Oman, Qatar, Syria and the UAE – in an effort to harmonise business arrangements with the rest of the world and their neighbours.

Since readers with a special geographic interest in this trading region very likely have colleagues with local experience they can refer questions to directly, I refer the reader to the section 'Arab currencies' of the web page http://www.londonfx.co.uk/valdates.html for further details.

To give an idea of the sort of potential complexity one may encounter, a T+2 spot FX transaction in a currency pair such as USDSAR effected on a Wednesday may well have a spot date of Monday the next week (the two business days counting forward from Wednesday being Thursday and Friday in the USA and Saturday and Sunday in Saudi Arabia), but one may even have split settlement where the dollars are settled on Friday and the Saudi Arabian riyal are settled separately on Monday.

The determination of the *spot date* for currency pair ccy1ccy2, from the *today date*, is best described by the following algorithm. Note that settlement can neither be on a USD holiday nor a holiday in either of the currencies in the currency pair – this arises when constructing such a spot trade via the crosses of ccy1 and ccy2 versus USD. For currency pairs with T+2 settlement, the interim date *can* be a USD holiday so long as it is not a holiday in any non-USD currencies in the currency pair – except for currency pairs involving 'special' Latin American currencies such as MXN, ARS and CLP, which impose the same restrictions for the interim date as the settlement date itself.

***Spot settlement rules – algorithm***

```
function SpotFromHorizonDate(horizonDate)


if T+2 settlement  // Step 1
{
if CCY1 or CCY2 is a ``special'' LatAm currency // MXN, ARS,
CLP (not BRL)
{
advance forward one good business day, skipping CCY1, CCY2
and USD holidays
}
else
{
advance forward one good business day, skipping holidays in
CCY1 (unless CCY1=USD) and similarly skipping holidays in
CCY2 (unless CCY2=USD)
}
}

// step 2
if T+1 or T+2 settlement, or T+0 settlement and today is either
not a good business day or a holiday in CCY1, CCY2 or USD
{
advance forward one good business day, skipping holidays in
CCY1, CCY2 and USD
}
```

***Spot settlement rules – examples***

(a) EURUSD: Today: Mon 28Sep09, Spot: Wed 30Sep09 `// T+2`
(b) USDTRY: Today: Thu 12Feb09, Spot: Fri 13Feb09 `// T+1`
(c) GBPUSD: Today: Sat 20Jun09, Spot: Tue 23Jun09 `// T+2 from a weekend`
(d) EURUSD: Today: Wed 29Apr09, Spot: Mon 04May09 `// 01May09 = EUR holiday`
(e) USDCAD: Today: Fri 31Jul09, Spot: Tue 04Aug09 `// 03Aug09 = CAD holiday`
(f) AUDNZD: Today: Thu 08Oct09, Spot: Tue 13Oct09 `// 12Oct09 = USD holiday`
(g) USDBRL: Today: Tue 10Nov09, Spot: Thu 12Nov09 `// 11Nov09 = USD holiday`
(h) USDMXN: Today: Tue 10Nov09, Spot: Fri 13Nov09 `// 11Nov09 = USD holiday`

The spot date for today and the spot date relative to any expiry date (see Section 1.5 below) are known as ***value dates***, and are by convention rolled forward to the next applicable value date at 5 pm New York time – with the exception of NZDUSD, which rolls forward at 7 am Auckland time. Note that these times are fixed in local time, and depending on whether the USA or New Zealand are currently observing daylight savings time, this will affect the exact time of the value date rollover in other financial centres, or even relative to UTC.

## 1.5 EXPIRY AND DELIVERY RULES

If we have an FX option that expires on a particular date (the expiry date), then, if exercised, it will (generally) be exercised as a spot FX transaction at the prearranged strike $K$. Consequently, the delivery date bears the same $\mathrm{T} + x$ relationship to the expiry date that the spot date bears to today – though it can be later, in which case the option is said to have a delayed delivery feature.

If an option is specified with a concrete expiry date, e.g. 22Sep09, then the determination of the delivery date follows from Section 1.4 above. However, it is more common for options to be quoted for a specific term, e.g. 9D, 2W, 3M or 1Y (days, weeks, months and years respectively). How do we interpret this?

The devil is once again in the detail. For terms expressed as $n$ months or $n$ years, we count forward from the spot date by $n$ or $12n$ calendar months respectively to obtain the delivery date, making sure that we don't inadvertently roll over the end of a month (i.e. using the *modified forward* convention, which is explained properly in Section 1.5.2 below). This aligns the delivery date for a 1M, 2M, ..., 1Y, ... FX option with the settlement date for an FX forward of the same tenor. We then count backwards to obtain the expiry date, which has *that* date as its value date. A simple example of this is given in Section II.B of the Master Agreement in ICOM (1997).

However, for terms expressed as $n$ days or $n$ weeks, as there is no liquid FX forwards market quoted with respect to tenors measured in an integral number of days or weeks, we count forward from today by $n$ or $7n$ calendar days respectively to determine the expiry date (if the date arrived at isn't a good expiry date, we adjust that expiry date forward if required so that it is a good business day). Finally, the delivery date is given by obtaining the spot date corresponding to that expiry date using the procedure described in Section 1.4 above.

In short, for expiries measured in days or weeks, we count forward from today to the expiry date and then forward to the delivery date – but for expiries measured in months or years, we count forward from the spot date to the delivery date and then back to the expiry date.

### 1.5.1 Expiry and delivery rules – days or weeks

The prescription here is to count forward $n$ or $7n$ calendar days from today to obtain the expiry date. If the candidate for the expiry date arrived at is a holiday in whichever of ccy1 and ccy2 is not USD (or both, if ccy1ccy2 is a cross), then we keep counting forward (going over the end of a month into the next month is perfectly permissible) until we have a good business day that is not a holiday in whichever of ccy1 and ccy2 is not USD. Note that USD holidays are completely disregarded for determination of the expiry date.

#### *Expiry rule – days or weeks – algorithm*

```
function ExpiryFromToday(today,term,units)

if units = 'D'  // Step 1 - obtain candidate expiry date
d = today + term
else if units = 'W'
d = today + 7*term
else
throw error
```

```
// Step 2 - obtain actual expiry date by avoiding impermissible
holidays
x1 = false
x2 = false
x1 = isCcy1Holiday(d)
x2 = isCcy2Holiday(d)
while (IsWeekend(d) or x1 or x2)
{
d = d + 1
x1 = isCcy1Holiday(d)
x2 = isCcy2Holiday(d)
}
expiry = d
delivery = SpotFromHorizonDate(expiry)
```

### 1.5.2 Expiry and delivery rules – months or years

The prescription for inferring the expiry and delivery dates when the tenor is provided in units of months or years is the following. We roll forward from today's spot date by that number of months or years, making sure that we don't inadvertently move over the end of a month end boundary. For example, if the spot date is Mon 31 Jan 2011, then the 1M delivery date obviously cannot be 31 February (February only has 29 days at most!) and commonsense requires that it be set to Monday 28 Feb 2011, and not roll over into any date in March.

Markets generally implement this using what is referred to as the *modified following* convention. Suppose the spot date is the $i$th day of month $X$ and we need to calculate the delivery date $n$ months forward from the spot date.

If this spot date is the last good business day of month $X$, then the delivery date is the last good business day of month $X + n$. Additionally, if month $X + n$ has less than $i$ days or the $i$th day of month $X + n$ is beyond the last good business day in that month, the delivery date is the last good business day in month $X + n$.

If none of these conditions hold, then the delivery date is set to the $j$th day of month $X + n$, where $j \geq i$ is the first good business day in month $X + n$, obtained by counting forward from the $i$th day of month $X + n$ until a good business day (possibly even the $i$th day itself) is found.

#### *Expiry rule – months or years – algorithm*

```
function DeliveryFromSpot(spotDate,term,units)

m = (units = 'Y') ? 12 : 1 // should allow only units of 'M' or 'Y'
convertDateToYearMonthDay(spotDate, theYear, theMonth, theDay)
theMonth = theMonth + m * term
while (theMonth > 12)
{
theMonth = theMonth - 12
theYear = theYear + 1
```

```
}
numberOfDaysInMonth = LastDayOfMonth(theMonth, theYear)
if spotDate is the last good business day in the month for
CCY1CCY2 (excluding holidays in CCY1 and CCY2)
{
theDay = numberOfDaysInMonth // ensure that month end sticks
to month end
}
if (theDay > numberOfDaysInMonth) theDay = numberOfDaysInMonth

d = DateSerial(theYear, theMonth, theDay)

switch (theDay)
case numberOfDaysInMonth:
while (!isValidFXDeliveryDay(d, ccy1, ccy2)) { d = d - 1 }
return d

default:  // i.e. theDay < numberOfDaysInMonth
while (!isValidFXDeliveryDay(d, ccy1, ccy2)) { d = d + 1 }
if (d.month() <> theMonth)
{
d = d - 1
while (!isValidFXDeliveryDay(d, ccy1, ccy2)) { d = d - 1 }
}
return d
```

Once we have the delivery date, we find the expiry date by selecting the furthest horizon date in the future, subject to being a good business day and a trading day in at least one centre, which has a spot date corresponding to the given delivery date.

```
function ExpiryFromDelivery(deliveryDate)


d = deliveryDate
while (d is a weekend or 01JAN or a CCY1 holiday (unless CCY1=USD)
or a CCY2 holiday (unless CCY2=USD) or SpotFromHorizonDate(d) >
deliveryDate) { d = d - 1 }
return d
```

## 1.6 CUTOFF TIMES

From Section 1.5 above, we are able to determine the expiry date. However, at what *time* on that date should the option be understood to expire? This is a particularly relevant question given the international 24-hour markets that FX trades in.

There are, in practice, only a few possibilities that arise – these are known as *cutoff times*. As a rule, the cutoff is 3 pm local time in the trading centre in question, with the exception of New York, for which it is 10 am local time. However, as Hicks (2000) describes, 10 am New York time usually coincides with 3 pm London time, except when one centre is on daylight saving time (DST) and the other is not. As a result, the London cutoff is extremely rarely used – the most common cutoff by far, and the one that is implicitly meant when a particular cutoff is not