Turbulent Flow

Analysis, Measurement, and Prediction

Peter S. Bernard
James M. Wallace

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To the memory of Foster Arnold Bernard

Peter S. Bernard

To my wife, Barbara, and son, Jaimie, who encouraged my participation in the writing of this book in spite of the time it took away from our family, and to my friends Bob Brodkey and Helmut Eckelmann, who made my first experience in turbulence research so much fun.

James M. Wallace

Contents

Preface

To the extent that many practical turbulent engineering flows are beyond our capacity to predict, it is clear that the turbulence "problem" is not yet solved. The need to predict turbulent fluid behavior, however, is not diminished even if finding a reliably accurate solution technique is elusive. In essence, the subject of turbulence is driven by its applications: answers to difficult turbulent flow problems are required, and engineers or physicists must supply the best possible answer. Judgments must be made between competing approaches whether experimental, numerical, or analytical. Whatever decision is made, it is subject to debate and justification.

Whether the reader is an engineer faced with turbulent flow prediction or a scientist or student intending to pursue research in the field, one needs to become familiar with what might at first glance appear to be a vast literature of loosely connected theories, numerical predictions, and experimental measurements of turbulence. A principal goal of this book is to help readers see the subject as a relatively coherent whole, so that they will be able to make informed decisions as to how best to study and predict turbulent flows of interest.

The book aims at a relatively wide coverage, but without being exhaustive. Our interest is to include those areas that are essential to making sense of currently available options for measuring or predicting turbulent flows. We believe that to be most useful, the book needs to familiarize the reader with current techniques used to measure, simulate, analyze, and predict turbulence, so we have attempted to provide meaningful discussions of many such techniques. If successful, the book will put the reader in a good position to understand more advanced and specialized books in the field. As a general rule, we have striven to keep the discussion focused on those results and theories that are relatively well established. More speculative ideas are often part of the research frontier and appropriate for independent study.

After giving some essential preliminary notation and concepts in Chapter 1, in Chapter 2 we provide a natural entry to the subject of turbulence through consideration of the averaged equations of motion and the basic physical processes that they encompass. Note that this volume will be concerned exclusively with incompressible turbulent flow. The focus of Chapter 2 is to provide an overview of the principal concepts and issues in understanding turbulence, so that the basic language of the subject is available to the reader when considering the major facets of turbulence in the subsequent chapters.

In Chapter 3 we survey the main techniques used in performing physical experiments, with a view toward making clear what is feasible and what is not. Included here is a discussion of how turbulent flow can be simulated on a computer. Such algorithms, considered to be "numerical wind tunnels," have become a very large part of turbulent flow "measurement." A number of the experimental and numerical techniques mentioned in Chapter 3 have been used for many years to gain fundamental knowledge of turbulence. A review of some of the major aspects of the knowledge acquired this way is presented in Chapter 4, covering bounded flows, and in Chapter 5, covering free shear flows. The hope here is to give the reader a feeling of what turbulent flow is and something of what is presently known about it. This is an important background to have when considering the predictive methodologies developed in later chapters.

In Chapters 6 and 7 we go into greater depth in exploring the essential physics of turbulence. In view of its position as a distinguishing characteristic of turbulence, the issue of turbulent transport is considered from a number of perspectives in Chapter 6. Following this, in Chapter 7, the properties of idealized turbulent flows are considered. This permits a relatively unobstructed view of such fundamental processes as energy dissipation. Many aspects of the theoretical analysis of such flows are useful to the development of predictive theories.

The stage is then set in Chapter 8 to begin a discussion of turbulence closures, which are the predominant means by which turbulent flows are predicted in engineering work. The relationship that closure models have to our understanding of the physics of turbulent flow will become evident. Once we have discussed many features of closure schemes, in Chapter 9 we give some examples of how they perform in the practical solution of turbulent flow problems.

In Chapter 10 we consider prediction methods known as large eddy simulations (LES), which are situated between closure schemes on the one hand and direct computation of the flow on the other. First discussed are traditional grid-based LES techniques, which have long been used by meteorologists and are now finding some application in engineering work. This is followed by consideration of the application of grid-free vortex methods to LES, which is just now beginning to become a significant new methodology for predicting turbulence.

A key part of many engineering flows is the occurrence of heat or mass transfer, and in Chapter 11 we give some basic background knowledge of this subject. Finally, in Chapter 12 we introduce several of the principal trends in the theoretical analysis of turbulence. To varying degrees, these theories, which tend to be mathematically difficult, have been developed with a view toward understanding turbulence at its most fundamental level. There remains considerable controversy in this area, and our purpose here is only to hint at some of the major developments in what is a very large field in and of itself.

This book represents material that has been successfully taught by the authors in two complementary graduate-level courses on turbulent flow given at the University of Maryland over the last 20 years. One of these covers the analysis of the physics of turbulent flow through its measurement and simulation, and the other concentrates on the prediction and theoretical analysis of turbulence. As suggested by the title of this volume, we bring together here material that would be sufficient for one or both of these courses. Depending on the inclination of the instructor, more focused readings of particular research articles related to topics in the book can be included to help extend the book into current research activities. In fact, our goal for this book will be met if students are brought by it to the point where they can read classical and current research articles with comprehension and an informed critical eye.

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1

Preliminaries

1.1 TURBULENCE

Every airplane passenger knows firsthand that the atmosphere contains something called *turbulence.* When airplanes enter into it, they shake and vibrate unpredictably, until, to every passenger's relief, the turbulent zone is passed and the flight returns to a reassuring calm. It is evident that turbulence must consist of a collection of disordered wind gusts, which in this case are capable of pushing an airplane around.

It is also possible that the reader has experienced turbulence on a hand or back placed over the jet of fluid entering a swimming pool or jacuzzi. Far from a constant flow, one has the sensation of an ever-changing eddying motion which is particularly pronounced at the edges of the jet. Although on a very much smaller scale than in the airplane flow and in a different fluid medium, the apparent random pulsations of the turbulent flow appear to be a common characteristic.

Countless other experiences of turbulent flow accompany us in our daily lives. Examples abound of turbulent flow in technological, environmental, and biological applications. The flow around an automobile, between buildings in a downtown street, and through a diseased artery are but three commonplace occurrences.

It is natural to wonder where turbulence comes from, why it occurs, and why it so prevalent. Furthermore, it is not hard to see that there are probably many situations where one wishes to be able to predict, a priori, either the occurrence of turbulence or its behavior. Our starting point for shedding light on these questions is to be precise about exactly what is meant by turbulence.

The apparent randomness of the buffeting of an airplane as it flies through turbulence appears to be a defining characteristic of such flows. However, since it is believed that fluid flows evolve deterministically according to the Navier–Stokes equation, evidently, a useful definition of turbulence must be more precise than the statement that the flow displays random characteristics. To be explicit about the role of determinism and randomness in turbulent flow, consider the flow in a controlled setting such as a laboratory wind tunnel. Imagine that the following experiment is repeatable at will: Starting from quiescent conditions, the wind tunnel is turned on to a particular speed setting. A fixed time after it is started, the fluid velocity is recorded at arbitrary points within the test section of the tunnel. Depending on the speed of the flow, one of two possible outcomes is possible: either the measured velocities are identical, within measurement accuracy, each time the experiment is repeated, or they vary. The former case is referred to as *laminar flow*, and the second, in which the velocity field is not repeatable in either the whole or in part of the flow domain, is what will formally be referred to as *turbulent flow*, or *turbulence*. By this definition, transitional flows in which the motion is undergoing a change from laminar to turbulent conditions qualify as turbulent.

The differences in velocity from realization to realization of a turbulent flow are explained by the sensitivity of the evolving flow to small uncontrollable perturbations in the initial and boundary conditions. These originate from slight thermal currents, from changing surface roughness at a small scale, from small variations in the input power of the tunnel, perhaps even from microscopic sources such as the subcontinuum molecular motions that cause Brownian motion [4]. In no way is the determinism of the evolving flow in doubt. Rather, it is the ability of the flow to amplify slight, unpredictable changes in boundary and initial conditions to measurable scale which is the defining characteristic of turbulence. The result is a randomly appearing flow structure over a sizable physical extent, such as is encountered by airplanes in the atmosphere. This contrasts with laminar flows in that the same small perturbations experienced in turbulent flow are present, but in this case they are damped out successfully as soon as they appear, and the flow velocity, if measured, is always the same.

Under normal circumstances, the ability of flows to remain laminar depends on the degree to which viscous damping, deriving from the molecular diffusion of momentum, can erase the influences of individual perturbations. Since the Reynolds number, $R_e = UL/\nu$, where ν is the kinematic viscosity, U is a typical velocity of the flow, and L a typical length scale, characterizes the relative strength of viscous and inertial forces, it is the key parameter used in deciding whether or not a flow is likely to be turbulent. For low values of R_e viscous forces are dominant and the flow tends to be laminar. For larger values, a point is reached where a transition occurs in which disturbances are no longer damped out but rather, are amplified. Beyond this point a fully turbulent state results. Figure 1.1 illustrates the transition process in a turbulent jet. The flow is seen to undergo a dramatic change from a laminar state, through one containing organized oscillatory motions, finally, to fully turbulent conditions. In a bounded flow such as in a pipe for which $R_e = U_m d/\nu$, where U_m is the average mass flow velocity and d is the diameter, transition begins as low as $R_e \approx 200$ in the form of decaying turbulent slugs passing through the pipe [3], and generally a fully turbulent state occurs for $R_e > 2300$. It is not possible to be completely precise about these critical values since they are affected by upstream flow conditions and the smoothness of the boundary. With sufficiently small input disturbances and smooth walls, laminar flow in a pipe can be maintained to at least $R_e = 10^5$ [2].

[Image not available in this electronic edition.]

Fig. 1.1 *Transition to turbulence in a jet. (Courtesy of J.-L. Balint and L. Ong.)*

It is a fact of our experience that the Reynolds number encountered in many practical problems of concern to engineers and physicists is large, in fact well beyond the transitional range. For example, in the case of a 100 m-long submarine in seawater ($\nu = 1.044 \times 10^{-6}$ m^2/s), traveling at 15 knots, at the stern $R_e \approx 7.4 \times 10^8$, while at the rear of a 6 m-long car in air ($\nu = 1.525 \times 10^{-6}$ m^2/s) at 60 mph, $R_e \approx 1.1 \times 10^8$. The inevitable turbulence occurring in these and other flows must be accommodated in the design process. In the case of a submarine, turbulent forces and acoustic fields are of significant concern as well as the exact position of turbulent separated flow regions, which might lead to undesirable flow patterns affecting the propeller. Turbulent drag and side forces affect the economy and stability of road vehicles, and turbulent buffeting of the driver side window by vortices shed off the rear view mirror generates noise (see Fig. 1.2). Turbulent flow in the engine compartment affects cooling, and turbulence in the engine cylinder is necessary for effective combustion and reduced pollutant emissions. Atmospheric turbulence formed from thermal currents and gravity waves shed off topological features in stratified flow lead to the turbulence encountered by airplanes flying at low altitudes. Countless more examples exist: it is the ubiquity of turbulence in high-Reynolds-number flow that creates the need for the study of turbulence and its prediction.

A common characteristic of the flows depicted in Figs. 1.1 and 1.2 is the presence of rotational motion in the form of vortices. Large-scale vortical structures are visible in the outer edge of the jet. In the wake of the automobile the scales of the vortices and other turbulent motions are so small that the smoke marker in this region appears as a diffused blur. The dynamics of the energetic large-scale vortices have a significant influence on the physics of turbulent flow. Moreover, the importance and presence of vortices extend through all scales of turbulent motion, including the smallest

[Image not available in this electronic edition.]

Fig. 1.2 *Visualization using smoke helps engineers see the flow patterns around vehicles, including the turbulence in their wakes. (Used with permission of the Volvo Car Corp., Aerodynamics Div.)*

scales. For example, the three-dimensional plot of vorticity vectors in a numerical simulation of turbulence shown in Fig. 1.3 suggests that embedded tubelike vortices form the essential fabric of turbulence at its smallest scale. It will often be seen in this book that the dynamics of the vorticity field represents the most convenient and succinct means for describing and understanding the behavior of turbulent fluid flow. Thus subsequent questions about the nature of turbulent transport phenomena and the physics of the energy field will find explanation in terms of vorticity dynamics.

The turbulent flow examples that have been mentioned thus far suggest that the data needed in analyzing turbulent fields can involve both mean statistics, as in the average forces on automobiles and submarines, or instantaneous quantities, such as the peak pressures responsible for sound generation on solid surfaces and the position of separated flow structures. In all cases, if the velocity and pressure fields in a turbulent flow can be obtained by one means or another, then besides knowing the instantaneous flow properties, it would be possible to compute average properties as well. As will be seen, however, it is usually a very difficult matter to predict turbulent fields, so much so that it is often the case that only mean statistics can be computed, and usually these are approximations produced by a partial analysis of the flow physics. It will thus become clear in the course of this book that knowledge of turbulence and how to predict it is incomplete. The boundary between the known and the unknown changes steadily, and the future can only bring improved insights and better predictive schemes.

Characterization of flows by their Reynolds number (e.g., between high and low Reynolds numbers) is an important factor to take into account when deriving solution strategies. There are a number of other fundamental categories into which flows may

[Image not available in this electronic edition.]

Fig. 1.3 *Vorticity vectors in a computer simulation of turbulent flow in a periodic cubic box. (From [5]. Reprinted with the permission of Cambridge University Press.)*

be divided in order to better suggest schemes for their measurement or prediction. One demarcation is between interior and exterior flows, as in the case of a pipe or engine cylinder versus the flow past an airplane. In the former, all parts of the flow generally are strongly affected by the presence of solid boundaries. For exterior flows, turbulence is often created at the boundary but then evolves downstream, free of its direct influence, while spreading into increasingly greater portions of the flow domain. Near boundaries, turbulent flow is often accompanied by high shear (i.e., large values of the mean velocity gradients). Such regions are a focal point of the physical processes governing the overall production and distribution of turbulence, and thus are worthy of extra attention in the discussions that follow. The opposite extreme from high-shear regions is *homogeneous turbulence*, where mean properties of the turbulence including mean velocity do not vary with position (i.e., are independent of translations of the coordinate axes). Little production of new turbulence takes place in these circumstances, and the flow is dominated by dissipation. In some instances, homogeneous turbulence is also *isotropic,* wherein, in addition to the independence of the mean turbulence properties to position, the turbulence displays no intrinsic directional preference.

The concept of isotropy implies that the turbulent flow must be homogeneous, since inhomogeneous flows have directional preferences, thus contradicting the assumption of isotropy. On the other hand, the condition of homogeneity does not imply that the flow must be strictly isotropic. In fact, another possibility is the case of *homogeneous shear flow,* in which there is a uniform mean velocity gradient everywhere, so that even though there is a directional preference, it is the same everywhere. In such flows, the statistical properties of the turbulence do not vary with position, with the sole exception of the mean velocity field. Homogeneous shear flows play a useful role in developing turbulence prediction schemes, since they allow attention to be focused on just specific parts of the governing equations. In addition to imposed shear, homogeneous flows with uniform rotations, plane strains, and combinations of these effects are also studied.

Finally, it should be noted that engineering flows are often *complex,* or *nonequilibrium,* in the sense that they contain unusual geometrical features or unsteadiness which causes the turbulent motion not to be explained easily in terms of relatively simple balances of physical effects, as would be the case, for example, in an attached boundary layer on a smooth wall. The latter type of flow is one of several that are often called *canonical,* since they are relatively simple and fairly well understood experimentally and numerically. Aspects of all these types of flows are discussed in subsequent chapters.

1.2 AVERAGING AND TURBULENT FIELDS

The average of a random field, such as the velocity in turbulent flow, is most naturally defined as an *ensemble average* over independent realizations of the flow field. Each realization may be viewed as one occurrence of an experiment, and an average over N of them is given by

$$\overline{\mathbf{U}}(\mathbf{x}, t) = \frac{1}{N}\sum_{j=1}^{N} \mathbf{U}^j(\mathbf{x}, t), \tag{1.1}$$

where, for example, $\mathbf{U} = (U_1, U_2, U_3)$ is the velocity field and $\mathbf{U}^j$ is the velocity measured in the jth experiment. $\mathbf{x} = (x_1, x_2, x_3)$ denotes position in three-dimensional space and t is the time. Throughout this book, vectors are denoted either in boldface, as in the case of $\mathbf{x}$ and $\mathbf{U}$ in (1.1), or equivalently, using index notation, as in $U_i,\ i = 1, 2, 3$. As the occasion arises, the notation (x, y, z) will also be used to represent (x_1, x_2, x_3) and (U, V, W) to denote (U_1, U_2, U_3).

Ensemble averaging conveniently commutes with time and space derivatives:

$$\overline{\frac{\partial \mathbf{U}}{\partial t}}(\mathbf{x}, t) = \frac{\partial \overline{\mathbf{U}}}{\partial t}(\mathbf{x}, t), \qquad \overline{\frac{\partial \mathbf{U}}{\partial x_i}}(\mathbf{x}, t) = \frac{\partial \overline{\mathbf{U}}}{\partial x_i}(\mathbf{x}, t), \tag{1.2}$$

which is of great benefit in theoretical analyses. In some instances, as in flow in an internal combustion engine cylinder, each four-stroke cycle of the engine may be considered to be one realization of the flow, in which case the average in (1.1) can readily be evaluated by experimental techniques.

It is more often the case, at least in experimental investigations, that (1.1) is impractical to implement because it would involve the laborious task of repeating the experiment many times. It thus is useful to consider alternatives. A particularly useful average is one over time, as in

$$\overline{\mathbf{U}}(\mathbf{x}, t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} \mathbf{U}(\mathbf{x}, s)\, ds, \tag{1.3}$$

where T is a sufficiently long time interval and $\mathbf{U}(\mathbf{x}, s)$ is obtained from a continuous sampling of the flow velocity at $\mathbf{x}$ over the interval $(t - T/2, t + T/2)$. Equation (1.3) is conveniently evaluated in many standard experiments, and is most useful when the random field is *stationary* (i.e., the time average is independent of the interval T over which it is taken). In this case, the integral in (1.3) is independent of t. Time averaging loses legitimacy in nonstationary flows where the underlying mean signal changes over time. The engine flow is a good example of this. In such circumstances, (1.3) is a strong function of t and T and the first of the formulas in (1.2) is not formally valid since the time derivative does not commute with the integral in (1.3).

Another possibility is spatial averaging:

$$\overline{\mathbf{U}}(\mathbf{x}, t) = \frac{1}{V} \int_{\mathcal{V}} \mathbf{U}(\mathbf{x}', t)\, d\mathbf{x}', \tag{1.4}$$

where V is generally the volume of a region $\mathcal{V}$ surrounding the point $\mathbf{x}$, although in some cases, particularly flows with geometrical symmetries, $\mathcal{V}$ may be taken to be particular lines or surfaces in the flow field. Often, (1.4) is used in the analysis of numerical simulations such as of a channel flow, where symmetries allow for averaging over planes. This reduces the number of time steps or realizations of the flow needed to get converged statistics. Similar to the case of time averaging, depending on the particular volume $\mathcal{V}$ and the spatial variability of the mean field, commutation with spatial derivatives in the second relation in (1.2) will sometimes be legitimate and sometimes not.

Finally, note that it is plausible that ensemble, time, and spatial averages are equivalent when circumstances permit, although this conclusion lacks formal mathematical proof. Throughout the book each of these methods of averaging will be employed depending on the circumstances, and which type is in effect will be made clear. For most theoretical discussions it will be assumed that ensemble averaging is employed. In some contexts it will also be seen that it is useful to define partial averages, such as conditional averaging in which averages are taken only over flow events or phases of processes satisfying some predetermined criteria. In a similar vein, the large eddy simulation method discussed in Chapter 10 requires specialized averaging in the form of a filtering process.

For either ensemble, time, or space averaging, a turbulent velocity field may be decomposed according to

$$\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}, \tag{1.5}$$

where $\mathbf{u} \equiv \mathbf{U} - \overline{\mathbf{U}}$ is referred to as the *velocity fluctuation vector*. Note that where no confusion can result, such as in this instance, we refrain from indicating the dependence of $\mathbf{U}$ and the other fields on $\mathbf{x}$ and t. The average of the fluctuation is always zero (i.e., $\overline{\mathbf{u}} \equiv 0$), although this may not be exactly true in the case of spatial or time averaging unless the flow is appropriately homogeneous or steady. In laminar flow, by definition, $\mathbf{U} = \overline{\mathbf{U}}$, so that $\mathbf{u} = 0$.

Besides the mean, higher-order moments of the fluctuating velocity field may also be of interest: for example, $\overline{u_1^2}, \overline{u_2^2}, \overline{u_3^2}$, which are the variances of the components of $\mathbf{u}$. The sum of these yields the turbulent kinetic energy per unit volume, $K \equiv \rho\overline{u_i^2}/2$, where ρ is the density, and here and henceforth it is understood that repeated indices are summed from 1 to 3. More generally, the tensor

$$R_{ij} \equiv \overline{u_i u_j} \tag{1.6}$$

is the covariance of the random vector field $\mathbf{U}$. The components of $\overline{u_i u_j}$ have a clear physical interpretation as momentum fluxes, as shown in Section 2.1.

In many circumstances where averages are computed directly from the random field, our knowledge of the turbulent physics can be deepened by examining the associated probability density function (pdf). For example, a velocity component such as U_1 will occur over a range of values, with each one having a different likelihood of occurring. The cumulative effect is the average, $\overline{U}_1$, but the pdf tells how often values of U_1 in a particular range are likely to occur. Thus, if $p(U_1)$ is the pdf of U_1, then by definition, $p(U_1)\, dU_1$ is the probability that U_1 takes on a value U such that $U_1 \leq U \leq U_1 + dU_1$. Clearly,

$$\int p(U)\, dU = 1, \tag{1.7}$$

since U_1 must take on some value for each experiment. It also follows that

$$\overline{U}_1 = \int U p(U)\, dU, \tag{1.8}$$

$$\overline{u_1^2} = \int (U - \overline{U}_1)^2 p(U)\, dU, \tag{1.9}$$

and so on, for higher moments.

Probability density functions can also be written for multiple variables, in which case they are referred to as *joint probability density functions*. For example, for velocity components U_1 and U_2, $p(U_1, U_2)\, dU_1\, dU_2$ is the probability that both $U_1 \leq U \leq U_1 + dU_1$ and $U_2 \leq V \leq U_2 + dU_2$ are satisfied by a realization (U, V) of the velocity field. A notable use of the joint pdf is in analyzing correlations such as

$$\overline{u_1 u_2} = \iint (U - \overline{U}_1)(V - \overline{U}_2) p(U, V)\, dU\, dV, \tag{1.10}$$

which are important to the analysis of turbulent momentum transport near boundaries.

An important example of a probability density function is

$$p(U) = \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-(U-\overline{U})^2/2\sigma^2}, \tag{1.11}$$

where σ is the standard deviation, which characterizes Gaussian random variables. Such variables make an appearance at several junctures in turbulent flow analyses; for example, it is often a good approximation to the fluctuating velocity field at a single point in turbulent flow, the main difference being that large fluctuation amplitudes are less likely to occur than in a true Gaussian. Equation (1.11) generalizes to accommodate processes that are jointly Gaussian and to Gaussian random fields (see Section 12.2) where the joint probability density function for variables at multiple locations in the flow field obeys a Gaussian form. It is one of the more profound attributes of turbulent flow fields that even though the velocity field may be approximately Gaussian at individual points in the flow, it does not constitute a Gaussian random field.

Besides second-order moments, other higher-order statistics of importance in turbulent flow analysis are the skewness and flatness, defined, respectively, via

$$S = \frac{\overline{u^3}}{\overline{u^2}^{3/2}} \tag{1.12}$$

and

$$F = \frac{\overline{u^4}}{\overline{u^2}^2}. \tag{1.13}$$

These quantities are particularly useful in helping to judge how far from Gaussianity a particular random variable might be, since for strict Gaussianity they satisfy $S = 0$ and $F = 3$.

In some circumstances it is useful to extend the averaging process to include the product of velocities at separate points, as in the two-point double and triple velocity correlation tensors, respectively, defined by

$$\mathcal{R}_{ij}(\mathbf{x}, \mathbf{y}, t) \equiv \overline{u_i(\mathbf{x}, t)u_j(\mathbf{y}, t)} \tag{1.14}$$

and

$$S_{ij,k}(\mathbf{x}, \mathbf{y}, t) \equiv \overline{u_i(\mathbf{x}, t)u_j(\mathbf{x}, t)u_k(\mathbf{y}, t)}. \tag{1.15}$$

Such correlations provide an opportunity to capture structural features of the turbulence which would be lost in one-point statistics. When $\mathbf{x} = \mathbf{y}$, $\mathcal{R}_{ij}(\mathbf{x}, \mathbf{x}, t) = \overline{u_i(\mathbf{x}, t)u_j(\mathbf{x}, t)} = R_{ij}(\mathbf{x}, t)$ [i.e., the single-point velocity covariance defined in (1.6)].

Letting $\mathbf{r} = \mathbf{y} - \mathbf{x}$ denote the position vector connecting $\mathbf{x}$ with $\mathbf{y}$, then as $\mathbf{r}$ varies, $\mathcal{R}_{ij}(\mathbf{x}, \mathbf{x}+\mathbf{r}, t)$ gives an indication of how the velocity field around $\mathbf{x}$ at a particular time t is "correlated." For small $\mathbf{r}$ the velocity field at $\mathbf{x}$ and $\mathbf{x}+\mathbf{r}$ will be quite similar and hence highly correlated, while for large $\mathbf{r}$, $\mathcal{R}_{ij}(\mathbf{x}, \mathbf{x}+\mathbf{r}, t)$ should be at or close to zero since there would not generally be a mechanism to create correlation over large distances. For $\mathbf{x}$ fixed it is convenient to simplify notation by defining $\mathcal{R}_{ij}(\mathbf{r}, t) \equiv \mathcal{R}_{ij}(\mathbf{x}, \mathbf{x}+\mathbf{r}, t)$. In homogeneous turbulence, correlations cannot depend on position, and in this case there is truly no $\mathbf{x}$ dependence in $\mathcal{R}_{ij}(\mathbf{r}, t)$.

$\mathcal{R}_{ij}(\mathbf{r}, t)$ is generally a bounded function of compact support, so its three-dimensional Fourier transform exists:

$$E_{ij}(\mathbf{k}, t) \equiv (2\pi)^{-3} \int_{\Re^3} e^{\imath \mathbf{r}\cdot\mathbf{k}} \mathcal{R}_{ij}(\mathbf{r}, t)\, d\mathbf{r}, \tag{1.16}$$

where $\imath \equiv \sqrt{-1}$, $\mathbf{k}$ is the wavenumber vector, and $E_{ij}(\mathbf{k}, t)$ is referred to as the *energy spectrum tensor*. Corresponding to (1.16) is the inverse transform

$$\mathcal{R}_{ij}(\mathbf{r}, t) = \int_{\Re^3} e^{-\imath \mathbf{r}\cdot\mathbf{k}} E_{ij}(\mathbf{k}, t)\, d\mathbf{k}, \tag{1.17}$$

where $d\mathbf{k} = dk_1\, dk_2\, dk_3$ is a differential volume in wavenumber space. Equations (1.16) and (1.17) represent one way to enter into a spectral analysis of turbulence in which the local fluid motion is decomposed according to the different scales which are acting, as represented by Fourier components $e^{\imath \mathbf{r}\cdot\mathbf{k}}$.

Setting $i = j$ in (1.17), summing over indices $i = 1, 2, 3$, dividing by 2, and setting $\mathbf{r} = 0$ gives the spectral decomposition of the turbulent kinetic energy

$$K(t) = \frac{1}{2} \int_{\Re^3} E_{ii}(\mathbf{k}, t)\, d\mathbf{k}, \tag{1.18}$$

where $E_{ii}(\mathbf{k}, t)$ is the density of energy in wavenumber space, and it is understood that all quantities here have an implied dependence on $\mathbf{x}$. It is useful to collect the energy, which is typically scattered throughout $\mathbf{k}$ space, onto shells of fixed distance $k = |\mathbf{k}|$ from the origin. This is done by first rewriting (1.18) using spherical coordinates in wavenumber space:

$$K(t) = \int_0^\infty dk \left[\frac{1}{2} \int_{|\mathbf{k}|=k} d\Omega\, E_{ii}(\mathbf{k}, t) \right], \tag{1.19}$$

where $d\Omega$ is an element of solid angle in $\mathbf{k}$ space and $d\mathbf{k} = d\Omega\, dk$. Next, the term in brackets is defined as the *energy density function* or *energy spectrum*:

$$E(k, t) \equiv \frac{1}{2} \int_{|\mathbf{k}|=k} E_{ii}(\mathbf{k}, t)\, d\Omega, \tag{1.20}$$

yielding

$$K(t) = \int_0^\infty E(k, t)\, dk. \tag{1.21}$$

In this formula, $E(k, t)$ shows how the kinetic energy is distributed among the different scales of the flow. The nature of $E(k, t)$ in turbulent flows has long played a major role in the theoretical analysis of turbulence and will be considered at length in later chapters.

Two useful quantities that can be formed from $\mathcal{R}_{ij}(\mathbf{r}, t)$ are the longitudinal and transverse velocity correlation functions defined by

$$\overline{u_1^2} f(r) \equiv \mathcal{R}_{11}(r\mathbf{e}_1, t) \tag{1.22}$$

and

$$\overline{u_2^2} g(r) \equiv \mathcal{R}_{22}(r\mathbf{e}_1, t), \tag{1.23}$$

respectively (see Fig. 1.4). Here $\mathbf{e}_i$ represents a unit vector in the x_i direction for $i = 1, 2, 3$. For simplicity of notation we refrain from indicating the time dependence of f and g. Note that our use of the x_1 and x_2 directions is meant as an example; similar functions can be defined for any direction.

From their definitions it is clear that $f(0) = g(0) = 1$. Moreover, they must decay to zero as $r \to \infty$. Locally, near $r = 0$ they can be expanded in a Taylor series as in

$$f(r) = 1 + r\frac{df}{dr}(0) + \frac{r^2}{2!}\frac{d^2 f}{dr^2}(0) + \cdots. \tag{1.24}$$

The first three terms on the right-hand side form a parabolic approximation to f whose intersection with the r axis, as shown in Fig. 1.5, is used to define a length scale, λ, termed the *microscale*. Thus, by definition,

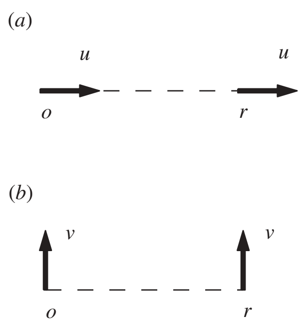

Fig. 1.4 *(a) Longitudinal and (b) transverse velocity correlations used in definitions of $f(r)$ and $g(r)$, respectively.*

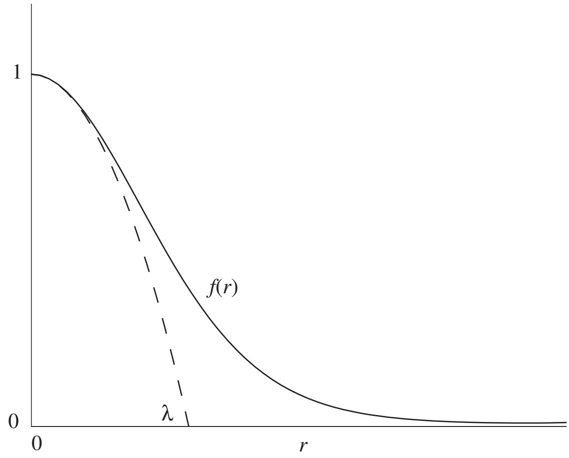

Fig. 1.5 *Microscale definition.*

$$0 = 1 + \lambda \frac{df}{dr}(0) + \frac{\lambda^2}{2!}\frac{d^2 f}{dr^2}(0), \tag{1.25}$$

from which λ can be computed. The microscale is a measure of the scales of motion at which turbulent energy dissipation takes place, as shown in Section 7.3.1.

While λ is representative of the smaller scales in turbulent flow, a measure of the largest scales is given by the integral scale, defined by

$$\Lambda \equiv \int_0^\infty f(r)\, dr. \tag{1.26}$$

The integral in (1.26) exists because $f(r)$ has bounded support. Λ is a measure of the size of the largest turbulent "eddies," meaning regions of the flow whose motion is in some sense interconnected. Ordinarily, events at the scale of Λ are very energetic since they are likely to be the direct result of the stirring mechanism that is responsible for turbulence generation.

In many circumstances it is most likely that time sequences of turbulent velocities at a given spatial point are known (e.g., from direct measurement using a fixed probe) rather than ensemble or volume information. In this case, it is not feasible to evaluate the energy density function per se. However, the data are sufficient to determine what is referred to as the one-dimensional energy spectrum, $E_{11}(\omega)$, where the angular frequency, ω, has units of hertz (i.e., cycles per second). $E_{11}(\omega)$ is defined via the Fourier transform of the time autocorrelation coefficient

$$\mathcal{R}_E(\tau) = \frac{\overline{u(t)u(t+\tau)}}{\overline{u^2}(t)}, \tag{1.27}$$

where $u(t)$ is a measured time sequence of velocities at a fixed point.

In practice, the right-hand side of (1.27) would be evaluated using a time average, although the following discussion is simplified somewhat if it is viewed as an ensemble average. $\mathcal{R}_E(\tau)$ is related to its Fourier transform, $\widehat{\mathcal{R}}_E(\omega')$, where $\omega' = 2\pi\omega$ is the frequency, via the transform pair

$$\widehat{\mathcal{R}}_E(\omega') = \int_{-\infty}^{\infty} e^{-\imath\tau\omega'} \mathcal{R}_E(\tau)\, d\tau \tag{1.28}$$

and

$$\mathcal{R}_E(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\imath\tau\omega'} \widehat{\mathcal{R}}_E(\omega')\, d\omega'. \tag{1.29}$$

Evaluating (1.29) at $\tau = 0$ and defining

$$E_{11}(\omega) = 2\overline{u^2}\,\widehat{\mathcal{R}}_E(2\pi\omega) \tag{1.30}$$

gives, after a change of variables,

$$\overline{u^2} = \frac{1}{2} \int_{-\infty}^{\infty} E_{11}(\omega)\, d\omega. \tag{1.31}$$

If $\mathcal{R}_E(\tau)$ is symmetric [i.e., $\mathcal{R}_E(-\tau) = \mathcal{R}_E(\tau)$], as it would be if the random process is stationary so that (1.27) is independent of t, then (1.28) implies that

$$\widehat{\mathcal{R}}_E(\omega') = 2 \int_{0}^{\infty} \cos\ \tau\omega' \mathcal{R}_E(\tau)\, d\tau \tag{1.32}$$

and then that

$$\widehat{\mathcal{R}}_E(-\omega') = \widehat{\mathcal{R}}_E(\omega'), \tag{1.33}$$

so that (1.29) gives

$$\mathcal{R}_E(\tau) = \frac{1}{\pi} \int_{0}^{\infty} \cos\ \tau\omega' \widehat{\mathcal{R}}_E(\omega')\, d\omega'. \tag{1.34}$$

Thus $\mathcal{R}_E(\tau)$ and $\widehat{\mathcal{R}}_E(\omega')$ form a Fourier cosine transform pair. It also follows from (1.30), (1.31), and (1.33) that

$$\overline{u^2} = \int_{0}^{\infty} E_{11}(\omega)\, d\omega. \tag{1.35}$$

The fact that $\mathcal{R}_E(\tau)$ is easily evaluated from data from a single probe means that it is a relatively simple matter to determine the one-dimensional spectrum $E_{11}(\omega)$. In contrast, it is nearly impossible to directly measure $E(k, t)$. In Section 3.2.3.3

it is shown that in at least some circumstances it is possible to link the temporal and spatial variations in u. In this case, as shown in Section 7.6.3, a relationship between $f(r,t)$ and $\mathcal{R}_E(\tau)$ can be established and from this a means of estimating $E(k,t)$ from $E_{11}(\omega)$. Thus the one-dimensional spectrum $E_{11}(\omega)$ is a particularly useful quantity in characterizing turbulent flow.

1.3 TURBULENT FLOW ANALYSES

Traditionally, direct physical measurement of turbulence by experimental techniques has provided a large part of our current knowledge of turbulent flows. Physical experiments provide both descriptions of specific flow fields as well as data needed in verifying theoretical ideas. In some cases, careful scrutiny of measured data suggests the existence of general laws requiring theoretical justification. It will become clear below that it is more than a simple matter to perform fully satisfying measurements in turbulent flow. The flow sensors available for measuring properties of turbulence are beset with intrinsic difficulties of resolution, reliability, precision, and cost. Moreover, the flow variables that can be measured are often limited, and the kinds of flows that pose relatively few problems for experimental study are often not the ones that are encountered in engineering investigations or are sought to verify theoretical predictions.

Despite these obstacles, experimentalists over the last 100 years have accumulated a large body of knowledge about the nature of many individual turbulent flows and facets of the general physical laws governing them. Limited only by the characteristics of the sensors, physical experiments can achieve extremes of Reynolds number, geometrical complexity, and unsteadiness which so far have not been attainable by other methods. Moreover, advances across a wide technical front in electronics, microfabrication, optics, and computer control have led to significant new advances in devising experimental techniques. The extensive use of powerful laboratory workstations for data acquisition and analysis has made possible simultaneous multisensor measurements at several flow locations. Impressive recent advances augur well for very important new accomplishments in the future. Some indication of these is given in Chapter 3.

It is only in the modern era beginning in the 1980s that advances in computational science permitted numerical solutions to the Navier–Stokes equation to be obtained for turbulent conditions. These, referred to as *direct numerical simulations (DNS)*, have become an important and growing source of new information and insights about turbulence [1]. Within their range of applicability, which depends on having sufficient computer storage and speed to accommodate the full range of length and time scales active in a turbulent flow, they are generally unsurpassed for accuracy and wealth of information obtainable. They allow for complete and satisfying analyses of the relative importance of the terms in the averaged equations of motion. They permit examination of the time evolution of the three-dimensional pressure field or the life history of turbulent eddies or the calculation of multipoint correlations. As DNS continue to evolve, they are applied to an ever-widening range of important flows, such as jets, wakes, and mixing and boundary layers, as well as flows with